Data-driven Computational Mechanics: Implementation and Application

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Final Presentation

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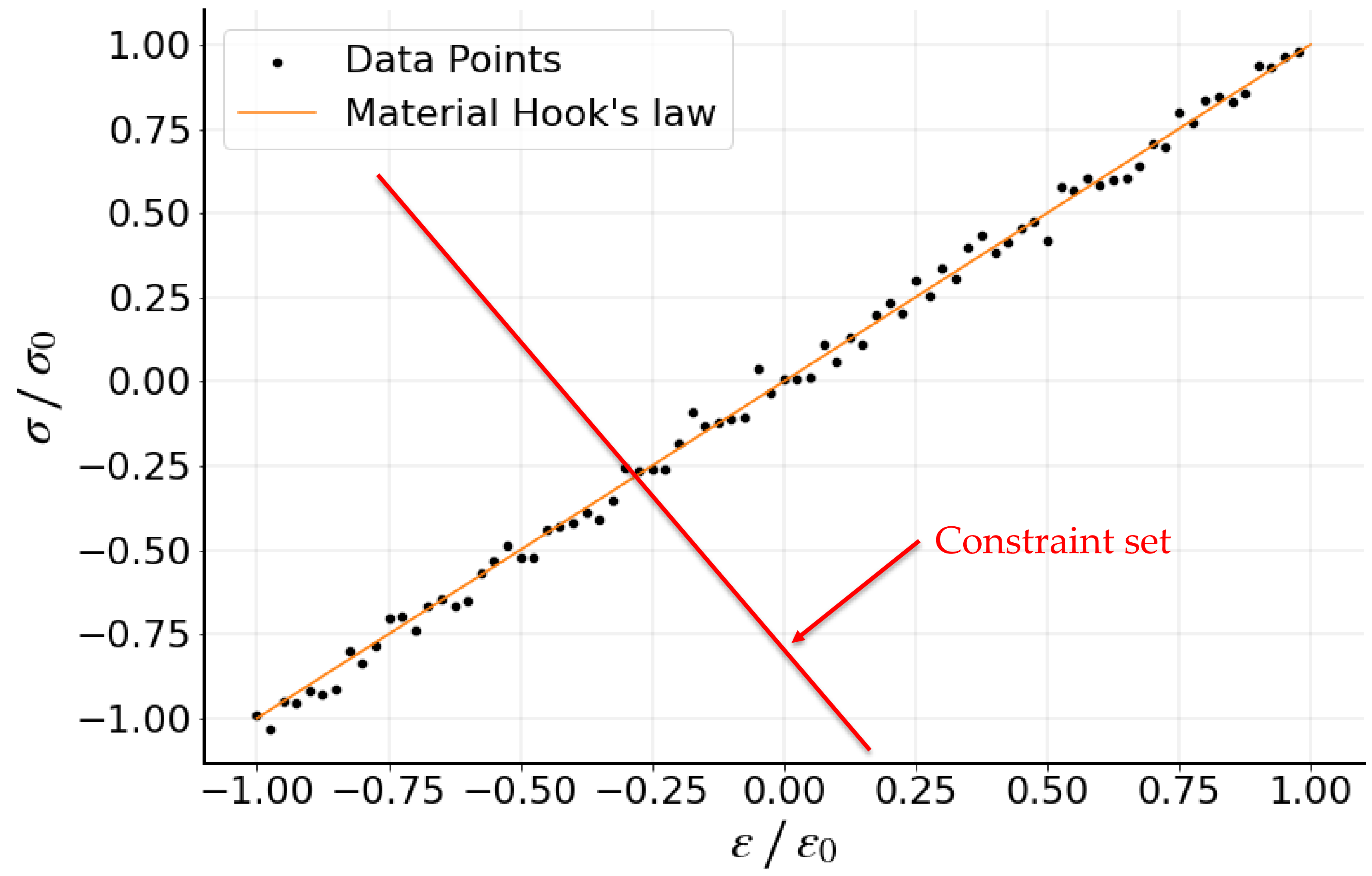


Figure 1: Example of normalised material dataset for a linear elastic truss bar

- Data-Driven Computational Mechanics (DDCM) is a new paradigm in computational mechanics that utilizes data as the primary input, rather than relying on traditional material laws.
- DDCM solvers aims at finding best possible local state, while satisfying compatibility and equilibrium.
- Exploring how DDCM can be used to model the dynamics of a system, and how it compares to traditional methods such as Finite Element Method (FEM).

II.1 Truss Structures

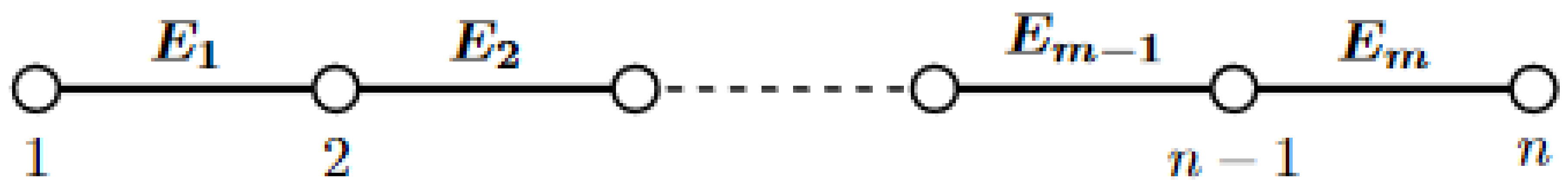


Figure 2: Example of truss structures with different datasets

Finding best possible local state i.e., Optimality of local state:

- Defined with a cost function that penalizes distance to the constraint set in phase space

The local penalty functions considered here are defined as:

$$\boldsymbol{F}_e(\epsilon_e, \sigma_e) = \min_{(\epsilon'_e, \sigma'_e) \in E_e} \boldsymbol{W}_e(\epsilon_e - \epsilon'_e) + \boldsymbol{W}^*_e(\sigma_e - \sigma'_e)$$

for each bar $e \in \{1, ..., m\}$ in the truss with,

$$\boldsymbol{W}_e(\epsilon_e) = \frac{1}{2} C_e \epsilon_e^2 \quad \text{and} \quad \boldsymbol{W}^*_e(\sigma_e) = \frac{1}{2} C_e \sigma_e^2$$

II.1 Truss Structures

Figure 2: Example of truss structures with different datasets

The constrained minimisation problem:

Minimise: $$\sum_{e=1}^{m} w_e \cdot \boldsymbol{F}_e(\epsilon_e, \sigma_e)$$

subject to: $$\epsilon_e = \sum_{i=1}^{n} B_{ei} \cdot u_i, \quad \forall \text{ bar } e \in \{1, ..., m\}$$

$$\sum_{e=1}^{m} w_e \cdot B_{ei} \cdot \sigma_e = f_i, \quad \forall \text{ node } i \in \{1, ..., n\}$$

where $\boldsymbol{u} = [u_1, ..., u_n]^T$ is the array of nodal displacements, $\boldsymbol{f} = [f_1, ..., f_n]^T$ is the array of applied nodal forces and the matrix $\boldsymbol{B}$, of size $m \times n$, encodes the connectivity and geometry of the truss.

Enforcing the constraint and solving the stationary problem by means of Lagrange multipliers, the optimal states define the main DDCM linear system:

$$\boldsymbol{K} \cdot \boldsymbol{u} = \boldsymbol{U}(\boldsymbol{\epsilon}^*)$$

$$\boldsymbol{K} \cdot \boldsymbol{\eta} = \boldsymbol{f} - \boldsymbol{H}(\boldsymbol{\sigma}^*)$$

- The stiffness matrix:

$$\boldsymbol{K} = \sum_{e=1}^{m} w_e C_e B_{ej} B_{ei} = \boldsymbol{B}^T \boldsymbol{W} \boldsymbol{C} \boldsymbol{B}$$

- The displacement constraint vector:

$$\boldsymbol{U}(\boldsymbol{\epsilon}^*) = \sum_{e=1}^{m} w_e C_e \epsilon_e^* B_{ei} = \boldsymbol{B}^T \boldsymbol{W} \boldsymbol{C} \cdot \boldsymbol{\epsilon}^*$$

- The Lagragian multiplier constraint vector:

$$\boldsymbol{H}(\boldsymbol{\sigma}^*) = \sum_{e=1}^{m} w_e B_{ei} \sigma_e^* = \boldsymbol{B}^T \boldsymbol{W} \cdot \boldsymbol{\sigma}^*$$

II.2 Convergence analysis

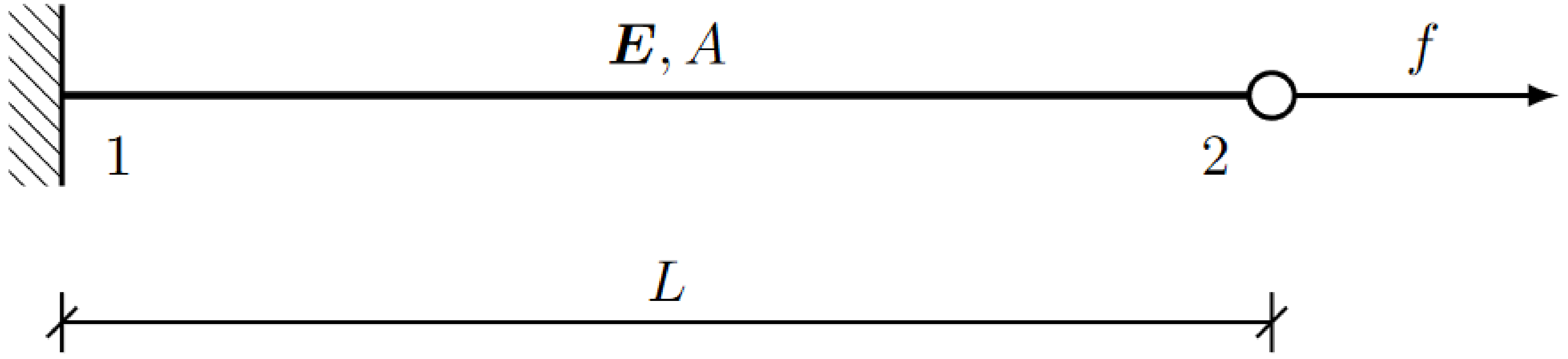


Figure 3: One dimensional bar under uniaxial load

- Can be easily solved by hand. In traditional approach, with all parameters set to 1, the displacement at the free node is 1 [mm].

- In Data-Driven Mechanics, the material properties are a set of strain-stress pairs. The datasets properties (size and noisiness) are crucial for achieving meaningful results.

II.2 Convergence analysis

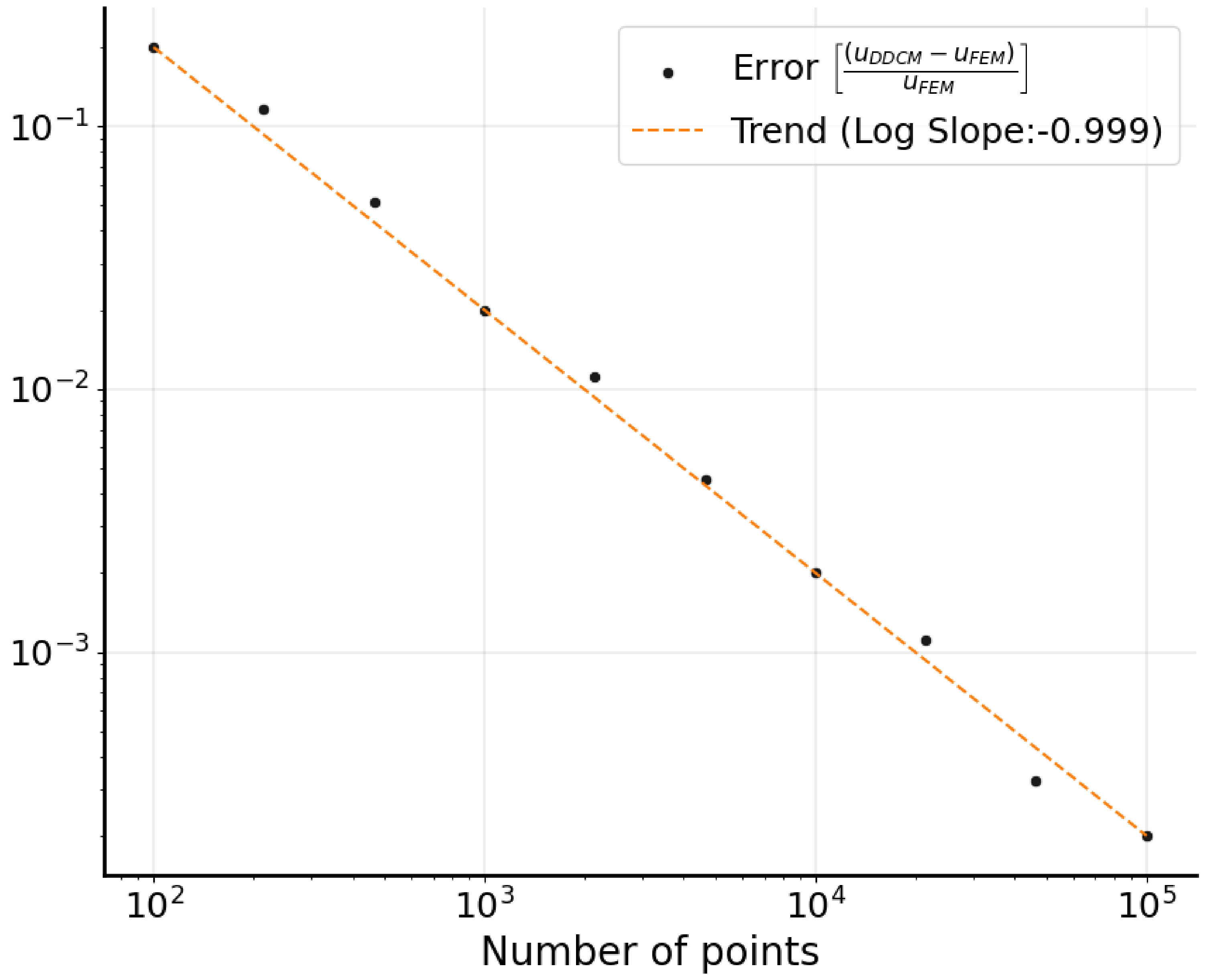


Figure 4: Effects of dataset size on the error on computed displacement

<u>Influence of the number of data points</u>

- A large number of data points increases the chances of finding the optimal state.

II.2 Convergence analysis

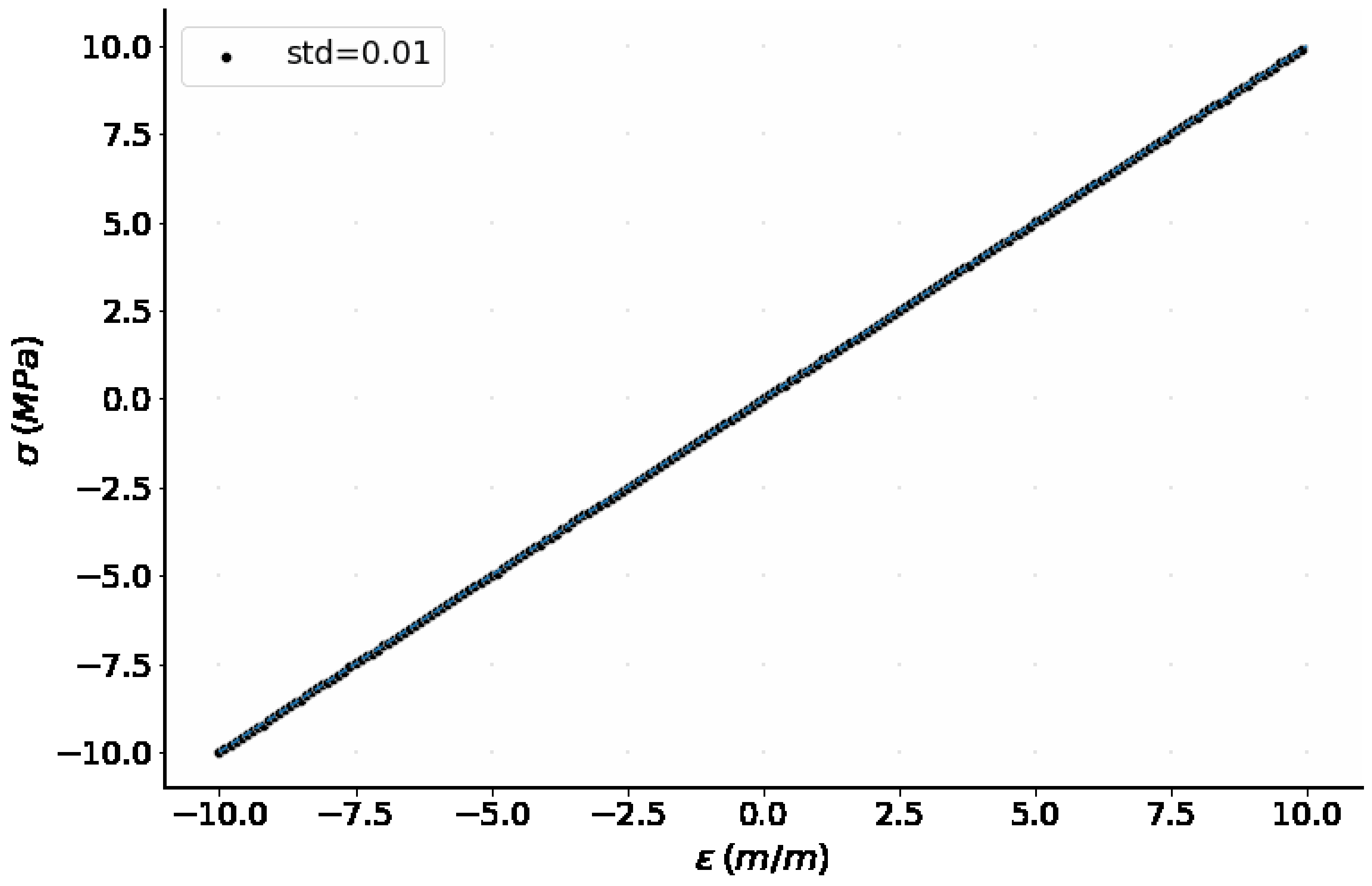


Figure 5: Effects of gaussian noise on the data points dispersion

Influence of noisiness

- Noise refers to the random dispersion in the dataset that deviates from the usual material law model.
- With more data points far from traditional law, results can stray far from the classical solution.

II.2 Convergence analysis

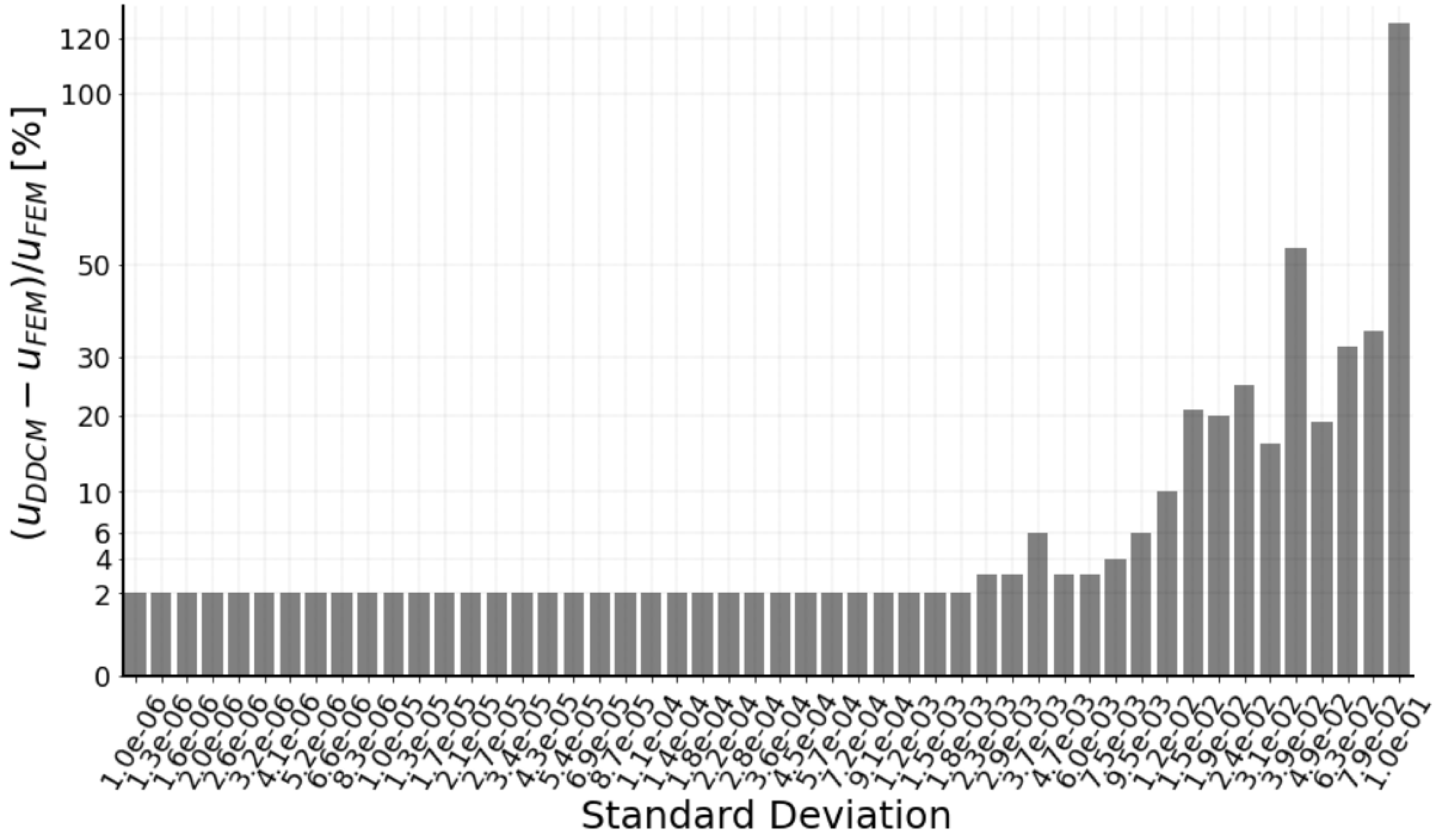


Figure 6: Effects of dataset noisiness on the error on computed displacement

III.1 Extension of data-driven solver to dynamics:

Equilibrium constraint at time t_j: $$\sum_{e=1}^{m} w_e \cdot B_{ei} \cdot \sigma_{e,j} = f_{i,j} - \boldsymbol{M} \cdot a_{i,j} \quad \forall i \in \{1, \ldots, n\}$$

Time discretisation based on the Newmark algorithm:

- the Newmark predictors

$$\boldsymbol{u}_j^{pred} = \boldsymbol{u}_{j-1} + \Delta t \cdot \boldsymbol{v}_{j-1} + \left(\frac{1}{2} - \beta\right) \Delta t^2 \cdot \boldsymbol{a}_{j-1}$$
$$\boldsymbol{v}_j^{pred} = \boldsymbol{v}_{j-1} + (1-\gamma)\Delta t \cdot \boldsymbol{a}_{j-1}$$

- the update for acceleration and velocity:

$$\boldsymbol{a}_j = \frac{1}{\beta \Delta t^2} \cdot (\boldsymbol{u}_j - \boldsymbol{u}_j^{pred})$$
$$\boldsymbol{v}_j = \boldsymbol{v}_j^{pred} + \gamma \Delta t \boldsymbol{a}_j$$

with the Newmark parameters β, γ and Δt being the time step.

III.1 Extension of data-driven solver to dynamics:

The two independent equations in static becomes coupled in dynamic and the system at time t_j becomes:

$$\begin{bmatrix} \boldsymbol{K} & -\frac{1}{\beta\Delta t^2}\boldsymbol{M} \\ \frac{1}{\beta\Delta t^2}\boldsymbol{M} & \boldsymbol{K} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{u}_j \\ \boldsymbol{\eta}_j \end{bmatrix} = \begin{bmatrix} \boldsymbol{U}(\boldsymbol{\epsilon}_j^*) \\ \boldsymbol{f}_j - \boldsymbol{H}(\boldsymbol{\sigma}_j^*) + \frac{1}{\beta\Delta t^2}\boldsymbol{M} \cdot \boldsymbol{u}_j^{pred} \end{bmatrix}$$

III.2 Comparison with traditional solvers

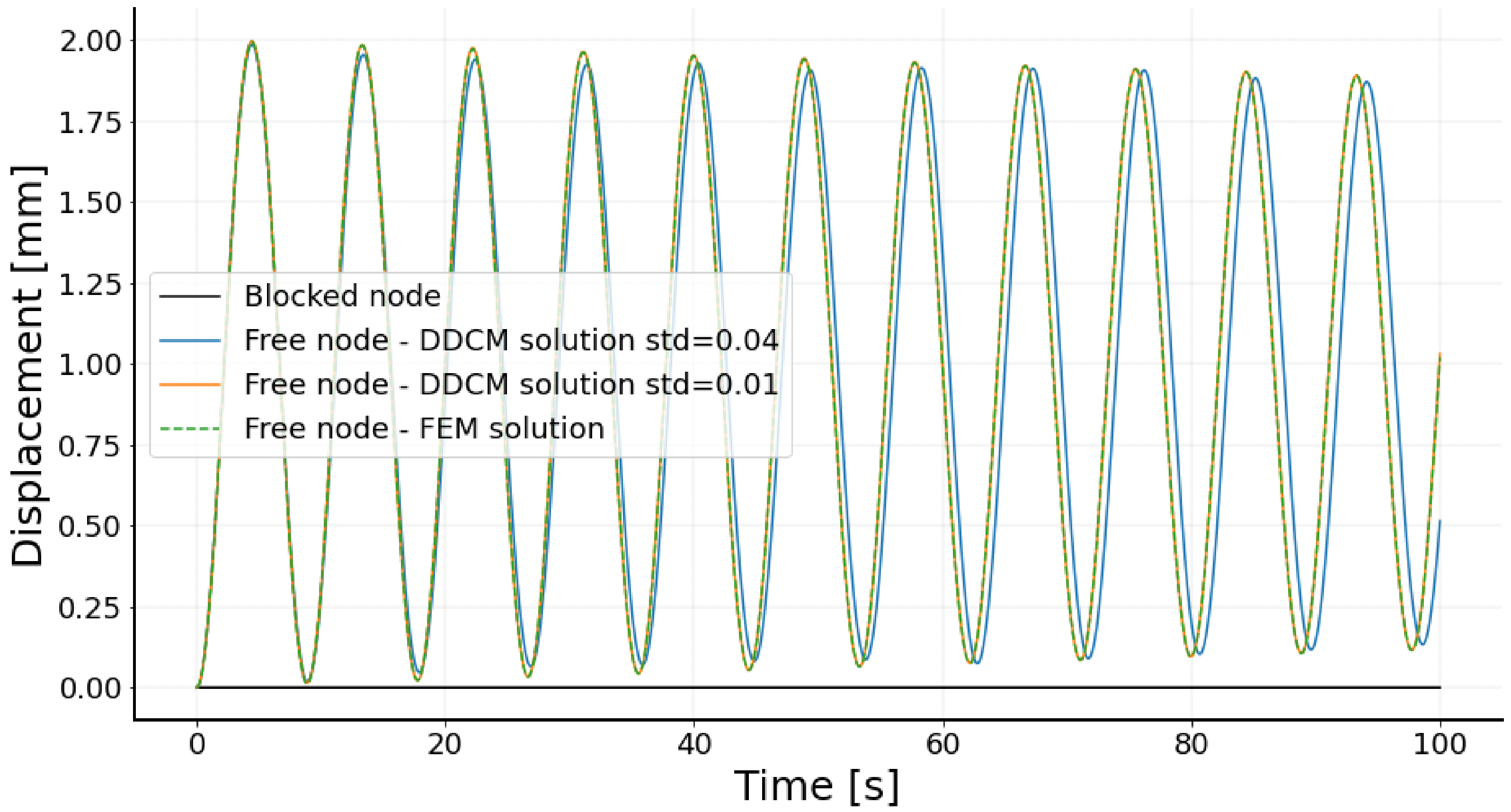


Figure 7: Evolution of displacements

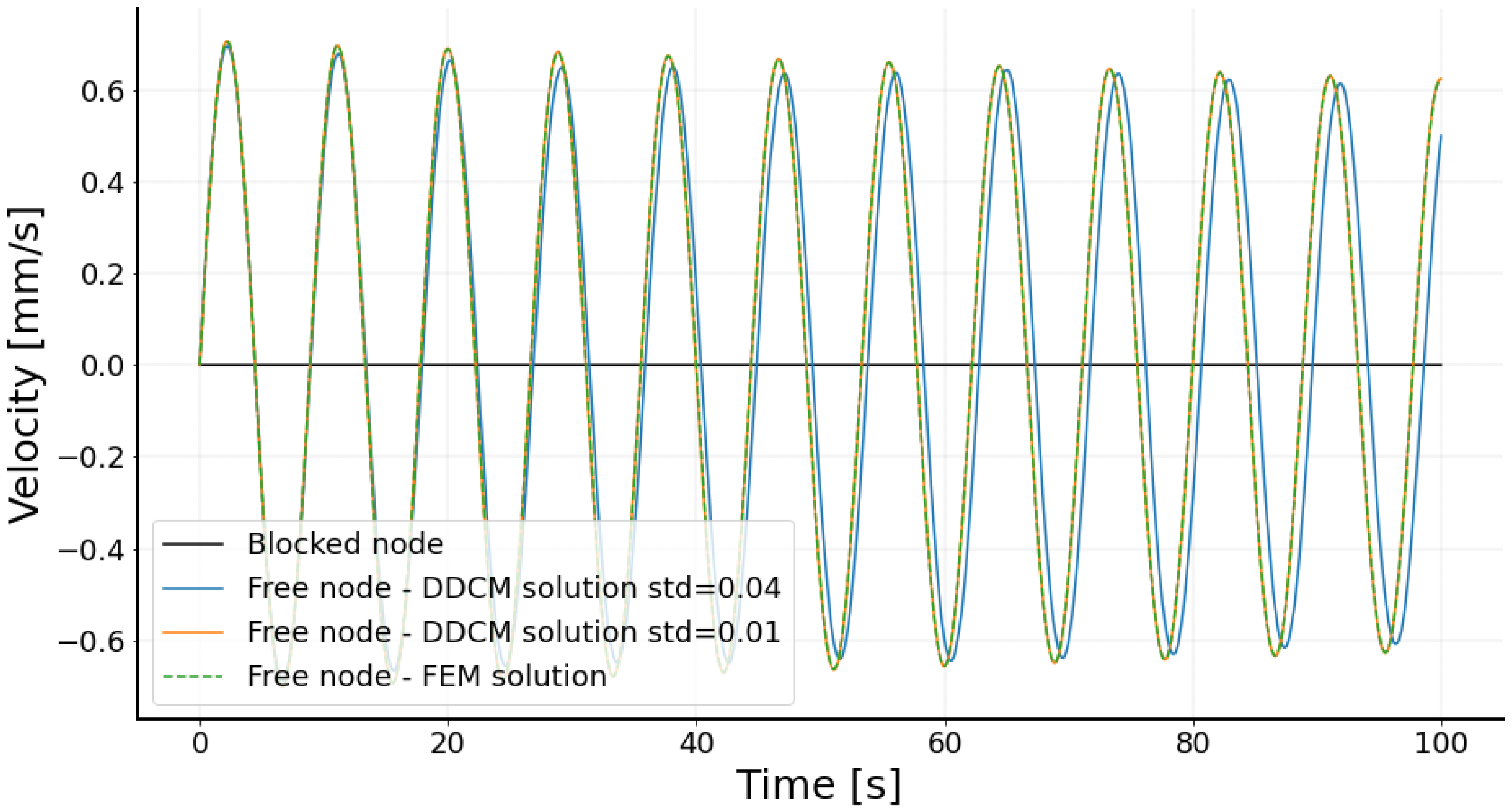
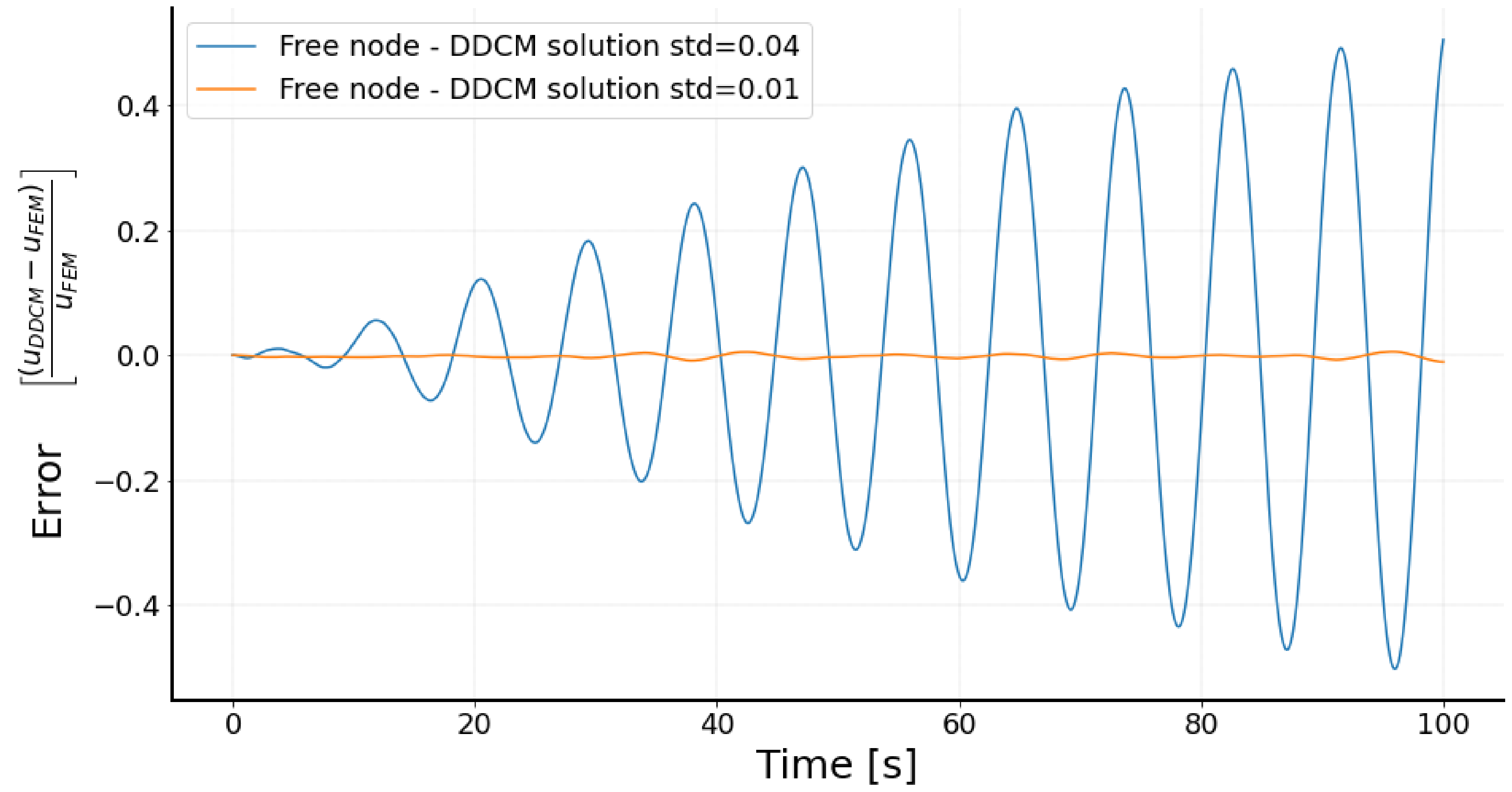


Figure 8: Evolution of velocities

III.2 Comparison with traditional solvers



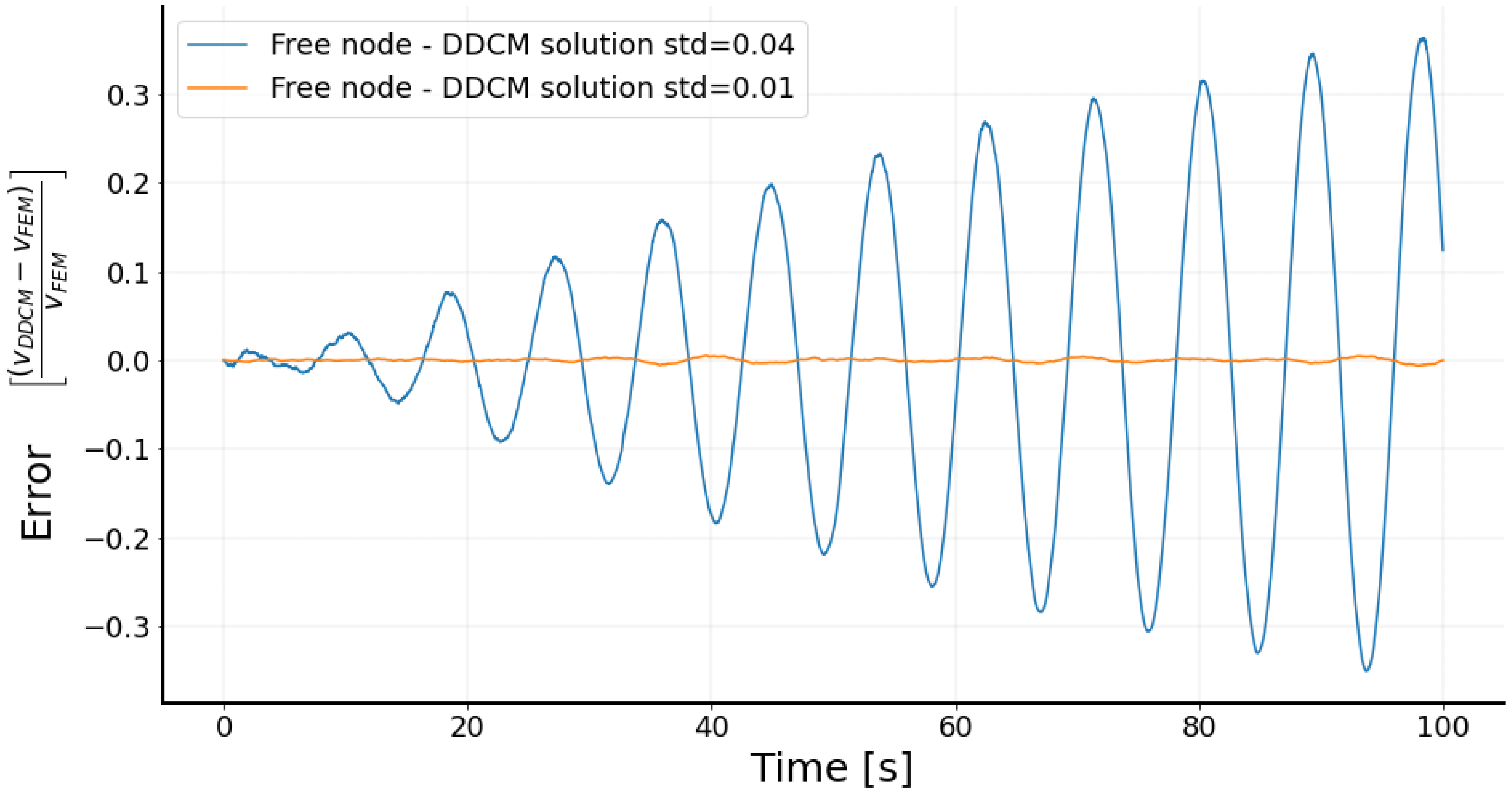


Figure 9: Evolution of errors on displacements (left) ,and velocities (right)

III.2 Comparison with traditional solvers

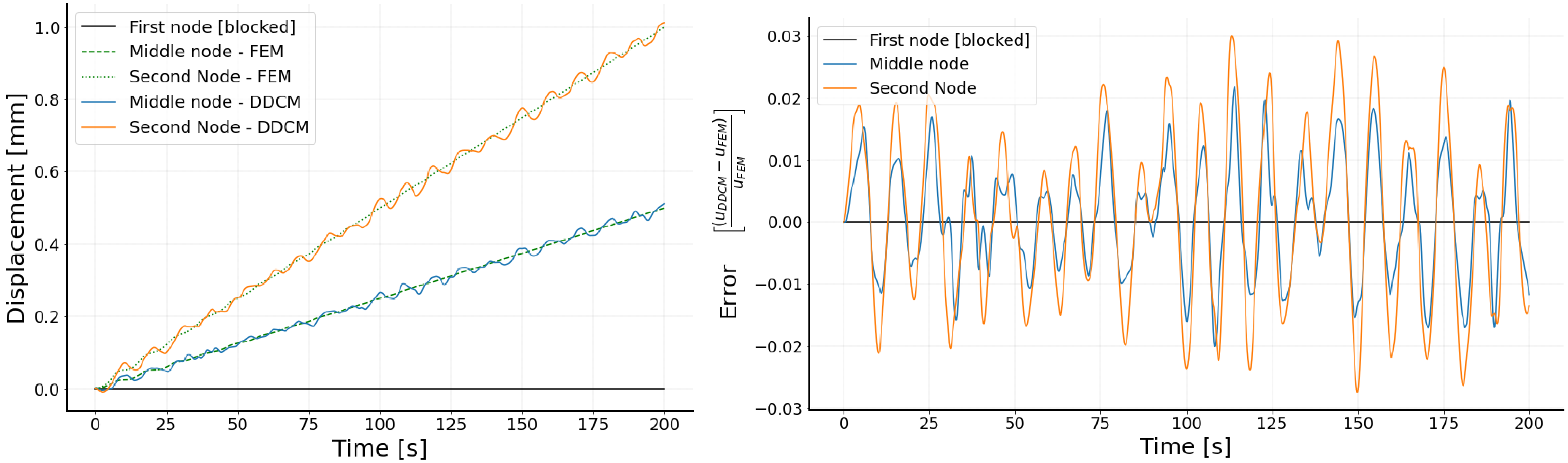
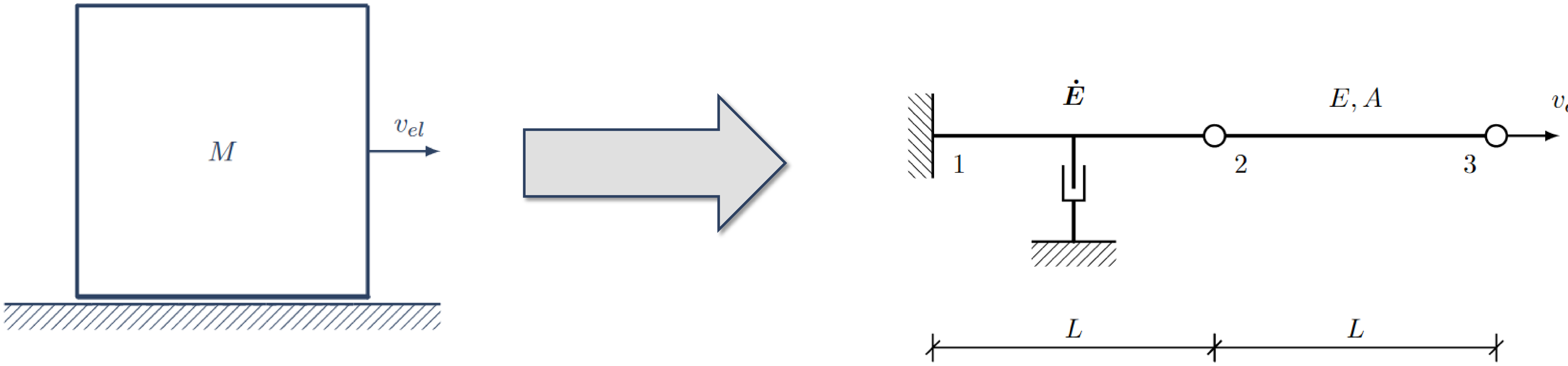


Figure 10: Evolution of displacements for a linearly increasing applied force

IV.1 Problem Description

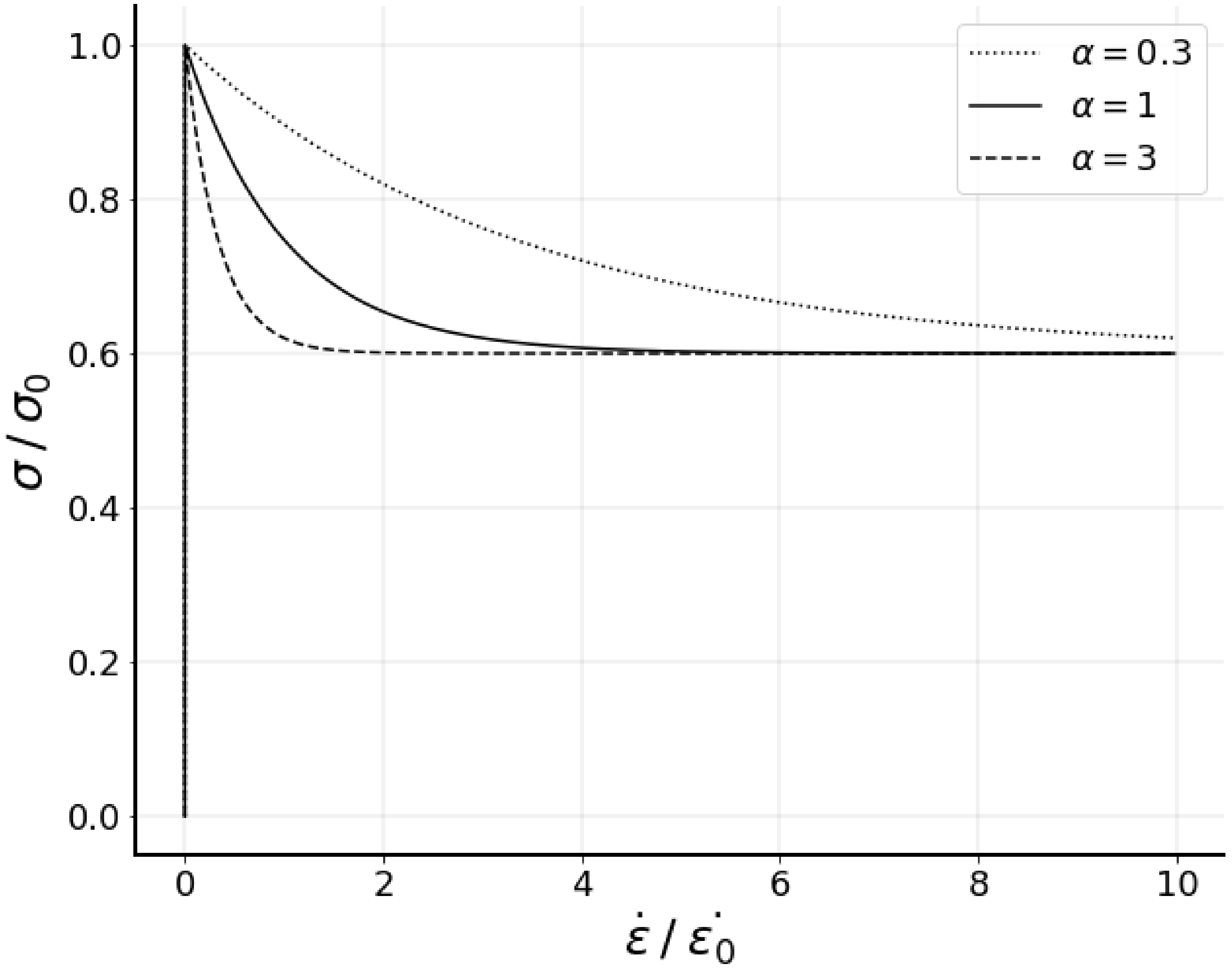
Goal: solving the problem of a sliding block on a horizontal frictional surface.

- Friction plays a crucial role in this system, as it affects the block's motion
- The problem is modeled in one dimension using truss elements to reuse DDCM pipeline



IV.2 Mathematical expression of friction law

To describe the frictional sliding behaviour, a mathematical expression is derived:



$$\sigma(\dot{\epsilon})/\sigma_0 = \begin{cases} \mu_s \cdot \dot{\epsilon}/\dot{\epsilon}_{sk} & \text{if } \dot{\epsilon} \leq \dot{\epsilon}_{sk} \\ \mu_k + (\mu_s - \mu_k)\, e^{-\alpha\cdot(\dot{\epsilon}-\dot{\epsilon}_{sk})} & \text{otherwise} \end{cases}$$

Figure 11: Example of frictional law in the stress-strain rate phase space with $\mu_s = 1$, $\mu_k = 0.6$ and $\dot{\epsilon}_{sk} \ll 1\ [s^{-1}]$

IV.3 Data-Driven Solver with damping

Goal: developing a DDCM procedure that can solve problems in the phase space associated to the frictional surface

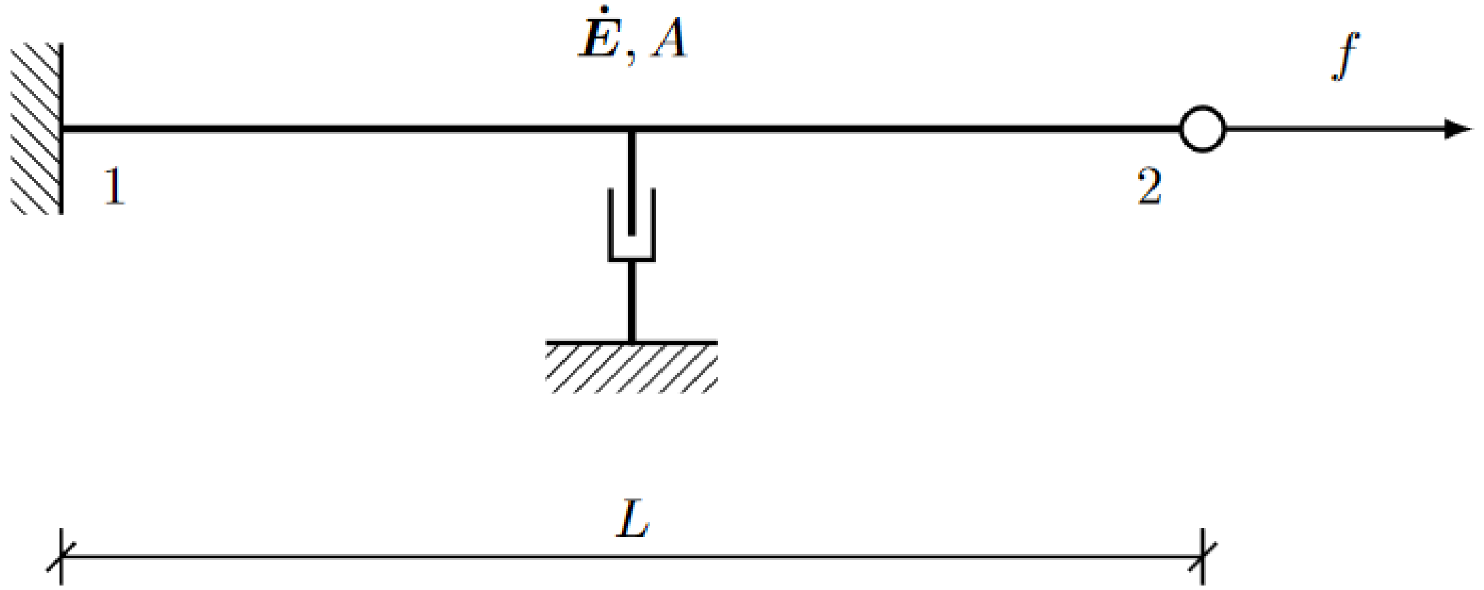


Figure 12: One dimensional bar with damping under uniaxial load

The corresponding minimisation problem:

$$\text{Minimise:} \quad \sum_{e=1}^{m} w_e \cdot \boldsymbol{F_{e,j}}(\dot{\epsilon}_{e,j}, \sigma_{e,j})$$

$$\text{subject to:} \quad \dot{\epsilon}_{e,j} = \boldsymbol{B_e v_j}, \quad \forall\, \text{bar } e \in \{1, \ldots, m\}$$

$$\sum_{e=1}^{m} w_e \boldsymbol{B_e^T} \sigma_{e,j} = \boldsymbol{f_j} - \boldsymbol{M a_j}$$

IV.3 Data-Driven Solver with damping

With the stress-strain rate phase space the DDCM main system at time t_j becomes:

$$\begin{bmatrix} \boldsymbol{K} & -\frac{1}{\beta \Delta t^2}\boldsymbol{M} \\ \frac{1}{\beta \Delta t^2}\boldsymbol{M} & \boldsymbol{K} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{u}_j \\ \boldsymbol{\eta}_j \end{bmatrix} = \begin{bmatrix} \boldsymbol{U}\left(\frac{\beta \Delta t}{\gamma}\dot{\boldsymbol{\epsilon}}_j^*\right) - \boldsymbol{K}\left(\frac{\beta \Delta t}{\gamma}\boldsymbol{v}_j^{\boldsymbol{pred}} - \boldsymbol{u}_j^{\boldsymbol{pred}}\right) \\ \boldsymbol{f}_j - \boldsymbol{H}(\boldsymbol{\sigma}_j^*) + \frac{1}{\beta \Delta t^2}\boldsymbol{M} \cdot \boldsymbol{u}_j^{\boldsymbol{pred}} \end{bmatrix}$$

IV.3 Data-Driven Solver with damping

Numerical Results: system with an initial velocity imposed v = 1 [mm/s] and a constant friction coefficient $\mu_k = 0.6$ [−]

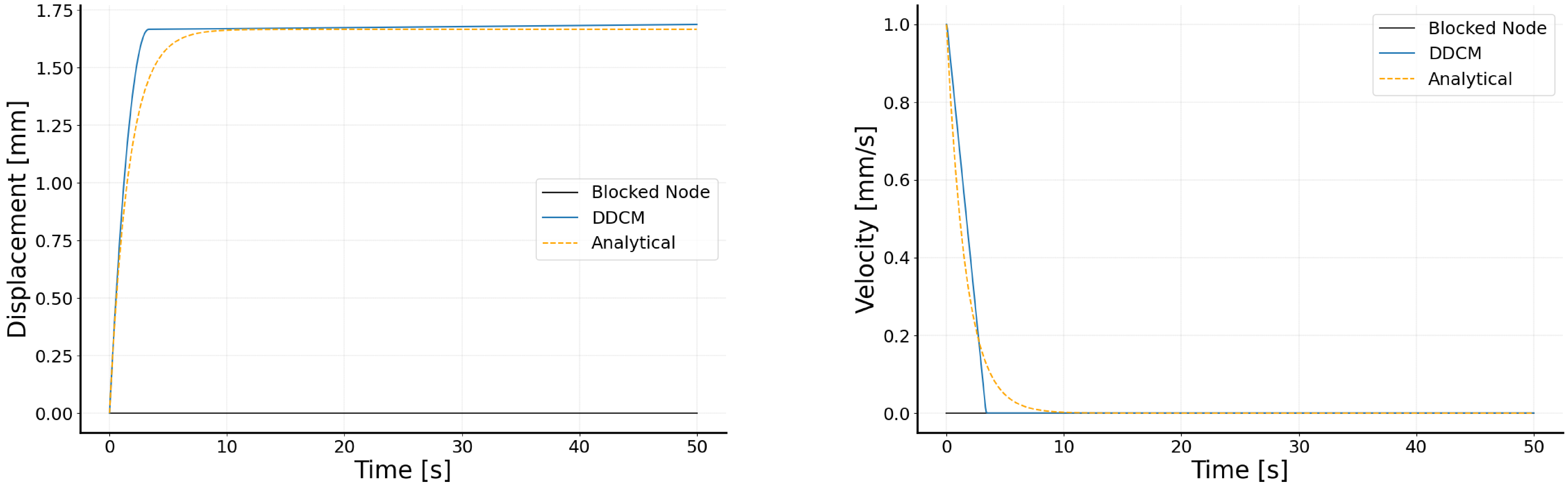


Figure 13: Results of DDCM damping solver

IV.4 Adaptation to Stick-Slip

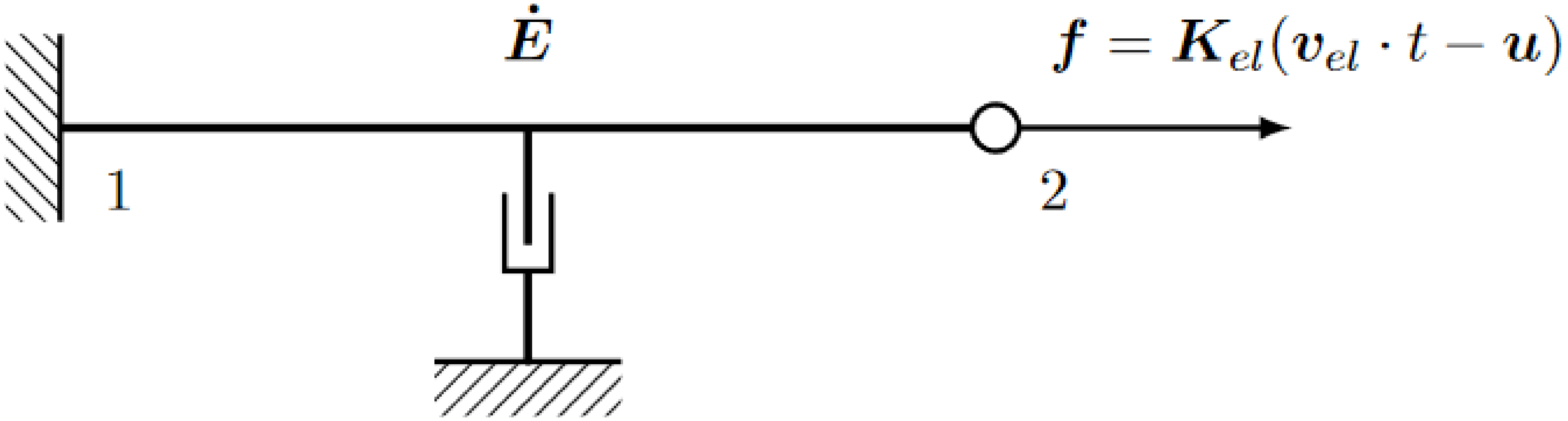


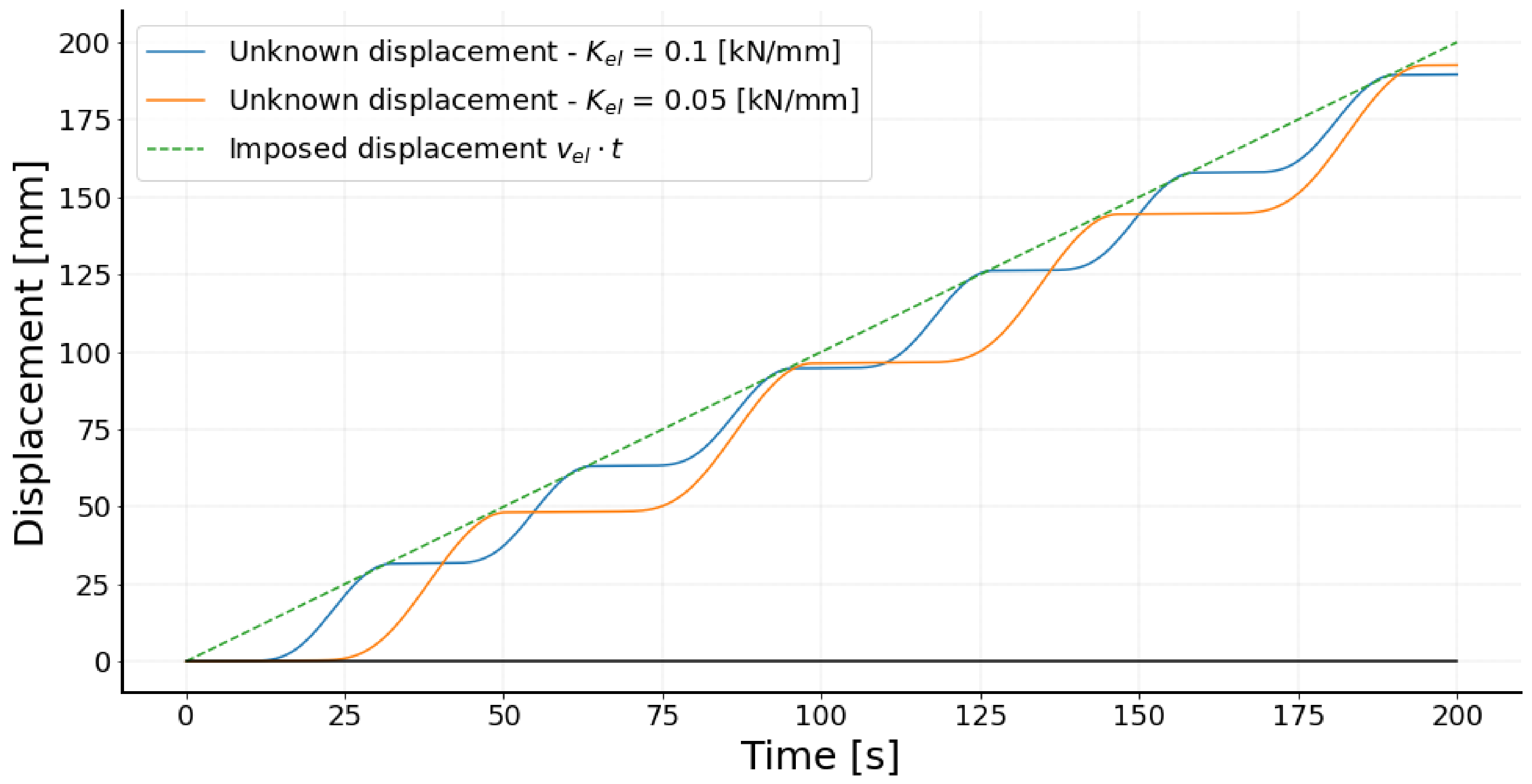
Figure 14: Modelisation of the sliding block as an equivalent force and the frictional surface as a truss bar with damping

Modification of the DDCM main system since the force now depends on the displacement

$$\begin{bmatrix} \boldsymbol{K} & -\left(\frac{\boldsymbol{M}}{\beta \Delta t^2} + \boldsymbol{K}_{el}\right) \\ \left(\frac{\boldsymbol{M}}{\beta \Delta t^2} + \boldsymbol{K}_{el}\right) & \boldsymbol{K} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{u}_j \\ \boldsymbol{\eta}_j \end{bmatrix} = \begin{bmatrix} \boldsymbol{U}\left(\frac{\beta \Delta t}{\gamma} \dot{\boldsymbol{\epsilon}}_j^*\right) - \boldsymbol{K}\left(\frac{\beta \Delta t}{\gamma} \boldsymbol{v}_j^{pred} - \boldsymbol{u}_j^{pred}\right) \\ \boldsymbol{K}_{el} \cdot \boldsymbol{v}_{el} t_j - \boldsymbol{H}(\boldsymbol{\sigma}_j^*) + \frac{1}{\beta \Delta t^2} \boldsymbol{M} \cdot \boldsymbol{u}_j^{pred} \end{bmatrix}$$

IV.4 Adaptation to Stick-Slip

- Displacement curve is stair-like, displacement increases during slipping phases and remains constant during sticking phases
- Velocity curve is sinusoidal with an amplitude of the oscillations being twice the imposed velocity value.



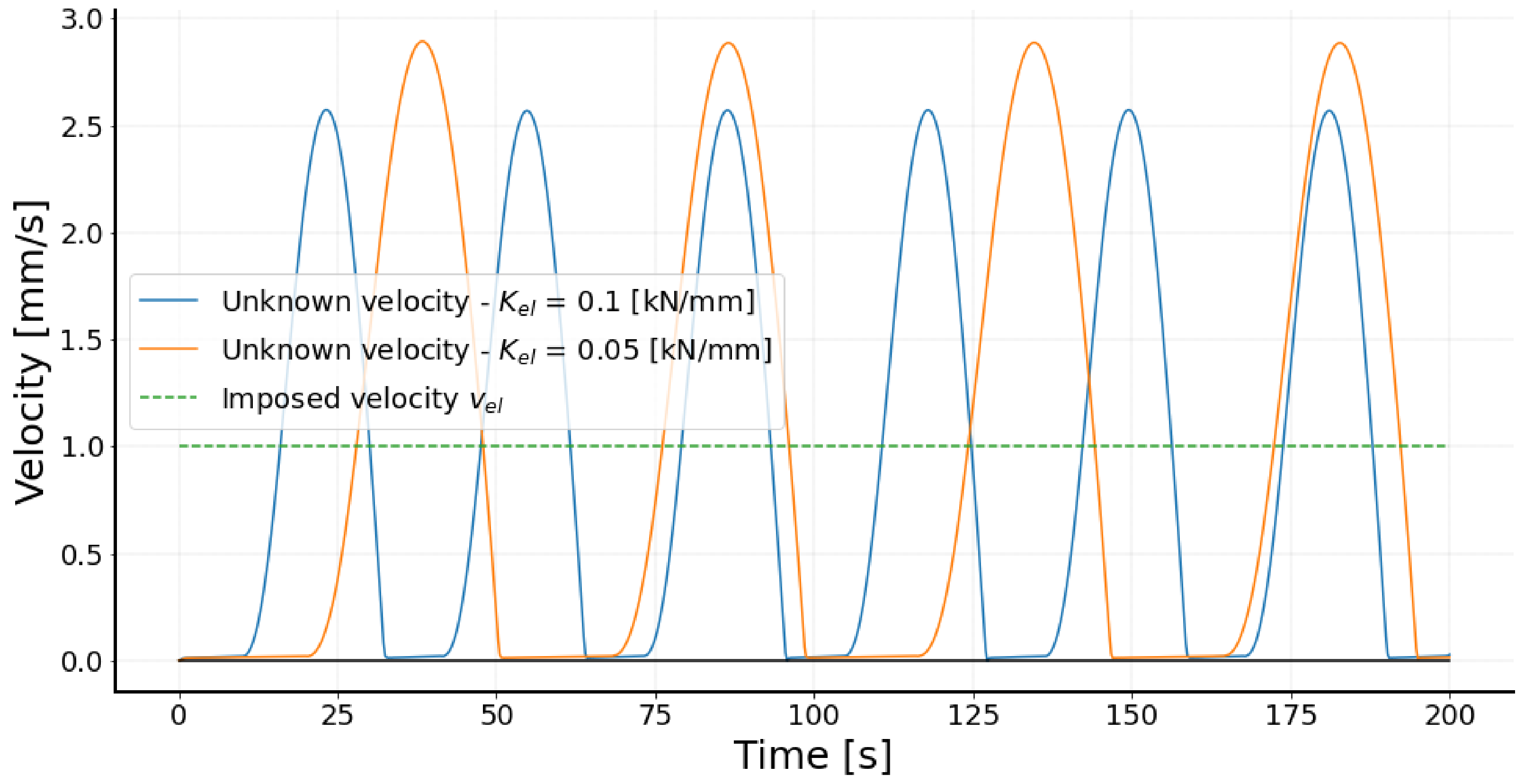


Figure 15: DDCM results for the sliding block

- Tested and compared data-driven solvers to traditional finite element method on various load and truss configurations
- Results showed accurate capture of dynamic behavior of simple trusses with increasing accuracy with more data points and decreasing noise
- Adapted basic solver to study stick-slip behavior between block and frictional surface
- Results in good agreement with expected behavior, indicating potential for modeling such systems in the future.

Thank you for listening