

CREEP-FATIGUE INTERACTION DAMAGE MODEL FOR GFRP LAMINATES BASED ON THERMODYNAMICS

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Abstract: *In this study, the fatigue behavior of FRP composites is analyzed based on the thermodynamics theory. An analytical model based on the thermodynamics of the irreversible process is utilized to evaluate the long-term behavior of the FRP composite laminates. The advantage of the proposed model is that it considers both time-dependent and cycle-dependent behaviors interacting during fatigue loading, by assuming the accumulated entropy generation at failure as a metric for assessing the fatigue life. In the developed model, the accumulated entropy generated from dissipated hysteresis energy and the corresponding damage energy resulting from that is attributed to the cycle-dependent fatigue damage. Moreover, due to the viscoelastic nature of polymer matrix composites, the time-dependent behavior participates even in constant amplitude fatigue loading with non-zero mean stress. Therefore, due to the energy stored in the material resulting from viscoelastic deformation (dissipated at failure), the corresponding entropy generation is considered to quantify the damage resulting from time-dependent behavior. The results of constant amplitude fatigue tests for GFRP composite laminates are used to evaluate the model's performance and quantify creep and fatigue damage contribution to the failure depending on the stress level and the stress ratio.*

Keywords: Fatigue, Composites, Creep-fatigue interaction; Thermodynamics theory; Fatigue Failure.

1. Introduction

The degradation of the material under loading is usually interpreted as the result of irreversible thermodynamic processes that disorder the material under loading [1]. Accordingly, entropy, the fundamental thermodynamic parameter characterized by disorder, can provide a reliable measure of material degradation [2,3]. Therefore, thermodynamics-based fatigue damage models have recently been developed to analyze and predict the fatigue behavior of different materials. By applying the first and second laws of thermodynamics, entropy generation-based models can be proposed to evaluate the fatigue behavior considering all the dissipation processes involved during fatigue loading.

Naderi et al. [1-3] firstly investigated the metals' low-cycle fatigue degradation using the entropy generation approach. Entropy generation was calculated using the hysteresis loop obtained during fatigue testing. It has been shown that a constant amount of entropy is generated and accumulated up to the failure under fatigue loading, called fracture fatigue entropy (FFE), [1,2,4]. Besides, it has been proposed that FFE is independent of loading amplitude, frequency,

and testing conditions. Naderi and Khonsari extended the application of the entropy generation model to analyze the FRP composite laminates' fatigue behavior [5], showing that the accumulation of entropy generation was an efficient measure for evaluating the fatigue life of composite laminates under cyclic loading, despite neglecting damage energy in FFE calculation [5]. In another study, they improved the analysis by also considering the damage energy for calculating the FFE and achieving better agreement with experimental data [6].

The present work aims to extend the thermodynamics-based analysis to include also the time-dependent behavior of FRP composites, i.e., the creep deformation under fatigue loading, to calculate FFE. Therefore, both hysteresis energy and damage energy, calculated using the first thermodynamics law, are considered for fatigue behavior analysis. Since the FRP composite materials would undergo creep deformation even under constant amplitude cyclic loading, a new methodology is presented to consider time-dependent behavior and its contribution to the failure. Thus, the second law of thermodynamics is modified compared to the literature to include the mechanical work done and stored in the material due to viscoelastic deformation. This time-dependent deformation causes an amount of work which could be stored in the material through primary and secondary stages of creep deformation. Although this work is stored in the material during loading, it will be dissipated at the failure in the tertiary stage of creep, causing entropy generation. Therefore, the entropy generation due to creep deformation can be estimated and compared with the entropy generation resulting from fatigue damage. As a result, the contribution of fatigue damage and creep deformation to fatigue failure can be evaluated for different loading conditions. The constant amplitude fatigue tests for different stress levels and $R=0.1$ of GFRP laminates reported in [7] are used to validate the proposed analysis in this work. Besides, the limited results obtained for the same material and $R=0.5$ [8] are used in the last part for comparison.

2. Thermodynamics-based creep-fatigue model

According to the mentioned literature on this topic, the specimen's gauge section between grips under fatigue loading can be considered as a system that can be studied thermodynamically. The mechanical energy inserted into the system in each cycle under fatigue loading can be obtained by calculating the area of the hysteresis loops [6]. According to the different energy dissipation processes involved during fatigue loading, the energy balance, as presented in Eq. (1), holds as a result of applying the first law of thermodynamics. Therefore, the mechanical energy input, which is called hysteresis energy (H), can be divided into volumetric heat dissipated energy ($E_{Heat\ dissipation}$), the energy associated with thermal capacity ($E_{Thermal\ capacity}$), and the energy consumed by fatigue damage ($E_{Fatigue\ damage}$) [6]. The thermal capacity energy is responsible for energy stored in the material resulting from the self-heating temperature rise of the specimen under cyclic loading. The damage energy is associated with the energy consumed for initiation and propagation of the different damage modes resulting from fatigue loading. The heat can be dissipated from the specimen (system) to the surrounding through convection, conduction, and radiation, as presented in Eq. (2), [6].

$$H = E_{Thermal\ capacity} + E_{Heat\ dissipation} + E_{Fatigue\ damage} \quad (1)$$

$$E_{Heat\ dissipation} = E_{convection} + E_{radiation} + E_{conduction} \quad (2)$$

Based on the energy balance equation resulted, the energy consumed to initiate and propagate fatigue damage during loading can be calculated. Besides the measurements of hysteresis loops (H), the terms representing the thermal capacity and heat dissipation energies should be estimated. Using the definition of thermal capacity, Eq. (3) can be adopted, where ρ is the density, c is the specific heat, T_s is the specimen's temperature, and t represents time. According to Eq. (4), the energy due to heat dissipation can also be estimated using the basic equations in thermodynamics theory for convection, radiation, and conduction processes. In Eq. (4), h is the convection heat transfer coefficient, T_s and T_a are specimen and ambient temperatures, e represents the surface emissivity, β is the Stefan–Boltzmann constant, k is the thermal conductivity coefficient of the GFRP laminate, $\frac{\Delta T}{\Delta z}$ is the temperature gradient between the ends of the specimen and the grips, V is the volume of the gage section, and finally, A_s and A_c are surface and cross-sectional area of the gage section (for conduction), respectively [6].

$$E_{Thermal\ capacity} = \rho c \frac{\partial T_s}{\partial t} \quad (3)$$

$$E_{Heat\ dissipation} = h(T_s - T_a) \frac{A_s}{V} + e\beta(T_s^4 - T_a^4) \frac{A_s}{V} + 2k \frac{\Delta T}{\Delta z} \frac{A_c}{V} \quad (4)$$

Based on thermodynamics theory, the variation of entropy of a system (dS) is the summation of two components according to Eq. (4).

$$dS = d_i S + d_e S \quad (5)$$

where $d_e S$ represents the entropy exchanging between the system and the surroundings and $d_i S$ is the entropy generation within the specimen, which must be non-negative, according to the second law of thermodynamics. Corresponding to the different energy dissipation processes mentioned, the entropy generation rate is the sum of entropy generation due to mechanical input ($S_{mechanical}$), internal variables evolutions or fatigue damage (S_{damage}), and heat dissipation (S_{heat}) as presented in Eq. (6) [6].

$$S = S_{mechanical} + S_{damage} + S_{heat} \quad (6)$$

Considering Eq. (6), the first term represents the work done on the system resulting from the inelastic deformation of the material. This term can be estimated based on hysteresis energy dissipation. However, apart from the cyclic inelastic deformation, the time-dependent deformation of GFRP composite laminates, i.e., the creep deformation under mean stress during cyclic loading, should be considered in the analysis. Therefore, the work done on the system due to this viscoelastic behavior is regarded as the contribution of the time-dependent behavior to failure. This part of the energy is stored in the material during the loading, which would be released and dissipated at the time of failure. Therefore, the entropy generation due to inelastic deformation is comprised of a component related to the hysteresis energy dissipation (H), another one attributed to the work done due to creep (W_{creep}) and other components related to the entropy generation of the internal variable evolution due to damage energy (E_{damage}), and the thermal dissipation due to heat conduction (E_{heat}) according to Eq. (3). However, many

works in the literature showed that entropy generation due to heat conduction is often negligible during a fatigue process [6], so Eq. (6) is derived for the total entropy generation.

$$S = \left(\frac{H}{T} + \frac{W_{creep}}{T} \right) + \frac{E_{damage}}{T} \quad (7)$$

The accumulation of entropy at the failure ($S_{acc.}$) is obtained by integration of Eq. (7), as derived in Eq. (8).

$$FFE = S_{acc.} = \int_0^{t_f} \left(\left(\frac{H}{T} + \frac{W_{creep}}{T} \right) + \frac{E_{damage}}{T} \right) dt \quad (8)$$

Eqs. (4) and (5) assert that the degradation and damage process in a composite laminate is a function of the temperature evolution, the damage energy, the hysteresis energy, as well as the creep energy. In the previous works, it has been shown that a large amount of hysteresis energy (50–70%) is directly responsible for creating damage in FRP composites [6]. In this work, we consider an additional term, namely the energy associated with creep deformation. In the following section, three sources of entropy generation (hysteresis energy, creep energy, and damage energy), as well as the accumulated entropy generation resulting from them, are considered to assess degradation in the GFRP composite laminate.

3. Results and discussions

As mentioned above, the constant amplitude fatigue test results reported in [7] and [8] are used for angle-ply GFRP laminates under $R=0.1$ and $R=0.5$, respectively. For the thermodynamics analysis, hysteresis areas are calculated based on strain measurements obtained by a video-extensometer. Temperature evolution recorded by the thermal camera has been used for heat dissipation analysis. Table 1 shows the thermal conductivity, surface emissivity, density, and specific heat coefficient of angle-ply GFRP laminates and also the fatigue testing ambient temperature.

Table 1: Thermal properties of angle-ply GFRP laminates and fatigue test temperature

$k_{45}(Wm^{-1}K^{-1})$	e	$\rho(kgm^{-3})$	$c(JkgK^{-1})$	$T_a(k)$
0.24	0.9	1800	1400	293

The results of dissipation analysis based on the first law of thermodynamics are obtained and presented in Figure 1 for two different stress levels and $R=0.1$. The assumptions considered in heat dissipation analysis include constant thermal properties with respect to the temperature and fatigue damage, the isothermal surface temperature of the gage section, which linearly varies from the end of the gage section to the grips (for conduction), and finally constant ambient temperature [6]. Therefore, the heat dissipation energy through convection, conduction, and radiation is estimated according to Eq. (4) and the properties reported in Table 1. Using the thermal capacity energy estimation (Eq. (3)) and the energy balance equation (Eq. (1)), the energy corresponding to the fatigue damage can be obtained by subtracting the energy associated to the thermal capacity and the heat dissipation from the hysteresis energy. The evolution of each term presented in the energy balance equation as a function of the normalized number of fatigue cycles (with respect to the fatigue life) is shown in Figure 1.

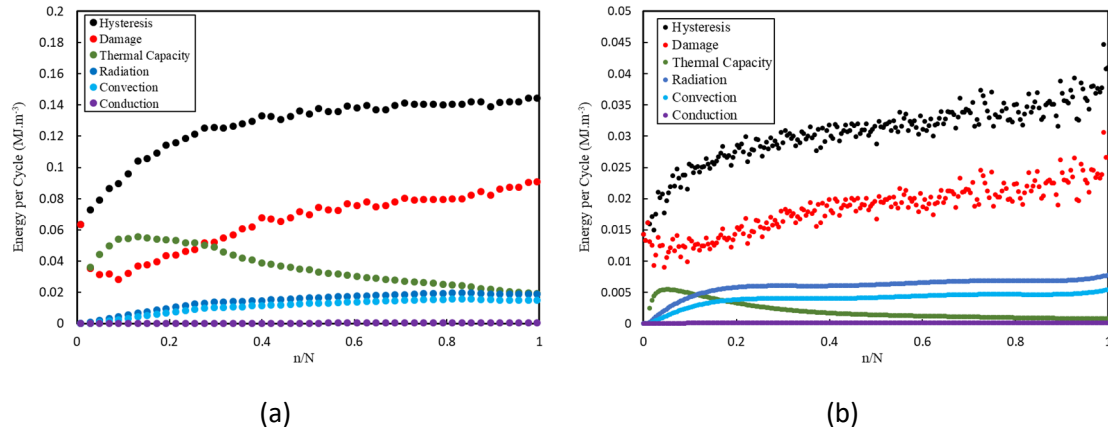


Figure 1. Energy dissipation analysis for $R=0.1$ and two maximum stress levels of (a) 70 MPa and (b) 58 MPa

According to Figure 1, radiation and convection are the dominant heat dissipation processes compared to conduction for both stress levels. The dissipation of heat is negligible in the early fatigue cycles, and most of the mechanical input (hysteresis energy) is allocated to increasing the temperature of the laminate. With increasing loading cycles, the energy associated with the thermal capacity decreases, as the material is allowed to dissipate energy through different heat processes, and finally reaches a steady state for the higher stress level, while it almost vanishes at lower stress. Conversely, during the initial cycles, the damage energy increases, resulting from the initiation of matrix micro-cracks in the laminate. As shown in Figure 1, more cyclic hysteresis energy and consequently more thermal capacity and damage energy can be observed for a higher stress level compared to the lower one.

The hysteresis energy, fatigue damage energy, and temperature profile are utilized to calculate the cyclic and accumulated entropy generation corresponding to hysteresis and damage energies based on Eqs. (7) and (8). The results of cyclic entropy generation for the same tests discussed earlier are plotted in Figure 2. The total cyclic entropy is the summation of the cyclic damage and the hysteresis entropy generation. The results show that the cyclic hysteresis entropy generation is increasing with fatigue cycles, as well as the cyclic damage entropy generation, except from a slight decrease during the early cycles due to the temperature rise under loading. The other component of the entropy generation resulting from the viscoelastic behavior is included for the estimation of the accumulated entropy generation. The mean stress evolution resulting from the creep behavior and that from the cyclic loading is used for the calculation of the creep energy storage. The resulting energy, which will be released at failure, is used to estimate the accumulated entropy generation due to creep deformation. The accumulated entropy generation due to hysteresis, fatigue damage, creep deformation, and the summation of all components is presented in Figure 4. The results show that the damage energy dissipated by different damage mechanisms resulting from fatigue loading is not negligible and can reach up to roughly 50% of the total accumulated entropy. Therefore, the total accumulated entropy can explain the long-term behavior of material more efficiently, considering all dissipation processes involved during loading and considering both creep and fatigue behaviors.

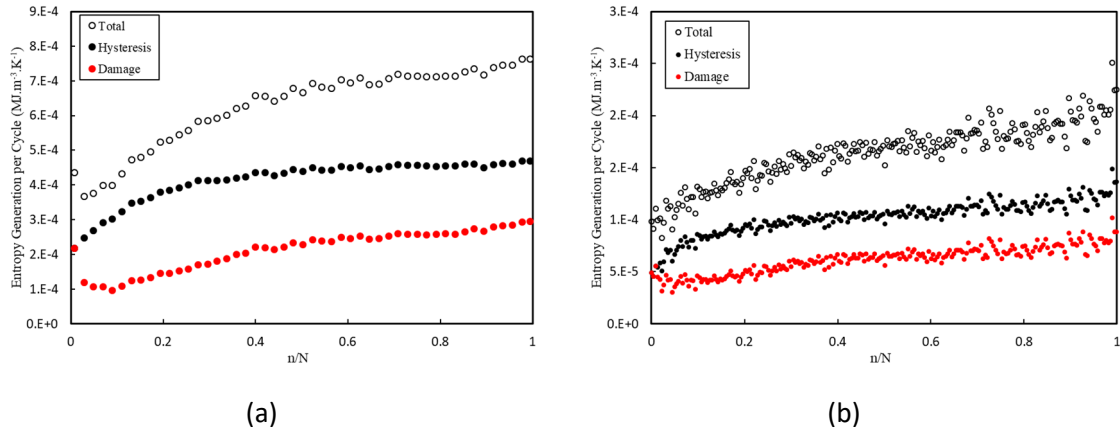


Figure 2. The cyclic entropy generation for $R=0.1$ and two maximum stress levels of (a) 70 MPa and (b) 58 MPa

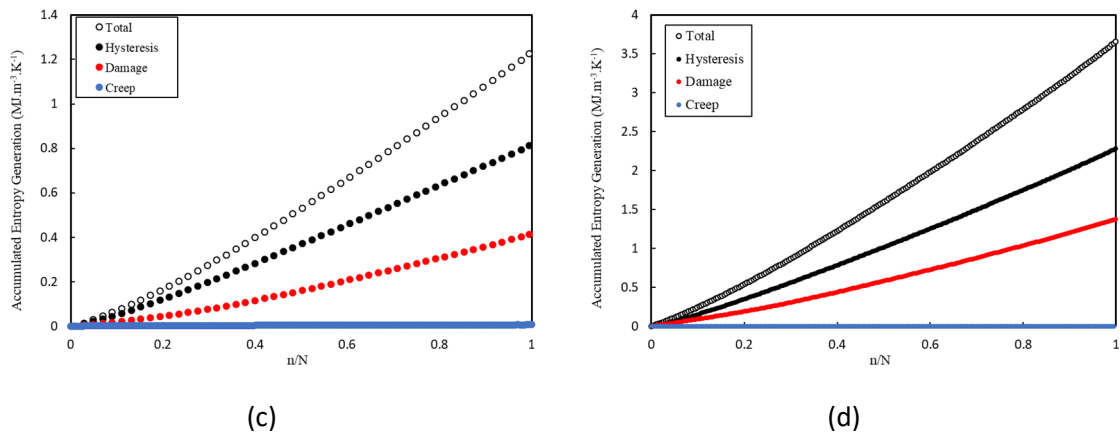


Figure 3. The accumulated entropy generation for $R=0.1$ and two maximum stress levels of (a) 70 MPa and (b) 58 MPa

As depicted in Figure 3, the entropy generation due to creep deformation is negligible compared to the fatigue damage as expected since the R -ratio is low ($R=0.1$). For further evaluation, as shown in Figure 4-(a), the entropy generation per cycle is presented for all the specimens tested under $R=0.1$ with various maximum stress levels. For higher stress levels, the entropy generation per cycle is higher, resulting from more damage and hysteresis energy. However, according to Figure 4-(b), the accumulated entropy generation for lower stress levels is much more than that generated at higher stress levels showing a more severe damage state in these specimens at failure. This behavior can be explained by considering the fact that at lower stress levels, the material under cyclic loading could tolerate more damage, more evenly distributed through the specimen. Conversely, the damage grows more rapidly for higher stress levels and would concentrate in some parts of the specimen up to failure [7]. Therefore, the conventional consideration of constant entropy generation for different stress levels is not accurate enough. The different material damage states at failure could explain this dependency of entropy generation on stress levels. Also, as depicted in Figure 4-(c), the normalized accumulated entropy generation is a linear function of the normalized fatigue life which could be simply formulated and proposed as a failure criterion for fatigue life prediction. According to Figure 4-

(d), the results obtained for all fatigue tests of $R=0.1$ are presented in comparison with the limited results provided for $R=0.5$. The evolution of the total accumulated entropy generation (black) and the contribution of creep (blue) and fatigue damage (red) are depicted with respect to the maximum stress level for both R -ratios of 0.1 (circle) and 0.5 (triangle). The accumulated entropy generation shows an exponential dependency to stress levels, indicating different damage states at the failure for different stress levels. Based on the results, the proposed entropy-based model can effectively quantify the contribution of fatigue damage and creep deformation to the final fatigue failure of GFRP composite laminates. The dependency of the creep and fatigue accumulated entropy generation on the stress level shows more contribution of creep compared to the fatigue damage for higher stress levels. Moreover, both trends show the convergence when stress level approaches the ultimate tensile stress (UTS), which could help interpreting different damage progress and failure modes resulting from different types of loading (fatigue, creep, and quasi-static loadings).

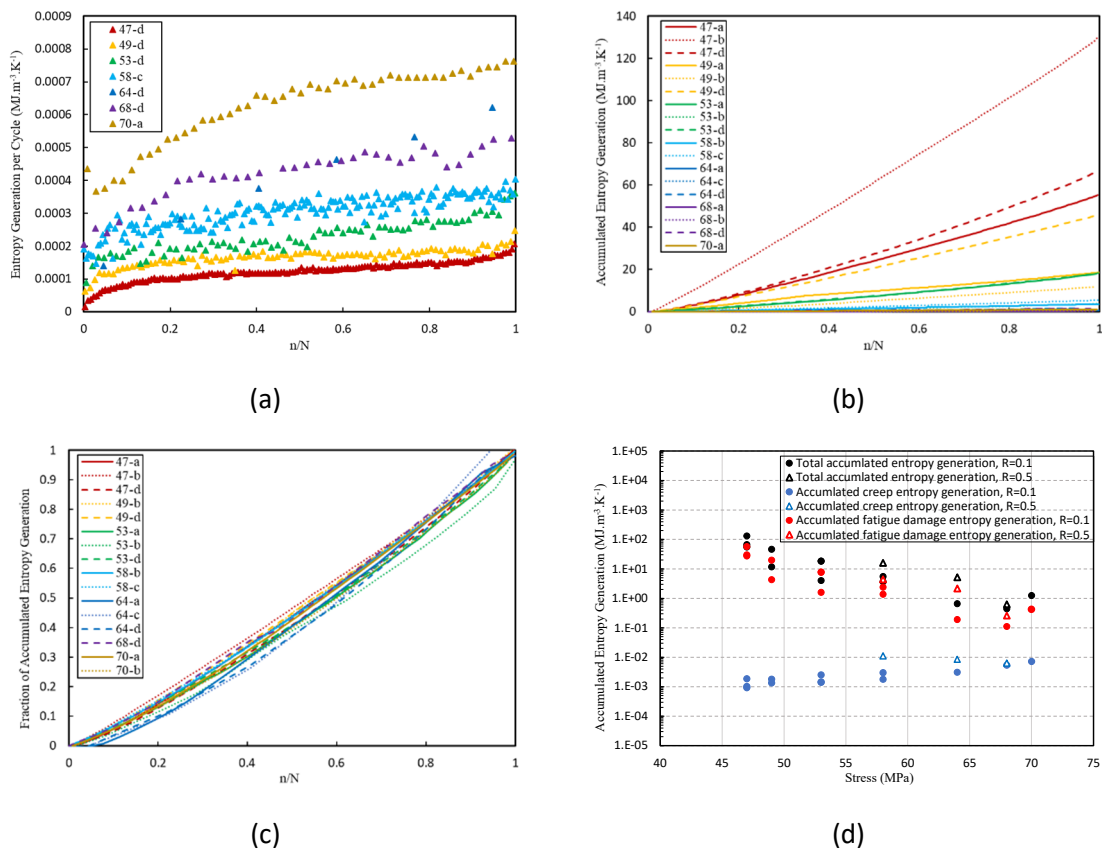


Figure 4. The result of cyclic and accumulated entropy generation for various maximum stress levels (a) Cyclic entropy generation, (b) Accumulated entropy generation, (c) Normalized accumulated entropy generation, (d) Accumulated entropy generation due to creep and fatigue damage for $R=0.1$ and 0.5 .

4. Conclusions

A thermodynamics-based damage model was proposed to evaluate the contribution of creep and fatigue damage in angle ply laminates under cyclic loading. The energy associated with

fatigue damage was obtained using the energy balance equation by applying the first law of thermodynamics. The creep contribution was evaluated by considering the mean strain evolution under mean stress and calculating the corresponding energy storage in the material. The stored energy will be dissipated at failure and causing the entropy generation due to creep behavior. The cyclic and accumulated entropy generation resulting from the different energy dissipation processes during loading was estimated as the metric for evaluating the FRP composite laminates. Therefore, the entropy generation due to hysteresis, fatigue damage, and creep energy were used to obtain the total accumulated entropy at failure. The fatigue tests for angle-ply GFRP laminates under $R=0.1$ and 0.5 were used to evaluate the performance of the model to interpret the degradation of FRP composite materials caused by creep and fatigue damage. By comparing the accumulated entropy generation results for low and high stress levels, it can be realized that contribution of creep entropy generation is much more at higher stress levels than at lower stress for both R -ratios. Therefore, entropy generation can be considered as a measure for the dependency of creep entropy on stress level for contribution to failure. Although the results show more contribution of creep damage for $R=0.5$ compared to $R=0.1$, the material's behavior and the failure were still dominated by the fatigue damage in both cases. This analysis should be applied for more CA fatigue tests under higher R -ratios to further evaluate the performance of the suggested model by considering both fatigue and creep behaviors.

Acknowledgments

The authors wish to acknowledge the support and funding of this research by the Swiss National Science Foundation (Grant No. 200020_185005).

5. References

1. Naderi M, Khonsari MM. Real-time fatigue life monitoring based on thermodynamic entropy. 2010 Jun 14;10(2):189–97.
2. Naderi M, Khonsari MM. An experimental approach to low-cycle fatigue damage based on thermodynamic entropy. Int J Solids Struct. 2010 Mar 15;47(6):875–80.
3. Naderi M, Amiri M, Khonsari MM. On the thermodynamic entropy of fatigue fracture. Proc R Soc A Math Phys Eng Sci. 2010;466114:423–38.
4. Liakat M, Khonsari MM. Rapid estimation of fatigue entropy and toughness in metals. Mater Des. 2014 Oct 1;62:149–57.
5. Naderi M, Khonsari MM. Thermodynamic analysis of fatigue failure in a composite laminate. Mech Mater. 2012 Mar 1;46:113–22.
6. Naderi M, Khonsari MM. On the role of damage energy in the fatigue degradation characterization of a composite laminate. Compos Part B Eng. 2013 Feb 1;45(1):528–37.
7. Movahedi-Rad AV, Keller T, Vassilopoulos AP. Fatigue damage in angle-ply GFRP laminates under tension-tension fatigue. Int J Fatigue. 2018 Apr 1;109:60–9.
8. Movahedi-Rad AV, Keller T, Vassilopoulos AP. Stress ratio effect on tension-tension fatigue behavior of angle-ply GFRP laminates. Int J Fatigue. 2019 Sep 1;126:103–11.