Spun fibres: a quasi circularly birefringent medium

Ana Gabriela Correa-Mena, Xu Cheng, and Luc Thévenaz

École Polytechnique Fédérale de Lausanne, Group for Fibre Optics, SCI-STI-LT Station 11, 1015 Lausanne, Switzerland

ABSTRACT

A simplified model describing the polarisation characteristics of spun fibres is proposed, aiming at determining how close to a circularly birefringent medium such a fibre is. This is of crucial importance regarding the interest of such a medium for magneto-optic sensing using optical fibres, mostly exploited in fibre optics current sensors (FOCS). The spun fibre is modelled as a stack of thin linearly birefringent plates, each experiencing a slight incremental rotation. The eigenvalues and eigenvectors of the full stack of plates are determined, enabling us to assess how far a given spun fibre is from an ideal circularly birefringent medium for any spinning rate and linear birefringence. Experimental validation is carried out to compare the model results with real fibres.

Keywords: Spun fibre, linear birefringence, spinning rate.

1. INTRODUCTION

Magneto-optic sensing (e.g. using the Faraday effect) requires that the state of polarisation (SOP) of the light can freely rotate during propagation. The unavoidable residual and/or bend-induced linear birefringence causes precession of the polarisation direction and entirely screens the weak Faraday effect, widely hindering the development of simple current sensing solutions. It turns out that the only suitable media are the so-called circularly birefringent (or optically active), in which a linear polarisation can freely rotate or, equivalently, the circular states of polarisation are maintained all along the propagation (eigenstates).

Historically, this has been achieved by mechanically twisting - so applying a shear-only stress - a fibre showing a low intrinsic birefringence, which is a strenuous process making the fibre eventually flimsy. More recently, a comfortable solution has been proposed by rapidly spinning during fibre drawing a preform of an ordinary highly linearly birefringent fibre (HiBi or polarisation-maintaining), the so-called *spun fibre*.

The question that arises is to determine to what extent such a fibre can be considered like an optically active medium, to evaluate its suitability for magneto-optic sensing. Former models - though describing correctly the polarisation properties - are not enough informative to evaluate this suitability.[1] More recently, a different approach to model the spun fibre has been proposed,[2] by identifying it as a stack of infinitesimally thin identical linear retardation plates that incrementally experience an infinitesimally small identical rotation, as illustrated in Fig.1.

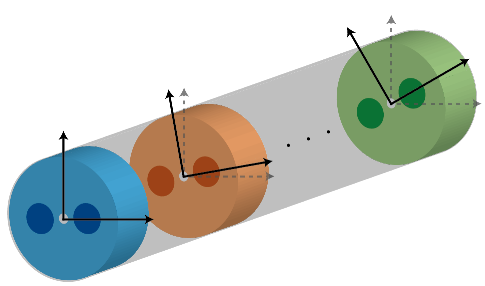

Figure 1. Schematic representation of the spun fibre as a stack of identical linearly birefringent retardation plates, each experiencing a slight incremental rotation.

Further author information:
Luc Thévenaz: E-mail: luc.thevenaz@epfl.ch, Telephone: +41 21 6934774

European Workshop on Optical Fibre Sensors (EWOFS 2023), edited by Marc Wuilpart, Christophe Caucheteur, Proc. of SPIE Vol. 12643, 126432P · © 2023 SPIE · 0277-786X · doi: 10.1117/12.2679782

This complete model can be actually simplified, as proposed in this paper, by considering the infinitesimal size of the plates, leading to simpler expressions and an easier interpretation in an experimental context. The eigenvalues and the eigen-SOPs of the entire stack can be analytically calculated and their similarity with the ideal circular SOPs can be quantitatively evaluated, establishing if a given spun fibre can be reasonably considered like circularly birefringent.

2. DESCRIPTION OF THE STACK MODEL

The description is based on Jones calculus. Let ϕ be the infinitesimal phase delay induced by the linear birefringence and α the infinitesimal rotation angle between each plate. Assuming the birefringence axes of the first plate to be aligned with the coordinate axes without loss of generality, the Jones matrix of the entire stack of N plates is expressed as Eq. 1, applying the usual rule for rotated elements in Jones calculus:

$$\begin{aligned}\overline{\overline{M_N}} = \; & \overbrace{\overline{\overline{\mathbf{R}}}\left[(N-1)\alpha\right]\begin{bmatrix}\exp^{-j\phi/2} & 0 \\ 0 & \exp^{j\phi/2}\end{bmatrix}\overline{\overline{\mathbf{R}}}^{-1}\left[(N-1)\alpha\right]}^{Plate\ N} \; \ldots \\ & \overbrace{\overline{\overline{\mathbf{R}}}\left[2\alpha\right]\begin{bmatrix}\exp^{-j\phi/2} & 0 \\ 0 & \exp^{j\phi/2}\end{bmatrix}\overline{\overline{\mathbf{R}}}^{-1}\left[2\alpha\right]}^{Plate\ 3}\overbrace{\overline{\overline{\mathbf{R}}}\left[\alpha\right]\begin{bmatrix}\exp^{-j\phi/2} & 0 \\ 0 & \exp^{j\phi/2}\end{bmatrix}\overline{\overline{\mathbf{R}}}^{-1}\left[\alpha\right]}^{Plate\ 2}\overbrace{\begin{bmatrix}\exp^{-j\phi/2} & 0 \\ 0 & \exp^{j\phi/2}\end{bmatrix}}^{Plate\ 1}\end{aligned} \tag{1}$$

where the traditional definition for the rotation matrix and its inverse is in Eq. 2, respectively:

$$\overline{\overline{\mathbf{R}}}\left[\alpha\right] = \begin{bmatrix}\cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha\end{bmatrix} \qquad \overline{\overline{\mathbf{R}}}^{-1}\left[\alpha\right] = \overline{\overline{\mathbf{R}}}\left[-\alpha\right] \tag{2}$$

Using this simple property of rotation matrices: $\overline{\overline{\mathbf{R}}}^{-1}\left[m\alpha\right]\overline{\overline{\mathbf{R}}}\left[(m-1)\alpha\right] = \begin{bmatrix}\cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha\end{bmatrix} = \overline{\overline{\mathbf{R}}}\left[-\alpha\right]$

Eq. 1 can be rewritten in a much compacter form as Eq. 3:

$$\overline{\overline{M_N}} = \left[\overline{\overline{\mathbf{R}}}\left[N\alpha\right]\right]\left[\begin{bmatrix}\cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\exp^{-j\frac{\phi}{2}} & 0 \\ 0 & \exp^{j\frac{\phi}{2}}\end{bmatrix}\right]^N = \left[\overline{\overline{\mathbf{R}}}\left[N\alpha\right]\right]\left[\overline{\overline{\mathbf{A}}}^N\right] \tag{3}$$

Therefore, the spun fibre is equivalent to applying N times the same matrix $\overline{\overline{A}}$, followed by a pure rotation over an angle $(N\alpha)$. Since the matrix $\overline{\overline{A}}$ is successively reapplied, a good approach is to determine its eigenvalues and eigenvectors, so that the eigenvalues of $\overline{\overline{\mathbf{A}}}^N$ are simply those of $\overline{\overline{\mathbf{A}}}$ to the power N and the eigenvectors are the same. To carry out this determination, in Eq. 4 we introduce a key simplification, considering that the angles α, ϕ are infinitesimal, and using a first-order expansion:

$$\overline{\overline{\mathbf{A}}} = \begin{bmatrix}\cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\exp^{-j\frac{\phi}{2}} & 0 \\ 0 & \exp^{j\frac{\phi}{2}}\end{bmatrix} \approx \begin{bmatrix}1 & \alpha \\ -\alpha & 1\end{bmatrix}\begin{bmatrix}1-j\frac{\phi}{2} & 0 \\ 0 & 1+j\frac{\phi}{2}\end{bmatrix} \approx \begin{bmatrix}1-j\frac{\phi}{2} & \alpha\left(1+j\frac{\phi}{2}\right) \\ -\alpha\left(1-j\frac{\phi}{2}\right) & 1+j\phi/2\end{bmatrix} \tag{4}$$

from which the eigenvalues can be easily calculated and identified to phase shifts using the same simplified logic:

$$\lambda_{1,2} = 1 \pm j\sqrt{\alpha^2+(\phi/2)^2} \approx \exp^{\pm j\sqrt{\alpha^2+(\phi/2)^2}} \tag{5}$$

Eq. 5 shows that the 2 eigenstates experience a phase shift only, combining the linear birefringence and the rotation as a geometric sum. This result is similar to the one established a long time ago for media presenting both intrinsic linear and circular birefringences.[3,4] It is remarkable here that the same result is obtained without intrinsic optical activity and by a simple geometrical rotation.

To determine the eigenvectors, we define the ratio $\gamma = (\phi/2)/\alpha$ which scales the relative importance of linear birefringence and spin rate of the spun fiber. Thus, if the spinning is dominating, $\gamma \to 0$; if the linear birefringence is strongest, $\gamma \to \infty$; and if both effects have the same magnitude, $\gamma = 1$.

Introducing the unitless factor $C = \frac{\phi/2}{\alpha} + \sqrt{1 + \frac{\phi^2/4}{\alpha^2}} = \gamma + \sqrt{1+\gamma^2}$, the eigenvector can be calculated for the first and second eigenvalues as Eq. 6, respectively:

$$\overline{V_1} = \frac{1}{\sqrt{1+C^2}} \begin{bmatrix} 1 \\ jC \end{bmatrix} \qquad \overline{V_2} = \frac{1}{\sqrt{1+C^2}} \begin{bmatrix} C \\ -j \end{bmatrix} \tag{6}$$

From this simple form for the eigenvectors, it can be immediately seen that, if $C = 1$, or equivalently if $\gamma = 0$, they correspond to circular SOPs like in a circularly birefringent medium. In this case, the spin pitch is much shorter than the linear birefringence beat length, or in other words, the rotation effect dominates the linear birefringence.

Furthermore, it enables us to evaluate how close to circular SOPs the eigenvectors are, too. Technical specs for a typical commercial spun fibre claim $\alpha = 4\phi$, so that $\gamma = 1/8$ and $C = 1.13$. Using elementary linear algebra it is found that each eigenvector is circularly polarised at 99.6% in terms of power.

By decomposing an arbitrary input linear SOP on the eigenvectors, it is possible to evaluate to what extent the linear polarisation is maintained and how much it rotates after propagation in the medium. Replacing the infinitesimal quantities α and ϕ by more sizable parameters, such as the spin rate S_R (in turns per metre) and the conventional birefringence Δn, and introducing the fibre length L, the total rotation θ_R can be expressed as:

$$\theta_R = 2\pi S_R L(\sqrt{1+\gamma^2} - 1) \overset{\gamma << 1}{\simeq} 2\pi S_R L \frac{\gamma^2}{2} = \frac{\pi\, \Delta n^2}{4\lambda^2 S_R} L \tag{7}$$

This expression shows that the rotation rate of the linear polarisation (turns per unit length) is actually dominated by the amplitude of the linear birefringence Δn and is inversely proportional to the spin rate S_R, which may sound at first glance counter-intuitive. In short, the linear birefringence is essential as an attracting centre for the linear polarisation and no rotation will be observed in absence of linear birefringence, even at a high spin rate. On the other hand, a too-high linear birefringence is detrimental to the quality of the overall circular birefringence and the rule is that the spin rate S_R is much larger than the linear birefringence beat length (or $\gamma << 1$). A deeper analysis shows that a linear polarisation gets periodically slightly elliptical during propagation, reaching a maximum when the polarisation has rotated by 45°, and then returns to linear after achieving a 90° rotation, and so on periodically. The maximum ellipticity amounts to:

$$\epsilon_{\max} = \arctan\left(\frac{C^2 - 1}{1 + C^2}\right) \tag{8}$$

showing that the ellipticy is logically vanishing for $C \to 1$ or $\gamma << 1$. These results show that by measuring the rotation per unit length and the maximum ellipticity, it is in principle possible to retrieve the actual values of Δn and S_R. This possibility has been tested and is reported in the next section.

3. EXPERIMENTAL VALIDATION

A very simple setup was used to carry out a primary validation of the results delivered by the model. It is simply made of a single-frequency laser in the 1300 nm wavelength range to avoid any depolarisation effect in the birefringent medium, a linear polariser placed in front of the fibre under test, and a polarisation analyser (ThorLabs PAX1000) to determine the SOP at the fibre output. Since the phase shift due to the linear birefringence depends on wavelength, it was decided to make the system non-destructive by scanning the laser wavelength over a narrow spectral range, instead of modifying the fibre length by physical fibre cuts. The test sample of total length $L = 10$m is from a spun fibre produced by the company iXblue (model IXF-SPUN-1310-80), delivered with these important technical specs: mode field diameter of 7 μm, round core shape, spin rate 400 turns/m, linear birefringence beat length of 8 ± 2mm. This gives a tentative γ factor of 0.156.

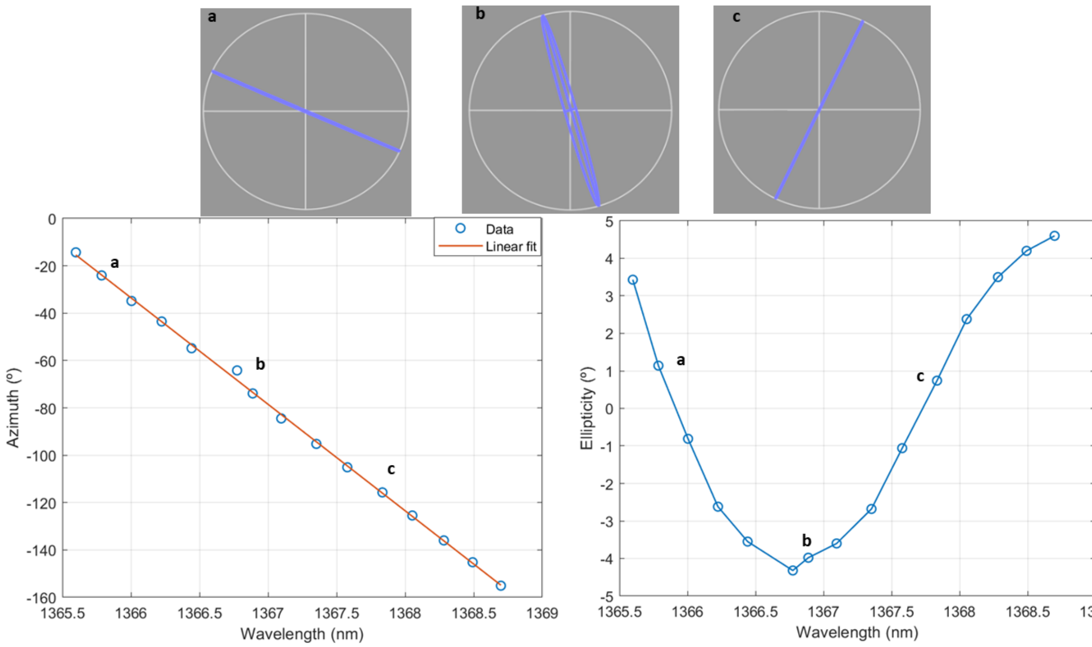

Figure 2. Screenshots of the SOP at the output of the spun fibre (a) linear polarisation, (b) elliptical polarization, and (c) linear polarisation (top); azimuth angle as a function of wavelength (left bottom); and ellipticity as a function of wavelength (right bottom).

By varying the wavelength we observed that the SOP at the fibre output was predominantly linear and its direction was continuously rotating with the wavelength. Moreover, the SOP was periodically turning from linear to slightly elliptical, to return to linear with a periodicity of 90°, exactly as expected from our simplified model. This is illustrated in Fig.2, where screenshots of the SOP are shown, as well as the azimuth angle of the linear polarisation and the ellipticity as a function of the wavelength.

From the data, it could be extracted an azimuth rotation of -45°/nm and a peak ellipticity of 5.5°. From this last quantity, we calculate an experimental value of $\gamma = 0.096$, which can be considered like securing a good maintaining of the circular SOP and definitely makes this fibre a good circularly birefringent medium. Using the mechanical spin rate $S_R = 400$ given in the technical specs by the manufacturer, considered as reliable since not instrumentally measured, the linear birefringence beat length is evaluated to be 13 mm, so some 50% longer than the manufacturer specs. Considering that the beat length in the technical specs is that of the unspun fibre, it can be reasonably accepted that the spinning process reduces the linear birefringence to 2/3 of its original value. The rate of azimuthal rotation per wavelength unit was not yet exploited in the calculation since it depends strictly on the group linear birefringence that may significantly differ from the phase linear birefringence in polarisation maintaining fibres. Cutback destructive tests will be carried out in a next measurement campaign.

Acknowledgements: The authors are grateful for the Innosuisse funding 51518.1 supporting this research.

REFERENCES

[1] Laming, R. and Payne, D., "Electric current sensors employing spun highly birefringent optical fibers," *Journal of Lightwave Technology* **7**(12), 2084–2094 (1989).

[2] Liu, Y. and Song, H., "Theoretical analysis on polarization characteristics of spun birefringent optical fiber based on an analytical Jones matrix model," *Optik* **228**, 166179 (2021).

[3] Tabor, W. J. and Chen, F. S., "Electromagnetic propagation through materials possessing both faraday rotation and birefringence: Experiments with ytterbium orthoferrite," *Journal of Applied Physics* **40**(7), 2760–2765 (1969).

[4] Ulrich, R. and Simon, A., "Polarization optics of twisted single-mode fibers," *Appl. Opt.* **18**, 2241–2251 (Jul 1979).