Symmetry for Architectural Design

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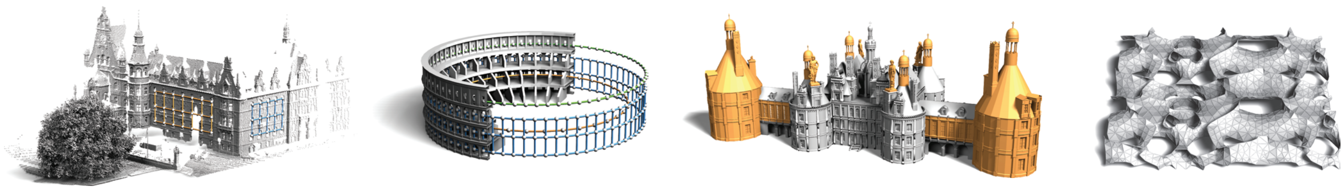

Figure 1: *Various architectural models analyzed and modified by exploiting information on shape symmetries and regular repetitive patterns.*

Abstract

Symmetry and regularity abound in architectural models, often as a result of economical, manufacturing, functional, or aesthetic considerations. We show how recent work on symmetry detection and structure discovery can be utilized to analyze architectural designs and real-world artifacts digitized using 3D scanning technology. This allows reverse engineering of procedural models that facilitate effective exploration of the underlying design space and the synthesis of new models by modifying the parameters of the extracted structures and symmetries. We demonstrate the effectiveness of such an approach on a number of example designs.

Keywords: shape analysis, symmetry, repetitive patterns, structural regularity, procedural modeling

1 Introduction

Architectural designs commonly exhibit significant symmetries or contain repetitive patterns. These types of structural regularity are not accidental, but often the result of economical, manufacturing, functional, or aesthetic considerations. Whether by evolution or design, symmetry implies certain economies and efficiencies of structure that make it universally appealing. Symmetry also plays an important role in human visual perception and aesthetics. Arguably much of the understanding of the world around us is based on the perception and recognition of shared or repeated structures, and so is our sense of beauty [Thompson 1992].

Symmetry is also fundamental in the laws of physics, hence optimality conditions in terms of statics often lead to symmetric configurations. In addition, structural regularity in architectural models allows pre-fabrication and mass-production of repetitive elements and can thus lead to significantly reduced production costs.

Recent work in 3D shape analysis has focused on detecting symmetries and regular structures in geometric models [Martinet et al. 2006], [Mitra et al. 2006], [Podolak et al. 2006], [Simari et al. 2006], [Mitra et al. 2007], [Li et al. 2008], [Pauly et al. 2008]. These research efforts offer a wealth of tools that can be employed to improve the architectural design process. In particular, explicit knowledge of symmetry and geometric regularity can be exploited to facilitate reverse-engineering of design rules for procedural modeling or symmetry-aware shape optimization. Symmetry information can also be beneficial for shape reconstruction from scanned data [Pauly et al. 2005], [Thrun and Wegbreit 2005], [Pauly et al. 2008] or images [Müller et al. 2007], [Liu et al. 2008].

In this paper, we summarize our previous work on symmetry detection [Mitra et al. 2006], symmetrization [Mitra et al. 2007], and structure discovery [Pauly et al. 2008] with special emphasis on potential applications in architectural design.

2 Symmetries and Regular Structures

Our approach is based on the techniques for finding symmetry information and repetitive structures introduced in [Mitra et al. 2006] and [Pauly et al. 2008], respectively. We briefly describe the central ideas of these methods, but refer to the papers for a more detailed discussion. Symmetry and structural repetitiveness can be formalized using the notion of invariance under transformations. We say that two parts $\mathcal{A}, \mathcal{B} \subseteq \mathcal{S}$ of a 3D model $\mathcal{S}$ are symmetric, if there exists a transformation T, e.g., a rotation, reflection, or translation, such that $\mathcal{B} = T(\mathcal{A})$. In general, we consider the space of similarity transformations composed of uniform scaling, rotation, translation, and possibly reflection. To find symmetry transformations of a given shape, we apply a sampling approach illustrated in Figure 2 that has been proposed in [Mitra et al. 2006] and, independently, in [Podolak et al. 2006].

The surface of the model is sampled uniformly with average sample spacing h. The user parameter h determines the scale of the smallest symmetric elements that we want to detect. For every sample

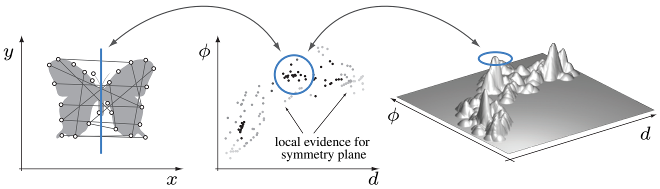

Figure 2: *To detect symmetries in geometric models, we uniformly sample the boundary of the shape (left). Every pair of samples with compatible local surface geometry provides local evidence for a symmetry transformation (center). In this example we consider reflections that are parameterized by an angle ϕ and the distance d to the origin. Accumulating such evidence using a clustering approach yields the dominant symmetries of the model (right).*

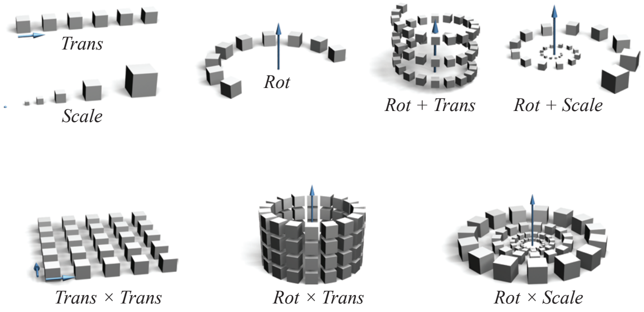

Figure 3: *Schematic illustration of regular structures. The helix and spiral are generated by transformations that combine rotation with translation and scaling, respectively. The bottom row shows the three types of commutative 2-parameter groups that can be detected with our approach.*

point we compute a local signature that compactly encodes local geometric properties at that point that are invariant under transformations of the specific transformation space under consideration. Sample points with similar signatures are paired and a canonical transformation that maps one sample to the other is computed and refined using local registration methods.

The key observation is the following: If a shape contains symmetries or repetitive structures, then the estimated transformations exhibit specific accumulation patterns when mapped to a suitable transformation space. These patterns can be extracted using clustering methods and grid fitting techniques. While the method of [Mitra et al. 2006] is mostly concerned with pairwise symmetries, the structure discovery method of [Pauly et al. 2008] in addition analyses the spatial relations among different symmetries. The underlying formulation is based on theory of transformation groups and thus allows a rigorous mathematical treatment of the concept of structural regularity. Different types of regular repetitive structures that can be detected by this method are shown in Figure 3.

The result of this analysis in transformation space is a set of symmetries and repetitive patterns that encode important medium and large scale structural information of the processed shape. Symmetries can often be represented in a hierarchy, while repetitive structures are described by a representative element, i.e., a patch $\mathcal{P} \subset \mathcal{S}$, a set $\mathcal{T}$ of generating transformations, and the number of repetitions in each dimension (see Figure 6, lower left).

3 Shape Analysis and Design

The analysis of digital 3D models using the methodology described above provides us with a compact representation of the symmetries and repetitive structures of a shape. We first show some examples and then discuss how this information can be utilized to provide effective tools for shape exploration and manipulation in the context of architectural design.

Figure 4 shows the dominant symmetries detected in a digital model of the Sydney opera. The underlying transformation space is the seven-dimensional space of similarity transformations whose elements are composed of uniform scaling, rotation, and translation.

Figure 5 shows an application of the structure discovery algorithm to raw scanner output. The point cloud has been acquired with a single-viewpoint laser scanner, which leads to gradually varying sample spacing due to perspective distortion. Despite the low sampling density and holes in the data caused by occlusion, the algorithm robustly finds two regular translational grids. The figure also illustrates how the detected symmetry information can be utilized for model repair.

Figure 6 illustrates the difference between top-down symmetry detection according to [Mitra et al. 2006] and bottom-up structure discovery using the method of [Pauly et al. 2008]. The former extracts mostly pairwise symmetries, such as the global reflective symmetry or the rigid motions mapping the towers or chimneys onto each other. The latter detects translational and rotational grids of windows and other structural elements, but ignores the chimneys, since their spatial arrangement does not match any of the repetitive patterns defined in Figure 3. On the other hand, this method is capable of discovering and compactly representing structures composed of very small elements such as the balustrade, which are not extracted by the top-down symmetry detection approach.

Procedural Modeling. A simple yet effective modeling operation is part replacement. Structural elements can be replaced or modified using standard modeling tools. The system then automatically replaces all symmetric copies to preserve the structural integrity of the model. This type of operation is illustrated in Figures 6 and 8. In addition, we can modify the parameters of the regular structures, e.g., the number of repetitions as illustrated in Figures 7 and 8. This type of procedural design allows the user to quickly create variations of an original design or scanned artifact that would be tedious to achieve with traditional modeling tools.

Symmetrization. The extracted symmetries are often not perfect in the sense that the transformed part $T(\mathcal{A})$ might not exactly match the corresponding part $\mathcal{B}$. This occurs, for example, when scanning a real-world object due to the discrete sampling process, or when the model itself is not perfectly symmetric, e.g., a partly preserved ruin. In addition, many physical architectural prototypes or design studies are often not build with high geometric accuracy, so that a digitized model might not possess all the intended symmetries. To enhance approximate symmetries we can employ the symmetrization approach of [Mitra et al. 2007]. As illustrated in Figure 9, this method can be used to generate symmetric meshes, which can be important if the mesh represents structural elements such as struts or beams, e.g., in a steel-glass construction.

4 Conclusion and Future Work

We discussed how symmetry and structure discovery algorithms can be exploited for shape analysis and synthesis in the context of architectural design. These tools provide a first step towards a more comprehensive framework for procedural modeling based on reverse-enginerring of shape design rules. The analysis of symmetry and repetitive structures can also be utilized in the classification of buildings from different historical periods and potentially provide insights into the style of a specific architect or designer.

The modeling operations of the above examples solely rely on geometric information and thus do not take into account semantic information that might be important to adequately represent the underlying design intent. An important avenue for future research concerns the development of a framework that allows combining symmetry information with other functional or semantic characteristics of digital 3D designs.

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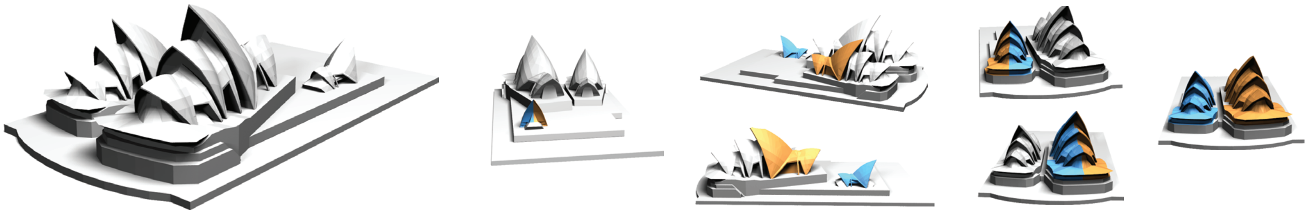

Figure 4: *Large-scale symmetries detected in a digital model of the Sydney opera. The extracted symmetries include reflections, as well as general similarities that involve uniform scaling, rotation, and translation.*

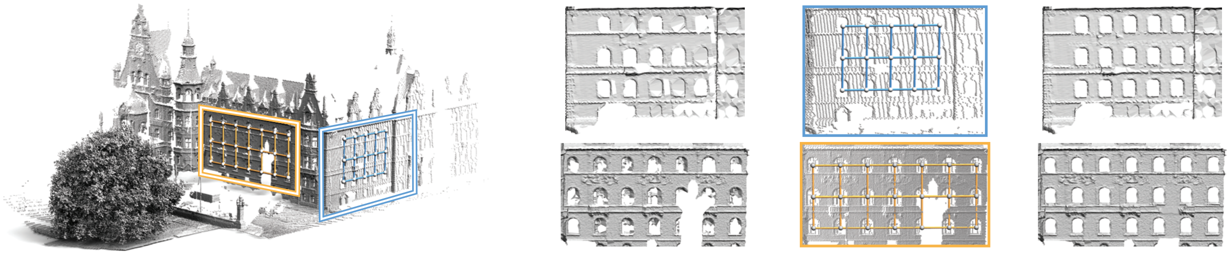

Figure 5: *Structure discovery and model repair on a laser scan of a complex outdoor scene. The algorithm fully automatically discovers two translational grids within the acquired point cloud. Standard surface reconstruction yields an incomplete and inconsistent triangulation shown in the zooms on the left. The models on the right have been created by augmenting the point set using replicated samples from the representative elements prior to reconstruction.*

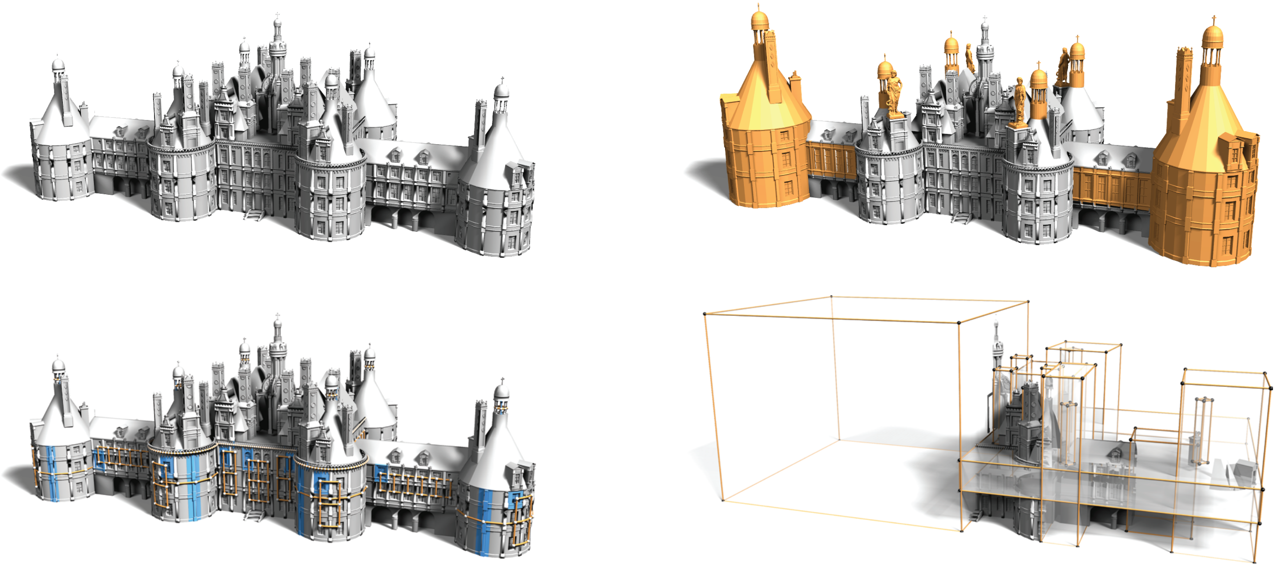

Figure 6: *Conceptual differences between bottom-up structure discovery and top-down symmetry detection. The transparent bounding boxes (lower right) show the extracted symmetry hierarchy that can be utilized for symmetry aware part replacement as shown in the top right. The repetitive translational and rotational structures shown on the lower left support more fine-grain edits to individual structural elements.*

Figure 7: *Structure discovery and procedural modeling on a building facade. The regularity patterns of the model on the left have been extracted automatically and can be modified by the user to alter the facade design as shown in the middle. For comparison, the image on the right shows the original model scaled along the horizontal axis.*

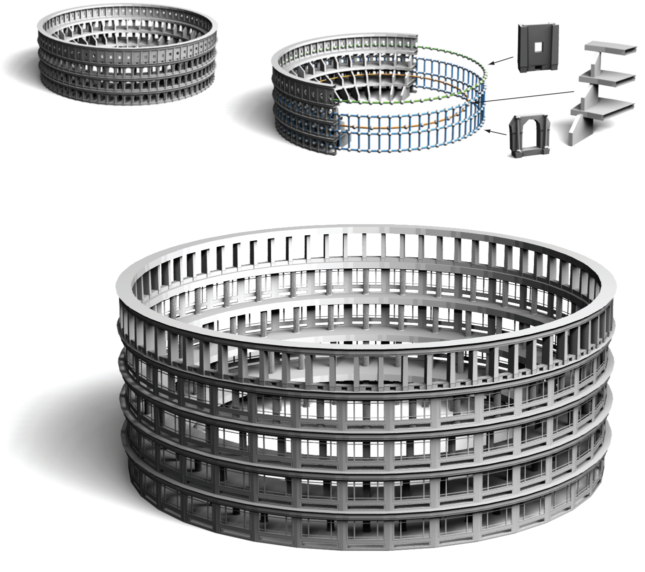

Figure 8: *The input model in the top left corner has been analyzed to reveal three dominant repetitive structures, illustrated in the top right. The zooms show the corresponding structural elements. A new design has been created by modifying the number of repetitions and replacing the repetitive elements with new geometry.*

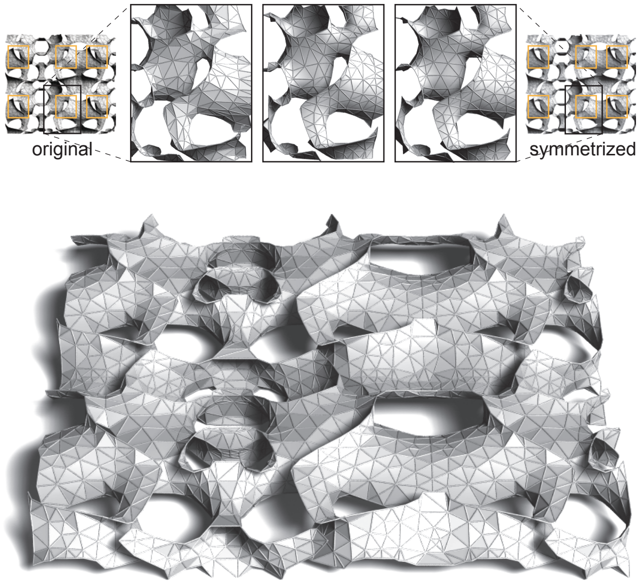

Figure 9: *Symmetrization illustrated on an architectural design study. The top row shows how one of the symmetric elements evolves during the optimization. After processing, the six-fold approximate symmetry of the original model is perfect both in terms of geometry and meshing.*

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