Gyrokinetic simulations using a delta-f approach with an evolving background Maxwellian

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Introduction & motivation

- The δf PIC scheme [1] [2] is useful when simulating plasma core as small deviations from equilibrium distribution is expected, thus satisfying the $|\delta f|/|f_0| \ll 1$ assumption, which leads to noise reduction when compared to the full-f scheme
- When simulating the **plasma edge**, steep profile gradients, low density levels and high fluctuation amplitudes lead to **violation of the δf assumption**
- Following a previous work [3], a **Maxwellian control variate as a function of the unperturbed collisionless invariants** is used to demonstrate the advantages gained

Adaptive control variate

- Assume the control variate f_0 to be:

$$f = f_0(t) + \delta f, \qquad f_0(\hat{\psi}_0, \mathcal{E}, t) = \frac{\hat{n}_0(\hat{\psi}_0, t)}{[2\pi \hat{T}_0(\hat{\psi}_0, t)/m]^{3/2}} \exp\left\{-\frac{\mathcal{E}}{\hat{T}_0(\hat{\psi}_0, t)}\right\}$$

with $\hat{\psi}_0$ the corrected canonical toroidal momentum [7] and $\mathcal{E} = mv^2/2 + m\mu B$

- The change in each species' background flux-surface-averaged (f.s.a.) density and kinetic energy is calculated with time-averaging at every N_α^{th} step by the ad-hoc relaxation equations [8]:

$$\frac{\partial}{\partial t}\langle n_0\rangle_{fsa}(\psi) = \alpha_n \overline{\left\langle \int \mathrm{d}^3 v \delta f \right\rangle_{fsa}}, \quad \frac{\partial}{\partial t}\langle E_{kin0}\rangle_{fsa}(\psi, t) = \alpha_E \overline{\left\langle \int \mathrm{d}^3 v \delta f \mathcal{E} \right\rangle_{fsa}},$$

which lead to the modification in f_0 via:

$$\hat{n}_0(\psi, t) \approx \langle n_0\rangle(\psi, t), \quad \hat{T}_0(\psi, t) \approx \langle E_{kin0}\rangle_{fsa}(\psi, t)/[3/2\langle n_0\rangle(\psi, t)].$$

- Once the control variate is changed, weights w_p are redefined to account for $\frac{\mathrm{d}f}{\mathrm{d}t} = 0$
- Modifications to the quasi-neutrality equation (QNE) to solve for potential ϕ,

$$\alpha_P \frac{e n_{e0}(\psi, t)}{T_{e0}(\psi, t)}(\phi - \langle\phi\rangle_{fsa}) - \nabla_\perp \cdot \left(\frac{m_i n_{i0}(\psi, t)}{eB^2}\nabla_\perp \phi\right)$$
$$= \int \mathrm{d}^3 v\, \mathrm{d}^3 R\{\delta f_i \delta[\boldsymbol{R} + \boldsymbol{\rho}_L - \boldsymbol{r}]\} - \delta n_{e,T} - \delta n_{e,P}|_{00} + \int \mathrm{d}^3 v\, \mathrm{d}^3 R\{\Delta f_{i0} \delta[\boldsymbol{R} + \boldsymbol{\rho}_L - \boldsymbol{r}]\} - \Delta n_{e0},$$

with red terms constitute the changes to the left- (LHS) and right- (RHS) hand-side due to background profile changes.

Test-bed: Physical assumptions, profiles, and cases

Code and profiles

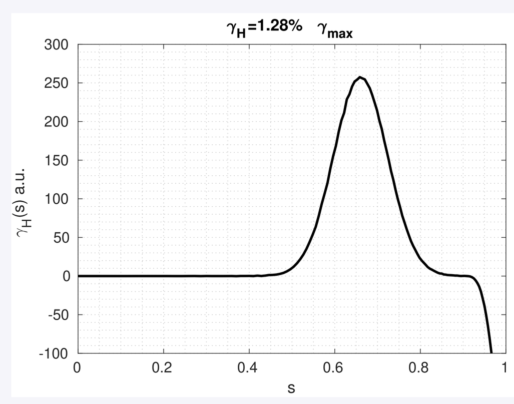
- ORB5 code [4] with following restrictions: single ion species, adiabatic/hybrid electrocs, electrostatic, collisionless, Krook-like noise control [5]
- Density and temperature profiles share the same form:

$$\rho_V = \sqrt{\frac{V(\psi)}{V(\psi_{edge})}} \qquad T(\rho_V) = \begin{cases} a_0 + a_2\rho_V^2 & 0 \le \rho_V \le \rho_{core} \\ T_{ped}\exp[-\kappa_T(\rho_V - 0.8)] & \rho_{core} < \rho_V < 0.8 \\ T_1 + \mu_T(1 - \rho_V) & 0.8 \le \rho_V \le 1 \end{cases}$$
$$T_{ped} = T_1 + 0.2\mu_T$$

- Magnetic equilibrium derived from CHEASE code [6] based on TCV shot #43516, with aspect ratio 3.64, elongation 1.44 and triangularity 0.20 at the last closed flux surface
- $\rho_\star(s_0) = 1/245$ with $s = \sqrt{\psi/\psi_{edge}}$ and $s_0 = 1.0$
- Safety factor: $q(s) = 0.78 + 2.51s^2$

ITG case

- Ion temperature gradient induced turbulence
- Adaptive ion temperature with adiabatic electrons
- 'Flux-driven' with source term $\gamma_H(s)\frac{1}{\langle T_{i0}\rangle_{fsa}(\psi,t=0)}\left[\frac{\mathcal{E}}{\langle T_{i0}\rangle_{fsa}(\psi,t=0)} - \frac{3}{2}\right] f_{i0}(\psi, \mathcal{E}, t=0) + S_{corr}^{(H)}$
- Ion and electron parameters: $\rho_{core} = 0.4431$, $T_1 = 1$, $\kappa_T = 2.3$, $\mu_T = 12$, $n_1 = 1$, $\kappa_n = 3.1$, $\mu_n = 5$



TEM case

- Trapped electron mode induced turbulence
- Adaptive and hybrid electron densities
- 'Temperature-gradient-driven' with source term $-\gamma_K(f - f_0(t)) + S_{corr}^{(K)}$
- Ion parameters: $N_p = 256\mathrm{M}$, $\rho_{core} = 0.4016$, $T_1 = 1$, $\kappa_T = 2.3$, $\mu_T = 6$, $n_1 = 1$, $\kappa_n = 2.3$, $\mu_n = 5$
- Electron parameters: $N_p = 256\mathrm{M}$, $\rho_{core} = 0.4016$, $T_1 = 1$, $\kappa_T = 2.5$, $\mu_T = 10$, $n_1 = 1$, $\kappa_n = 2.3$, $\mu_n = 5$

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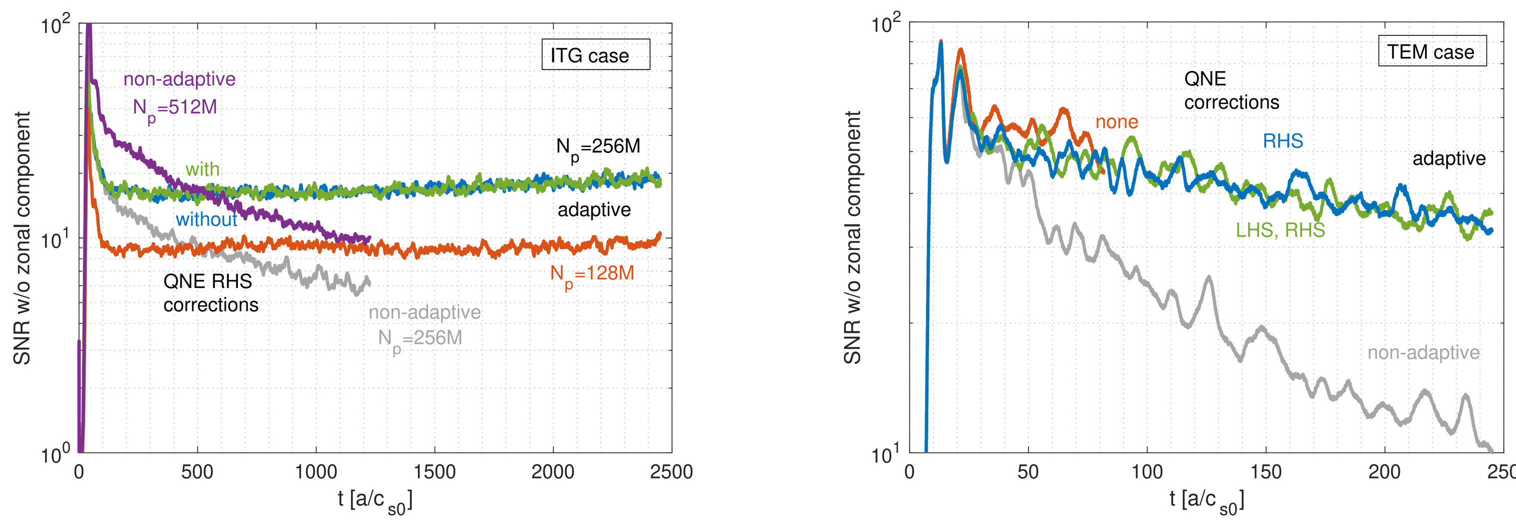
Summary

- All physical assumptions, profiles and cases studied to test the adaptive δf scheme are introduced
- Mechanism for the adaptive scheme using a canonical Maxwellian control variate with time-dependent density and temperature profiles is explained
- Results of all adaptive cases converged, even for simulations with lower number of markers, with improved SNR values and greater profile relaxation

Signal-to-noise ratio diagnostic

- Consistent with gyrokinetic ordering, a Fourier filter [9] is applied to the amplitudes of the DFT of the spline coefficients of QNE allowing only $(m, n) \neq (0, 0)$ modes satisfying $|m + nq(x)| < \Delta m$ to be resolved, which constitute the signal



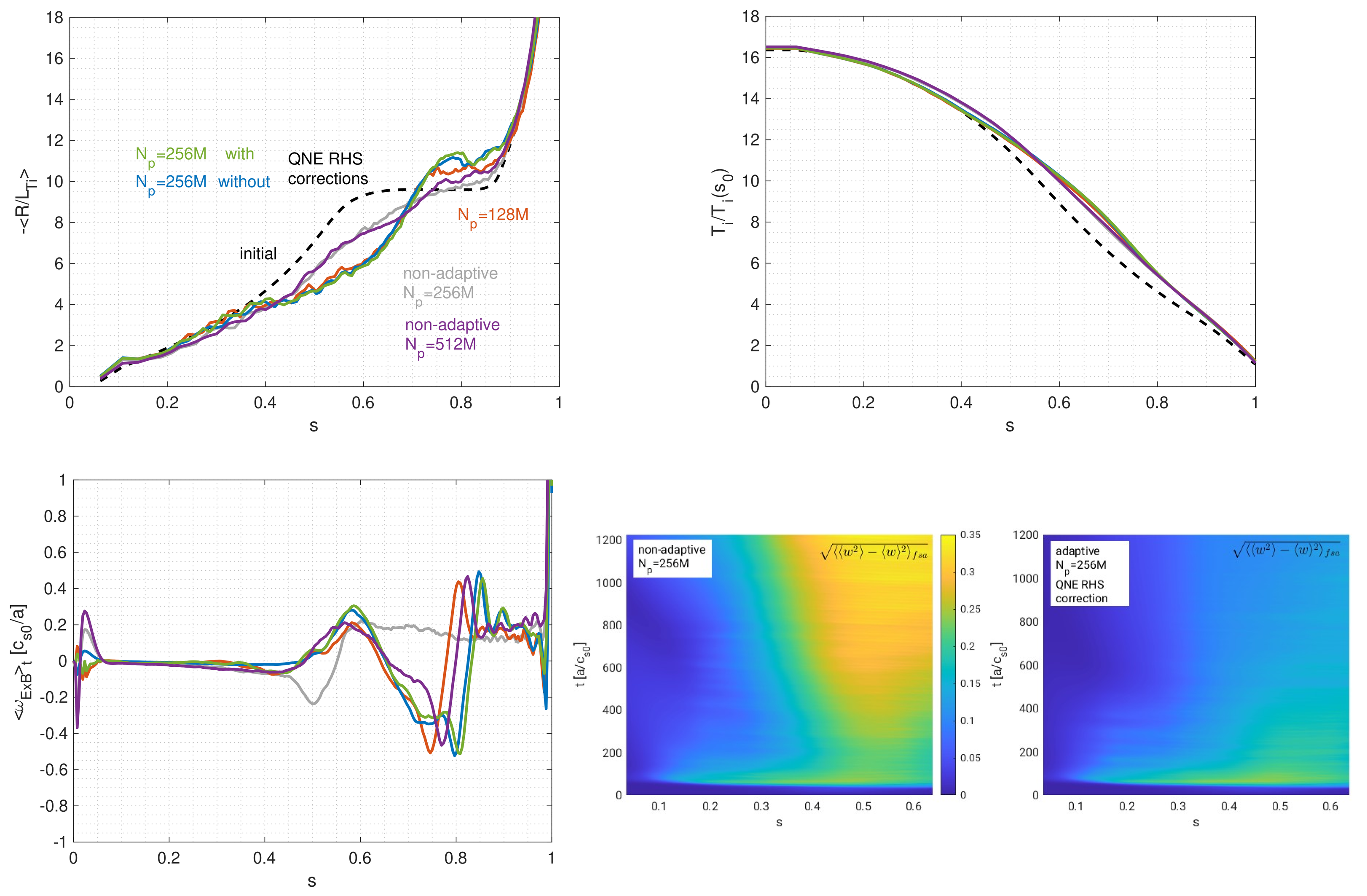
ITG case: 1 species temperature adaptation

- Looking at the time-averaged $c_{s0}t/a \in [1000, 1200]$ ion temperature and zonal flow shearing rate [10] $\omega_{E\times B}$ profiles,

$$\omega_{E\times B}(s, t) = \frac{s}{2\psi_{edge}q}\frac{\partial}{\partial s}\left(\frac{1}{s}\frac{\partial\langle\phi\rangle_{fsa}}{\partial s}\right),$$

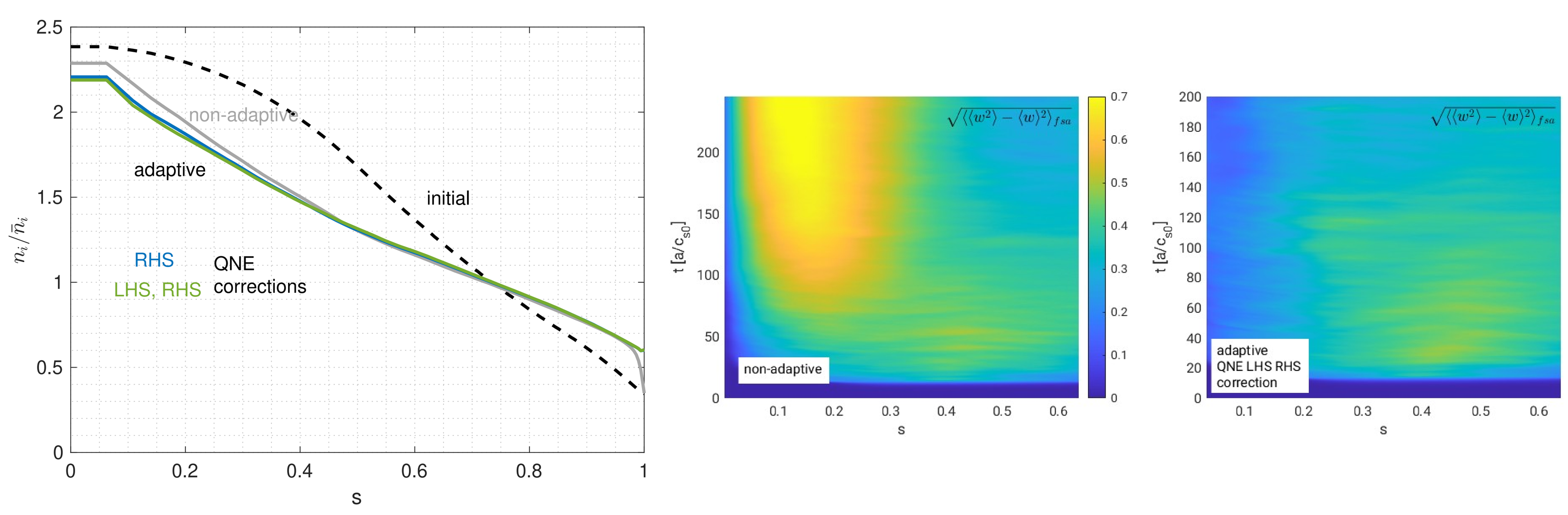
all adaptive cases converged, while the non-adaptive case stopped relaxing

- Comparison between non-adaptive and adaptive cases can also be made via the **f.s.a. δf weight standard deviation**; higher values of this quantity leads to poorer evaluation of the gyrocenter density in the QNE from the δf contribution
- Correction to RHS of QNE seems to have minimal effect, as it is of order $\mathcal{O}(\rho_\star^2)$ for background temperature changes



TEM case: 2 species density adaptation

- Looking at the time-averaged $c_{s0}t/a \in [200, 250]$ ion density profiles, all cases converged
- Due to ambipolarity, ion and electron densities differ locally by 2%
- Nonetheless, adaptive cases **greatly reduce f.s.a. δf weight standard deviation**
- Correction to LHS of QNE shows no difference in results despite $\mathcal{O}(1)$ deviation



Future work and generalisation

- Perform simultaneous density and temperature adaptation for 'flux-driven' TEM-case
- Perform marker number convergence studies at quasi-steady state
- Consider a more general control variate: $f_0(\hat{\psi}_0, \hat{\mathcal{E}}, t) = \sum_{ij} a_{ij}(t)\Lambda_i(\hat{\psi}_0)\Lambda_j(\hat{\mathcal{E}})e^{-\hat{\mathcal{E}}}$

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