Rate Performance Objectives of Multihop Wireless Networks

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Abstract—We consider the question of what performance metric to maximize when designing ad hoc wireless network protocols such as routing or MAC. We focus on maximizing rates under battery-lifetime and power constraints. Commonly used metrics are total capacity (in the case of cellular networks) and transport capacity (in the case of ad hoc networks). However, it is known in traditional wired networking that maximizing total capacity conflicts with fairness, and this is why fairness-oriented rate allocations, such as max-min fairness, are often used. We review this issue for wireless ad hoc networks. Indeed, the mathematical model for wireless networks has a specificity that makes some of the findings different. It has been reported in the literature on Ultra Wide Band that gross unfairness occurs when maximizing total capacity or transport capacity, and we confirm by a theoretical analysis that this is a fundamental shortcoming of these metrics in wireless ad hoc networks, as it is for wired networks. The story is different for max-min fairness. Although it is perfectly viable for a wired network, it is much less so in our setting. We show that, in the limit of long battery lifetimes, the max-min allocation of rates always leads to strictly equal rates, regardless of the MAC layer, network topology, channel variations, or choice of routes and power constraints. This is due to the "solidarity" property of the set of feasible rates. This results in all flows receiving the rate of the worst flow, and leads to severe inefficiency. We show numerically that the problem persists when battery-lifetime constraints are finite. This generalizes the observation reported in the literature that, in heterogeneous settings, 802.11 allocates the worst rate to all stations, and shows that this is inherent to any protocol that implements max-min fairness. Utility fairness is an alternative to max-min fairness, which approximates rate allocation performed by TCP in the Internet. We analyze by numerical simulations different utility functions and we show that the proportional fairness of rates or transport rates, a particular instance of utility-based metrics, is robust and achieves a good tradeoff between efficiency and fairness, unlike total rate or maximum fairness. We thus recommend that metrics for the rate performance of mobile ad hoc networking protocols be based on proportional fairness.

Index Terms—System design, mathematical programming/optimization, wireless, max-min, utility fairness, best-effort.

1 Introduction

1.1 Rate-Based Performance Metrics with Power Constraints

We consider the question of what metric to use when evaluating the performance of ad hoc wireless network protocols such as routing or MAC. We focus on maximizing rates under battery-lifetime and power constraints. Typical application examples are networks of wireless laptops and PDAs; this is also the framework used by many papers analyzing various models of physical layers (purely information theoretic approach [29], [32], CDMA [7], UWB [8]). In contrast, some sensor networks put more emphasis on minimizing energy under minimum rate constraints. Here, we study the former and leave the latter to a companion paper.

For cellular wireless networks, a frequently chosen performance metric is total capacity, i.e., the sum of the rates of all flows. An extension that maximizes a weighted sum of the rates is applied in CDMA/HDR [24]. In multihop wireless networks, the same metrics are used, but also used is transport capacity, a variant popularized by Gupta and Kumar in [10]. This is in fact a weighted sum of rate, where weights are the distances between the source and the destination of each flow.

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Manuscript received 30 Apr. 2004; revised 14 July 2004; accepted 16 July 2004.
For information on obtaining reprints of this article, please send e-mail to: tmc@computer.org, and reference IEEECS Log Number TMCSI-0150-0404.

1.2 The Tension between Efficiency and Fairness

Traditionally, wired networking has also focused on performance metrics that incorporate some form of fairness. Indeed, it is known that considering only total capacity yields gross unfairness if implemented in a wired network [17]. The unfairness has been observed in the framework of multihop ad hoc networks in [8]. Hence, different performance metrics that account for fairness have been developed. A typical example is max-min fairness [3], which is used in many existing networking protocols, including the ABR mode of ATM [5]. This is an egalitarian approach by which the rate of a flow can be increased only when it is not possible to increase the rate of an already smaller flow. Max-min fairness is often viewed as an extreme fairness; this justifies using a *fairness index* that measures the departure from max-min fairness (it is a slight variant of the fairness index defined by Jain in [14]; see Section 4.4).

Max-min fairness is also used, often implicitly, in many existing wireless multihop network protocols (e.g. [30], [12]). In fact, as we show, 802.11 essentially implements max-min fairness. However, in wireless networks, there is still no tradition of evaluating a system in light of both total rate and fairness. It turns out that the issue is significantly different than in wired networking, due to the peculiarities of the mathematical models for wireless networks. This phenomenon has been first observed in [2]. In particular, we find that the allocations that implement max-min fairness have fundamental efficiency problems. This is due to the "solidarity" property of the set of feasible rates (Section 5.1).

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Another way to reduce the tension between efficiency and fairness is to use a weighted sum of the rates as a design objective. The most well-known example of this type of criteria in wireless networks is transport capacity [10] where each flow is assigned a weight equal to the distance between the source and the destination of the flow. We show in Fig. 7 in Section 7 that this approach does not reconcile the tension.

1.3 Utility Fairness

Utility fairness is often used as an alternative, a less egalitarian approach to max-min fairness. It corresponds to the "utility" metric $\sum_j U(x_j)$, where x_j is the rate of flow j and $U()$ is a concave function (called the utility function). The concept of utility is a convenient way to represent user preferences and a utility function $U()$ can be interpreted as a user satisfaction [19]. The Internet congestion control performed by TCP approximates some form of utility fairness.

The properties of utility fairness depend on the choice of utility functions. It is known in wired networking that, for a large class of strictly concave utility functions, maximizing the utility metric is fairer than maximizing the total capacity, but less egalitarian than a max-min fair allocation.

The most often used class of utility functions is of form

$$U(x,\xi) = \begin{cases} (1-\xi)^{-1} x^{1-\xi} & \text{if } \xi \neq 1 \\ \log(x) & \text{if } \xi = 1, \end{cases} \tag{1}$$

proposed in [21]. This class of utility function is general enough to incorporate the most often used objectives: rates maximization (for $\xi = 0$), *proportional fairness* [16] (for $\xi = 1$), *minimum potential delay* [20] (for $\xi = 2$), and max-min fairness (for $\xi \to \infty$). We consider this form of utility in order to evaluate if and how the tradeoff between efficiency and fairness can be tuned through parameter ξ of the utility function.

Note that the utility approach can easily be extended to account for power and energy—not in the form of constraints as we do here, but through a cost function subtracted from the utility metric. This is explored, for example, by Baldi et al. [1]. We leave such metrics out of the scope of this paper, as we focus on rate-based metrics with power constraints.

1.4 Modeling of Ad Hoc Wireless Networks

We are interested in a model of a wireless network in order to analyze efficiency and fairness of different design criteria for various network technologies. We define a model of an ad hoc wireless network that allows for the most general assumptions on a physical layer (including variable rate 802.11, UWB, or CDMA), MAC, and routing protocols. And, for a given network topology, channel and noise statistics and traffic demand, we characterize a set of feasible long-term average end-to-end rate and transport rate allocations. Next, we find the optimal allocations on the two sets, with respect to the three design criteria considered. In some numerical examples, where it is not possible to find an exact solution of the optimization problem due to its nonconvexity, we consider an approximation that is close to the optimal solution and that allows us to accurately characterize the efficiency and fairness of the optimum.

1.5 Our Findings

We prove that, under a general model of an ad hoc wireless network, in the limit of long battery lifetime, max-min fairness leads to equal long-term average rates of all flows, regardless of network topology, channel and noise variations, routing, or power constraints. This means that all rates are equal to the rate of the worst flow, making the network very inefficient. The same happens when considering long-term average transport rates. We show numerically that the problem persists with battery-lifetime constraints. This conclusion is in sharp contrast with the findings from the framework of wired networks, where max-min fairness is widely used. Also, this generalizes the result in [2]; it shows that their finding is not a unique property of 802.11 and that any protocol that strives for max-min fairness will have the same problem.

We also show that a protocol that maximizes the total capacity starves flows with bad channel conditions for sufficiently high powers. We prove analytically in this setting that if we consider a network with no random fading, only the most efficient flow gets a positive rate and all other flows have a zero rate. The same holds when maximizing transport capacity.

We verify numerically, on a large number of random networks, that this unfairness occurs in networks with or without random fading, and not only at the limit, but also with realistic transmission power constraints. This generalizes the results in [8], showing that this unfairness property is not a problem of UWB, but rather of the design criteria. We also show that the use of transport capacity, although fairer than total capacitys, does not completely alleviate unfairness, and can also assign zero rates to the worst flows.

We further show that for very small battery lifetimes, the max-min fair, proportionally fair, and rate maximizing allocations are almost equivalent. In this case, fairness is not an issue and any of these metrics can be used in a design. However, we find that this, in general, does not hold for realistic power constraints.

Finally, we analyze the general form of the utility fairness, described by (1). We show that for values of ξ around one, the utility fairness represents a robust trade-off between fairness and efficiency, insensitive to different transmission power and Long-Term Average Power constraints, and network topologies. Thus, the proportional fairness (the sum of the logarithms of the achieved rates over all source destination pairs; corresponds to $\xi = 1$) is an ideal candidate metric when designing or evaluating performance of an ad hoc wireless network.

We also present detailed simulation results for different network topologies and different power constraints. These results can be used as a guideline for choosing a performance metric for a given design objectives. One can use the results to fine-tune the utility metric through parameter ξ in order to achieve the desired trade-off between efficiency and fairness for a given power limitations.

The findings also suggests that 802.11 should be redesigned with proportional fairness as a design objective, in order to avoid inefficiencies observed in [2].

1.6 Organization of This Paper

The outline of the paper is as follows: In the next section, we give an overview of the related work in this field. In Section 3, we describe system assumptions. In Section 4, we give a mathematical formulation of the model of a network. In Sections 5, 6, and 7, we present findings related to max-min fairness, maximizing total capacity, and proportional fairness objectives, respectively. In Section 8, we discuss the influence of Long-Term Average Power constraints. In the last section, we give conclusions and directions for further work. Proofs of propositions can be found in the Appendix.

2 Related Work

2.1 Efficiency and Fairness

The tension between efficiency and fairness for a cellular network was reported by Tse and Hanly in [32]. A strategy that maximizes the total capacity is such that a node with the best channel conditions in a given slot should send data. Nodes that are farther away will less frequently satisfy this constraint, but will still have a positive throughput, due to the random part of fading. However, if a node is very far away from the base station, its average rate is going to be very small and, essentially, it will not be able to communicate. In [32], a remedy is found by assigning weights to node rates, such that a level of fairness is assured. The implicit assumption in this type of network is that an area with mobile nodes is well covered with base stations, so there is no big variation in distances from the mobile nodes to the closest base stations.

However, variations in the distances between sources and destinations in the case of ad hoc networks are typically much higher since a node does not talk to the closest base-station but to an arbitrary destination in a network. This makes it difficult to remedy fairness with weights, and longer flows risk low or zero throughput. Indeed, it has been observed in the context of Ultra Wide Band by Cuomo et al. [8] that the unfairness of total capacity persists in wireless networks and some long distance flows obtain zero throughput.

A performance anomaly was reported by Berger-Sabatel et al. in [2] for 802.11. There, several nodes talk to a base station. One of them is far away and codes for 1 Mb/s while others are near and code for 11 Mb/s. Still, on average, all nodes achieve the same throughput of approximately 1 Mb/s. We show later in this paper that this anomaly is in fact not abnormal behavior, rather a fundamental property of max-min fairness for wireless networks, regardless of any underlying physical, MAC, or routing protocol.

2.2 Max-Min and Utility Fairness

Max-min fairness is originally proposed in [3]. Its use within the framework of wireless networking is presented in [30], [12].

Different approaches to the use of utility fairness in wired networking is presented in [21], [16], [20]. Variants of utility fairness are used in existing wireless multihop network protocols. In [22], per-link proportional fairness is considered. An algorithm that achieves end-to-end proportional fairness using hop-by-hop congestion control is presented in [34]. In [18], a general framework for finding a utility-fair rate allocation in a multihop wireless network is presented. The influence of parameter ξ from (1) on the tradeoff between efficiency and fairness in wired networks is considered in [28]. An approach that combines power and rates through cost and utility functions is given in [1].

Another type of fairness for ad hoc networks is defined in [9]. Instead of considering a fair rate sharing, the authors consider a fair time sharing. They assume a physical model similar to 802.11. Two links can either transmit concurrently, or collide with each other forming a contention region. If several links compete for the same contention region, then time sharing is necessary and a form of max-min fair time sharing is proposed. Although this approach alleviates the inefficiency of max-min fairness, it is difficult to generalize it to more general wireless physical models that include rate or power adaptations.

2.3 Modeling Wireless Networks

General models of wireless networks that incorporate various physical layers, and MAC and routing protocols are discussed in [7], [31], [15], [23], [18], [34]. Maximizing a weighted sum of the rates as a criteria is considered in [7], [31] for a very general model of a network. Proportional fairness and maximizing the minimal rate in a network (a weaker version of max-min fairness) are analyzed in [15]. However, the latter considers only a subset of possible routing and MAC protocols, those that can be transformed to convex problems.

3 System Assumptions

We analyze an arbitrary ad hoc wireless network that consists of a set of nodes, and each two nodes that directly exchange information are called a link. Next, we give properties of the physical model of communications on links.

3.1 Physical Model Properties

3.1.1 Power Attenuation and White Noise

For each pair of nodes, we define a signal attenuation (or fading). If a signal on link l is transmitted with power P_l, then it is received with power $P_l h_l$, where h_l is the attenuation on link l. This attenuation is usually a decreasing function of a link size due to power spreading in all directions.

The attenuation is also random, due to mobility and variability in the transmission medium. We assume the system is in the state that varies over time according to some stationary stochastic process $\{(S(t)), t > 0\}$, where $S(t) \in \mathcal{S}$ is the state at time t. The attenuation of link l is a function $h_l(S(t))$ of the state of the system.

We assume the network is located on a finite surface and that all attenuations are always strictly positive ($h_l(s) > 0$ for all $s \in \mathcal{S}$), hence every node can be heard by any other node in the network at all times, and there is no clustering.

We also assume a presence of a white Gaussian background noise in the system. This noise may differ in space and time. Similarly to attenuations, we model it as a function of the system state: $N_l(s)$ is the white noise on link l when system state is s ($N_l(s) > 0$ for all $s \in \mathcal{S}$).

3.1.2 Rate Function

All physical links are point-to-point, that is, each link has a single source and a single destination. There are more advanced models such as broadcast channel and relay channel [6] that attain higher performances, but they are not used in most of the contemporary networks, and their performances are in general not known and are still an open research issue. Broadcast is used by 802.11, by MAC layer control packets, but this is an aspect we do not model here.

A node can either, at one given time, send to one destination or receive from one source. There are more complex transmitter or receiver designs that can overcome these limitations. An example is a multiuser receiver that could receive several signals at a time. This would change the performance of links with a common destination, but would not change the interactions over a network. However, these more complex techniques are not used in contemporary multihop wireless networks (like 802.11, UWB, bluetooth, or CDMA) due to high transceiver complexity, and we do not analyze them here.

We model rate as a *strictly increasing* function $r(\mathrm{SNR})$ of the signal-to-noise ratio at the receiver, which is a ratio of received power by the total interference perceived by the receiver including the ambient noise and the communications of other links that occur at the same time. This model corresponds to a large class of physical layer models, for example:

- Shannon capacity of a Gaussian channel [6]:

$$r(\mathrm{SNR}) = 1/2 \log_2(1 + \mathrm{SNR}).$$

- Ultra-wide band [8]: $r(\mathrm{SNR}) = K \times \mathrm{SNR}$.
- CDMA HDR [24]: $r(\mathrm{SNR})$ is a stair function of SNR.
- Variable rate 802.11 [11]: $r(\mathrm{SNR})$ is a stair function of SNR.

We note that in the last two models, rate is not a strictly increasing function of SNR, but in most applications can be approximated as such. On the contrary, in the basic model of 802.11 (e.g. [10]), the rate is assumed to be constant, hence this model does not fit this framework.

3.2 MAC Protocol

We further assume a slotted protocol. In each slot, a node can either send data, receive, or stay idle, according to the rules defined in Section 3.1.2. Each slot has a power allocation vector associated with it, which denotes what power is used for transmitting by the source of each link. If a link is not active in a given slot, its transmitting power is 0. An example of a schedule is depicted in Fig. 1.

For each slot, we define its relative frequency, which represents a fraction of time of the schedule it occupies. Consider the example from Fig. 1b. There, we have two slots, slots 1 and 2. Suppose we first schedule slot 2 during a time interval T_1^2, then we schedule slot 1 during a time interval T_1^1, then again slot 2 for another T_2^2 and, finally, slot 1 for another T_2^1. This schedule than repeats in time. The relative frequency of slot 1 is thus $(T_1^1 + T_2^1)/(T_1^1 + T_2^1 + T_1^2 + T_2^2)$ and the relative frequency of slot 2 is $(T_1^2 + T_2^2)/(T_1^1 + T_2^1 + T_1^2 + T_2^2)$.

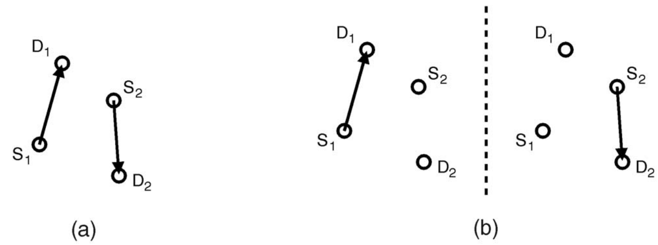


Fig. 1. Two examples of schedules: Consider a network with two links, (S_1, D_1) and (S_2, D_2). (a) One possible schedule is to have both links sending all the time. (b) Another schedule is to have two slots. In the first slot link, (S_1, D_1) transmits, and in the second, slots link (S_2, D_2) transmits.

In general, although durations of slots may vary depending of the available data at nodes, or packet sizes, the average link rate depends only on the relative frequency. A schedule can consist of an arbitrary number of slots of arbitrary durations and we consider only frequencies of the slots and power allocations within slots as a parameter of a schedule. There is a schedule assigned to every possible system state, and these parameters are a function of the system state.

We assume an ideal MAC protocol that always knows the state of the system and disposes of an ideal control plane with zero delay and infinite throughput to negotiate power allocations. Given the state, it calculates the optimal transmission power of each link in each slot, as well as the optimal slot frequencies, in a centralized manner and according to a predefined metric. It then schedules a random slot from the schedule that corresponds to the system state, with the probability equal to the relative frequencies of the slots (as in [23]).

The assumptions on a MAC protocol, with an ideal control plane and the complete knowledge of the system, might seem overly simplified for a wireless multihop network scenario. However, as shown in [23], [7], there exist decentralized protocols whose performances come close to the optimal one. Also, by considering an ideal protocol, we focus our analysis on the properties of performance metrics, and not artifacts of leaks in protocol design. Our assumption corresponds to neglecting the overhead (in rate and power) of the actual MAC protocol.

3.3 Routing Protocol and Traffic Flows

We assume an arbitrary routing protocol. Flows between sources and destinations are mapped to paths, according to some rules specific to the routing protocol. At one end of the spectrum, nodes do not relay and only one-hop direct paths are possible. At the other end, nodes are willing to relay data for others and multihop paths are possible. There can be several parallel paths. The choice of routes may also depend on the current state of the system. All these cases correspond to different constraint sets in our model, as explained in Section 4.1.

Flow control is assumed to use all available links' capacities in order to maximize performance while keeping the network stable without dropping packets. Hence, it can be TCP, UDP (with with optimally choosen rates), or some other best-effort flow control.

3.4 Power Control

There are three types of power constraints in a wireless network: peak constraint, short-term average constraint, and long-term average constraint. Here, we describe them in detail:

Peak Power Constraint. Given a noise level on a receiver, a sender can decide which codebook it will use to send data over the link during one time slot. Different symbols in the codebook will have different powers. The maximum power of a symbol in a codebook is then called *the peak power*. It depends on the choice of the physical interface and its hardware implementation and we cannot control it. It limits the choice of possible codebooks and it puts restrictions on the available rate, For example, the rate of an UWB link, given the average SNR on the receiver, depends on the shape of the pulse, thus on the peak power level of the pulse [33]. In our model, the peak power constraint is integrated in a rate function, given as an input.

Transmission Power. We assume a slotted system. In each slot, a node chooses a codebook and its average power and sends data using this codebook within the duration of the slot. We call *the transmission power* the average power of a symbol in the codebook. This is a short-term average power within a slot since a codebook is fixed during one slot. We assume that this transmission power is upper-bounded by P^{MAX}. This power limit is implied by technical characteristics of a sender and by regulations, and is not necessarily the same for all nodes. For example, this is the only power constraint that can be set by users on 802.11 equipment.

Long-Term Average Power. While transmitting a burst of data (made of a large number of bits), a node uses several slots, and possibly several different codebooks. Each of these codebooks has its transmission power. We call *the Long-Term Average Power* the average of transmission powers during a burst, and we assume it is limited by $\overline{P}^{MAX}$. Long-Term Average Power is related to the battery lifetime in the following way:

$$T_{\text{lifetime}} \approx \frac{E_{\text{battery}}}{\overline{P}^{MAX} \times u},$$

where T_{lifetime} is the battery lifetime, E_{battery} is the battery energy, $\overline{P}^{MAX}$ is the Long-Term Average Power constraint and u is the fraction of time a node has data to send (or activity factor, measured in Erlangs). The approximation corresponds to ignoring overhead spent managing the sleep/wakeup phases, etc. $\overline{P}^{MAX}$ is thus set by a node to control its lifetime; it can vary from node to node.

We incorporate explicitly in our model the transmission power and the Long-Term Average Power constraints. The peak power is incorporated implicitly through the choice of the rate function.

4 Mathematical Formulation of the Feasible Sets and of the Metrics

4.1 Feasible Set of Rates

We model the wireless network as a set of I flows, L links, and N time-slots. At every time instant, the system is in some state belonging to the set of possible states $\mathcal{S}$ (depicting different channel conditions and noise levels). There is a finite (possibly very large) number of states $|\mathcal{S}|$. Every flow can use one or several paths. There are P paths ($P \leq 2^L$). We also assume that there exists a schedule for each system state, and that the schedule consists of time slots $n = 1, \cdots, N$ of relative frequency $\alpha_n(s)$. We are interested in the set of feasible long-term average end-to-end rates.

Here, we give a list of notations used throughout the paper:

- $S \in \mathcal{S}$ is a random variable representing the system state. We assume system state is a continuous, stationary random process, and due to stationarity, we drop the time argument (the model can be easily extended to a larger class of ergodic processes, as in [23], [4]).
- $h_{l_1 l_2}(s)$ is the attenuation of a signal from the source of link l_1 to the destination of link l_2 when the network is in state s.
- $\alpha_n(s)$ for $n \in \{1, \cdots, N\}$ is the relative frequency of time slots in the schedule assigned to system state s. We have $\sum_{n=1}^{N} \alpha_n(s) = 1$.
- $\mathbf{p}_l^n(s)$ is the power scheduled to link l in slot n and state s.
- $\mathbf{SNR}_l^n(s)$ is the signal-to-noise ratio at the receiver of link l in slot n and system state s.
- $r(\mathrm{SNR})$ is the rate function that defines the rate of a link given the signal-to-noise ratio at the receiver of the link.
- $\mathbf{x}_l^n(s) = r(\mathbf{SNR}_l^n(s))$ is the rate of link l in slot n and state s.
- $\overline{\mathbf{x}}_l(s) = \sum_{n=1 \cdots N} \alpha_n(s) \mathbf{x}_l^n(s)$ is the average rate achieved on link l in state s.
- $\mathbf{y}(s) \in \mathbb{R}^P$ is the vector of average rates used on paths in state s.
- $\overline{\mathbf{f}}(s) \in \mathbb{R}^I$ is the vector of average rates achieved by flows when the network is in state s.
- $F(s)$ (flow matrix) is such that $F_{i,p}(s) = 1$ if path p belongs to flow i in the state s, and 0 elsewhere. We have $\overline{\mathbf{f}}(s) = F(s)\mathbf{y}(s)$.
- $R(s)$ (routing matrix) is such that $R_{l,p}(s) = 1$ if path p uses link l in state s. We have $\overline{\mathbf{x}}(s) \geq R(s)\mathbf{y}(s)$. The matrix $R(s)$ is defined by the routing algorithm and may depend on the state s.
- $\overline{\mathbf{f}} \in \mathbb{R}^I$ is the vector of long-term average rates achieved by flows. We have $\overline{\mathbf{f}} = \mathbb{E}\{\overline{\mathbf{f}}(S)\}$.
- $\mathcal{F}$ is the set of feasible average rates.
- $\mathcal{T}$ is the set of feasible average transport rates.

Since $\mathbf{x}^n(s)$ has dimension L (where L is a number of links), by virtue of the Carathéodory theorem, it is enough to consider $N = L + 1$ time slots of arbitrary lengths $\{\alpha_n(s)\}_{n=1 \cdots N}$ in order to achieve any point in the convex closure of points $\mathbf{x}^n(s)$.

We are interested in the set $\mathcal{F}$ of feasible long-term average flow rates, which is the average over the system state S, and over slots allocations in each system state. It is the set of $\overline{\mathbf{f}} \in \mathbb{R}^I$ such that for each system state $s \in \mathcal{S}$ there exist a schedule $\alpha_n(s)$, a set of power allocations $\mathbf{p}^n(s)$, a corresponding set of rate allocations $\mathbf{x}^n(s)$, and average rates and powers $\overline{\mathbf{x}}(s)$ and $\overline{\mathbf{p}}(s)$, such that the following set of equalities and inequalities are satisfied:

$$\overline{\mathbf{f}} = \mathbb{E}\{\mathbf{f}(S)\}, \tag{2}$$

$$\mathbf{f}(s) = F(s)\mathbf{y}(s), \tag{3}$$

$$\overline{\mathbf{x}}(s) \geq R(s)\mathbf{y}(s), \tag{4}$$

$$\overline{\mathbf{x}}(s) = \sum_{n=1}^{L+1} \alpha_n(s)\mathbf{x}^n(s), \tag{5}$$

$$\mathbf{x}_l^n(s) = r(\mathbf{SNR}_l^n(s)), \tag{6}$$

$$\mathbf{SNR}_l^n(s) = \frac{\mathbf{p}_l^n(s)h_{ll}(s)}{N_l(s) + \sum_{k\neq l}\mathbf{p}_k^n(s)h_{kl}(s)}, \tag{7}$$

$$\mathbf{p}_l^n(s) \leq P_l^{MAX}, \tag{8}$$

$$\mathbb{E}\{\overline{\mathbf{p}}_l(s)\} \leq \overline{P}_l^{MAX}, \tag{9}$$

$$\overline{\mathbf{p}}(s) = \sum_{n=1}^{L+1} \alpha_n(s)\mathbf{p}^n(s), \tag{10}$$

$$1 = \sum_{n=1}^{L+1} \alpha_n(s), \tag{11}$$

$$1 \geq \sum_{l:l.\mathrm{src}=o} 1_{\{p_l^n(s)>0\}} + \sum_{l:l.\mathrm{dst}=o} 1_{\{p_l^n(s)>0\}}, \tag{12}$$

where $l.\mathrm{src} = o$ and $l.\mathrm{dst} = o$ are true if node o is the source or the destination of link l, respectively. Function $1(x)$ denotes the identity function; it is 1 if x is true, or else it is 0.

Equations (6) and (7) give SNRs and rates for all links in all slots and all states. The average rates of links during a schedule, and in a given state, are given by (5). Routing for a given state is defined by (3) and (4). Equations (8), (9), and (10) define power constraints. Medium access constraints are given by (12). The end-to-end rates, average with respect to the system states, are given in (2).

We note that if $\overline{\mathbf{x}}(s) = R(s)\mathbf{y}(s)$, there might be buffering problems. However, as shown in [23], any point in the interior of the set $\mathcal{F}$ is feasible, and the system is stable, for a large class of traffic patterns and system state distributions, thus we can approach arbitrarily close to the equality in (4). To avoid an additional complexity of the presentation, we will allow $\overline{\mathbf{x}}(s) = R(s)\mathbf{y}(s)$.

4.2 Feasible Set of Transport Rates

In [10], the transport rate of a flow is defined as the rate of a flow multiplied by the distance covered by the flow between the source and destinations (call this $\mathrm{len}(i)$ for flow i). Therefore, we define the set of feasible long-term average transport rates $\mathcal{T}$ as

$$\mathcal{T} = \{\overline{\mathbf{t}} \in \mathbb{R}^I \mid (\exists \overline{\mathbf{f}} \in \mathcal{F})\ \overline{\mathbf{t}}_i = \overline{\mathbf{f}}_i\mathrm{len}(i)\}. \tag{13}$$

4.3 Design Criteria

Given network technology, for each topology and traffic demand there is a given set of feasible rates $\mathcal{F}$ and a set of transport rates $\mathcal{T}$. We consider optimizing the system according to one of the following criteria:

4.3.1 Rate Criteria

- **capacity**: Maximize $\sum_{i=1}^I \overline{\mathbf{f}}_i$ over all $\overline{\mathbf{f}} \in \mathcal{F}$.
- **max-min fairness**: Find the max-min fair rate vector $\overline{\mathbf{f}}^*$ in $\mathcal{F}$.
- **utility fairness**: Maximize $\sum_{i=1}^I U(\overline{\mathbf{f}}_i, \xi)$ over all $\overline{\mathbf{f}} \in \mathcal{F}$.

It is easy to verify that $\mathcal{F}$ and $\mathcal{T}$ are convex and compact. The first and the third criteria are defined by concave maximization problems over $\mathcal{F}$, thus they always have a solution. Max-min fairness is defined as follows [3]: We say a point $\mathbf{x}^*$ is max-min fair on some set $\mathcal{X}$ iff for all $\mathbf{x} \in \mathcal{X}$ and all index i $\mathbf{x}_i > \mathbf{x}_i^*$ implies that there exists an index j such that $\mathbf{x}_j < \mathbf{x}_j^* \leq \mathbf{x}_i^*$, i.e., increasing some component $\mathbf{x}_i^*$ must be at the expense of decreasing some already smaller component $\mathbf{x}_j^*$. The max-min fair allocation does not always exist, but, if it exists, it is unique. It always exists if $\mathcal{X}$ is convex and compact [25], which is the case here.

The max-min fair allocation does not have $\mathbf{x}_i^* = \mathbf{x}_j^*$ in general for $i \neq j$, even on convex sets (see [3] for some examples).

In general, the rate vectors that satisfy each of the three criteria are significantly different, as illustrated by the examples in the following sections.

4.3.2 Transport Rate Criteria

Similarly, we define:

- **transport capacity**: Maximize $\sum_{i=1}^I \overline{\mathbf{f}}_i\mathrm{len}(i)$ over all $\overline{\mathbf{f}} \in \mathcal{F}$.
- **transport-max-min fairness**: Find the max-min fair transport rate vector $\overline{\mathbf{t}}^*$ in $\mathcal{T}$.
- **transport-utility fairness**: Maximize

$$\sum_{i=1}^{I} U(\overline{\mathbf{f}}_i\mathrm{len}(i), \xi)$$

over all $\overline{\mathbf{f}} \in \mathcal{F}$.

A nice feature of the proportional fairness criterion ($\xi = 1$) is that the transport proportional fairness and proportional fairness lead to the same objective (up to a constant) and need not be considered separately. In contrast, the rates that maximize utility for any $\xi \neq 1$ differ from the rates that maximize the corresponding transport utility. Existence and uniqueness hold for transport criteria in the same way as for rate criteria.

4.4 Performance Indices

In the rest of this paper, we evaluate the properties of the optimal rates that correspond to each of the criteria above. It is convenient to use indices that quantify efficiency and fairness.

The **efficiency index** of a feasible rate $\overline{\mathbf{f}}$ in a given feasible set $\mathcal{F}$ is

$$\frac{\sum_{i=1}^{I} \overline{\mathbf{f}}_i}{\sum_{i=1}^{I} \overline{\mathbf{f}}_i^c},$$

where $\overline{\mathbf{f}}^c$ is the rate vector that maximizes capacity in $\mathcal{F}$. It is always between 0 and 1.

Similarly, the **transport efficiency index** of $\overline{\mathbf{f}}$ in $\mathcal{F}$ is

$$\frac{\sum_{i=1}^{I} \overline{\mathbf{f}}_i \mathrm{len}(i)}{\sum_{i=1}^{I} \overline{\mathbf{f}}_i^t \mathrm{len}(i)},$$

where $\overline{\mathbf{f}}^t$ is the rate vector that maximizes transport capacity in $\mathcal{F}$.

The **max-min fairness index** of $\overline{\mathbf{f}}$ in $\mathcal{F}$ is thus defined as

$$\frac{(\sum_i \overline{\mathbf{f}}_i^* \overline{\mathbf{f}}_i)^2}{(\sum_i \overline{\mathbf{f}}_i^{*2})(\sum_i \overline{\mathbf{f}}_i^2)},$$

where $\overline{\mathbf{f}}^*$ is the max-min fair element of $\mathcal{F}$. The max-min fairness index is defined as $\cos^2 \alpha$, where α is the angular deviation from $\overline{\mathbf{f}}$ to the max-min fair allocation $\overline{\mathbf{f}}^*$ in $\mathcal{F}$. The max-min fairness index is between 0 and 1; it is equal to 1 if $\overline{\mathbf{f}}$ is proportional to the max-min fair allocation of rates. The smaller it is, the less fair the allocation is. When the number of flows I is large, the minimum value of the max-min fairness index is close to 0.

Our max-min fairness index coincides with Jain's definition of fairness index [14] in the case where the max-min fair allocation $\overline{\mathbf{f}}^*$ has all components equal. Otherwise, it differs.

Similarly, the **transport max-min fairness index** of $\overline{\mathbf{f}}$ in $\mathcal{F}$ is

$$\frac{(\sum_i \overline{\mathbf{t}}_i^* \overline{\mathbf{f}}_i \mathrm{len}(i))^2}{(\sum_i \overline{\mathbf{t}}_i^{*2})(\sum_i (\overline{\mathbf{f}}_i \mathrm{len}(i))^2)},$$

where $\overline{\mathbf{t}}^*$ is the max-min fair element of $\mathcal{T}$.

4.5 Performance Metrics

The indices defined above require computing the reference rate vector that is optimal with respect to a design criterion, and they depend on the set of rate vectors that is being considered. In contrast, metrics are defined as a function of the rate alone, independent of any set of rate vectors. For completeness, we now give the metrics that correspond to the design criteria defined above. They may be useful in practical situations where, unlike in this paper, the computation of the reference rates is not feasible. This occurs, for example, when a protocol is given by its implementation in a simulator and the feasible set is hard to define explicitly.

For a rate vector $\overline{\mathbf{f}}$, the **capacity metric** is $\sum_{i=1}^{I} \overline{\mathbf{f}}_i$ and the **transport capacity metric** is $\sum_{i=1}^{I} \overline{\mathbf{f}}_i \mathrm{len}(i)$. They both measure the efficiency of $\overline{\mathbf{f}}$.

A metric that corresponds to max-min fairness is more difficult to define. Many authors use Jain's fairness index defined above, but this is not always appropriate. Indeed, it measures the deviation from an ideal rate vector where all components are equal, and this is not necessarily the fairest vector. A more accurate, but more complex, metric uses leximin ordering [19], [27]. It is not a real number in the usual sense. Instead, the **fairness metric** $f(\overline{\mathbf{f}})$ of a rate vector $\overline{\mathbf{f}}$ is the list of all its components in increasing order, and we say that a rate vector $\overline{\mathbf{f}}^1$ is fairer than a rate vector $\overline{\mathbf{f}}^2$ if $f(\overline{\mathbf{f}}^1)$ is larger than $f(\overline{\mathbf{f}}^2)$ in lexicographic order. The max-min fair vector is the fairest, in the sense of this metric. Similarly, the **transport fairness metric** is defined as the order statistic of the vector of transport rates $(\overline{\mathbf{f}}_i \mathrm{len}(i))_i$.

The ξ-**utility** is $\sum_{i=1}^{I} U(\overline{\mathbf{f}}_i, \xi)$. This defines a family of metrics, obtained through a different choice of parameter ξ. One example is logarithmic utility, corresponding to the proportional fairness. The family also includes capacity metric ($\xi = 0$). However, the metric is not defined for $\xi \to \infty$ and we cannot use this approach to derive a metric for max-min fairness.

Analog to that, we defined the ξ-**transport utility metric** as $\sum_{i=1}^{I} U(\overline{\mathbf{f}}_i \mathrm{len}(i), \xi)$. As noted in the discussion about performance objectives, the ξ-transport utility and ξ-utility of a rate allocation differ, in general, except for $\xi = 1$ where they differ by a constant additive factor, thus they can be regarded as the same metric.

By using the utility performance metric, one can assess both utility and fairness. The importance of one or another is controlled by varying parameter ξ. We show that both $\xi = 0$ and very large ξ are not appropriate since they completely ignore fairness and efficiency, respectively. We also show that the design criteria based on values of ξ around 1 are the best, in the sense of robustness, against efficiency or fairness anomalies, for various power limitations. This suggests using logarithmic utilities ($\xi = 1$) as a metric of choice for evaluating ad hoc wireless networks.

5 MAX-MIN FAIRNESS

In this section, we analyze properties of the max-min fair allocation. We show that there exists a class of convex sets with the property that a max-min fair vector on such a set has all components equal. We then show that a set of feasible long-term average rates in any wireless network without Long-Term Average Power constraints, modeled by (2)-(12), admits this property thus implying that the rates in max-min fair allocation have to be equal.

5.1 Solidarity Property and Equality

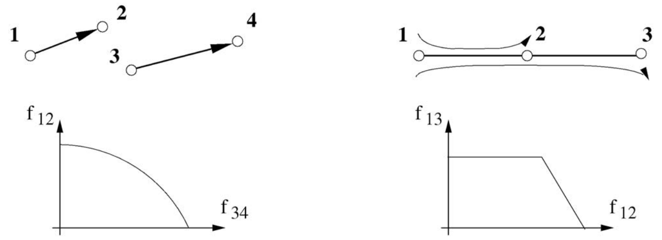
Let us consider a class of sets in $\mathbb{R}^n$ with a property that for any feasible point, we can trade a sufficiently small value of one component for a sufficiently small value of other components. More precisely, we define the solidarity property as follows:

Definition 1. *A subset $\mathcal{X}$ of $\mathbb{R}^n$ has* the solidarity *property iff for all $i, j, i \neq j$, for all $\mathbf{x} \in \mathcal{X}$ such that $\mathbf{x}_i > 0$, and for all $\epsilon > 0$ small enough, there exist positive $0 \leq \alpha_i < \epsilon$, $0 < \alpha_j < \epsilon$ and for all $k \neq i, k \neq j$, there exists $0 \leq \alpha_k < \epsilon$ such that $\mathbf{y} = \mathbf{x} - \alpha_i \mathbf{e}_i + \alpha_j \mathbf{e}_j + \sum_{k \neq i, k \neq j} \alpha_k \mathbf{e}_k$ belongs to $\mathcal{X}$.*

Not all sets have the solidarity property. In particular, not all convex sets have the solidarity property. Simple examples of networks with feasible rate sets with and without the solidarity property are given in Fig. 2.

A characteristic of a set with the solidarity property is that all components of the max-min fair vector are equal. This is formulated in the following proposition:

Proposition 1. *If a set $\mathcal{X}$ has the solidarity property, then the max-min fair allocation $\mathbf{x}$ on $\mathcal{X}$ has all components equal: $\mathbf{x}_i = \mathbf{x}_j$ for all i, j, if the max-min fair allocation on $\mathcal{X}$ exists.*



(a) (b)

Fig. 2. (a) Example of a wireless network whose set of feasible rates has the solidarity property. Rate of flow 12 is constrained by $f_{12} \leq r(P_{12}h_{12}/(N + P_{34}h_{32}))$. Rate of flow 34 is constrained by $f_{34} \leq r(P_{34}h_{34}/(N + P_{12}h_{14}))$. It is always possible to increase the rate of one flow at the expense of the other. (b) Example of a wired network whose feasible rate set does not have the solidarity property. Flows f_{12} and f_{13} are constrained by $f_{12} + f_{13} \leq x_{12}$ and $f_{13} \leq x_{23}$. When flow 13 hits limit on link 23 it cannot be further increased by decreasing rate of flow 12.

The proofs of this and the other propositions can be found in the Appendix.

5.2 Solidarity of the Feasible Rate Set of a Multihop Wireless Network

The feasible set of a wired network is given with a set of linear constraints. It is convex, but, in general, it does not have the solidarity property, as can be seen on the right of Fig. 2. In the case of an ad hoc wireless network, defined under the framework from Section 4, we show that the feasible rate set of any such network without Long-Term Average Power constraints, has the solidarity property.

Proposition 2. *Any feasible rate set $\mathcal{F}$ given by a set of equalities and inequalities (2)-(12), assuming $\overline{P}_l^{MAX} > P_l^{MAX}$ for all links l, has the solidarity property. Also, a feasible transport rate set $\mathcal{T}$ given by (13) has the solidarity property.*

The proofs of this and the other propositions can be found in the Appendix.

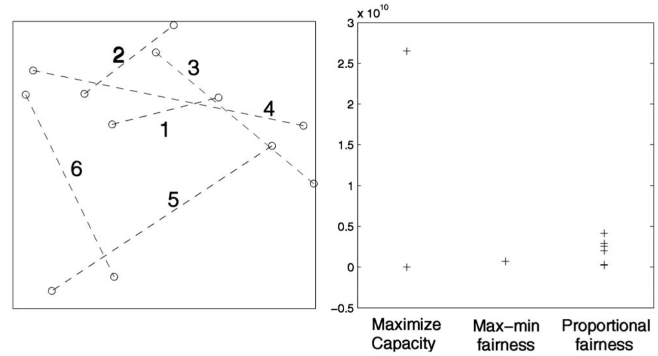
5.3 Equality of Max-Min Fair Rates

Consider an arbitrary network where Long-Term Average Power constraints are larger than transmission power constraints. It is easy to verify that the feasible set given by constraints (2) - (12) is convex, hence according to [25] it has the max-min fair allocation. Since this set also has the solidarity property, we have the following:

Corollary 1. *The max-min fair average rate allocation of any network given by constraints (2) - (12), with no Long-Term Average Power constraints ($\overline{P}^{MAX} \geq P^{MAX}$), has all rates equal. The max-min fair transport rate allocation has all transport rates equal.*

Equality of rates implies that all flows, including the most inefficient ones, have an equal rate. This can be very inefficient in a heterogeneous network. For example, if one node is almost disconnected, then it will receive a rate close to zero. According to Corollary 1, all other flows will have the same rate.

Another example is given in Fig. 3. Fig. 3a shows an example of a network where 12 nodes (siz flows) are randomly placed on a square of $100\text{m} \times 100\text{m}$. The source and the destination of each flow are joined with a line. Each flow can use either the direct route or the minimum energy route. In this example, we assume no random fading, and we set all transmission power constraints to be equal to $P^{MAX}/N = 90\text{dB}$, where N is a white background noise. The actual SNR on each receiver depends on the distance between the source of the link and the destination of the link. For example, according to the UWB indoor path loss model [13], if a source sends to a destination that is 10 m away with maximum power and $P^{MAX}/N = 90\text{dB}$, we have SNR at the receiver around 10 dB. This in turn leads to the rate of 100 Mb/s within the framework of [8], [33].



(a) (b)

Fig. 3. (a) Example of random network topology. Twelve nodes are randomly placed on a $100\text{m} \times 100\text{m}$ grid. Nodes are depicted with circles, and source-destination pairs are joined with lines. Each flow uses the optimal routing (the direct or the minimum-energy route). (b) Corresponding rate distribution (total capacity, max-min fairness and proportionally fairness).

In Fig. 3b, we see the optimal rate allocations with respect to the three metrics, for this example. We see that, when maximizing total capacity, only flow 1, which is the shortest flow, has a positive rate, and the rates of other flows are zero. In the case of max-min fairness, all rates are the same. Proportional fairness exhibits larger variation in rates than max-min fairness, but it does not starve the least efficient flows. But, it is more efficient than max-min fairness. We also illustrate the Corollary 1 on more random examples in Fig. 4 in Section 6.2.

From Corollary 1, we also see that in the case of the max-min fair transport rate allocation, all transport rates are equal. Obviously, the rates themselves are not equal as the flow lengths differ. Still in this case, as can be seen in the numerical examples from Fig. 4 in Section 6.2, the corresponding rate allocation suffers from the same inefficiency problem.

5.4 Influence of Long-Term Average Power Constraint

Corollary 1 holds when there are no Long-Term Average Power constraints (which is equal to having Long-Term Average Power constraints greater or equal to transmission power constraints). When Long-Term Average Power constraints are smaller than the transmission power constraints, the max-min fair rates are

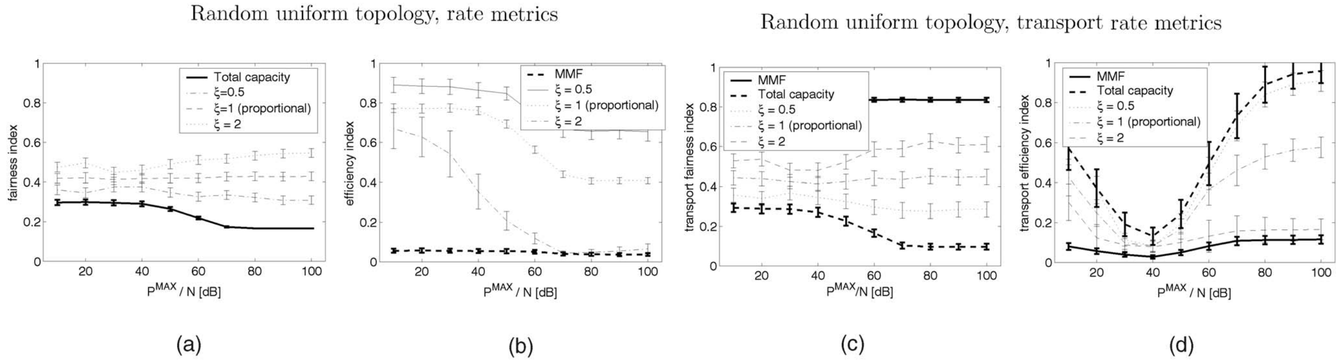


Fig. 4. The fairness and efficiency indices of optimal rates with respect to different performance objectives ($\xi = 1$ means proportional fairness), versus the ratio of maximal transmitting power and noise (note that both the fairness index of the max-min fair allocation and the efficiency index of the maximal total capacity are one). (a) and (b) Indices for the three performance objectives applied to the set of feasible rates. (c) and (d) Indices for the three performance objectives applied to the set of feasible transport rates. In both cases, nodes are uniformly spread over the entire $100\text{m} \times 100\text{m}$ square. Values of P^{MAX}/N on the x axis represent a realistic values that can be found on existing UWB or CDMA systems. In all cases, we put no constraints on Long-Term Average Power. All figures show 95 percent confidence intervals. Notice that the minimal value of fairness index is $1/I$, where I is the number of flows; in this example, the smallest fairness index is 0.17 and represents that only one flow has a positive rate and all others have the zero rates.

not equal anymore. However, we see that for high transmission power constraints ($P^{MAX} \geq 40\text{dB}$, see Fig. 4 in Section 6.2) and high Long-Term Average Power constraints ($\overline{P}^{MAX}/P^{MAX} \geq 0.5$, see Fig. 8, Section 8) the max-min fair rate allocation is still inefficient.

In Fig. 8, we also see that for very small Long-Term Average Power constraints, the optimal allocation becomes independent of the choice of the metric.

Overall, these arguments show that max-min fairness is not an appropriate metric even when Long-Term Average Power constraints exist.

5.5 An Application to an 802.11 Network

An example of the above findings can be seen in [2]. Consider an 802.11 network where several nodes send data directly to a single destination (base-station). Assume node 1 is far away and it codes for 1Mb/s, and the others are close enough to code for 11Mb/s. One would expect that node 1 achieves a smaller rate than the other nodes. However, as shown in [2], this is not the case, and all nodes achieve an effective throughput of around 1Mb/s.

According to the analysis done in [2], when a node gets an access to the network, it sends a packet of a fixed size, thus the occupancy time is inversely proportional to the coding rate. In other words, a node sends the same amount of data during a channel use, regardless of its coding rate. Let us consider a discrete random process X_t representing a user that occupies a channel during the tth channel use. According to (7) and (8) from [2], X_t is an i.i.d. uniform random process, and all nodes have an equal probability to obtain network access when the network is idle. This leads us straightforward to the following proposition:

Proposition 3. *An 802.11 network in DCF mode where all nodes talk directly to a single destination (hence, there is no hidden terminal problem) implements max-min fairness.*

In other words, the equality of rates observed in [2] is not solely a property of the 802.11 physical layer, but rather of max-min fairness that is obtained in this specific example. This means that any other protocol that would implement max-min fairness, and would fit in the framework of (2)-(12), would have the same inefficiency problem.

5.6 When Max-Min Fairness Does Not Lead to Equality

We note that the assumptions of Corollary 1 are not true in general for any convex set, but only for those that have the solidarity property. To illustrate this, we give a few counter examples:

- *Wired Networks:* The corollary does not hold for a class of wired networks. For an example, see Fig. 2b.
- *Clustered Networks:* The corollary does not hold for a clustered wireless network. Assume a simple network of two links, link $(1, 2)$ and link $(3, 4)$, and assume it is clustered such that nodes 3 and 4 do not hear nodes 1 and 2 and vice versa (meaning that $h_{13} = h_{14} = h_{23} = h_{24} = 0$). Then, rates $\bar{f}_{12}$ and $\bar{f}_{34}$ are not going to be equal.
- *Long-Term Average Power Constraint:* The corollary does not hold if the Long-Term Average Power constraint is smaller than Transmission Power constraints, as shown in Section 8.

6 Maximizing Total Capacity

6.1 Asymptotic Results

As previously discussed, maximizing the total capacity metric is efficient but may lead to high unfairness, especially in the case of large transmission power constraints. In order to demonstrate this, we first look at the asymptotic case. We start by considering a network without variable noise power and channel conditions (that is, with only one state), and we show that total capacity metric becomes totally unfair as transmission power tends to infinity.

At this point, we need an additional assumption on the rate function $\lim_{\text{SNR}\to\infty} r(\text{SNR}) = \infty$, that is we can increase the rate of a link arbitrarily high by sufficiently increasing the signal-to-noise ratio on this link. We also assume here no Long-Term Average Power constraint, hence $\overline{P}^{MAX} \geq P^{MAX}$.

In order to simplify the presentation, we assume that all transmission power constraints are the same, that is for all l, $P^{MAX} = P_l^{MAX}$. This can be generalized for nonuniform power constraints, assuming that, when P^{MAX} goes to infinity, there exist fixed positive numbers γ_l such that $P_l^{MAX}/P^{MAX} > \gamma_l$.

Proposition 4. *Consider an arbitrary network with one system state ($|\mathcal{S}| = 1$). Assume that, when the signal-to-noise ratio at a receiver $\mathbf{SNR}_i$ tends to infinity, the rate of link i, $r(\mathbf{SNR}_i)$ also tends to infinity, and assume there is no Long-Term Average Power constraint (or equivalently, $\overline{P}^{MAX} \geq P^{MAX}$). In a limiting case when $P^{MAX} \to \infty$, there will be one or more flows that have the same rate $f = O(r(P^{MAX}/N))$ and all the others will have a rate that is $o(r(P^{MAX}/N))$. The same happens when considering transport rates.*

The proofs of this and the other propositions can be found in the Appendix. Proposition 4 tells us that if a signal-to-noise ratio is high enough, then only the most efficient flows are going to divide all the capacity of the medium, whereas all other flows will starve. An example is shown on Fig. 3. In this example, all flows are single hop, and the most efficient flow is the shortest one, which is flow 1. Therefore, all flows, except for flow 1, have zero rates. In the following, we illustrate that the same problem occurs for a large range of realistic signal-to-noise settings.

The proposition will not hold for a network with variable channel fading or noise power. Consider a single-hop flow over a long link (that is, a link with high average signal attenuation). As suggested in [32], when maximizing total capacity, it is always optimal to schedule the flow with the best channel conditions. In most cases, the long flow will not be scheduled. However, it can still happen that, due to variability in the network medium, the long flow has the best channel in some slots and the long-term average rate of this flow is larger than zero. Nevertheless, in a very asymmetric multihop network, the probability of the flow, with a poor average performance, to have the best realization is very small. A typical example is a multihop network that has very short and very long links, and single-hop and multihop flows. In those cases, as Proposition 4 indicates, the rates of flows with poor average conditions will be close to zero, as is illustrated on Fig. 6.

6.2 Numerical Results

In the above section, we have seen that an increase in transmission power constraints will eventually lead to all but some flows having zero rates. It is not clear what the realistic values of the constraints for which this phenomenon occurs are. From [8], we see the phenomenon has been observed in a realistic network and, in this section, we investigate in which transmission power region it occurs. Another issue we evaluate is the effect of variable channel fading and noise power on fairness of maximizing total capacity metric.

6.2.1 Simulation Setting

In order to analyze the behavior of the total capacity performance metric for a realistic power setting, we numerically evaluated it on random network topologies. We adapted the framework from [7], which assumes a rate is a linear function of the signal-to-noise ratio at a receiver (this also corresponds to an UWB model from [8]). It is shown in [7] that the optimal power allocation strategy that maximizes total capacity is either to send with maximal power or not to send at all. Nevertheless, the optimization problem is still exponentially complex so it was not possible to run simulations for more than 12 nodes (six flows) and with no random fading (the effect of random fading and noise power is evaluated latter in this section).

For each flow, we consider a multipath routing with a set of routes that comprise nodes that are on the shortest path between the source and the destination. This is a suboptimal set of routes since, in the case of high congestion in one area of a network, the optimal path may avoid that area even if it is not the shortest one. However, in most cases, this heuristic is a good approximation, and it simplifies our calculation. Furthermore, by running tests on several random topologies, we concluded that in all cases the optimal heuristic among those is either the minimum energy route (relaying over intermediate nodes that minimizes total dissipated power), or the direct route (send directly to the destination without relaying). Since constraining on these two routes for each flow further reduces the complexity of optimization, we used these heuristics to produce the results. In our example of networks with 12 nodes (six flows), average number of hops per route is 2.19.

6.2.2 Uniform Topologies

We first consider uniform random network topologies with 12 nodes uniformly distributed on a square of $100\mathrm{m} \times 100\mathrm{m}$. Half of them are sources sending data, each to its own destination among the other half. All nodes are assumed to have the same transmission power constraints. We are looking for routing, scheduling, and power control that maximize the total capacity. An example of such a network, together with the optimal end-to-end rates with respect to different objectives, is given in Fig. 3.

In Fig. 4a, we show average fairness indices of the optimal rates with respect to total capacity and utility metrics, as well as the confidence intervals. On the x-axis, a ratio between maximal transmitting power and noise in dB is given.

From the numerical results depicted in Fig. 4a, we see that maximizing total capacity leads to an acceptable fairness in the case of small transmission power limits. However, for large transmission power limits, we see that maximizing total capacity exhibits high unfairness. The minimal value of fairness index is $1/I = 0.17$ and corresponds to the case when only one flow has a nonzero rate. The fairness index of total capacity drops to minimum for high power limits, as predicted by Proposition 4. We can see that this happens already for $P^{MAX}/N > 70dB$, and it happens for smaller power constraints when topology is nonuniform. These results confirm the unfairness observations made in [8], and show they are a consequence of the performance metrics rather than physical layer particularities. All these results are for unlimited battery lifetime constraints. However, the unfairness exists for a limited battery lifetime; for details, see Section 8.

Next, we used the three metrics to find the transport-optimal solutions on the set of rates $\mathcal{F}$. We calculated the transport fairness and the transport efficiency indices of the optimal rate allocations. This can be seen in Figs. 4c and 4d.

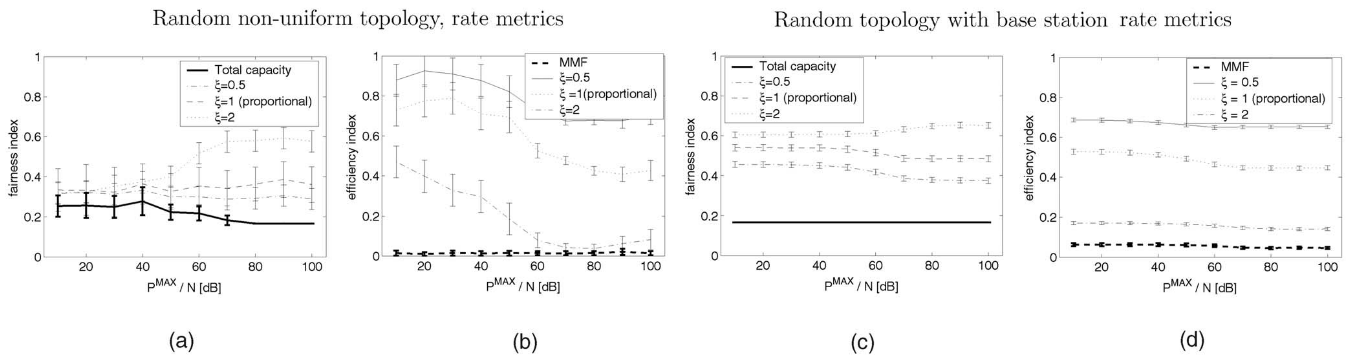


Fig. 5. Nonuniform topologies. (a) and (b) We see the indices for the performance objectives ($\xi = 1$ means proportional fairness) applied to the set of feasible rates in the case when nodes are distributed only in the upper-left and lower-right quarters of a $100\mathrm{m} \times 100\mathrm{m}$ square. (a) The fairness index is given and (b) is the efficiency one (note that both the fairness index of the max-min fair allocation and the efficiency index of the maximal total capacity are one). (c) and (d) The same indices are given for the case of uniformly distributed nodes talking to a base-station in the center. In all cases, we put no constraints on Long-Term Average Power. All figures show 95 percent confidence intervals. Notice that the minimal value of fairness index is $1/I$, where I is the number of flows; in this example, the smallest fairness index is 0.17 and represents that only one flow has a positive rate and all others have the zero rates.

We see that, when considering the transport fairness index, we have similar results as for the fairness index: max-min fairness is the most transport-fair metric and maximizing total capacity is the least. Similarly, maximizing total capacity is the most transport-efficient metric, and max-min fairness is the least. We also see a drop of the transport-efficiency of rate based metrics for $P^{MAX}/N = 40\mathrm{dB}$. Thus, there is a major transport-inefficiency of the optimal rate allocations, but only in the case of intermediate transmission power limits. Intuitively, when powers are very small, there is little interference and interactions between links. Hence, maximizing capacity yields similar performances to maximizing transport capacity. When powers are high, whether maximizing capacity or transport capacity, radio resources are allocated to the most efficient link, and the two performances are similar again. For intermediate powers, the two metrics differ, and we see a large dip.

6.2.3 Nonuniform Topologies

We also analyzed the fairness index of the optimal rates in a case of random nonuniform networks. We consider two nonuniform scenarios. The first one has a nonuniform node distribution but a uniform source-destination distribution. We again considered a square area of $100\mathrm{m} \times 100\mathrm{m}$ and we divided it into four equal subsquares of $50\mathrm{m} \times 50\mathrm{m}$ each. We placed, in total, 12 uniformly distributed nodes in the upper left and lower right subsquares. Each node chose uniformly one destination among all other nodes. We thus had several short and several long flows, and a hot-spot in the center of the big square. The results are depicted in Figs. 5a and 5b.

The second scenario has a uniform node distribution and a nonuniform source-destination distribution. A base station is placecd in the middle of a $100\mathrm{m} \times 100\mathrm{m}$ square. Nodes are randomly distributed over the square and they all talk to the base station. In this scenario, we consider six flows, in order to maintain the same number of flows as in the other simulations. The results are depicted in Figs. 5c and 5d.

6.2.4 Variable Channel Fading and Noise Power

Finally, we analyze the case with random fading and random noise powers. We assume that the fading between the source of link i and the destination of link j belongs to the set of possible fadings H_{ij}^s, and that the power of the white noise at the destination of link i belongs to the set of possible powers N_i^s. Then, the set of possible states of the system is $\mathcal{S} = H_{11}^s \times \cdots \times H_{1L}^s \times \cdots \times H_{LL}^s \times N_1^s \times \cdots \times N_L^s$. This means that the number of possible system states $|S|$ grows exponentially with the number of links in the network and the number of possible states for each link. Since we have to find the optimal schedule for each system state, it is difficult to numerically evaluate this problem, even for small networks. In order to show the effect of randomness in a network, we consider a case of networks with two single-hop flows. We assume that the set of possible fadings is $H_{ij}^s = \{h_{ij}, 0.75h_{ij}, 0.5h_{ij}\}$ with probabilities $\{0.75, 0.2, 0.5\}$, respectively, where h_{ij} is the attenuation that depends on the distance, as used in the fixed fading case. We assume the same distribution for the noise power at receivers. We then construct the set of possible system states $\mathcal{S}$ with the corresponding probabilities. Using the same numerical approach as in the case of fixed fading, this time for every possible state, we calculate the optimal rates with respect to different metrics.

The results are depicted in Fig. 6. Max-min fair rate allocation is as inefficient as it is in the case of constant fading and noise power. Total capacity is less unfair than it is in the case of constant fading. However, we still see that for high power limits ($P^{MAX}/N > 70dB$), one flow will have the zero rate (that is, fairness index will be equal to 0.5).

7 Utility Fairness

As seen in the previous sections (e.g., Fig. 3), both maximizing total capacity and max-min fairness suffer from either inefficiency or unfairness. In this section, we analyze utility fairnesses in detail. We numerically evaluated the efficiency and fairness of the proportional fairness metric using the same setting as in Section 6.2. We show

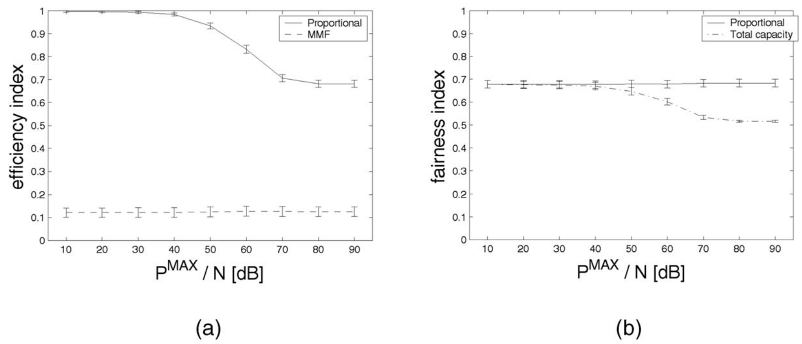


Fig. 6. Random fading and noise power: (a) Efficiency index and (b) fairness index of maximum total capacity, max-min fair and the proportionally fair ($\xi = 1$) rates. Results are obtained on random uniform network topologies with four nodes (two flows). Both fading and noise power take a fraction of $\{1, 0.75, 0.5\}$ of the constant fading case, with probabilities $\{0.75, 0.2, 0.5\}$, respectively. Note that in this example, fairness index of 0.5 means the one flow has zero rate.

that proportional fairness ($\xi = 1$) in particular represents a robust compromise between efficiency and fairness.

As explained in Section 6, the optimal power allocation strategy that maximizes total capacity is either to send with maximal power or not to send at all. It is shown in [26], that, within the same framework, the same findings hold in the case of utility fairness. Therefore, we use the same numerical approach as explained in Section 6, to calculate different utility fairnesses. We also use the same routing heuristics.

The fairness and the efficiency indices of utility fair rate allocations for different values of ξ are depicted in Figs. 4a and 4b, respectively. We give the results for frequently used proportional fairness ($\xi = 1$) and minimum potential delay ($\xi = 2$), and also for $\xi = 0.5$. We see that the difference between the design objectives are smaller when power limits are small (except for max-min fairness). This is due to the fact that the interference, hence the mutual interactions among nodes transmitting in parallel, become smaller when power is small.

We next analyze the case of large power limits ($P^{MAX}/N > 70$dB). For $\xi = 0.5$, the fairness index of the optimal rate allocation is close to the fairness index of the one that maximizes sum of rates, hence it exhibits the same unfairness problems. For $\xi = 2$, the efficiency index of the optimal rate allocation becomes very close to the one of max-min fair rate allocation, hence the optimal allocation is inefficient. The proportionally fair rate allocation ($\xi = 1$) is the most robust. Its efficiency remains constant for all values of transmission power constraint, and it is up to 10 times more efficient than the max-min fair allocation for high transmission power constraints.

We analyzed the efficiency index of the optimal rates in a case of random nonuniform networks, as above, and the results are depicted in Fig. 5. In Figs. 5a and 5b, we considered a nonuniform node distribution, and, in Figs. 5c and 5d, we considered a nonuniform traffic distribution. The fairness indices are given in the top row and the efficiency indices are given in the bottom row. The results are similar to those from the symmetric case, and the same conclusions hold.

In Figs. 4c and 4d, we depict the transport fairness and transport efficiency properties of the optimal rates on set $\mathcal{F}$. Max-min fairness is again much less transport-efficient than utility fairnesses and maximizing total capacity is much less transport fair than utility fairnesses. Like in the case of fairness and efficiency indices, proportional fairness ($\xi = 1$) is the most robust, and balanced with respect to transport-efficiency and transport-fairness.

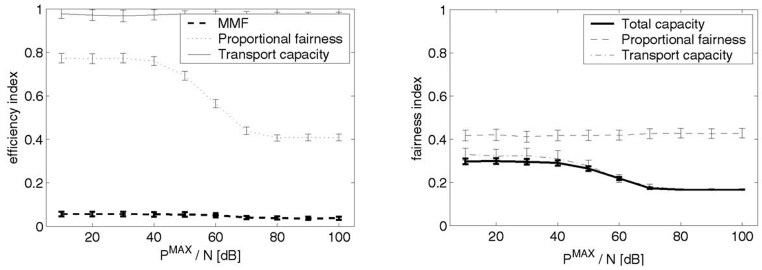
In some existing work, like [10], maximum transport capacity was used as a design objective for a multihop network. An interesting question is how appropriate is this metric with respect to the rate efficiency and the rate fairness indices. In other words, can maximizing transport capacity reconcile the rate unfairness of the total capacity objective? According to Proposition 4, maximizing transport capacity also exhibits high unfairness for large transmission power constraints.

We give numerical examples for realistic transmission power constraints on random uniform network topologies in Fig. 7. We see that the rate that maximizes transport capacity is only marginally more fair and marginally less efficient than the one that maximizes total capacity. The unfairness becomes the same in both cases for high powers, as suggested by Proposition 4. Again, utility fairnesses represents a much better compromise between efficiency and fairness than the total capacity-based metrics.

To see why transport capacity does not alleviate the fairness problem, consider a simple example with two links of distances l_1 and l_2. Suppose that a transmitter of the second link is close to the receiver of the first link, thus it is not optimal to have them both sending together, but we need to do time divisioning: the first link is scheduled α fraction of the time, and the second on $1 - \alpha$. For simplicity, we assume the rate is a linear function of SNR: $R = K\ SNR$, and assume attenuation is constant, and equal to bl^{-a}, where l is the distance between nodes and $a > 2, b$ are some constants. The optimization problem now becomes

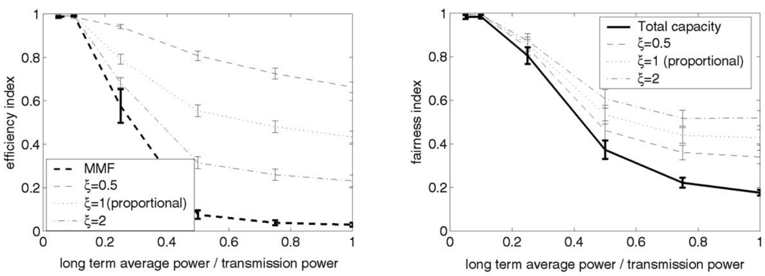
$$\max_{\alpha} \quad \alpha\, l_1\, K\, \frac{Pbl_1^{-a}}{N} + (1-\alpha)\, l_2\, K\, \frac{Pbl_2^{-a}}{N}.$$

Clearly, if $l_1^{-a+1} < l_2^{-a+1}$, which happens whenever $l_1 > l_2$, the optimal is to let the second link transmit all the time. We see that the unfairness properties of linear objective function cannot be compensated through weights. Although this is a very simple example, it is often found as a part of a larger network, thus we can easily find unfairness examples in arbitrary large networks. As we can see from Fig. 7 for larger networks and longer flows,



(a) (b)

Fig. 7. (a) Efficiency index and (b) fairness index of the rates that maximizes total capacity and transport capacity, and the proportionally fair ($\xi = 1$) rates. Results are obtained on random uniform network topologies.



(a) (b)

Fig. 8. (a) Efficiency index and (b) fairness index of max-min and proportional fairness for finite Long-Term Average Power constraint.

transport capacity brings only a minor improvement in fairness.

We conclude by saying that the choice of optimal ξ depends on the system design constraints and the desired tradeoff between efficiency and fairness, as can be seen from the numerical results presented in this section. However, we observe that in most cases the proportional fairness ($\xi = 1$) is the most robust tradeoff between efficiency and fairness.

8 Influence of Long-Term Average Power Constraint

In the previous sections, we have seen that when we do not put constraints on battery lifetime, we have no Long-Term Average Power constraints; in the case of max-min fairness, this leads to the equal rates of all flows and in the case of maximizing total capacity to zero rates of some flows.

This is not the case when the Long-Term Average Power constraint is smaller than the Transmission Power constraint, as it is illustrated in Fig. 8. When the Long-Term Average Power limit is very small, the optimal allocation is almost the same, regardless of the choice of the performance metrics. As the Long-Term Average Power limit grows, the difference becomes more significant and the unfairness of total capacity and inefficiency of max-min fairness are visible.

9 Conclusion

We analyzed three rate-based performance metrics: total capacity, max-min fairness, and different utility fairnesses, within the framework of ad hoc wireless networks. We defined a general model of such a network, which incorporates all existing physical layers (CDMA, UWB, variable rate 802.11, etc.), and allows for arbitrary scheduling, routing, and power control policy. We then evaluated the three metrics on this model.

We found that max-min fairness yields equal average rates to all flows, when users are not implying battery-lifetime constraints. In a heterogeneous network, this means that the rates of all flows are equal to the rate of the smallest flow, which makes a network very inefficient. This is true for both constant and variable channel fading and noise power.

The finding confirms and generalizes the observations from [2]. In the presence of Long-Term Average Power constraints, the max-min fair rate does not necessarily have this property, but the inefficiency persists. Also, the rate allocation of all flows depends on Long-Term Average Power constraints of a single user, which is an undesirable property of a performance objective.

We proved that for constant channel fading and noise power, and for large enough power constraints, maximizing total capacity gives zero rates to all but the most efficient flows. We showed that this type of unfairness occurs on most of the networks for realistic power constraints, even in presence of variable channel fading and noise power. This is a confirmation and a generalization of the findings from [8]. Like in the case of max-min fairness, this phenomenon is somewhat remedied in the case of a small Long-Term Average Power constraint, but it remains. We also showed that in the case of small Long-Term Average Power constraints, the optimal rate allocation depends more on these constraints than on the choice of the performance metrics.

Finally, we analyzed the utility fair rate allocations for different utility functions (different values of ξ) and on a large number of arbitrary networks with variable transmission power and Long-Term Average Power constraints. We found that the tradeoff between efficiency and fairness can be fine-tuned by the choices of parameter ξ. This choice depends to a large extent on Transmission Power and Long-Term Average Power limits. We also found that proportional fairness ($\xi = 1$) is particularly robust with respect to changes in topology and power constraints. In all cases, it maintains fairness and it achieves relatively high efficiency. These properties make proportional fairness a suitable performance metric when evaluating or designing a MAC or a routing protocol for an ad hoc wireless network.

We analyzed a model of a wireless network, which includes mobility (which is implicit in the model), fading, routing, power control, scheduling, and rate adaptation. Although fading, routing, and flow control may be arbitrary, we do assume that scheduling, rate adaptation, and power control are optimal (which corresponds to a network without protocol overhead). The main reason for doing so is that we want to analyze the fundamental trade offs in the choice of a performance metric, and we want the analysis to be independent of the choice of the MAC protocol. Our results suggest the kind of behavior that can be expected of a performance metric with an arbitrary MAC. The results do not apply directly to a specific MAC and they greatly depend on the implementation of scheduling, rate, and power control. However, as we show in the example on 802.11, the results are likely to hold for realistic protocols. Our findings thus give guidelines for choosing a performance metric when designing or evaluating a wireless network.

All the metrics analyzed in this paper are rate-based performance metrics. The power constraints were considered explicitly, rather than through performance metrics. Still, powers can be incorporated in all three types of metrics analyzed here. Future work would be to analyze the ideal power-based and combined performance metric for an ad hoc wireless network.

Appendix A

A.1 Proof of Proposition 1

Let us denote by $\mathbf{x}$ a max-min fair allocation on $\mathcal{X}$ and let us assume the contrary, that there exists i and j such that $\mathbf{x}_i - \mathbf{x}_j > 2\epsilon$ for some $\epsilon > 0$. Then, according to the solidarity property, there exists $\mathbf{y}$ such that

$$\mathbf{x}_i \geq \mathbf{y}_i > \mathbf{x}_i - \epsilon > \mathbf{x}_j + \epsilon > \mathbf{y}_j > \mathbf{x}_j,$$

and $\mathbf{y}_k \geq \mathbf{x}_k$ for all $k \neq i, k \neq j$ which contradicts with the definition of max-min fairness. □

A.2 Proof of Proposition 2

Let us denote by $(\mathbf{y}(s), \bar{\mathbf{x}}(s), \alpha(s), (\mathbf{p}^i(s))_{n=1\cdots N})_{s\in\mathcal{S}}$ the values of slack variables, used in the constraint set given by (2)-(12), that satisfies these constraints for rates $\bar{\mathbf{f}}$. We want to show that for every i and j, such that $\bar{\mathbf{f}}_i > 0$, it is possible to increase $\bar{\mathbf{f}}_j$ by an arbitrary small value, by decreasing $\bar{\mathbf{f}}_i$.

We first show that the average power limit is always satisfied if $P_l^{MAX} \leq \overline{P}^{MAX}$. We have that for every link l, slot n, and state s, the transmission power is limited by $p_l^n(s) \leq P_l^{MAX}$. We then have $\mathbb{E}\{\bar{\mathbf{p}}_l(S)\} \leq P_l^{MAX} \leq \overline{P}^{MAX}$, and we conclude that further on we do not have to explicitly consider the average power constraint.

Since $\bar{\mathbf{f}}_i > 0$, there exist a nonempty set $\mathcal{S}^+ \subseteq \mathcal{S}$ such that for all $s \in \mathcal{S}^+$, we have $\mathbf{f}_i(s) > 0$. We will first show by contradiction that for every $s \in \mathcal{S}^+$ we can increase $\mathbf{f}_j(s)$ by decreasing $\mathbf{f}_i(s)$. Then, we will show that this will in turn increase $\bar{\mathbf{f}}_j$.

We first choose an arbitrary $s \in \mathcal{S}^+$, and we proceed by contradiction. Consider a feasible rate $\mathbf{f}(s)$ such that we cannot increase $\bar{\mathbf{f}}_j(s)$ by decreasing $\bar{\mathbf{f}}_i(s) > 0$.

For every state $s \in \mathcal{S}$, there exist a set $K(s,j) \neq \emptyset$ of links (rows in matrice $R(s)$) such that, for each $k \in K(s,j)$, there exists path m with positive rate ($y_m(s) > 0$), that belongs to flow j ($F_{mj}(s) = 1$), and passes over link k ($R_{km}(s) = 1$), and there is a strict equality $\bar{\mathbf{x}}_k(s) = (R(s)\mathbf{y}(s))_k$ (else, we can increase $\mathbf{f}_j(s)$, at no cost). In plain words, $K(s,j)$ is a nonempty set of bottleneck links for flow j.

Suppose that for each link k from $K(s,j)$ there exist path m that has a positive rate ($y_m(s) > 0$), belonging to flow i, that passes over link k. If we decrease rates of all such paths by some ϵ, we decreased $\mathbf{f}_i(s)$, and in the newly obtained rate allocation, we have for all k, $\bar{\mathbf{x}}_j(s) < (R(s)\mathbf{y}(s))_j$. This in turn means that we can increase $\bar{\mathbf{x}}_j(s)$ that leads to contradiction. Therefore, we can find a link l that is not a bottleneck for flow j ($l \notin K(s,j)$) and on which flow i has a positive throughput (there exists path m such that $y_m(s) > 0, F_{mi}(s) = 1, R_{km}(s) = 1$).

Let us denote with

$$\begin{aligned}\mathrm{Com}(k) = \{m \in 1 \cdots L \ : \ &k.\mathrm{src} = m.\mathrm{src} \vee k.\mathrm{src} \\ &= m.\mathrm{dst} \vee k.\mathrm{dst} = m.\mathrm{src} \vee k.\mathrm{dst} = m.\mathrm{dst}\}\end{aligned}$$

a set of links that cannot be scheduled at the same time as link k since they share a common node. We pick an arbitrary $k \in K(s,j)$ (we have $l \neq k$), a slot n when link l is active, and divide it in three slots, n_1, n_2, and n_3 of lengths $\alpha_{n_1}(s), \alpha_{n_2}(s), \alpha_{n_3}(s) > 0$, respectively, such that $\alpha_{n_1}(s) + \alpha_{n_2}(s) + \alpha_{n_3}(s) = \alpha_n(s)$. In the first slot, we keep the same power allocation as in slot n. In the second slot, we turn off the link l, and leave the rest as it is. In the third slot, we turn off link l and all of the links from $\mathrm{Com}(k)$, and increase, if necessary, the power of link k such that $0 < p_k^{n_3}(s) \leq P_k^{MAX}$ and the interferences perceived by other active users is smaller than in the original scheduling of slot n. As we have shown above, this is always possible since $\overline{P}_k^{MAX} \geq P_k^{MAX}$ for all k, and all links from $\mathrm{Com}(k)$ are silent.

In the new scheduling, links belonging to $\mathrm{Com}(k)$ have the same rates in slot n_1, higher rates in slot n_2, and lower rates in slot n_3. It is easy to see that there exists a small $\epsilon_k > 0$ and a choice of $\alpha_{n_1}(s), \alpha_{n_2}(s), \alpha_{n_3}(s)$ such that all links have the same or higher rates, except for link k whose rate has decreased by at most ϵ^l and link k whose rate has increased by at least ϵ^k. We repeat the same for all $k \in K(s,j)$.

We thus have a new average link rate allocation $\bar{\mathbf{x}}'(s)$ such that $\bar{\mathbf{x}}_l(s) - \sum_{k\in K(s,i)} \epsilon_k < \bar{\mathbf{x}}'_l(s) < \bar{\mathbf{x}}_l(s)$ and $\bar{\mathbf{x}}_k(s) < \bar{\mathbf{x}}'_k(s) < \bar{\mathbf{x}}_k(s) + \epsilon_k$, for all $k \in K(s,i)$. Now, we can increase $\mathbf{f}_j(s)$ by at least $\min_{k\in K(s,j)} \epsilon_k$, by decreasing $\mathbf{f}_i(s)$ by at most $\sum_{k\in K(s,i)} \epsilon_k$.

This leads to a contradiction, and we have proved that for every $s \in \mathcal{S}^+$ we can increase $\mathbf{f}_j(s)$ by some positive $\epsilon(s)$.

We repeat the same for every $s \in \mathcal{S}^+$. We then have the new average rate of flow j,

$$\begin{aligned}\bar{\mathbf{f}}'_j &= \sum_{s\in\mathcal{S}^+} \mathbb{P}\{S=s\}\ (\mathbf{f}_j(s)+\epsilon(s))\\ &= \bar{\mathbf{f}}_j + \sum_{s\in\mathcal{S}^+} \mathbb{P}\{S=s\}\ \epsilon(s),\\ &> \bar{\mathbf{f}}_j,\end{aligned}$$

since $\sum_{s\in\mathcal{S}^+} P\{S=s\}\epsilon(s) > 0$. This proves the solidarity property of set $\mathcal{F}$.

The same reasoning holds for a set of transport rate, hence the second part of the statement. □

A.3 Proof of Proposition 4

Since in the theorem we consider a network with only one system state, we will omit the state S from the notation throughout the proof.

We first propose a lemma that characterizes the optimal schedule and power allocation when transmission power limit tends to infinity.

Lemma 1. *Let $\mathbf{p}^n$ be the optimal power allocation in slot n, that maximizes sum of rates, given transmission power limit P^{MAX}. For all slots n there exists link i such that both are true:*

1. *There exists $\epsilon_i > 0$ such that for all Ω_i, there exists $P^{MAX} > \Omega_i$ such that $\mathbf{p}_i^n / P^{MAX} > \epsilon$.*
2. *For all $j \neq i$ and for all $\epsilon_j > 0$, there exists Ω_j such that for all $P^{MAX} > \Omega_j$, we have $\mathbf{p}_j^n/\mathbf{p}_i^n < \epsilon$.*

Proof. We begin by showing that first statement is true using contradiction. Suppose that for some slot n and for each link i and all $\epsilon_i > 0$ there exists Ω_i such that for some $P^{MAX} > \Omega_i$, we have $\mathbf{p}_i^n / P^{MAX} < \epsilon$. Let us choose an arbitrary link j and increase its power allocation in slot n to $\mathbf{p}_i^{\prime n} = P^{MAX}$. We then have the following:

$$\frac{\mathbf{SNR}'^n_j}{\mathbf{SNR}'^n_i} = \frac{P^{MAX} h_{jj}}{\mathbf{p}_i^n h_{ii}} \frac{N + \sum_{k\neq i} \mathbf{p}_k^n h_{ki}}{N + \sum_{k\neq j} \mathbf{p}_k^n h_{kj}} \tag{14}$$

$$= \frac{P^{MAX}}{\mathbf{p}_i^n} K > \epsilon_i K, \tag{15}$$

where K is a fixed constant. Therefore, we can make new $\mathbf{SNR}'^n_j$ arbitrary higher than any signal-to-noise ratio in slot n. Due to the assumption on the rate function, in the same way we can make a rate of link j in slot n arbitrary larger than rates of other links in slot n, as well as the sum of rates of all links in slot n. In particular, if link j connects a source and a destination of a flow, by increasing $\mathbf{p}_j^n$ to P^{MAX} we increased the total rate, which contradicts with the initial assumption.

Next, we show the second part of the statement, again by contradiction. We suppose that in some slot n, there exists link j such that for some ϵ_j and for all Ω_j there exists P^{MAX} such that $\mathbf{p}_j^n/\mathbf{p}_i^n > \epsilon_j$. Again, we consider a new power allocation where $\mathbf{p}_l^{\prime n} = P^{MAX}$ and all the other powers are zero. We have the following

$$\begin{aligned}\frac{\mathbf{SNR}'^n_l}{\mathbf{SNR}^n_i} &= \frac{P^{MAX} h_{ll}}{\mathbf{p}_i^n h_{ii}} \frac{N + \mathbf{p}_j^n h_{ji} + \sum_{k\neq i,j} \mathbf{p}_k^n h_{ki}}{N}\\ &> \frac{P^{MAX}\mathbf{p}_j^n}{\mathbf{p}_i^n} K > P^{MAX}\epsilon_j K.\end{aligned}$$

Here K and ϵ_j are fixed constants and for an arbitrary Ω_j there exists $P^{MAX} > \Omega_j$ that satisfies the above inequality. This in turn means that we can make $\mathbf{SNR}'^n_l$ arbitrary larger than $\mathbf{SNR}^n_i$. The same applies for $\mathbf{SNR}^n_j$. We can do similarly for a link $k \neq i, k \neq j$ by virtue of (15). As we have shown above, if l is a link between a source and a destination of a flow, the new allocation increases total rate which contradicts with the initial assumption. □

Intuitively, Lemma 1 shows that in the optimal power allocation for very large power constraints, there should be only one link with a very large power active at a time. All the other links should be allocated very small powers. We proceed to the proof of the proposition.

Proof of Proposition 4. Consider a link i. From (2)-(12), we have the following inequality $\sum_{p\ni i} y_p \leq \sum_n \alpha_n \mathbf{x}_i^n$. By Lemma 1, we know that in the optimal power allocation, in each slot there is exactly one link whose power is $O(P^{MAX})$ and all other links have powers $o(P^{MAX})$. Therefore, we can assign all time to the power allocation achieving the highest rate $\sum_{p\ni i} y_p \leq \left(\sum_n \alpha_n\right) \max_n \mathbf{x}_i^n$. We might assume equality, since we otherwise can assign all extra time to other power allocations. Also, we can divide the new slot into subslots, each serving only one path, hence we can write $y_p = \alpha_{n(i,p)} \mathbf{x}_i^{n(i,p)}$.

Suppose we have an additional time $\Delta\alpha$ to serve path y_p. We need to spread it on all links belonging to path y_p such that each link i gets $\alpha_{n(i,p)}/\sum_{j\in p} \alpha_{n(j,p)}$ fraction of it, and the overall increase in rate of p is

$$\Delta\alpha\left(\alpha_{n(i,p)}\mathbf{x}_i^n / \sum_{j\in p} \alpha_{n(j,p)}\right).$$

Now, since the total capacity is a sum of the rates on all paths, in order to maximize total capacity, we will assign time only to links of those paths that have the highest increase factor, and will not serve the other paths letting them have zero rate.

The same happens in the case of transport rates, since the increase factor is the same as above, multiplied by a length of the corresponding flow. Consequently, the corresponding rates will also tend to zero. □

Acknowledgments

The work presented in this paper was supported (in part) by the National Competence Center in Research on Mobile Information and Communication Systems (NCCR-MICS), a center supported by the Swiss National Science Foundation under grant number 5005-67322.

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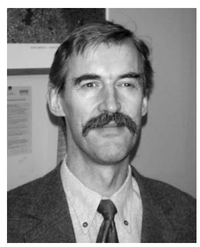

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