Fatigue life prediction in viscoelastic materials

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Abstract: *This paper introduces a new fatigue life prediction methodology for viscoelastic materials in the tension-tension fatigue loading region. The model was established based on the total amount of energy dissipated during fatigue loading, and offers two main advantages with respect to existing models in the literature, i.e. it considers the creep effect on fatigue behavior and requires less input data. The model was applied to three different materials, an angle ply- and a cross ply glass/epoxy fiber-reinforced polymer composite as well as an epoxy adhesive. It was observed that the model predicted the fatigue life of the studied materials well at different stress ratios including those close to 1.0 where the creep effect was considerable. The model was used to plot constant life diagrams (CLDs), as well as to simulate cyclic-creep interaction and determine the cyclic- and creep-dominated regions in CLDs.*

Keywords: Fatigue; creep; energy dissipation; life prediction; cyclic-creep interaction;

1. Introduction

It is widely accepted that fatigue is one of the most common failure mechanisms in structural components, and pure static failure is rarely observed [1]. Among the different types of structural materials, viscoelastic materials such as polymers and polymer based-composites have attracted great attention on the part of researchers and designers because of their high stiffness-to-weight ratio, high corrosion resistance, and low thermal expansion coefficient. These materials are subjected to different cyclic loading patterns during their service life, including different loading levels and loading amplitudes, while their fatigue behaviors are highly dependent on the type of applied loading pattern [2]. The sensitivity of viscoelastic materials to the type of cyclic loading pattern is due to the fact that they possess two different groups of mechanical properties; namely, cyclic- and time-dependent mechanical properties [3,4,5,6,7]. It is well documented that the cyclic- and time-dependent phenomena can interact during cyclic loading, especially in polymers and matrix-dominated composites, even at room temperature, and the extent of their interactions depends on the material type, loading conditions, and environmental situations [4,5,6].

There are two approaches to model the fatigue behavior of viscoelastic materials. In the first case, which covers the majority of the existing models, they were formulated in a way that only the cyclic-dependent mechanical properties were taken into account. A main drawback of these models is that the effect of the time-dependent mechanical properties on fatigue life was ignored, and pure creep was thought of as fatigue with the stress ratio ($R = \sigma_{min} / \sigma_{max}$) of 1.0, which led to inaccurate fatigue life predictions as the stress ratio increased towards 1.0 [8]. In the second approach, both cyclic- and time-dependent mechanical proprieties were considered in order to enhance the accuracy of the fatigue life prediction. Miyano et al. [3] introduced a model to predict the fatigue life at different stress ratios. To implement this model, two sets of *S-t* (stress-time) curves were required, one at the stress ratio of 1.0, obtained from a pure creep experiment, and another from a set of fatigue experiments at a stress ratio of 0.0. Reifsnider et al. in 2000 [9] suggested a new formulation based on the concept of the strength evolution integral (SEI) to calculate the remaining strength and life of composite materials and structures under mechanical, thermal, and environmental conditions that produce combinations of fatigue, creep, and stress rupture. Seven years later, Guedes [4] suggested a solution for SEI to predict the fatigue life for an arbitrary stress ratio. To make this model operational, the same two *S-t* curves as in Miyano's model are required as input data. In addition to *S-N* or *S-t* curves, a material's fatigue life can also be represented using constant life diagrams (CLDs) [8]. CLDs show the concurrent effect of mean stress (σ_m) and stress amplitude (σ_a) on the fatigue life of the examined material at different stress ratios. Existing CLD models offer a predictive tool for the estimation of the fatigue life of a material under loading patterns for which no experimental data exist; however, they have all been formulated based on only the cyclic-dependent mechanical properties [8].

According to the above review, the time-dependent mechanical properties should be considered in fatigue prediction methodologies for viscoelastic materials to enhance the prediction accuracy of the fatigue life at different stress ratios, especially those close to 1.0. In addition, it is well documented that many viscoelastic materials do not follow a linear cumulative damage law when cyclically loaded. Furthermore, due to the nonlinear nature of the interactions between time- and cyclic-dependent mechanical properties, fatigue behavior is nonlinearly dependent on the stress ratio [4]. Another unsolved aspect in the literature is the lack of knowledge regarding how much time- and cyclic-dependent mechanical properties participate in the overall fatigue behavior of the viscoelastic material, which is an important metric for a more application-dependent and reliable design in engineering structures. Therefore, a new method is needed to model the cyclic-creep interaction more accurately and consequently determine the creep- and cyclic-dominated stress levels and stress ratios.

The objective of this study is to present a novel model to predict the fatigue life of viscoelastic materials at different stress levels and stress ratios by introducing a new methodology based on the concept of the total dissipated energy (TDE). This model considers both cyclic- and time-dependent mechanical properties to predict *S-t* curves at different stress ratios including those close to 1, as well as pure creep. Unlike Miyano's model and the SEI model in which two sets of *S-t* curves were needed as input data, the proposed model requires only one *S-t* (or *S-N*) curve to predict the fatigue life at different stress ratios. The model is applied to different types of structural viscoelastic materials; namely, angle-ply composite, cross-ply composite, and epoxy adhesive. For each material, reference experimental results at the stress ratio of 0.1 were used as input data to predict fatigue behavior at two other stress ratios, i.e. 0.5 and 0.9 or 1.0. The accuracy of the predictions was evaluated by comparing them with experimental results obtained at the same stress ratios. The developed model is subsequently extended to simulate the cyclic-creep interaction at different stress ratios. Ultimately, the model serves to establish a new CLD formulation in which the creep- and cyclic-dominated regions are also determined.

2. Fatigue life prediction methodology

When cyclic loading is applied to a viscoelastic material, hysteresis loops are formed in the strain-stress coordinate system, see Figure 1a. The hysteresis area is a measure of the total dissipated energy per cycle (w_{diss}). During a load-controlled fatigue experiment, the hysteresis loops can shift under fatigue mean stress (σ_m), even at room temperature, indicating the presence of creep, and the evolution of the average strain per cycle (ε_m) can be monitored to describe this creep behavior [6].

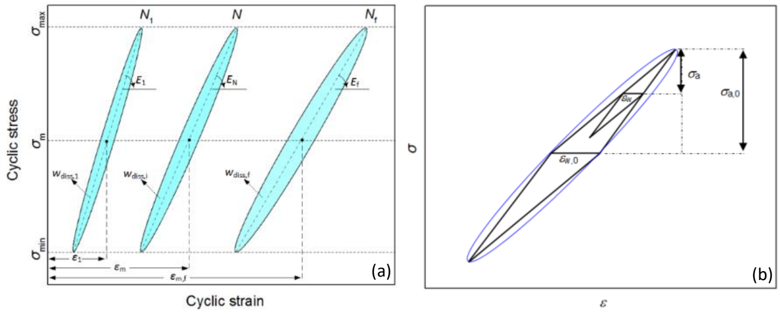

Figure 1. *(a) Schematic representation of hysteresis loops during tension-tension fatigue experiment, (b) Representation of hysteresis loop area and corresponding approximation of area by two triangles.*

The proposed prediction methodology was established based on the amount of the specimen's total dissipated energy (TDE), *W,* during its life under loading at the different stress levels and stress ratios. In fatigue experiments at the stress ratio of -1.0, energy dissipation was attributed to only the cyclic loading and therefore W was set equal to W_{cyclic}. At the stress ratio of 1.0, the origin of the energy dissipation was attributed to pure creep and TDE was set equal to W_{creep}. At other stress ratios, W was set equal to the algebraic sum of W_{cyclic} and W_{creep}, as follows:

$$W = W_{\text{cyclic}} + W_{\text{creep}} \tag{1}$$

W_{cyclic} was defined by the summation of all the individual stress strain hysteresis areas measured throughout the lifetime of the specimen [6,10].

$$W_{cyclic} = \sum_{i=1}^{N_f} w_{diss,i} \tag{2}$$

where $w_{diss,i}$ denotes the hysteresis area per cycle for the i^{th} cycle, and N_f is the number of cycles to failure. Since during a fatigue experiment, all the fatigue cycles cannot normally be recorded, W_{cyclic} was obtained here by calculating the area under the graph of w_i versus N from the first to the last cycle according to Eq. (3).

$$W_{\text{cyclic}} = \int_1^{N_f} w_{\text{diss}}(N)dN \tag{3}$$

The total creep energy in a viscoelastic material, E_{vis}, is equal to [11]:

$$E_{\text{vis}} = \int_0^t \sigma(\tau)\frac{\partial\varepsilon(\tau)}{\partial\tau}d\tau \tag{4}$$

where τ is the characteristic time. By solving this equation for a specimen loaded under σ_0 with a creep strain of ε_f at the moment of failure, $t = t_f$, W_{creep} was calculated by multiplying σ_0 by ε_f.

$$W_{\text{creep}} = \sigma_0\, \varepsilon_{\text{f}} \tag{5}$$

Due to the inherent power-law nature of fatigue-related phenomena in relation to fatigue life, the evolution of W_{cyclic} as a function of the fatigue maximum stress, σ_{max}, can be described by Eq. (6) in which α and β were the model parameters, as follows:

$$W_{\mathrm{cyclic}} = \alpha\sigma_{\mathrm{max}}^{\beta} \quad (6)$$

Another power-law equation was employed to show the variation of W_{cyclic} with respect to the fatigue life in which η and γ are model parameters, and $t_{\mathrm{f,cyclic}} = N_f/f$, where f denotes the fatigue experiment frequency.

$$W_{\mathrm{cyclic}} = \eta t_{\mathrm{f,cyclic}}^{\gamma} \quad (7)$$

The variation of W_{creep} versus the constant applied stress and creep life was expressed by two separate power-law equations as shown in Eq. (8) and Eq. (9) respectively in which *a*, *b*, *g*, and *k* were model parameters:

$$W_{\mathrm{creep}} = a\sigma_{\mathrm{m}}^{b} \quad (8)$$

$$W_{\mathrm{creep}} = g t_{\mathrm{f,creep}}^{k} \quad (9)$$

To simulate the stress ratio effect on W_{cyclic}, a new function, $\psi_{\mathrm{cyclic}}(R)$, was introduced in Eq. (7) in which $\psi_{\mathrm{cyclic}}(R)$ was defined as the ratio of the hysteresis loop area at the stress ratio of *R* over the hysteresis loop area at a reference stress ratio (R_0).

$$W_{\mathrm{cyclic}} = \psi_{\mathrm{cyclic}}(R)\, \eta t_{\mathrm{f,cyclic}}^{\gamma} \quad (10)$$

$$\psi_{\mathrm{cyclic}}(R) = \frac{w_{\mathrm{diss,i}}(R)}{w_{\mathrm{diss,i}}(R_0)} \quad (11)$$

Hahn and R.Y. Kim [12] suggested an approximation to model the hysteresis loop area by using two triangles as shown in Figure 1b. They showed that the width of the triangle base, ε_{w}, was proportional to the stress amplitude, σ_a, and proposed the following equation to describe the fatigue hysteresis loop area:

$$w_{\mathrm{diss}} = \varepsilon_{\mathrm{w}}\, \sigma_{\mathrm{max}}(1 - R)/2 \quad (12)$$

By implementing Eq. (12) into Eq. (11) for two stress ratios of R and R_0 at constant σ_{max}:

$$\psi_{\mathrm{cyclic}}(R) = \frac{\varepsilon_{\mathrm{w}}}{\varepsilon_{\mathrm{w,0}}}\,\frac{(1-R)}{(1-R_0)} \quad (13)$$

By assuming that the cyclic stiffness would be comparable at different stress ratios and changes in cyclic stiffness would not significantly affect the hysteresis loop area, $\frac{\varepsilon_{\mathrm{w}}}{\varepsilon_{\mathrm{w,0}}}$ was calculated using Thales's theorem (Figure 3), and substituted in the previous equation, which yielded:

$$\psi_{\mathrm{cyclic}}(R) = \left[\frac{(1-R)}{(1-R_0)}\right]^2 \quad (14)$$

To simulate the effect of the stress ratio on W_{creep}, a new function of $\psi_{creep}(R)$ was introduced in Eq. (9) in which $\psi_{\mathrm{creep}}(R)$ is the ratio of W_{creep} at the stress ratio of *R* over W_{creep} at the stress ratio R_0 ($W_{creep,0}$).

$$W_{\text{creep}} = \psi_{\text{creep}}(R)\, g t_{\text{f}}^{k} \tag{15}$$

$$\psi_{\text{creep}}(R) = \frac{W_{\text{creep}}}{W_{\text{creep,0}}} = \frac{\sigma_{\text{m}}\varepsilon_{\text{m,f}}}{\sigma_{\text{m,0}}\varepsilon_{\text{m,f,0}}} \tag{16}$$

In viscoelastic materials, a power function of stress has been widely used by various authors to express the stress-strain relationship, e.g. [13]:

$$\varepsilon_{\text{f}} \propto \sigma_{\text{m}}^{n} \tag{17}$$

where *n* is a material constant. Knowing $\sigma_{\text{m}} = \sigma_{\max}(1+R)/2$, $\psi_{\text{creep}}(R)$ was introduced as:

$$\psi_{\text{creep}}(R) = \left[\frac{(1+R)}{(1+R_0)}\right]^{1+n} \tag{18}$$

For the cyclic part, by equalizing Eq. (6) and Eq. (10), as shown in Eq. (19), the relationship between $t_{\text{f,cyclic}}$ and $\sigma_{\max}$ was obtained as a function of the stress ratio:

$$\alpha\sigma_{\max}^{\beta} = \psi_{\text{cyclic}}(R)\, \eta t_{\text{f,cyclic}}^{\gamma} \text{ or } t_{\text{f,cyclic}} = \left(\frac{\alpha}{\eta\, \psi_{\text{fatigue}}(R)}\right)^{1/\gamma} (\sigma_{\max})^{\beta/\gamma} \tag{19}$$

Concerning the creep part, $t_{\text{f,cyclic}}$ was determined by equalizing Eq. (8) and Eq. (15) (see Eq.(20)):

$$a\sigma_{\text{m}}^{b} = \psi_{\text{creep}}(R)\, g t_{f}^{k} \text{ or } t_{f,creep} = \left(\frac{a}{g\psi_{\text{creep}}(R)}\right)^{1/k} \sigma_{\text{m}}^{b/k} \tag{20}$$

To predict the *S-t* curve at an arbitrary stress ratio, Eq. (21) is proposed in which both $ln(t_{\text{f,cyclic}})$ and $ln(t_{\text{f,creep}})$, were taken into account via a linear interpolation:

$$ln(t_{\text{f}}) = \left(\frac{1-R}{2}\right) ln(t_{\text{f,cyclic}}) + \left(\frac{1+R}{2}\right) ln(t_{\text{f,creep}}) \tag{21}$$

where 0<R<1, and temperature as well as loading rate or frequency are constant.

3. Experimental data

The application and prediction accuracy of the suggested model were evaluated under tension-tension fatigue loading for three different materials: 1) a fully-cured angle-ply glass/epoxy fiber-reinforced polymer (FRP) composite with $[\pm 45]_{2s}$ layout (Material 1), 2) a cross-ply glass/epoxy composite with $[90,0]_{2s}$ layout (Material 2), both with a fiber content of 62% by volume, and 3) an epoxy-based resin with the commercial name SikaDur 330 (Material 3). Typical fatigue hysteresis loops of the studied materials at the selected stress levels are presented in Figure 2.

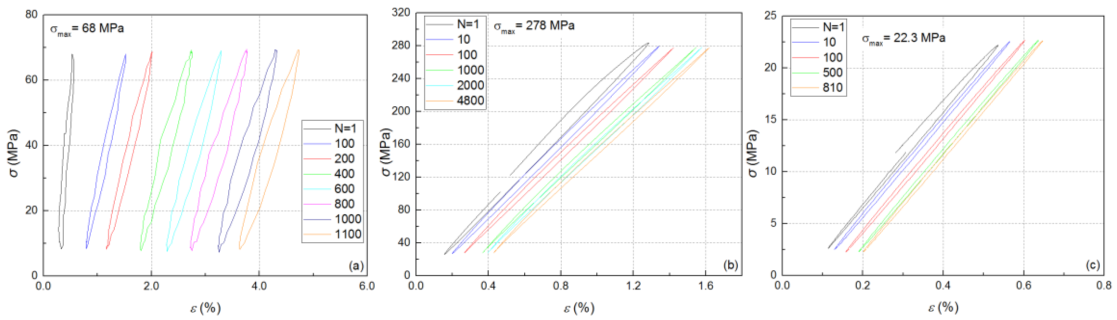

Figure 2. *Variation of hysteresis loops under cyclic loading at one selected stress level in different materials (a) Material 1, (b) Material 2, and (c) Material 3.*

4. Model application

The experimental data obtained at the stress ratio of 0.1 was used as the input data to determine the nine model parameters. Firstly, W_{cyclic} was calculated for all the experiments according to Eq. (3). Subsequently, the variation of the obtained W_{cyclic} was plotted against σ_{max} for all three materials. By fitting Eq. (6) to the experimental data points for each material, the two model parameters α, β were determined. Two other model parameters, η and γ, were estimated by fitting Eq. (7) to the experimental data points. W_{creep} was calculated at the stress ratio of 0.1 for all materials according to Eq. (5). By fitting Eq. (8) to the experimental data points, the two model parameters *a* and *b* were estimated. In order to determine the model parameters *g* and *k*, first the evolution of the W_{creep} against t_f in a double-logarithmic scale was plotted, and then Eq. (9) was fitted to the experimental data points. The last model parameter, *n*, was obtained by fitting Eq. (17) into the experimental data points. The model parameters of α, β, η, γ, *a*, *b*, *k*, *g*, and *n* obtained for Material 1 are 9.35×10^{22}, -11.48, 0.81, 0.82, 3.35×10^{-4}, 2.21, 2.22, -0.14, 1.21, respectively. Similarly, in Material 2, the model parameters are 6.88×10^{16}, -5.85, 2.06, 0.72, 1.03×10^{-2}, 1.01, 3.65, -0.11, 0.014, and finally in Material 3 they are 1.03×10^{7}, -4.72, 1.40×10^{-2}, 0.72, 3.11×10^{-4}, 1.99, 0.27, -0.22, 0.99.

5. Model validation

Figure 3 shows the *S-t* curve of the studied materials at the stress ratio of 0.1, which served as the input data, together with the predicted *S-t* curves at two selected stress ratios, i.e. 0.5 and 0.9 or 1.0. The predictions were validated by comparing them with the experimental results. Figure 3a and show that the proposed model predicted the fatigue life well at the stress ratio 0.5 for Material 1. At the stress ratio of 1.0, i.e. pure creep loading, the predicted *S-t* curve was in a similar good agreement with the experimental data points, although slightly overestimated. Figure 3b shows that the predicted *S-t* curves followed the experimental results well at both stress ratios of 0.5 and 0.9, although they were slightly overestimated at higher stress levels. In Material 3, the predicted *S-t* curves again corresponded well with the experimental data points at both stress ratios of 0.5 and 0.9 as shown in Figure 3c. It is noted that at stress ratio 0.9, runouts were observed at low stress levels, indicated with an arrow.

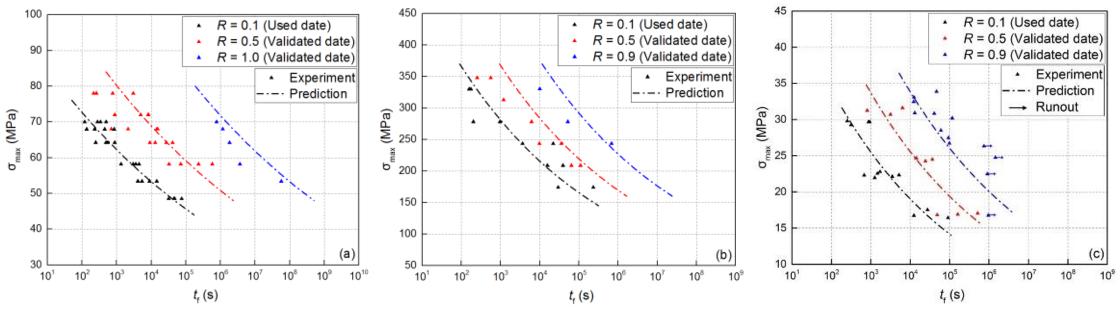

Figure 3. *S-t_f curves showing experimental results and predicted lifetime at different stress ratios in different materials (a) Material 1, (b) Material 2, and (c) Material 3.*

6. Modeling of CLD

The CLDs of the studied materials in the tension-tension sector were predicted according to Eq. (21) for the three studied materials and are shown in Figure 4. In each material, the constant life lines ranged between 10^3s and 10^6s. To implement the experimental data into the predicted

CLDs, power curve fitting was performed on all available experimental fatigue data at each stress ratio. In Material 1 (Figure 4a), at the stress ratio 0.5, there was a good agreement between the simulated constant life lines and the experiments; however, the prediction slightly underestimated the fatigue life at the low constant life line and overestimated it at the high constant life line. At the stress ratio 1.0, the predicted constant life lines slightly underestimated the fatigue life. In Material 2 (Figure 4b), at stress ratios of 0.5 and 0.9, the predicted constant life lines were in good agreement with the experiments at the high constant life line while they were underestimated at the low constant life line. The prediction results for Material 3 (Figure 4c) were in acceptable agreement with the experiments at the stress ratio of 0.5. At the stress ratio 0.9, at low constant life, the fatigue life prediction underestimated the fatigue life while at high constant life it was overestimated.

The participation of the cyclic and creep parts in the cyclic-creep interaction was set as equal to the total amount of energy dissipated by each, W_{cyclic} (Eq. (10)) and W_{creep} (Eq. (15)), respectively at different stress levels and stress ratios. Accordingly, if W_{cyclic} greater than W_{creep}, the fatigue behavior was cyclic-dominated while it was creep-dominated when W_{creep} was greater than W_{cyclic}. In this case, at the stress ratio R, W_{cyclic} might be equal to W_{creep} at $t_{f,\text{trans}}$, which was concluded that the fatigue behavior was equally governed by cyclic-dependent mechanical properties as well as time-dependent mechanical properties. In Figure 4, it can be seen that a major part of the CLDs was located in the cyclic-dominated region while the creep-dominated region was limited to the zone with higher stress ratios and lower constant life lines.

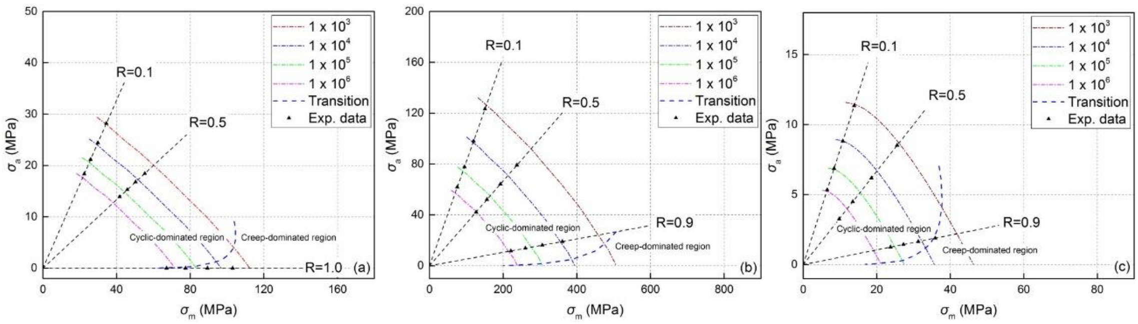

Figure 4. *Constant life diagrams (CLDs) for* $t_f = 10^3 s – 10^6 s$ *for different materials; (a) Material 1, (b) Material 2, (c) Material 3.*

7. Conclusions

In this work, a novel methodology to predict the fatigue life of viscoelastic materials in the tension-tension loading state was presented based on the total amount of energy that the materials dissipated during the cyclic loading (W_{cyclic}) and creep loading (W_{creep}). This model offers two main advantages with respect to existing models in literature, i.e. 1) the cyclic-creep interaction is considered to more accurately predict the number of cycles to failure at the different stress ratios, especially those close to 1.0, and 2) it requires less experimental effort. The experimental data obtained at the stress ratio of 0.1 was used as the input data to predict the fatigue life at other stress ratios. To evaluate the validity of the proposed model, the predicted results were compared with experimental results at the same stress ratios. It was observed that the predicted *S-t* curves were in good agreement with the experimental results. The developed model served as a new CLD formulation. The simulated constant life lines were compared with the experimental results and they were in good agreement at different stress ratios, including those close to 1.0, as well as for pure creep. A new definition of cyclic-creep interaction was proposed, in which the participation of the cyclic and creep parts in the cyclic-creep interaction was set equal to the amounts of W_{creep} and W_{cyclic}, respectively. Finally, the cyclic- and creep-dominated regions were determined, and it was observed that the creep-dominated region was limited to the zone with higher stress ratios and lower constant life lines.

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