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Maintenance scheduling of manufacturing systems based on optimal price of the network

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ABSTRACT

Goods can exhibit positive externalities impacting decisions of customers in social networks. Suppliers can integrate these externalities in their pricing strategies to increase their revenue. Besides optimizing the prize, suppliers also have to consider their production and maintenance costs. Predictive maintenance has the potential to reduce the maintenance costs and improve the system availability. To address the joint optimization of pricing with network externalities and predictive maintenance scheduling based on the condition of the system, we propose a bi-level optimization solution based on game theory. In the first level, the manufacturing company decides about the predictive maintenance scheduling of the units and the price of the goods. In the second level, the customers decide about their consumption using an optimization approach in which the objective function depends on their consumption, the consumption levels of other customers who are connected through the graph, and the price of the network which is determined by the supplier. To solve the problem, we propose the leader–multiple-followers game where the supplier as a leader predicts the strategies of the followers. Then, customers as the followers obtain their strategies based on the leader's and other followers' strategies. We demonstrate the effectiveness of our proposed method on a simulated case study. The results demonstrate that knowledge of the social network graph results in an increased revenue compared to the case when the underlying social network graph is not known. Moreover, the results demonstrate that obtaining the predictive maintenance scheduling based on the proposed optimization approach leads to an increased profit compared to the baseline decision-making (perform maintenance at the degradation limit).

1. Introduction

Social networks can have a significant influence on the economic system [1,2]. In a social network, users connect and communicate with each other. Hence, the decision of a user has the potential to affect the decisions of other users, particularly for goods that exhibit externalities. In such cases, users receive different extents of externalities from the consumption of other users to whom they are connected in their social network. The information about the consumption behavior of users spreads in the network. If supplier knows the graph network topology and the consumption behavior of the users, it can try to integrate this information in their optimization. Aiming to maximize their revenue, suppliers determine the price of the good for each user such that their revenue is maximized.

Besides considering the prizing of the manufactured goods, manufacturers also need to consider the production and the maintenance costs. For example, deterioration of manufacturing units can significantly decrease the production quality [3]. The quality of the production depends, among others, on the maintenance strategy of the manufacturing equipment to prevent production disruptions [4,5]. In fact, the goal of maintenance scheduling is to maximize the availability of the system, while fulfilling the demand of the users or customers. Traditionally, preventive maintenance is scheduled regularly to restore the units to their healthy states. However, preventive maintenance does not consider the system's condition and the dynamic changes [6]. Predictive maintenance has the potential to significantly reduce the maintenance costs. Furthermore, maintenance scheduling does typically not consider the consumption behavior of its customers which influences the prizing but also the produced amount which in turn influences the requirements on the availability of the system but also the degradation of the system and the demand for maintenance actions. Hence, in order to maximize the availability and reward function, it is advisable for a supplier to schedule the maintenance of its manufacturing units using the underlying graph of the social network of the users and the users' consumption function.

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In this paper, we address the maintenance scheduling problem of a manufacturing system of goods with positive externalities. In this context, the goal of the manufacturer is to find the optimal price of the goods and the optimal maintenance scheduling for the manufacturing units such that its revenue and availability are maximized. In order to obtain the maintenance scheduling, we consider the framework of predictive maintenance in which the remaining useful lifetime (RUL) of the units and the associated uncertainties can be predicted. We define the optimization model for the manufacturer which comprises two main elements: (1) the revenue that can be obtained by selling the produced goods and (2) maintenance cost of the manufacturing units. To model the positive externality effects, we assume that users or customers are connected through a graph and that they choose their consumption level of the good based on their utility function and the price of the good. Hence, we face a bi-level optimization problem with the two levels representing the supplier and the users. In order to solve the optimization problem, we propose to formulate the problem as a leader–follower game between the supplier and the users. In this game, the supplier is the leader and can predict the strategies (consumption function) of the followers. Then, based on this prediction, it determines the maintenance scheduling and optimal price of the network and sets the price for each of the users. We investigate the efficiency of the proposed approach based on a case study.

To the best of our knowledge, this is the first research that obtains the maintenance scheduling of different units in the manufacturing system by considering the price of the network and the positive externality effects of the goods using a leader–follower game among the manufacturer and the customers.

The rest of the paper is organized as follows. The review of related work is presented in Section 2. The preliminaries on predicting the remaining useful lifetime (RUL) and the leader–multiple-followers game are introduced in Section 3. The supplier's and customers' objective functions are formulated in Section 4. The solution method based of the leader–follower game is proposed in Section 5. The simulation results are presented in Section 6. The conclusion remarks are made in Section 7.

2. Related work

Network price of goods with positive externalities. Several approaches have been proposed to determine the optimal price in networks with positive externalities. The authors of [2] study the optimal pricing strategies of a supplier of goods with positive externalities assuming that consumers are connected by a social network graph. Their usage levels depend directly on the usage of their neighbors within a social network [2]. Since the users' consumption behavior affects that of other users, it has been modeled based on the game theory concept [7,8]. The optimal contracting between the seller and buyers is proposed in [7]. The authors develop the solution method using the Stackelberg game where the seller acts as a leader and the buyers are followers. The strategic interaction and networks have been addressed using game theory in [8]. In this study, the decisions of some suppliers affect those of other suppliers in the network.

In all of these works, the manufacturers solely consider the pricing of the goods and do not take the impact of the production on the operation and maintenance costs into account. In fact, sudden failures causing an interruption of the production can severally impact the ability of the supplier to respond to the demand. Furthermore, not considering the interruptions caused by planned maintenance in the pricing will result in sub-optimal prices and revenues. To maximize their revenue, the suppliers should strive to perform maintenance on their manufacturing units when the price of the network is low and they sell less.

Maintenance scheduling. Maintenance scheduling with the aim of increasing the system reliability and availability has been addressed in many applications such as electrical market [9,10], network transportation [11,12], and manufacturing system [13,14].

Several studies have proposed different approaches for maintenance scheduling and optimal management of manufacturing systems. The authors of [13] propose a method based on a Genetic algorithm to obtain the maintenance scheduling of manufacturing equipment using the predicted level of degradation. The maintenance scheduling for manufacturing systems proposed in [15] considers the product quality and the maintenance cost including the repair and preventive maintenance cost. The inventory planning, pricing, and maintenance are addressed in [16]. In this paper, the authors show that the maintenance scheduling can increase the profit and reduce the cost of product. The authors of [17] obtain the predictive maintenance policy for a machine by considering the economic dependency of the system.

Dynamic maintenance scheduling which obtains the optimal predictive maintenance using dynamic programming method is developed in [18,19], and [20]. In these studies, the deterioration process of the units is formulated as a Markov decision process. An optimization-based approach for dynamic maintenance scheduling of different units which considers the operating conditions is proposed in [14]. The authors of [21] propose reinforcement learning approach to obtain the optimal joint production, maintenance and product quality control policies by considering that the maintenance and repair duration time are random variables. The multi-deterioration state is modeled for the machine in [22], and then the production scheduling and maintenance planning are proposed. The dynamic maintenance and production scheduling for manufacturing system are addressed in [23] using model predictive control. In all of these studies, the maintenance scheduling is obtained only based on the degradation level of different units without considering the price of the goods and the effect of maintenance on the revenue of the manufacturer.

Notation:. Given $D \in \mathbb{R}^{n\times n}$, D^{-1} denotes the inverse of D. We define the column augmentation of $Z_n(t)$ for $t = 1,\dots,T$, as $Z_n = \mathbf{col}(Z_n(1),\dots,Z_n(T)) := [Z_n(1),\dots,Z_n(T)]$.

3. Preliminaries

3.1. Remaining useful life (RUL) and predictive maintenance

The remaining useful life (RUL) is defined as the amount of time that an asset will continue to satisfy its desired operating conditions [24]. Predicting RUL is one of the core tasks in predictive maintenance applications. Predicting the RUL enables on the one hand to perform maintenance before the failure occurs and by that also to improve the availability of the system while on the other hand, the lifetime of the system can be fully exploited resulting in less frequent replacements or repairs. Several methods have been proposed in the literature to estimate RUL. These methods can be categorized in three categories: (1) model-based approaches, where the physics principle models of the degradation of the asset are applied [25]; (2) data-driven methods, in which the RUL is estimated based on the condition monitoring data only [26]; (3) knowledge-based approaches that depend on the domain knowledge of an expert [27]. RUL estimations are always subject to uncertainties. While not all of the RUL prediction methods also estimate the uncertainty of the predictions, uncertainty quantification is an integral part of decision support systems and is particularly desirable by domain experts using such systems. For many systems with multiple-units that are jointly fulfilling the demand, performing predictive maintenance at the end of life of the system may not the optimal. Since, the maintenance decision of each unit also depends on the decision of other units and the production requirements. Hence, predictive maintenance scheduling is pivotal for fulfilling the demand while minimizing the maintenance costs. In our problem formulation, we assume that we are able to predict the RUL and can quantify the associated uncertainty. We assume that the distribution function of

RUL is given. The manufacturer should, therefore, take into account the uncertainty of RUL and schedule the maintenance based on the production requirements and the associated costs of the unavailability, while considering that a part of the lifetime of the unit may be lost when replacing the unit before the end of life.

3.2. Leader–multiple-followers game

Let us consider N-players, $i = 1, \dots, N$, as followers and one player as a leader. Let x_i denote the follower i's strategy and y the leader's strategy, respectively. Let $U_i^f(x_i, x_{-i}, y)$ denote the utility function of the follower i where x_{-i} is the strategy of all followers except follower i. Let $U^l(y, x)$, in which $x = \mathbf{col}(x_1, \dots, x_n)$, denote the utility function of the leader. The equilibrium point of the leader–multiple-followers game can be defined as follows:

Definition 1. Let (x^*, y^*) be the equilibrium point among the leader and the followers, then,

$$\begin{aligned} &U_i^f(x_i^*, x_{-i}^*, y^*) \geq U_i^f(x_i, x_{-i}^*, y^*), \quad i = 1, \dots, N, \\ &U^l(y^*, x^*) \geq U^l(y, x^*) \end{aligned} \tag{1}$$

In the leader–follower game, the leader predicts the followers' strategies and implements its strategy first, while the followers react to the leader's decisions.

4. Problem formulation

In this section, we propose the optimization model for obtaining the maintenance scheduling of the units in the manufacturing system based on the optimal price of the network.

Let us define $\mathcal{J} = \{1, \dots, J\}$ as the set of units in the manufacturing system that are manufacturing the good that has positive externalities to the customers. Let us define $\mathcal{N} = 1, \dots, N$ as the set of customers who are connected through the graph matrix W in the social network. The il element of matrix W denoted by $w_{il} \geq 0$ represents the strength of the connection between the two customers in the network and represents concurrently also the influence of customer l on customer i. The manufacturer aims to obtain the optimal price of the network while maintaining a high availability of the manufacturing units. We assume that the manufacturer has implemented a predictive maintenance strategy to improve the availability of the system and is able to predict the RUL of the system and the associated uncertainty. Predicting the RUL enables the manufacturer to schedule the maintenance before the end of life and, thereby, prevent unscheduled down times while fully exploiting the lifetimes of the units. The manufacturer aims on the one hand to maintain a high availability of the units and exploit the useful lifetime of the units as much as possible, while performing the maintenance at points in time when it has the least possible impact on its revenue. The manufacturer is, therefore, seeking to obtain the maintenance scheduling of manufacturing units for the decision horizon time $\mathcal{T} = \{1, \dots, T\}$ while maximizing the revenue that can be obtained by selling the products to the customers. In the following, we explain the proposed optimization model.

4.1. Manufacturer's optimization model: joint optimization of maintenance schedule and price of the network

In the following, we propose the manufacturer's optimization problem which concurrently optimizes the price of the network and the maintenance scheduling of the units.

It should be mentioned that there are many factors that can affect the good's price including factors on the demand side and also factors that affect the production costs. We consider one specific aspect of demand: goods with externalities and one specific aspect of the production costs: predictive maintenance. Supplier can integrate these externalities in its pricing strategies to increase its revenue. With respect to the production costs, we consider predictive maintenance. The combination of these two factors: pricing with network externalities and predictive maintenance scheduling based on the condition of the system have been an overlooked problem in the literature and is in the focus of the current paper.

Maintaining: In this study, we assume that the deterioration state of the unit increases with time. The maintenance restores the state of the unit to its initial condition (as good as new). Hence, we model the deterioration and the restoration as follows:

$$s_j(t+1) = \lambda_j(1 - x_j(t))s_j(t) + 1, \tag{2}$$

where λ_j denotes the aging rate of unit j, $s_j(t)$ and $s_j(t-1)$ denote the deterioration state of unit j at instants t and $t-1$, respectively, $x_j(t)$ is a binary variable and $x_j(t) = 1$ denotes that unit j performs maintenance at instant t.

When the deterioration state reaches the degradation threshold, it reaches the end of life which is equivalent to the state where the remaining useful life (RUL) reaches zero. If the unit has not been maintained before the end of life, it will fail. As discussed above, the prediction of the RUL is always associated with uncertainties. It can, therefore, be considered as a random variable with a known distribution function. We assume that the predicted RUL and the associated uncertainty are given. To avoid failure, the units should perform maintenance before the end of life. To model this aim, we apply the chance constraint problem as follows:

$$\mathbb{P}\Big(S_{2,j} \in \mathcal{A} | \quad s_j(t) - S_{2,j} \leq 0\Big) \geq 1 - \alpha, \tag{3}$$

where $\mathbb{P}$ is a probability measure defined over $\mathcal{A}$. $S_{2,j}$ is the degradation threshold of the deterioration state of unit j. $s_j(t)$ which satisfies the chance constraint (3) can be considered as the α-level feasible solution.

Another constraint of this problem is that the total consumption of all customers should be less than the total production of all units. We assume that the demand cannot be postponed and is satisfied at all times. Furthermore, we assume that the good is perishable and cannot be stored to cover the demand during the maintenance down time. Moreover, we assume that when the unit performs maintenance, its production is zero. Hence, we can model this constraint as follows:

$$\sum_{i \in \mathcal{N}} q_i(t) \leq \sum_{j \in \mathcal{J}} (1 - x_j(t)) q_{j,max}, \tag{4}$$

where $q_i(t)$ is the amount of the consumption of customer i at instant t, $q_{j,\max}$ is the maximum production output by manufacturing unit j.

The supplier seeks to obtain the maintenance scheduling which satisfies (3) and (4), while concurrently minimizing the maintenance cost of the units which is modeled as follows:

$$C = \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} c_j x_j(t), \tag{5}$$

where c_j is the maintenance cost of unit j.

Pricing: The supplier aims to obtain the price of the network which maximizes the following reward:

$$R = \sum_{i \in \mathcal{N}} \sum_{t \in \mathcal{T}} \phi_i(t) q_i(t), \tag{6}$$

where $\phi_i(t) \geq 0$ is the price of the consumed good of customer i at instant t, $q_i(t) \geq 0$ is the consumption of customer i at instant t.

Supplier's objective function: Using (5) and (6), the utility function of the supplier can be expressed as follows:

$$U = \sum_{i \in \mathcal{N}} \sum_{t \in \mathcal{T}} \phi_i(t) q_i(t) - \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} c_j x_j(t), \tag{7}$$

Hence, by applying (7), and by considering constraints (2), (3), and (4), we can formulate the supplier's objective function as follows:

$$\begin{aligned} \max_{\phi,x,s} & \sum_{i\in\mathcal{N}}\sum_{t\in\mathcal{T}} \phi_i(t)q_i(t) - \sum_{j\in\mathcal{J}}\sum_{t\in\mathcal{T}} c_j x_j(t) \\ \text{S.b.}\quad & \mathrm{C}_1: \quad s_j(t+1) = \lambda_j(1-x_j(t))s_j(t)+1, \\ & \mathrm{C}_2: \quad \mathbb{P}\Big(S_{2,j}\in\mathcal{A} \,|\, \quad s_j(t) - S_{2,j} \le 0\Big) \ge 1-\alpha, \\ & \mathrm{C}_3: \quad \sum_{i\in\mathcal{N}} q_i(t) \le \sum_{j\in\mathcal{J}} (1-x_j(t))q_{j,max}, \\ & \mathrm{C}_4: \quad q_i(t)\ge 0, \quad \phi_i(t) \ge 0, \end{aligned} \tag{8}$$

where $\phi = \mathbf{Col}\{\phi_1,\dots,\phi_N\}$, $\phi_i = \mathbf{Col}\{\phi_i(1),\dots,\phi_i(T)\}$, $x = \mathbf{Col}\{x_1,\dots,x_N\}$, $x_i = \mathbf{Col}\{x_i(1),\dots,x_i(T)\}$, $s = \mathbf{Col}\{s_1,\dots,s_N\}$, $s_i = \mathbf{Col}\{s_i(1),\dots,s_i(T)\}$, $i\in\mathcal{N}$.

The problem (8) is a nonlinear mixed-integer programming problem due to the constraints C_1 and C_2. We convert (8) to the mixed-integer linear programming by using the big-M method [28] and scenario-based approach [29]. In the following, we explain these two approaches.

First, let us introduce a new variable $y_j(t) = x_j(t)s_j(t)$. By using the big-M method, we can convert constraint C_1 to the following mixed-integer linear constraints:

$$\begin{aligned} & s_j(t+1) = s_j(t) - y_j(t) + 1, \\ & y_j(t) \ge s_j(t) - (1-x_j(t))M, \\ & y_j(t) \le s_j(t) + (1-x_j(t))M, \\ & 0 \le y_j(t) \le x_j(t)M. \end{aligned} \tag{9}$$

Now, we can substitute the chance constraint (3) with the finite number of constraints using the scenario based approach. Each constraint corresponds to a different realization of $S_{2,j}^k$, $k=1,\dots,K$, for the uncertain parameter $S_{2,j}$. Thus, we have:

$$s_j(t) - S_{2,j}^k \le 0, \quad k=1,\dots,K. \tag{10}$$

Remark 1. The number of the scenarios K should be chosen sufficiently large such that the feasible solution of (10) is an α-level feasible solution of (3).

Hence, by using (9) and (10), the optimization problem (8) leads to the following mixed-integer linear programming problem:

$$\begin{aligned} \max_{\phi,x,s,y} & \sum_{i\in\mathcal{N}}\sum_{h\in\mathcal{H}} \phi_i(t)q_i(t) - \sum_{j\in\mathcal{J}}\sum_{t\in\mathcal{T}} c_j x_j(t) \\ \text{S.b.}\quad & \mathrm{C}'_1: \quad s_j(t+1) = \lambda_j s_j(t) - \lambda_j y_j(t) + 1, \\ & \mathrm{C}'_2: \quad y_j(t) \ge s_j(t) - (1-x_j(t))M, \\ & \mathrm{C}'_3: \quad y_j(t) \le s_j(t) + (1-x_j(t))M, \\ & \mathrm{C}'_4: 0 \le y_j(t) \le x_j(t)M. \\ & \mathrm{C}'_5: \quad s_j(t) - S_{2,j}^k \le 0, \quad k=1,\dots,K. \\ & \mathrm{C}'_6: \quad \sum_{i\in\mathcal{N}} q_i(t) \le \sum_{j\in\mathcal{J}} (1-x_j(t))q_{j,max}, \\ & \mathrm{C}'_7: \quad q_i(t)\ge 0, \quad \phi_i(t)\ge 0, \end{aligned} \tag{11}$$

where $y = \mathbf{Col}\{y_1,\dots,y_N\}$, $y_i = \mathbf{Col}\{y_i(1),\dots,y_i(T)\}$.

The consumption $q_i(t)$ is obtained by the customers' objective function, which is explained in the following.

4.2. Customers' model: consumption

The customers seek to obtain the optimal consumption which maximizes their reward function. Let us model the customer i objective function as follows:

$$\max_{q_i} \sum_{t\in\mathcal{T}} \Big(v_i(q_i(t)) + \sum_{l\in\mathcal{N}} w_{il}q_l(t)q_i(t) - \phi_i(t)q_i(t)\Big), \tag{12}$$

where $q_i = \mathbf{Col}\{q_i(1),\dots,q_i(T)\}$. The objective function consists of three terms. The first term denotes the reward that customer i can obtain by consuming the good. Inspired by the studies in [2] and [1], we model the first reward term as follows:

$$v_i(q_i(t)) = -\frac{1}{2}a_i q_i^2(t) + b_i q_i(t), \tag{13}$$

where $a_i \ge 0$ and b_i are the customer's i model. The second term expresses the effect of other customers which are connected through the social network graph to customer i. This term implies that the consumption $q_k(t)$ of customer k, who is connected to customer i through the social network graph with the weight w_{ik}, influences the objective function of customer i. The third term formulates the price that customer i is willing to pay to the supplier for obtaining the consumed good $q_i(t)$.

Remark 2. The following equations justify that the second term in (12) can increase the consumption level of customer i:

$$\max_{q_i} \sum_{t\in\mathcal{T}} \Big(-\frac{1}{2}a_i q_i^2(t) + b_i q_i(t) + \sum_{l\in\mathcal{N}} w_{il}q_l(t)q_i(t) - \phi_i(t)q_i(t)\Big). \tag{14}$$

In this case, the consumption level of customer i at time t can be obtained as follows:

$$q_i(t) = \frac{1}{a_i}(b_i + \sum_{l\in\mathcal{N}} w_{il}q_l(t) - \phi_i(t)), \tag{15}$$

where the second term is the effect of the decisions of other customers on the decision of customer i. In fact, this term means that the consumption level of other customers in the network who are connected to customer i increases the consumption level of this customer.

5. Proposed solution: leader and multiple-followers game

In this section, we propose the solution method using the concept of leader–multiple-followers game [30,31] for solving the bi-level optimization problem ((11) and (12)). In this problem, we consider the customers as the followers who are seeking to obtain their consumption and the supplier as the leader who is responsible to obtain the price of the network and maintenance scheduling of its manufacturing units. The schematic of the proposed framework is shown in Fig. 1.

5.1. Consumption equilibrium of multiple-followers game

In the proposed solution, we assume that the customers are the followers of the supplier. According to (12), since the customers' objective function depends on the strategies of other customers, we face a game theory problem among the customers which can be described as $G = (\mathcal{N}, \mathcal{Q}, \mathcal{U})$:

(1) $\mathcal{N}$ denotes the set of customers as the players.

(2) $\mathcal{Q} = \prod_{i\in\mathcal{N}} q_i$ denotes the strategy space of the players.

(3) $\mathcal{U} = \{U_1,\dots,U_N\}$ is the set of utilities, where the utility of player i is defined as follows:

$$U_i(q_i, q_{-i}, \phi_i) = \sum_{t\in\mathcal{T}} \Big(v_i(q_i(t)) + \sum_{l\in\mathcal{N}} w_{il}q_l(t)q_i(t) - \phi_i(t)q_i(t)\Big) \tag{16}$$

where $q_{-i} = \{q_1,\dots,q_{i-1},q_{i+1},\dots,q_N\}$ are the strategies of all players except player i.

The Nash equilibrium (NE) is one appropriate output solution of the game. Using Definition 1, the NE among customers can be defined as follows:

$$U_i(q_i^*, q_{-i}^*, \phi_i^*) \ge U_i(q_i, q_{-i}^*, \phi_i^*) \tag{17}$$

In order to find the NE of the game among customers, let us define the following assumption:

Assumption 1. $a_i \ge \sum_{l\in\mathcal{N}} w_{il}$, $i\in\mathcal{N}$.

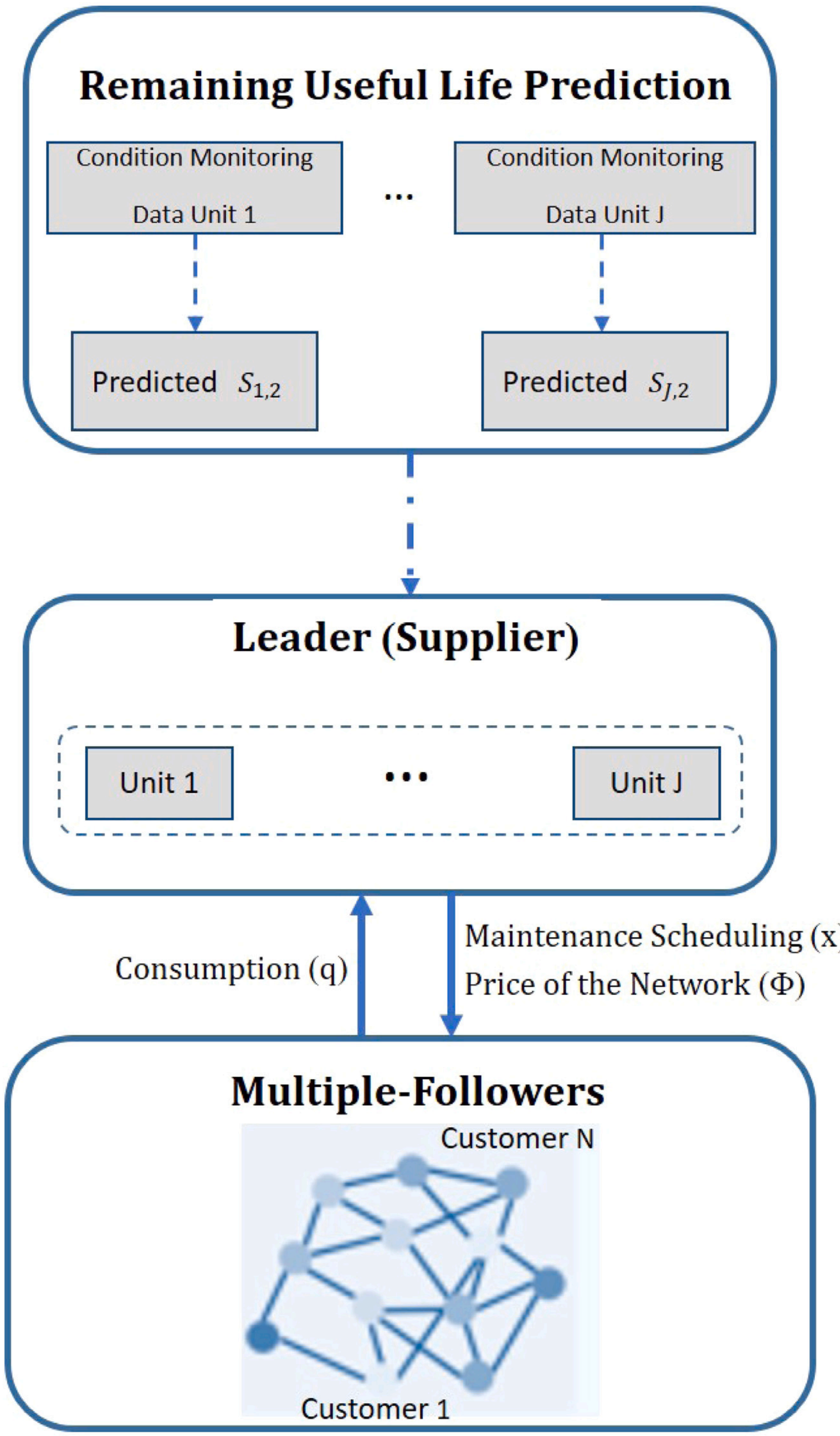

Fig. 1. Schematic of the leader–multiple-followers game for obtaining the maintenance scheduling based on the price of the network.

Theorem 1. *Under Assumption 1, the game $G = (\mathcal{N}, \mathcal{Q}, \mathcal{U})$ has a unique NE which is defined as follows:*

$$q^*(t) = (A - W)^{-1}(B - \Phi^*(t)), \tag{18}$$

where $A = \mathrm{diag}(a_1, \dots, a_N)$, $W = \begin{pmatrix} w_{11} & \cdots & w_{1N} \\ \vdots & \cdots & \vdots \\ w_{N1} & \cdots & w_{NN} \end{pmatrix}$, $B = \begin{pmatrix} b_1 \\ \vdots \\ b_N \end{pmatrix}$, $\Phi^* = \begin{pmatrix} \phi_1^* \\ \vdots \\ \phi_N^* \end{pmatrix}$

Proof. The best response of player i is as follows:

$$q_i(t) = \beta_i(q_{-i}(t)) = \frac{b_i - \phi_i^*}{a_i} + \frac{\sum_{l \in \mathcal{N}} w_{il} q_l(t)}{a_i}, \quad i \in \mathcal{N}, \tag{19}$$

Hence, (19) can be written in the matrix form as follows:

$$Aq(t) = B - \Phi^*(t) + Wq(t) \tag{20}$$

Under Assumption 1, matrix $A - W$ is invertible. Hence, the NE of the game can be obtained as follows:

$$q^*(t) = (A - W)^{-1}(B - \Phi^*(t)). \quad \square \tag{21}$$

Table 1
Deterioration and maintenance parameters of the manufacturing units.

Unit	Mean of deterioration state thresholds ($S_{2,j}$)	Standard deviation of deterioration state thresholds ($S_{2,j}$)	Maintenance cost (c_j) [MU]
1	12	1.4	20.48
2	10	3.2	21.39
3	11.2	2.5	22.73
4	9.4	1.1	24.78
5	11.8	2.1	24.82

5.2. Price and maintenance scheduling of the leader

As mentioned above, in the proposed model, we assume that the supplier can predict the strategies of the followers. Based on this prediction, the supplier optimizes its strategy. Moreover, the supplier acts first, in the sense that it sends its strategy to the followers, which then decide about their actions. We assume that the pricing is performed at the level of an individual customer. Customers who are more connected within the social network (and are, therefore, impacted by the network) can than be better targeted by the supplier. The strategy of the supplier comprises the price of the network and the maintenance scheduling of the manufacturing units. Hence, in the following, we obtain the optimal price and maintenance scheduling of the supplier relying on the fact that it knows the NE of the game (18) among the customers.

Let us define the matrix $(A-W)^{-1}$ as follows:

$$(A-W)^{-1} = (R_{il})_{i,l\in\mathcal{N}}. \tag{22}$$

In view of equality (18) and using (24), the optimization problem of supplier (11) can be defined as the following mixed-integer quadratic programming problem:

$$\begin{aligned}
&\max_{\phi,x,s,y} \sum_{t\in\mathcal{T}} \phi^{\top}(t)(A-W)^{-1}(B-\Phi(t)) - \sum_{j\in\mathcal{J}}\sum_{t\in\mathcal{T}} c_j x_j(t)\\
&\text{S.b.}\quad C_1',\quad C_2',\quad C_3',\quad C_4',\quad C_5',\\
&\qquad C_6'':\quad \sum_{l\in\mathcal{N}} R_{il}(b_l-\phi_l) \le \sum_{j\in\mathcal{J}}(1-x_j(t))q_{j,max},\\
&\qquad C_7'':\quad (A-W)^{-1}(B-\Phi(t)) \ge 0,\quad \phi(t)\ge 0.
\end{aligned} \tag{23}$$

By solving (23), the optimal price ϕ_i^*, $i\in\mathcal{N}$, can be obtained and sent to the customers. Based on the price, they can obtain their NE using (18).

6. Case study and evaluation

In this section, we implement the proposed method on a manufacturing system where the supplier has five manufacturing units and aims to obtain its maintenance scheduling for 30 days. We assume a normal distribution for the deterioration threshold value. The mean and the standard deviation of the deterioration state thresholds and the maintenance costs of each of the manufacturing units are displayed in Table 1.

Moreover, we assume 10 customers who are connected through a social network graph. The profile of the customers' demand is depicted in Fig. 2. We consider the following graph weights among the customers in the network:

$$W = \begin{pmatrix}
0 & 0.54 & 0.82 & 0.28 & 0.11 & 0.41 & 0.16 & 0.05 & 0.76 & 0.86\\
0.54 & 0 & 0.79 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0.82 & 0.79 & 0 & 0.36 & 0 & 0 & 0 & 0 & 0 & 0\\
0.28 & 0 & 0.36 & 0 & 0 & 0 & 0 & 0 & 0 & \\
00.11 & 0 & 0 & 0.08 & 0 & 0.62 & 0 & 0 & 0 & 0\\
0.41 & 0 & 0 & 0 & 0.62 & 0 & 0 & 0 & 0 & \\
0.16 & 0 & 0 & 0 & 0 & 0.63 & 0 & 0.75 & 0 & 0\\
0.05 & 0 & 0 & 0 & 0 & 0 & 0.75 & 0 & 0.54 & 0\\
0.76 & 0 & 0 & 0 & 0 & 0 & 0 & 0.54 & 0 & 0.25\\
0.86 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.25 & 0
\end{pmatrix} \tag{24}$$

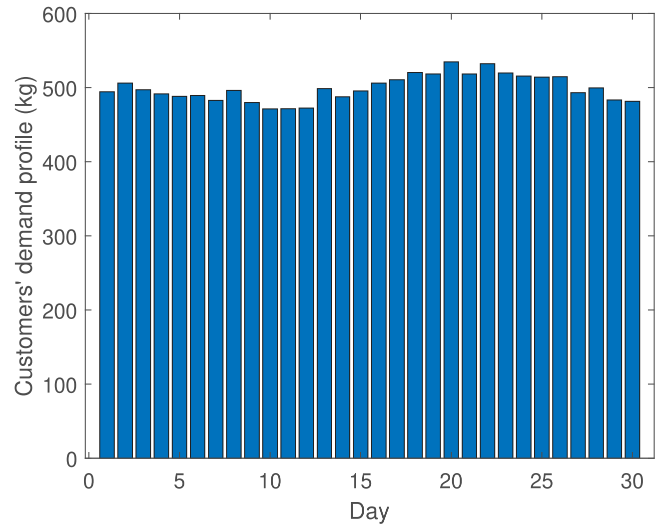

Fig. 2. The total demand of customers during 30 days.

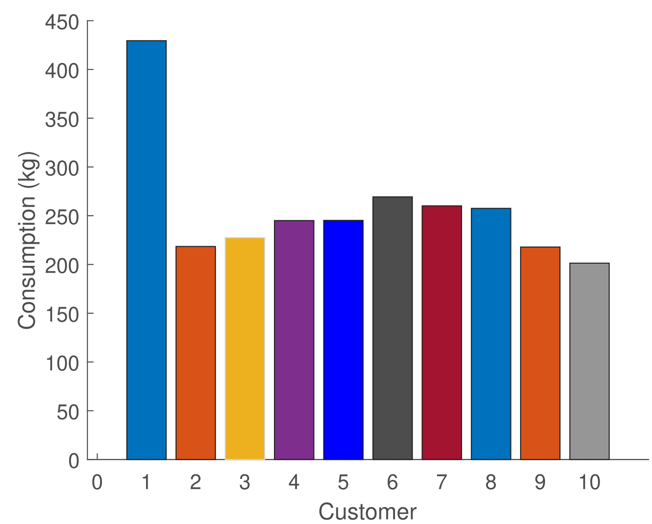

Fig. 3. The total consumption of customers during 30 days.

The weight matrix shows that customer 1 is connected to all other customers. Hence, his/her decisions have a high impact on the decisions of other customers. Fig. 3 shows the consumption level of each of the customers. As we can see, the consumption level of customer 1 is higher than that of other customers since the supplier offers a lower price to this customer to encourage him/her to consume more goods which has a high impact on other customers' strategies.

The network price and maintenance scheduling of the units are shown in Figs. 4 and 5. Moreover, the exact underlying values of Fig. 3 is provided in Appendix in Table A.8, and the exact underlying values of Figs. 2 and 4 are shown in Table A.9, again in Appendix.

As we can see from Figs. 4 and 5, the manufacturing units perform maintenance more frequently when the price of the network is low (in the first 15 days) compared to the time periods when the network price is high (between day 15 and 25).

The deterioration state of the units is shown in Fig. 6. The figure shows that the units perform maintenance before their failure time. Hence, the proposed model is feasible and also implicitly imposes conditions that prevent the model from explicitly imposing failure cost.

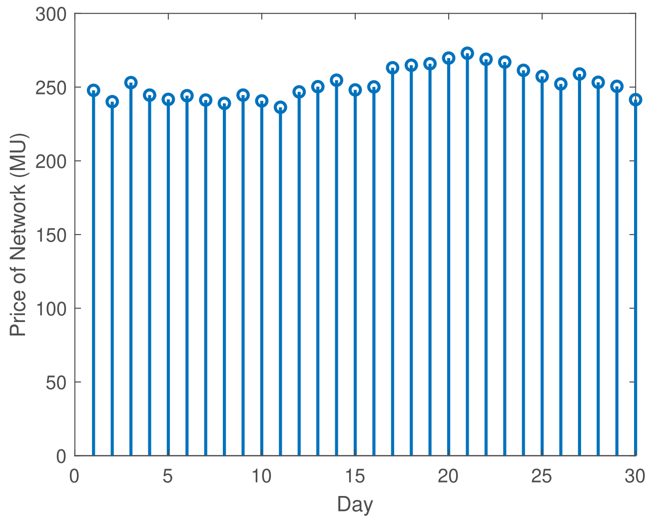

Fig. 4. The profile of the network price during 30 days.

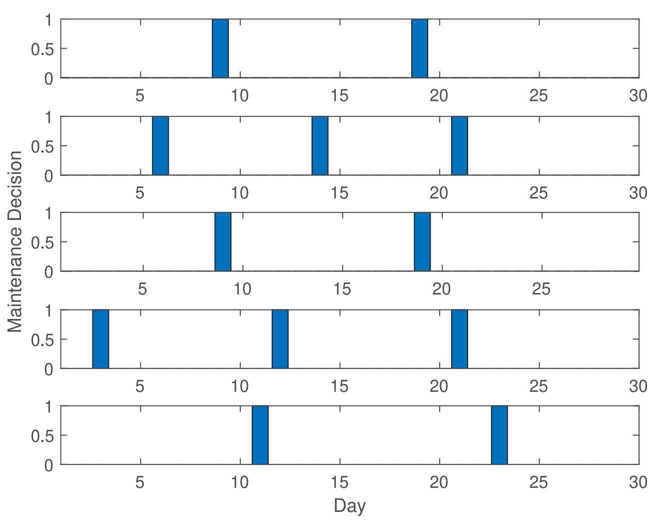

Fig. 5. Maintenance scheduling of all supplier's manufacturing units during 30 days.

Table 2
Computational time of computing the policy of supplier [Sec].

Number of customers	Computational time
10	9.45
100	50.77
1000	349.89

6.1. Computational time of the proposed method

In this section, we compare the computational time of the proposed solution for small, medium, and large sizes of the network. The computational experiments are performed with MATLAB R2019a on a Desktop PC with 16 GB of RAM and a 2.11 GHz processor. We consider 10, 100, and 1000 customers for small, medium, and large size-networks. Table 2 shows the computational time for the different sizes of the networks.

As we can see from Table 2, the computational time increases with the increasing number of the customers. However, even with a network size of 1000 connected customers, the computational time is still comparably small: ca. 350 s.

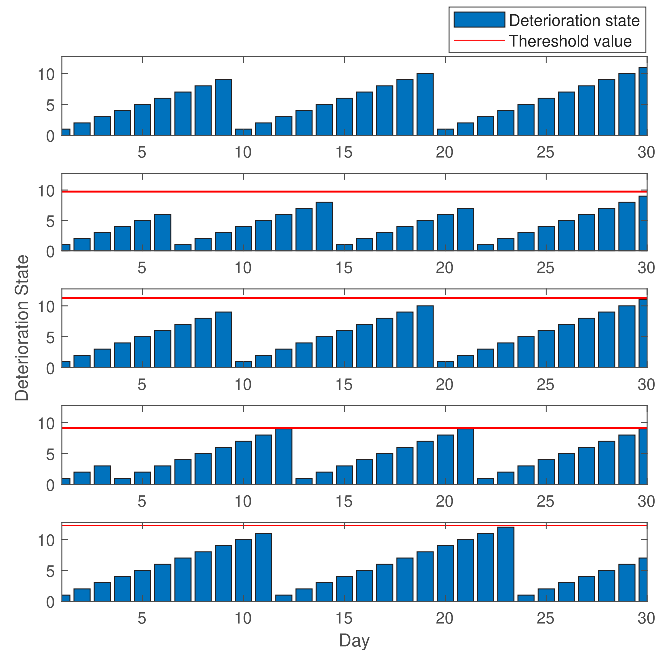

Fig. 6. Deterioration state of all supplier's units during 30 days.

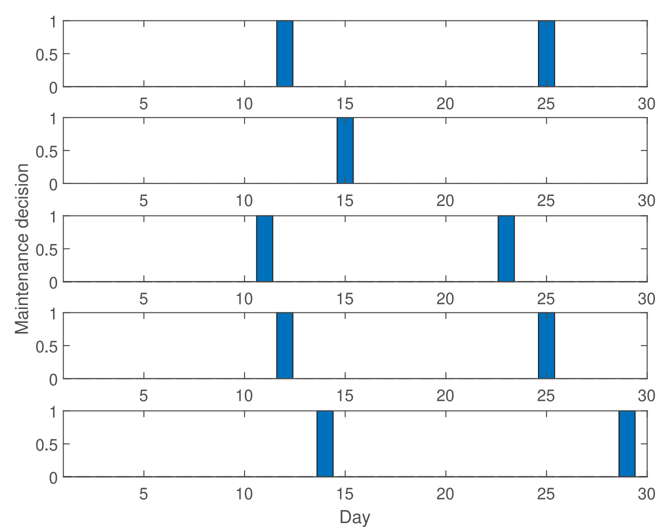

Fig. 7. Baseline maintenance scheduling of all supplier's manufacturing units with 10 customers during 30 days.

6.2. Comparison to a baseline decision

The results of the reward function for networks with a small, medium, and large size (with 10, 100, and 1000 customers) are compared to the baseline solution where the units perform maintenance at the deterioration threshold $\min_{k=1,\dots,K} S^k_{2,j}$, $j \in \mathcal{J}$. This baseline decision makes sure that the units can perform maintenance before they fail. The maintenance decision and the degradation cost for the small size network are shown in Figs. 7 and 8.

In Table 3, we compare the reward of the supplier when the units decide to perform maintenance when the threshold is reached with the proposed approach when they obtain their maintenance decision using (11).

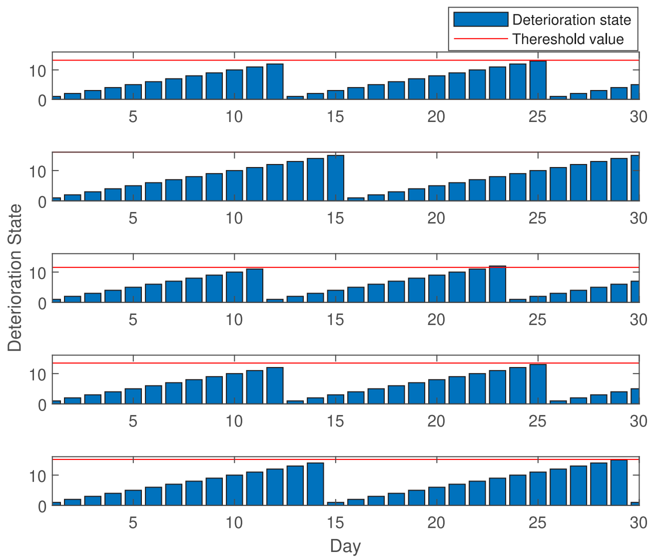

Fig. 8. Deterioration state of all supplier's units with 10 customers for baseline maintenance decision during 30 days.

Table 3
Comparison of the supplier objective function during 30 days for base-line and proposed method [MU].

Number of customers	Baseline	Proposed method
10	7.4919e+04	1.2893e+05
100	28.9074e+04	4.5432e+05
1000	3.3108e+05	8.3778e+05

The profit of the supplier in all three cases is less than the profit when it obtains its maintenance decision using (11). In this case, the maintenance scheduling of units does not depend on the price of the network but solely on the deterioration state.

6.3. The effect of the topology of the social network and the knowledge about it

In this section, we investigate the effect of the knowledge on the topology of the social network on the supplier's objective function. Moreover, we investigate the effect of changing the topology of the network on the reward of the supplier. In case that the network topology is not known to the supplier (the supplier does not know how the customers are connected and how strongly they are influencing each other), the supplier predicts the NE of the network as follows:

$$q^*(t) = A^{-1}(B - \phi^*(t)) \tag{25}$$

Then, the price and maintenance scheduling are obtained as follows:

$$\begin{aligned} &\max_{\phi,x,s,y} \sum_{t\in\mathcal{T}} \phi^\top(t)(A)^{-1}(B-\Phi(t)) - \sum_{j\in\mathcal{J}}\sum_{t\in\mathcal{T}} c_j x_j(t) \\ &\text{S.b.}\quad C'_1,\quad C'_2\quad ,\quad C'_3,\quad C'_4,\quad C'_5, \\ &\qquad C''_6:\quad \sum_{l\in\mathcal{N}} R_{il}(b_l-\phi_l) \le \sum_{j\in\mathcal{J}}(1-x_j(t))q_{j,max}, \\ &\qquad C''_7:\quad (A)^{-1}(B-\Phi(t)) \ge 0,\quad \phi(t)\ge 0. \end{aligned} \tag{26}$$

Table 4 shows the profit of the supplier for three different network topologies: (1) random topology, (2) topology with one dominant customer who is connected to all other customers, (3) fully connected network where all the customers are connected to each other through a graph. In this table, we compare the profit that the supplier can obtain for the entire duration of the period over 30 days under two different scenarios: (1) Under the assumption that the supplied knows the network topology, which corresponds to solving (23) to (2) Under the assumption that the supplier does not know the network topology which corresponds to solving (26).

Table 4
Comparison of the supplier objective function during 30 days for different network topologies under the condition that the topology [MU] is either known and unknown.

Number of customers	Network topology	Known network topology	Unknown network topology
10	Random topology 1	9.1813e+04	7.4529e+04
	Random topology 2	1.6021e+04	1.1566e+04
	Dominant customer	1.2893e+05	3.5673e+04
	Fully connected	3.3644e+05	1.6274e+05
100	Random topology 1	3.8013e+05	2.8190e+05
	Random topology 2	3.6197e+05	2.8540e+05
	Dominant customer	4.5432e+05	1.2942e+05
	Fully connected	6.9901e+05	4.9077e+05
1000	Random topology 1	7.2343e+05	5.8764e+05
	Random topology 2	6.7653e+05	5.3245e+05
	Dominant customer	8.3778e+05	2.1206e+05
	Fully connected	3.2342e+06	1.4324e+06

Table 5
Effect of changing supplier model parameter on the supplier objective function [MU].

Number of customers	Value of maintenance cost (c_1)	Profit
10	20.48+50%	8.1925e+04
	20.48+20%	8.1925e+04
	20.48	8.1946e+04
	20.48−20%	8.1953e+04
	20.48−50%	8.1966e+04
100	20.48+50%	3.1495e+05
	20.48+20%	3.1515e+05
	20.48	3.1525e+05
	20.48−20%	3.1536e+05
	20.48−50%	3.1545e+05
1000	20.48+50%	8.3346e+05
	20.48+20%	8.3358e+05
	20.48	8.3374e+05
	20.48−20%	8.3388e+05
	20.48−50%	8.3395e+05

Table 4 shows that by knowing the network topology, the supplier can obtain a strategy that results in a higher profit for small, medium, and large networks. Moreover, in the fully connected network, the supplier objective function is higher compared to the other two considered types of network topologies. This is due to the fact that the customers can affect the decision of each other and this effect is further reinforced in the fully connected topology. The results demonstrate that it is always desirable to know the network topology. In all the considered cases, the profit is higher under the assumption that the network topology is known. For example, for the small network case, the knowledge on random, dominant, and fully connected topologies increases the profit of the supplier by 18%, 72%, 51%, respectively. As we can see, knowing the network topology has the highest impact on the supplier objective function for the dominant case. This result is expected since in this case, by knowing who the dominant customer is and how he/she is connected to other customers, the supplier can offer a lower price and encourage him to increase his demand which consequently increases the consumption level of other customers.

6.4. Sensitivity analysis

In this section, we investigate the effect of varying selected parameters of the supplier and customer model on the objective function of the supplier in small, medium, and large networks.

Table 6
Effect of changing customer model parameter on the supplier objective function [MU].

Number of customers	Customer model (a_1)	Profit
10	3.6235+50%	7.6542e+04
	3.6235+20%	8.5926e+04
	3.6235	9.7492e+04
	3.6235−20%	1.1734e+05
	3.6235−50%	2.1280e+05
100	3.6235+50%	2.3125e+05
	3.6235+20%	2.5912e+05
	3.6235	2.8526e+05
	3.6235−20%	3.0510e+05
	3.6235−50%	3.2513e+05
1000	3.6235+50%	7.3421e+05
	3.6235+20%	7.5532e+05
	3.6235	7.8526e+05
	3.6235−20%	8.1123e+05
	3.6235−50%	8.2532e+05

Table 7
Comparison of the supplier objective function during 30 days between similar and proposed method [MU].

Methodology	Supplier reward
Proposed model	1.2893e+05
Model without price optimization [23]	6.6781e+04

In the following, we study the effect of changing the maintenance cost (c_j) and the parameters of the customer's model (a_i) on the supplier's reward. We consider two levels of variations ±20% and ±50% for each parameter. Table 5 provides an overview of the results of the sensitivity analysis.

As we can see from Table 5, the sensitivity of the objective function with respect to the maintenance cost is low. For example, for the small network case, by changing the maintenance cost by ±50%, the objective function changes only by around 0.02%, which is very low.

However, Table 6 shows that the objective function of the supplier is highly sensitive to the customer's objective function. For example, for a small system, when the objective function parameter changes by ±50%, the objective function changes by around 22%.

6.5. Comparison to a similar algorithm

In this section, we compare the results of the proposed methodology to the most similar prior work that considers predictive maintenance scheduling. Contrary to the current research focus, the algorithm does not consider customer demand. The model is derived from [23] where the authors propose a framework for obtaining predictive maintenance scheduling and production planning for manufacturing systems without considering the effect of customer demand. In other words, the authors assume that the price of the goods is considered as being given and is not part of the optimization problem. The focus of the algorithm is purely on maintenance scheduling.

For this comparison, we focus on the small network size (10 customers) and use the case study outlined in Section 6 with the corresponding parameters displayed in Table A.8, the weight matrix (24). Table 7 compares the profit of the supplier obtained with the algorithm proposed in [23] (without price optimization) and the algorithm proposed in this research.

Table 7 shows that the profit of the supplier with the proposed algorithm is higher compared to that obtained by the algorithm proposed in [23]. Hence, we can conclude that the proposed algorithm which considers the demand of the customers and obtains the price of the network using the leader–followers game leads to a higher profit compared to that of the model which does not consider the demand of the customers and does not integrate the price in the optimization problem.

6.6. Implications and recommendations

The proposed approach and the research results obtained in this study, reveal several insights and implications that can be useful both for asset management as well as for pricing strategies and have an impact on managerial decisions.

The most important insight is that jointly optimizing the price and predictive maintenance scheduling lead to an improved profit compared to the case when the pricing is not integrated in the maintenance optimization.

The revenue of the system can be increased by predicting the behavior of the customers who are connected through the social network graph and controlling the price of the goods. We demonstrate the effectiveness of our proposed method on a simulated case study. The results demonstrate that knowledge of the social network graph results in an increased revenue compared to the case when the underlying social network graph is not known. Moreover, the results show that obtaining the predictive maintenance scheduling based on the proposed optimization approach leads to an increased profit compared to the baseline decision-making (perform maintenance at the degradation limit) which also helps the managers to use their resources optimally.

The results also demonstrate that it is always beneficial to know the topology of the network, irrespective of type of the network topology. The gain of the knowledge on the network topology is particularly pronounced if there is a dominant customer who has a comparably large influence on the consumption behavior of other customers connected in the network.

7. Conclusion

In this paper, we address the problem of maintenance scheduling for manufacturing units of goods with positive externalities by concurrently optimizing the pricing and the maintenance schedule. The customers are connected through a social network graph. In order to solve the problem, we propose a bi-level optimization approach where at the first level, the supplier obtains its maintenance scheduling and the price at the level of individual customers. At the second level, the customers obtain their strategies based on the price offered by the supplier. In order to solve the bi-level optimization problem, we propose a solution based on a leader–multiple-followers game where the customers are the followers and the supplier acts as a leader who optimizes its decisions based on the predicted strategies of the customers. While we evaluated the performance of the proposed algorithm on different sizes of the network, it can be extended and implemented to the case of even larger and more complex networks. The numerical results of the case study show that when the network topology is known to the supplier, the obtained profit of the supplier is higher compared to the case that the connections between the customers in the network are not known. Moreover, we demonstrated that obtaining the maintenance decision using the proposed bi-level optimization method leads to a more profitable solution compared to the baseline solution where the units perform maintenance when they reach to their failure time. We also demonstrated that concurrently optimizing the price and the predictive maintenance scheduling results in higher profits compared to the case when the price is considered as given and only the maintenance scheduling is optimized.

As future research, multiple suppliers can be considered in the proposed framework where the strategies of the suppliers are related to each other and the interaction between the suppliers is also defined as a game. Moreover, a further promising extension of the research is to consider uncertainties in the RUL prediction and the customers' models. This will enable to develop a robust optimization method which can take the uncertainties into account. Furthermore, extending the proposed framework to dynamically changing network topologies which will result in dynamic maintenance scheduling and dynamic prices is also subject for further research.

CRediT authorship contribution statement

Pegah Rokhforoz: Conceptualization, Methodology, Software, Validation, Data curation, Visualization, Investigation, Formal analysis, Writing – original draft. **Olga Fink:** Conceptualization, Validation, Formal analysis, Writing – review & editing, Project administration, Supervision, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix. Tables

See Tables A.8 and A.9

Table A.8
The total consumption of customers during 30 days [Kg].

	Agent number									
	1	2	3	4	5	6	7	8	9	10
Total consumption	429	218	227	245	245	269	260	257	217	201

Table A.9
Demand of customers and network price during 30 days.

	Network demand [Kg]	Network price [MU]
1	494.3	247.81
2	506	240.12
3	497.1	253.07
4	491.5	244.57
5	488.1	241.69
6	489.3	244.18
7	482.7	241.22
8	496.2	238.97
9	497.8	244.59
10	471.2	240.65
11	471.5	236.32
12	472.3	246.77
13	498.6	250.36
14	487.6	254.69
15	495.5	248.11
16	506	250.16
17	510.6	263.13
18	520.3	264.85
19	516.4	265.87
20	534.7	269.69
21	518.3	272.97
22	532.2	268.88
23	519.7	267.00
24	515.5	261.38
25	514.1	257.25
26	514.7	252.16
27	493.1	258.92
28	499.6	253.27
29	483.3	250.51
30	481.4	251.43

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