Gyrokinetic simulations using a delta-f approach with an evolving background Maxwellian

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Summary

- Noise control alone is not sufficient to reduce noise accumulation to acceptable levels
- Code used for the adaptive δf scheme is introduced, along with all physical assumptions and profiles used
- Mechanism for the adaptive scheme using a local Maxwellian with time-dependent temperature profile as control variate is explained (also submitted to PoP)
- Results show reduction in noise accumulation in the zonal component and improved signal-to-noise ratio values for the same simulation with lower marker numbers

Introduction & motivation

- The δf PIC scheme [1] [2] is useful when simulating plasma core as small deviations from equilibrium distribution is expected, thus satisfying the $|\delta f|/|f_0| \ll 1$ assumption, which leads to noise reduction when compared to the full-f scheme
- When simulating the **plasma edge**, steep profile gradients, low density levels and high fluctuation amplitudes lead to **violation of the δf assumption**
- Benefits retained when using an adaptive time-dependent background distribution $f_0(t)$ as the control variate [3]
- A simplified model involving a **flux-surface-averaged time-dependent background temperature** is used to demonstrate the advantages gained

Test-bed: Physical assumptions and profiles

Code: GKengine[4]

- single ion species with adiabatic electrons
- electrostatic
- collisionless
- potential represented by (cubic) B-splines
- sheared-slab geometry
- highly paralellised involving MPI, OpenMP, OpenACC

- δf represented by markers

$$\delta f = \frac{1}{2\pi}\Sigma_p^{N_p}\frac{w_p(t)}{B_\parallel^\star/m_i}\delta[\boldsymbol{R}-\boldsymbol{R}_p(t)]\delta[v_\parallel - v_{\parallel p}(t)]\delta[\mu-\mu_p(t)]$$

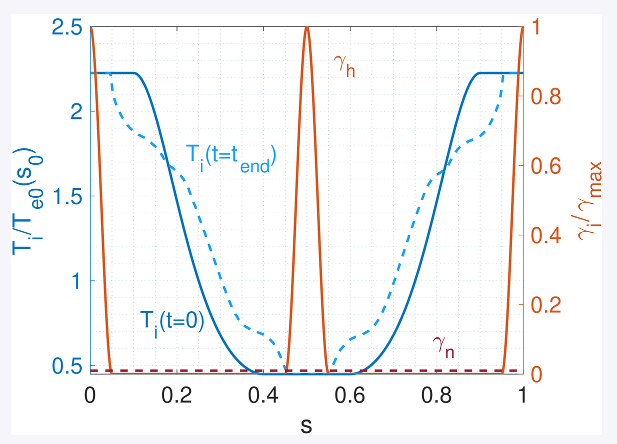
- quasi-neutrality equation for the perturbed distribution:

$$\frac{en_0}{T_e}(\phi - \langle\phi\rangle_{fsa}) - \nabla_\perp\cdot\left(\frac{m_i n_0}{eB^2}\nabla_\perp\phi\right) \quad (1)$$

$$= \int d^3R d\alpha dv_\parallel d\mu \frac{B_\parallel^\star}{m_i}\delta[\boldsymbol{R}+\rho_L(\mu,\alpha)-\boldsymbol{r}]\delta f(\boldsymbol{R},v_\parallel,\mu,t)$$

Symmetrised profiles

- Profiles are symmetrised to mitigate problem of markers exiting radial domain
- Periodic boundary conditions applied to all (x,y,z) directions
- $R_0 = 243.5\rho_s$, $\rho^\star = \frac{1}{66.4}$, $s_0 = 0.25, 0.75$
- $\eta_i(s_0) = 10$, $\left|\frac{d\log T_i(s_0)}{dx}\right| = \frac{4}{a} \rightarrow R_0/L_T = 14.6$
- Krook-like noise control [5] and heat source $\frac{df}{dt} = -\gamma_h(x)[f - f_0(t=0)] + S_{h,corr} - \gamma_n(f-f_n) + S_{n,corr}$



Adaptive control variate

- Assume the control variate f_0 to be:

$$f = f_0(t) + \delta f$$

$$f_0(X, v_\parallel, \mu, t) = f_M(t) = \frac{n_0(X)}{[2\pi T_{i0}(X,t)/m_i]^{3/2}}\exp\left\{-\frac{m_i v_\parallel^2/2 + \mu B(X)}{T_{i0}(X,t)}\right\}$$

- The change in background kinetic energy:

$$\delta E_{kin0}(x,t) = \frac{3}{2}n_0(x)\delta T_{i0}(x,t) = \Sigma_k \xi_k(t)\Lambda_k(x)$$

is calculated at every N_α^{th} step by the ad-hoc relaxation equation [6]:

$$\frac{\partial}{\partial t}\left(\frac{3}{2}n_0(x)\delta T_{i0}(x,t)\right) = \alpha_E\left\langle\int dv_\parallel d\mu\frac{2\pi B_\parallel^\star}{m_i}\delta f\left(\frac{m_i v_\parallel^2}{2} + \mu B\right)\right\rangle_{fsa},$$

which leads to the linear system:

$$\Sigma_k\dot{\xi}_k(t)M_{kj} = \alpha_E\Sigma_p^{N_p}w_p\Lambda_j(X_p)\left(\frac{m_i v_\parallel^2}{2} + \mu B\right)$$

with $M_{kj} = \int\Lambda_k(x)\Lambda_j(x)dx$.

- Once background is changed, weights w_p are redefined to account for $\frac{df}{dt} = 0$, and an additional term of order $(\rho^\star)^2$:

$$\int d^3R d\alpha dv_\parallel d\mu\frac{B_\parallel^\star}{m_i}\delta[\boldsymbol{R}+\rho_L-\boldsymbol{r}][\ \underbrace{\delta f(\boldsymbol{R},v_\parallel,\mu,t)}_{\text{markers}} + \underbrace{f_0(X,v_\parallel,\mu,t) - f_0(X,v_\parallel,\mu,t=0)}_{\text{analytic function}}] \quad (2)$$

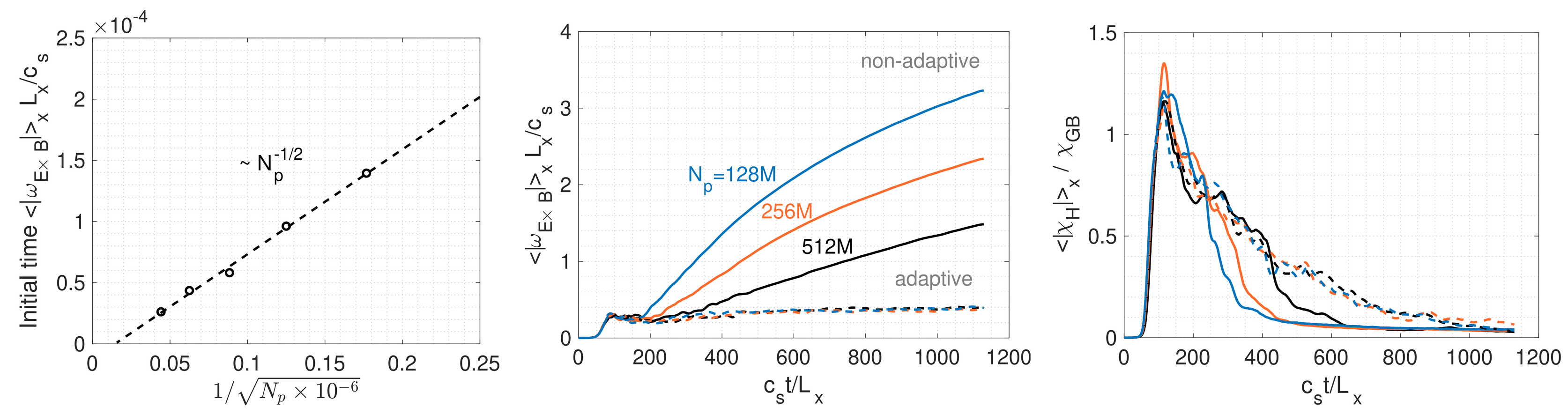
must be appended to the RHS of Eq.(1) to account for deviation from quasi-neutrality for the ion background; as this term is analytic, it is integrated via quadratures

Zonal flows and turbulence quenching

- Zonal flow strength is estimated by the radially averaged $E\times B$ shearing rate

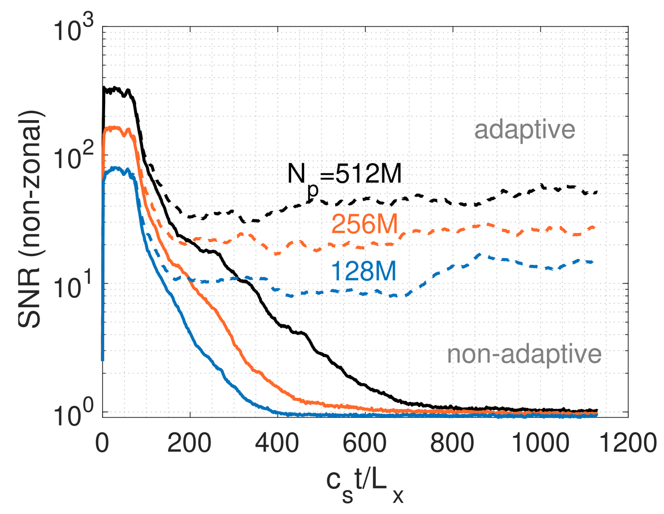
$$\omega_{E\times B}(t) = \left\langle\left|\frac{1}{B}\frac{\partial^2\phi_{00}}{\partial x^2}\right|\right\rangle_x$$

- As the zonal $(m,n) = (0,0)$ mode is not physically damped, it is corrupted by noise accumulation with time
- Spurious zonal flow creation resulting from **sampling error** can be seen from the magnitude of $\omega_{E\times B}(0)$, despite initialing simulations with non-zonal modes
- Strong zonal flow shearing rate **quenches turbulence** [7], as can be seen by radially averaged heat diffusivity χ_H



Signal-to-noise ratio diagnostic

- Consistent with gyrokinetic ordering of $k_\parallel\rho_s \sim \rho^\star$ [8], a Fourier filter is applied to the amplitudes of the DFT of the spline coefficients of (2)
- The **filter** [9] allowing only (m,n) modes satisfying $|m + nq(x)| < \Delta m$ to be resolved
- Definition of 'signal' and noise, represented by square amplitudes inside and outside the filter respectively

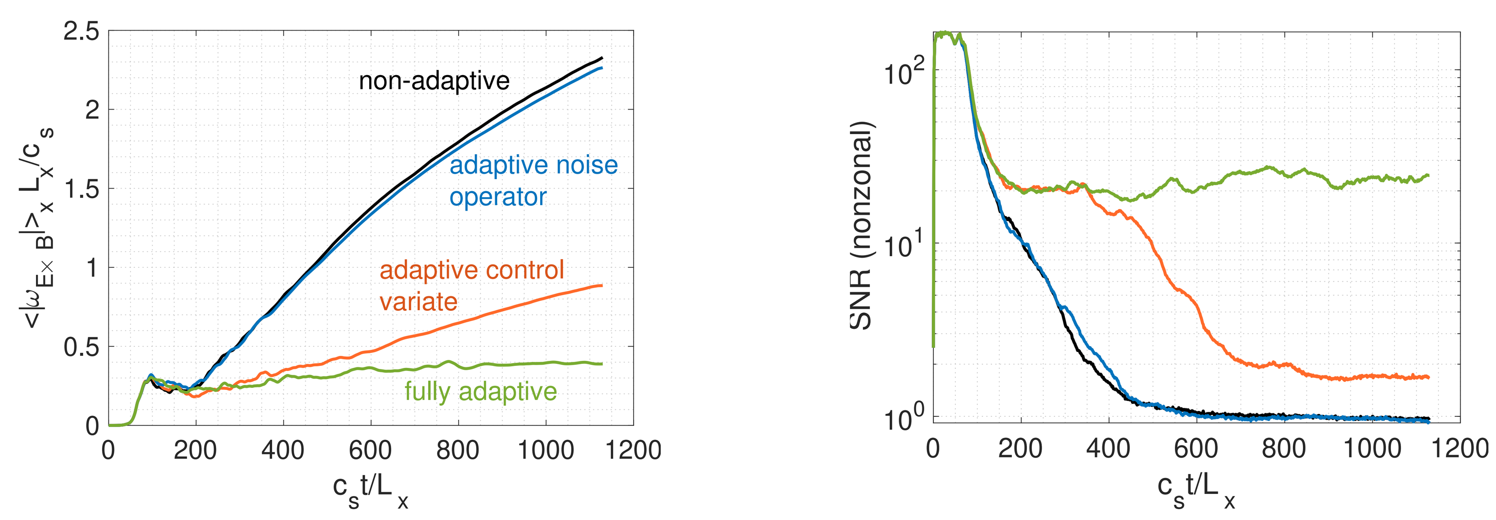


Importance of time-dependent control variate

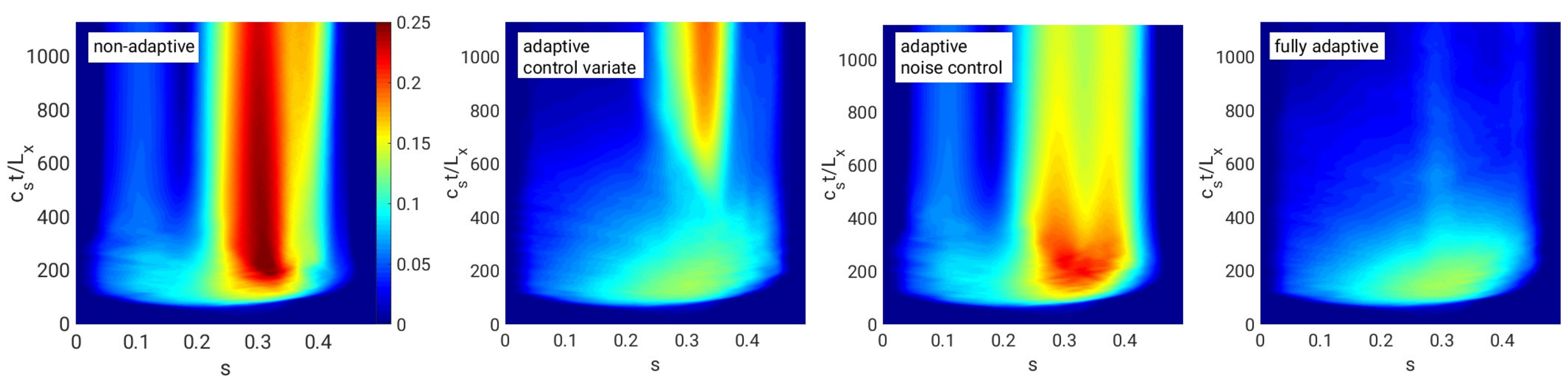
- One in principle could use a time-dependent control variate $f = f_M(t) + \delta f$ and a time-independent noise control operator $-\gamma_n(f - f_M(0))$
- Or vice versa, with $f = f_M(0) + \delta f$ and $-\gamma_n(f - f_M(t))$; for this case, the temperature profile of $f_n = f_M(t)$ adapts according to

$$\frac{\partial}{\partial t}\left(\frac{3}{2}n_0(x)T_{i0}(x,t)\right) = \alpha_E\left\langle\int dv_\parallel d\mu\frac{2\pi B_\parallel^\star}{m_i}[\delta f - (f_M(t) - f_M(t=0))]\right\rangle_{fsa}$$

- The end-time result of $\omega_{E\times B}$ and SNR are qualitatively **similar to the non-adaptive case**



- Comparison between cases can also be made via the **flux-surface-averaged weight standard deviation**



- A **time-dependent control variate and noise control** are necessary to reduce noise accumulation

Future work and generalisation

- Implement adaptive scheme in ORB5 [10] with realistic geometry (on-going)
- Generalise the adaptive scheme to include background density adaptation with its own relaxation rate α_n; useful even in the core when simulating e.g. kink ballooning, tearing, internal kink, modes
- Consider a more complicated control variate:
  - non-Maxwellian function
  - non-flux-surface-averaged function: to capture poloidal deviations

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