The effect of in-situ linear stress gradient on the frictional shear rupture growth in 2D

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1. Motivation

The hydro-mechanical response of fractures during hydraulic stimulations depends strongly on the initial stress conditions. Tractions acting on pre-existing discontinuities depend on both depth and orientation of the fracture planes. It is known that within certain length scales, the in-situ principal stresses vary linearly with depth. In this work, we investigate the effect of a linear stress gradient on the growth of injection-induced frictional ruptures under 2D plane-strain conditions. We notably compare a semi-analytical solution with fully-coupled numerical simulations using a numerical solver developed at EPFL's Geo-Energy Lab.

2. Problem formulation

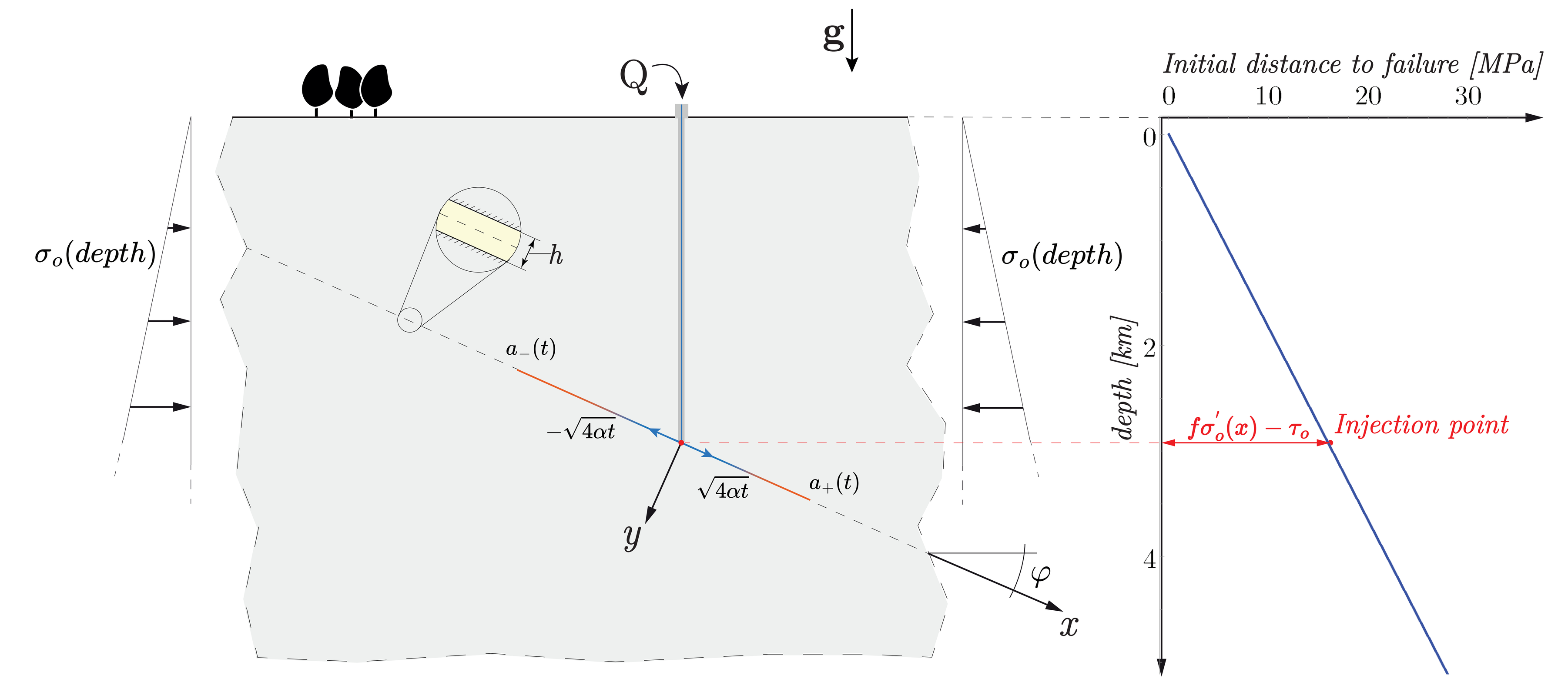


Figure 1. Problem set-up

We consider a planar fault embedded in a homogeneous isotropic linearly elastic solid under plane strain conditions (Fig. 1). The fault is oriented arbitrarily with regard to the gravitational acceleration and is characterized by a dip angle φ, constant friction coefficient f, and constant hydraulic properties (permeability k and hydraulic diffusivity α). The injection of fluid reactivates the fault in shear and produce a frictional rupture that propagates asymmetrically due to the stress gradient. The fluid flow inside the fault is modelled as a flow in the porous media from a line source with a constant injection overpressure.

Initial stresses resolved on the fault. We write the initial effective stresses with respect to the fault coordinate system (see Fig. 1): $\sigma'(x) = \sigma_o' + m_{\sigma'}x$ and $\tau(x) = \tau_o + m_\tau x$, where both the effective normal stress σ_o' and shear stress τ_o are calculated at the injection point and $m_{\sigma'}$, m_τ are the corresponding stress gradients.

Quasi-static elastic equilibrium. Under quasi-static conditions, the shear stress $\tau(x,t)$ acting on the fault plane is equal to the sum of the initial shear stress $\tau_o(x)$ and the changes due to the propagation of the slipping patch. During propagation, the shear stress $\tau(x,t)$ inside the slipping patch must equal the current value of the local shear strength $\tau_s(x,t)$. Therefore, the quasi-static elastic equilibrium within the slipping patch can be written as:

$$\tau_s(x,t) = \tau_o(x) + \frac{\mu^*}{2\pi}\int_{-a_-(t)}^{a_+(t)} \frac{\partial\delta(\xi,t)/\partial\xi}{\xi - x}\, d\xi, \tag{1}$$

where $\tau_s(x,t) = f(\sigma_o'(x) - p(x,t))$ is the fault strength equal to the product of the constant friction coefficient f and the current effective normal stress $\sigma_o'(x,t) = \sigma_o'(x) - p(x,t)$, with $p(x,t)$ the current distribution of pore pressure.

Finiteness condition for the stresses near the rupture tips. The right-hand side of the elastic equilibrium (1) does not prevent the shear stresses at the rupture tips to be infinite. In fact, the shear stress at the rupture fronts must be equal to the shear strength which is finite. Thus, the stress intensity factors must be zero

$$K_{-a_1} = 0, \quad K_{a_2} = 0$$

These two conditions provide equations for the propagating tips. To obtain expressions for the stress intensity factors we invert (1) for the derivative of the slip distribution and consider the slip behaviour near the propagating tips $x \to -a_1$ and $x \to a_2$.

Fluid flow. Fluid flow along the interface in the limiting case of an impermeable rock matrix reduces to a one-dimensional diffusion equation. The solution for the pore-pressure in the case of a point injection performed under a constant over-pressure Δp is

$$p(x,t) = p_o + \Delta p \,\mathrm{Erfc}\left(\frac{|x|}{\sqrt{4\alpha t}}\right)$$

3. Scaling

Two dimensionless numbers govern the propagation of such a fluid-induced frictional rupture

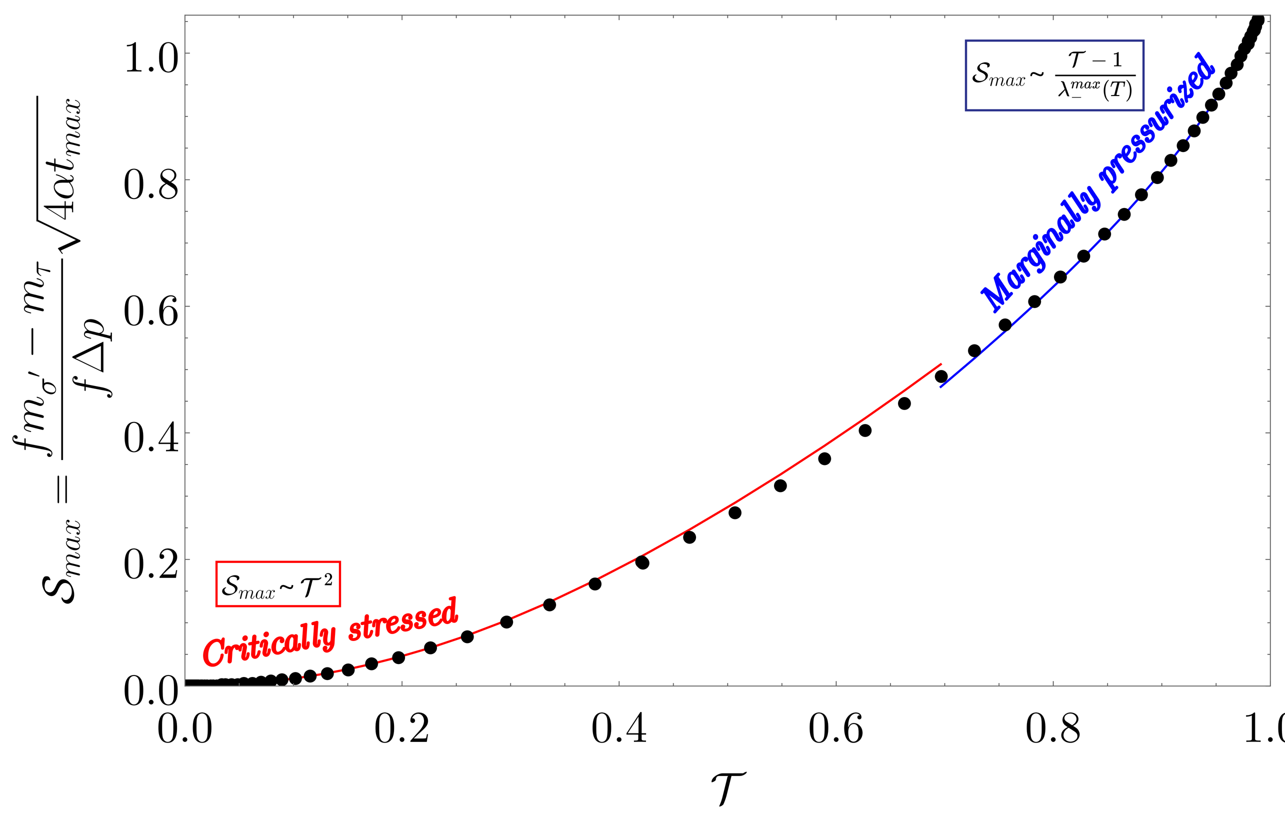
$$\mathcal{T} = \frac{f\sigma_o' - \tau_o}{f\Delta p} = \frac{distance\ to\ failure}{strength\ reduction}, \qquad \mathcal{S}(t) = \frac{f m_\sigma - m_\tau}{\sigma_*}\ell_*(t)$$

where the length scale $\ell_*(t)$ and stress scale σ_* depend on the limiting regime.

The so-called stress-injection parameter $\mathcal{T}$ previously introduced in [1] for a uniform in-situ stress field is here evaluated at the exact depth of the injection. In the case where $\mathcal{T}_o \to 0$ (which we refer to as the critically-stressed limit), the fault is initially close to failure (or alternatively the over-pressure is large). The corresponding length scale in this limit is $\ell_* = a_-(t) + a_+(t)$ and the stress scale $\sigma_* = f\sigma_o' - \tau_o$. On the other hand, when $\mathcal{T}_o \to 1$ (marginally pressurized limit) the fluid pressure is "just sufficient" to activate slip on the fault by reducing the shear strength $(f(\sigma_o - (p_o + \Delta p)) \to \tau_o)$. In this limit $\ell_* = \sqrt{4\alpha t}$ and $\sigma_* = f\Delta p$. In general, the distance to failure in the numerator of $\mathcal{T}$ increases with depth making the fault more marginally pressurized at the injection point. On the contrary, $\mathcal{T} \to 0$ at lower depths: the fault becomes more critically stressed closer to the surface.

The dimensionless parameter $\mathcal{S}(t)$ encapsulates the influence of in-situ effective stress gradient on fault strength. In other words, $\mathcal{S}(t)$ is related to the development of the rupture asymmetry. At early times $\mathcal{S}(t) << 1$ (or equivalently $\ell_* << 1/\nabla\mathcal{T}(x)$), the effect of the stress gradient is negligible. The solution corresponds to the uniform stress case and the rupture stays symmetrical. When $\mathcal{S}(t) \approx 1$, the linear dependence of the in-situ effective stress starts to play a role and an asymmetry of the rupture is expected to develop. $\mathcal{S}(\bar{t}) = 1$ defines the characteristic time-scale of the problem - which differs in the critically stressed vs the marginally pressurized regimes.

It is important to point out that the problem, as formulated here, eventually results in the nucleation of a dynamic rupture. Indeed, the initial distance to failure is equal to zero at the surface. As the fluid pressure increases along the fault and the propagating frictional rupture transfers elastic stress ahead of its tip, the fault shear strength decreases ahead of the rupture. The combination of these perturbations with the decreasing initial distance to failure at lower depths leads to a acceleration of the upper tip and ultimately the nucleation of a dynamic rupture. In terms of dimensionless time $\mathcal{S}(t)$, (see the figure below) for critically stressed faults the solution reaches this instability faster such that no significant rupture asymmetry has time to develop. With increasing values of $\mathcal{T}$, the rupture can propagate longer before this instability and "feel" the in-situ stress gradient.

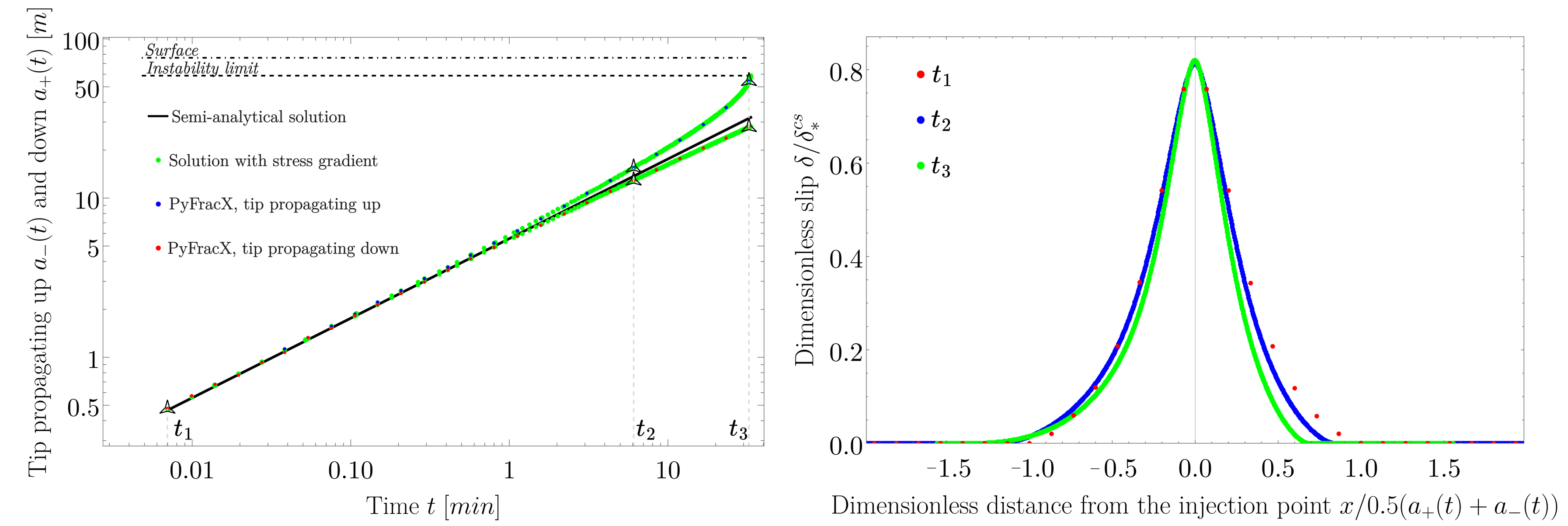


4. Results

Critically stressed fault. In the critically stressed limit the main role in propagating the rupture is played by pre-injection conditions and stress transfer phenomena. The proper scaling is

$$\sigma_* = f\sigma_o' - \tau_o, \quad \delta_*^{cr} = \frac{f\Delta p\sqrt{4\alpha t}}{\mu'}$$

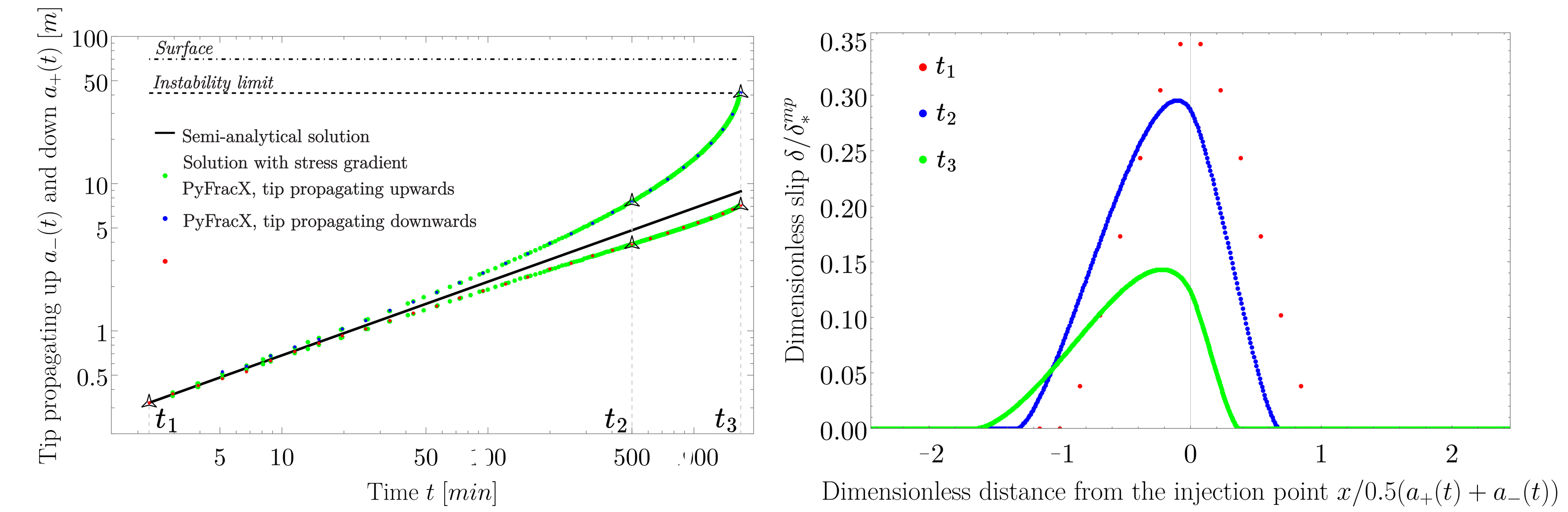
and the second dimensionless parameter in this limit (or equivalently dimensionless time $\hat{t}$) is $\hat{t} = \mathcal{S}(t) = \frac{fm_{\sigma'} - m_\tau}{f\sigma_o' - \tau_o}(a_-(t) + a_+(t))$. The rupture does not undergo significant influence of stress gradient and develops small asymmetry.



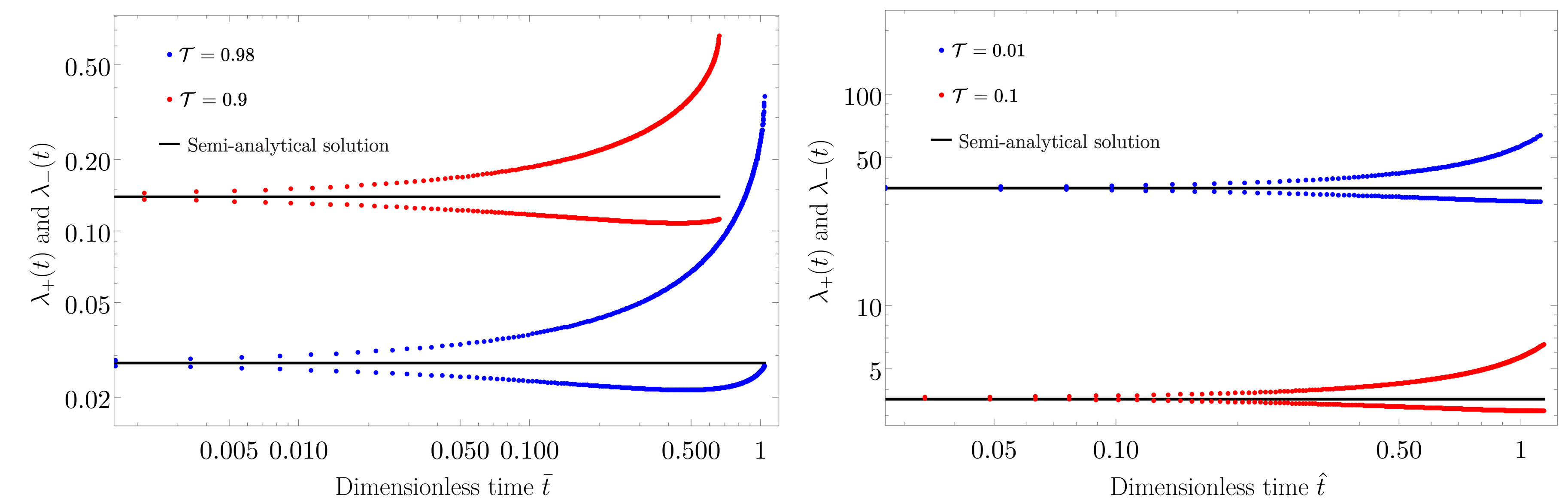
Marginally pressurized fault. In the marginally pressurized case, the correct scaling is

$$\sigma_* = f\Delta p, \quad \delta_*^{mp} = \frac{f\Delta p(a_-(t) + a_+(t))(\lambda_- + \lambda_+)}{4\mu'}$$

and the second dimensionless parameter (or equivalently dimensionless time $\bar{t}$) in this limit is $\sqrt{\bar{t}} = \mathcal{S}(t) = \frac{fm_\sigma - m_\tau}{f\Delta p}\sqrt{4\alpha t}$. Introduction of stress gradient leads to significant acceleration of the tip propagating upwards until it becomes unstable.



Will the rupture outpace/lag behind the diffusion front? We will follow [1-2] and introduce $\lambda_\pm = a_\pm(t)\ell_d$ where $\ell_d = \sqrt{4\alpha t}$ is the diffusion length scale. If the rupture outpaces the fluid front then $\lambda_\pm \geq 1$ and $\lambda_\pm \leq 1$ if it lags the fluid front, respectively. In the marginally pressurized limit, the fault will always lag the diffusion front whereas in the critically stressed limit it will always propagate faster than the fluid.



5. Conclusions

A linear variation of initial stresses along the fault introduces an additional time scale into the propagation of fluid-induced frictional ruptures. At early times the solution is self-similar and the rupture fronts propagate symmetrically. When the dimensionless time approaches the characteristic time, we observe the onset of rupture asymmetry. The tip propagating up accelerates with time which results in dynamic rupture nucleation even within constant friction assumption.

References

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