Application of random fields in geotechnical reliability analysis

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Abstract

The goal is to find the difference between two types of probabilistic slope stability analysis. The first admits a uniform soil and takes into account uncertainty of the soil properties. The second includes spatial variability of the soil properties. Both analyses are performed using a Monte Carlo framework with the help of a meta-model, in particular a Polynomial Chaos Expansion (PCE). The PCE is built based on N_{ED} 2D finite element realisations and allows a quick evaluation of new ones. This provides an efficient reliability analysis when performing the Monte Carlo Simulation method. A comparison between the probabilities of failure between the two approaches is shown. Furthermore, the effect of the correlation lengths of the random field on the probabilities of failure is evaluated.

Reliability analysis using PCE

A reliability analysis consists of estimating the probability that the system of interest will fail to meet a performance criterion [3]. To efficiently compute a large number of samples, the FE-model is approximated and replaced by a meta-model, the Polynomial chaos expansions (PCE). Its mathematical definition is as follows [1]:

$$\mathcal{M}(\mathbf{U}) \approx \mathcal{M}^{PC}(\mathbf{U}) = \sum_{\alpha \in A} a_\alpha \psi_\alpha(\mathbf{U})$$

where $\psi_\alpha(\mathbf{U})$ are multivariate polynomials orthonormal with respect to the vector of the joint probability density function of the random independent input variables f_U, and $a_\alpha \in \mathbb{R}$ are the corresponding polynomial coefficients to indices α in the truncated set $A \in \mathbb{N}^K$ [1]. In this context, the failure probability can be estimated by:

$$P_f = \frac{N_{Fail}}{N_{Tot}}$$

Random Fields

A random field $H(x,\omega)$ is a set of random variables whose geographical location is given by the spatial variable $x \in D_x$ and the associated outcome $\omega \in \Omega$ in the space of elementary events [1]. Gaussian random fields are defined by their mean $\mu(x)$, their standard deviation $\sigma^2(x)$, and their auto-correlation function $\rho(x, x'|l)$ with spatial correlation length l. The latter describes the distance over which properties tend to be spatially correlated [2]. The continuous field is discretized using the Karhunen-Loève expansion method:

$$H(x,\omega) \approx \widehat{H}(x,\omega) = \mu(x) + \sum_{i=1}^{M} \sqrt{\lambda_i}\xi_i(\omega)\varphi_i(x)$$

References

[1] Schöbi R. Sudret B: *Application of conditional random fields and sparse polynomial chaos expan- sions in structural reliability analysis*

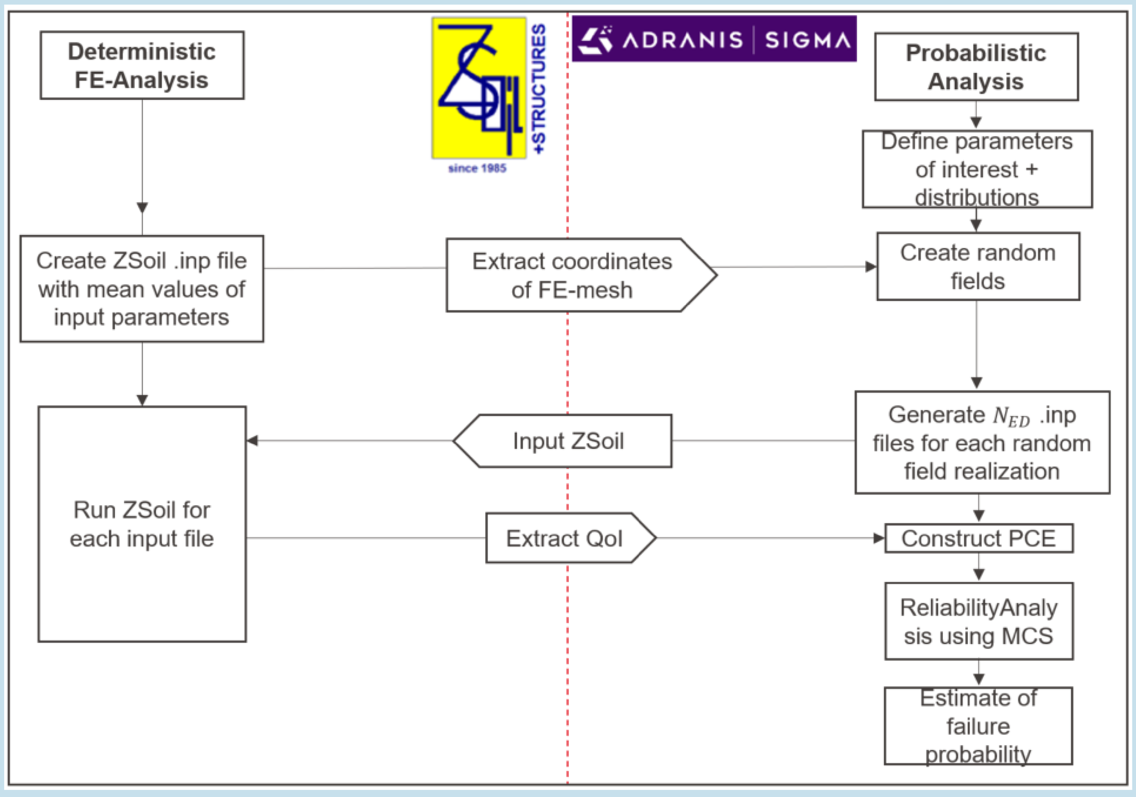
[2] Griffiths, Huang Fenton: *Probabilistic infinite slope analysis*

[3] Bruno Sudret: *Stochastic Finite Element Methods and Reliability: A State-of-the-Art Report.*

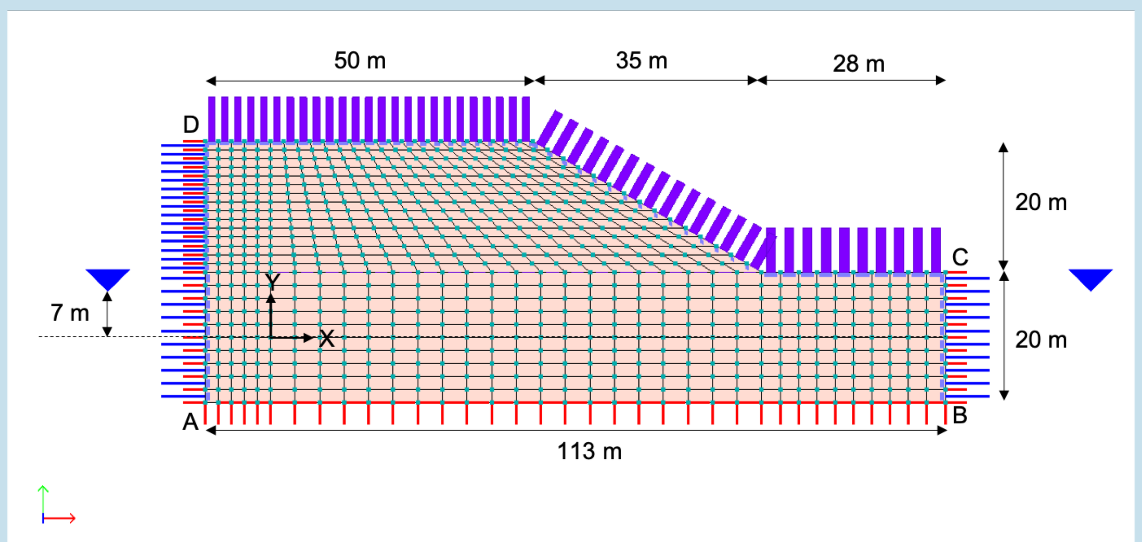
Workflow

The procedure applied to perform reliability analysis on a slope's stability is illustrated in the figure below. The left side of the figure, includes all steps which are performed using the finite element software ZSOIL. In this part of the approach, a finite element model of the considered slope is created with the mean values of the soil properties. All relevant information, such as the finite element mesh and the boundary conditions, are stored in a ZSOIL input file. The right side represents the steps followed in the probabilistic approach using the uncertainty quantification tool Adranis Sigma. The white arrow boxes between the two sides depict the coupling between the approaches, permitting the reliability analysis.

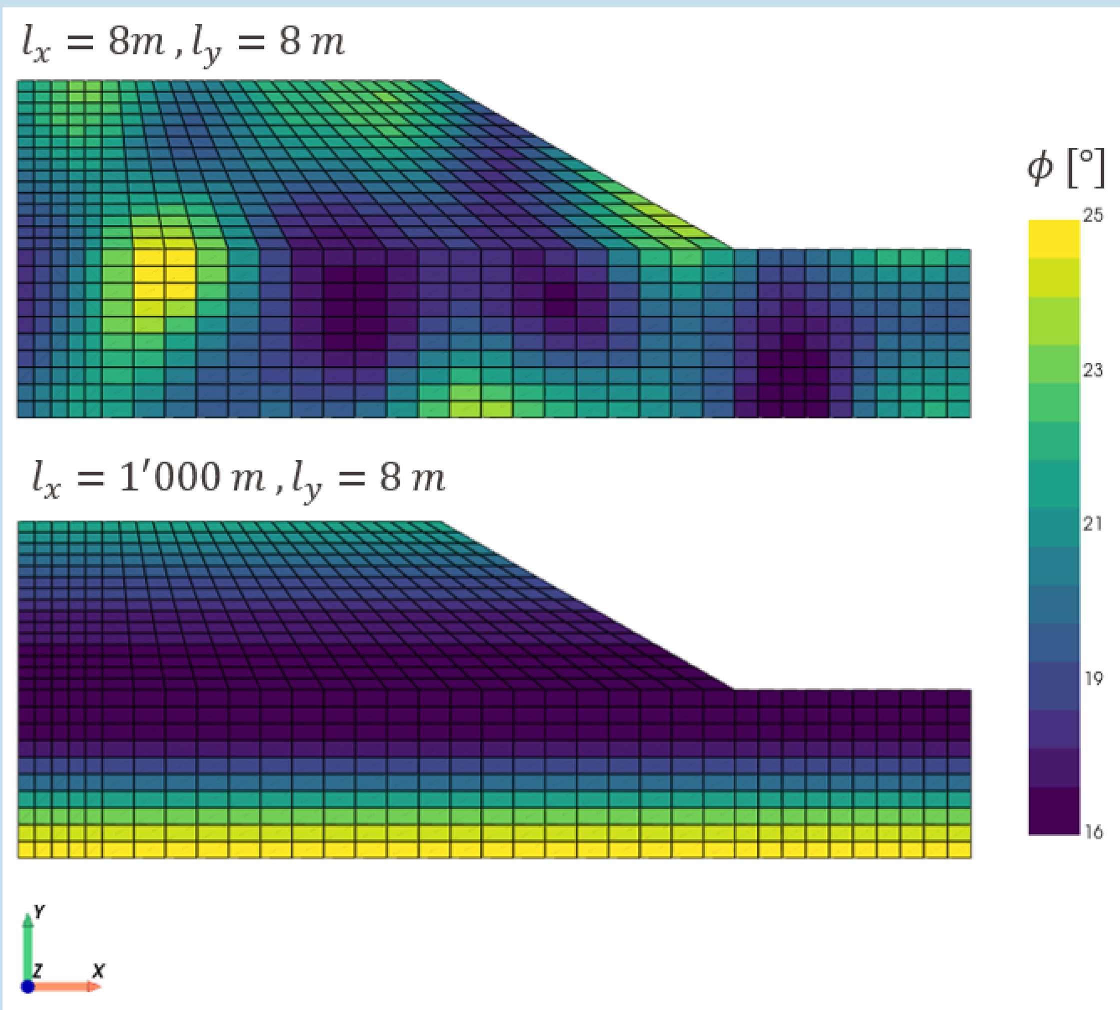
- First the uncertain input parameters along with their distributions and statistical moments are identified
- To create the random fields, the coordinates of the centroids $(x_{c,i}, y_{c,i})$ of each element $i = 1, 2, ..., n_{elem}$ (n_{elem} being the total number of elements in the FE-mesh) are extracted. An independent standard normal sample matrix of dimension $N_{ED} \times M$ is generated. The matrix is then transformed to the physical, correlated space and N_{ED} ZSOIL input files are created with different soil properties for each element.
- These samples are further evaluated by the finite element model and the quantities of interest (QoI) are extracted.
- A PCE is constructed on the basis of the computed samples and finally the failure probability is given through a Monte-Carlo Simulation.



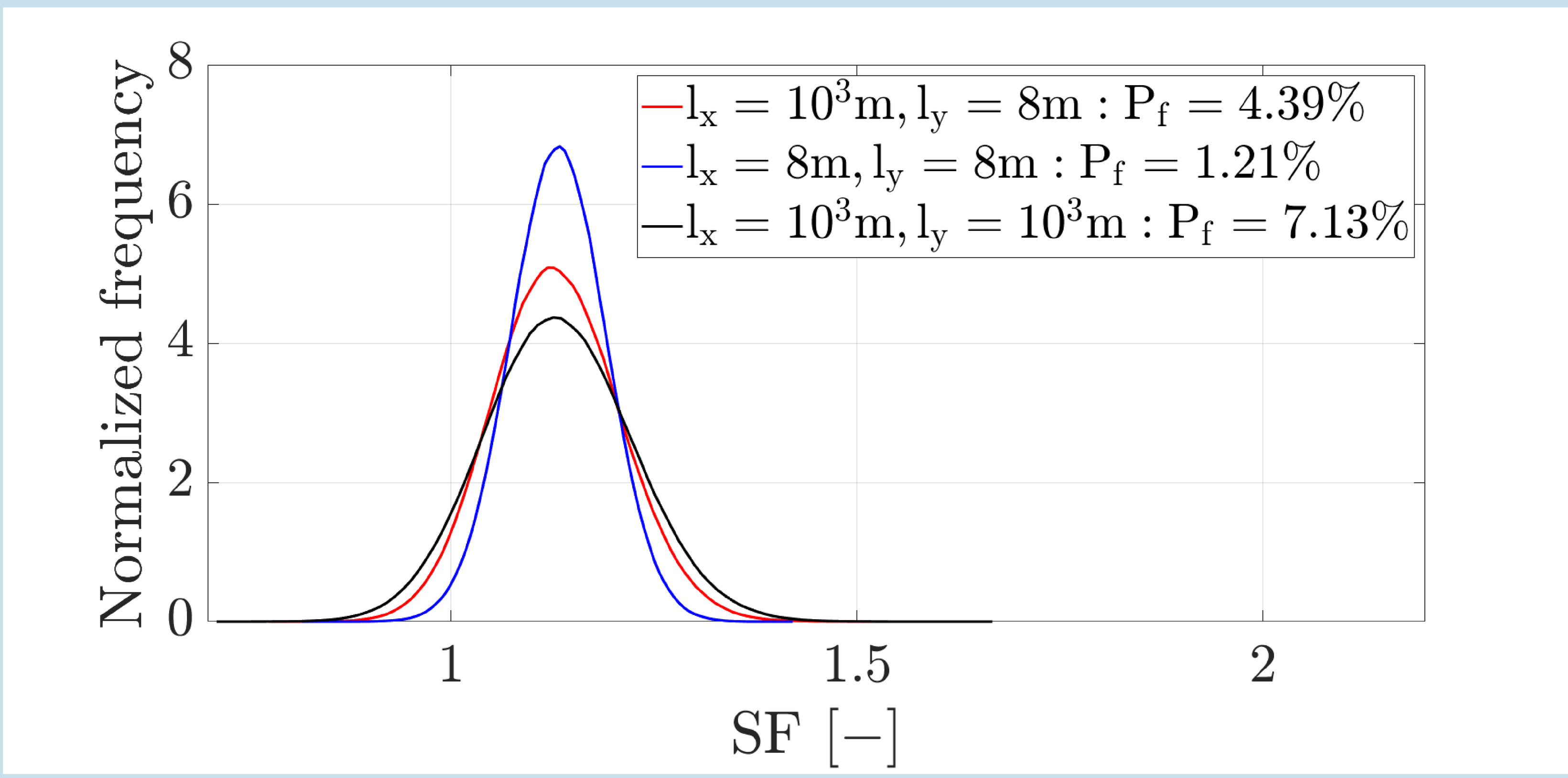
Slope reliability analysis with spatially variable friction angle



Finite element model of the considered slope



Exemplary random field realizations for a heterogenous field with $l_x = 8\ m$, $l_y = 8\ m$ and a horizontal stratiphication field with $l_x = 10^3\ m$, $l_y = 8\ m$



Probability density function PDF of the safety factor SF of the slope for the heterogenous random field in blue, for the horizontal layered random field in red and for the uniform field in black