

Exploration of static equilibrium representations; policies and genetic algorithms

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ABSTRACT: Design exploration consists in browsing the design space and through the incremental generation of diverse design candidates increases creativity and diversity. When integrated into the early stages of the structural design process, the task is expected to facilitate designers to balance loosely defined criteria with well-defined constraints like static equilibrium. Addressing the lack of tools to support designers during such crucial process, the authors developed a new parametric, policy-based workflow for exploring equilibrium representations: (–) that incrementally grows and transforms bar networks within specified geometric domains; (–) that maintains their static equilibrium at every intermediate transformation step; (–) that is based on parametric, equilibrium-aware policies, controlled by a choice of four low- or high-level rules; (–) and that is not constrained to precedent typologies or recursing topology patterns (e.g. triangles). In this paper, the exploratory power of the presented workflow is augmented by coupling it with interactive genetic algorithms. Its capacity to unveil unprecedented, unexpected, but statically valid, structural forms is illustrated through a case study.

1 INTRODUCTION

Architectural design, whether supported by digital means or not, usually only explores a tiny fraction of all possible design candidates and is often constrained by premature design fixation and lack of diversity and/or creativity (Purcell 1996). Design exploration consists in browsing the design space and increases diversity and creativity. Nevertheless, analogue design exploration is cumbersome and tedious, as changes require lots of effort. For this, specifically dedicated digital design tools are constantly developed for more than two decades, not only to analyze and evaluate, but also to generate and explore alternative design candidates (Janssen et al. 2002). Woodbury et al. (2000), broadly acknowledges such tools as *design space explorers* intended to provide novel and effective design candidates.

Same goes for structural design, namely the design of structurally-informed forms. Mainstream approaches consist either in adapting well-known and catalogued conventional types or searching for a seemingly optimum solution of well-defined problems. Additionally, exploration of non-resembling structural forms specifically is constrained by the lack of computational methods capable of unveiling structures in static equilibrium, free from precedent typologies or recursing topology patterns.

This paper presents the fusion of an equilibrium-based design space explorer, that builds on high-level policies (Mirtsoopoulos et al. 2022), with interactive genetic algorithms (IGA).

2 STATE-OF-THE-ART

Design being an ill-structured process (Simon et al. 1958, Simon 1973), it requires careful handling. Precisely, as design cannot be formulated explicitly in a mathematical way or solved by existing

methods easily, knowledge from other fields has been loaned frequently: e.g., *shape grammars* supported by recursive rules, *graphic statics* when static equilibrium is required, or *simulation-based algorithms* often implemented along with interactive interfaces.

2.1 *Shape grammars and rules*

First incepted by Stiny and Gips (1971), inspired by Noam Chomsky's theories (1957), grammar and rules have often been proposed as possible support mechanisms for architectural designers (McKay et al. 2012). Sets of design rules assembled in various configurations can generate numerous design variants. Many shape grammars and corresponding implementations have been proposed, but shape grammars are not widely adopted by architectural designers (Pauwels et al. 2015). Nevertheless, until nowadays they continue to attract the attention of scholars: Stiny and Mitchell (1978); Duarte (2005); Economou (2006); Chakrabarti (2011). Selected projects from other researchers include: Shea et al. (1997,1999) who applied shape grammars to the synthesis of triangulated trusses; Geyer (2008) who applied grammar rules at a component level for the design of buildings; Mueller (2014) who applied structural grammars randomly and manually to generate diverse sets of structural systems; Stouffs and Janssen (2016) who suggested a rule-based approach to generate building data for urban planning analysis; and Cascone et al. (2021) who proposed a structural grammar for the design of diagrid-like structures.

2.2 *Graphic statics*

A handy way to manipulate form and forces simultaneously, graphic statics builds on the reciprocal relationship between two figures: a *form diagram* and a *force diagram*. In the 19th century, graphic statics gained large popularity as a means to analyze 2D, or projected 3D structures. In the 21st century, its understanding and extension into three dimensions, together with representational and computational advances, has promoted it into a valuable exploration tool. Lee et al. (2016) have coupled vector-based graphic statics with grammar rules; Hartz et al. (2017) have enabled form-finding of structures that have both compression and tension forces through graphic statics; Ohlbrock et al. (2020) introduced the use of a topological graph, that visualizes nodal relationships, as a manipulative design input; Ochoa et al. (2020) combined Combinatorial Equilibrium Modeling (CEM) with machine learning as a bottom-up form-finding method of conceptual structures.

2.3 *Simulation-based algorithms*

Addressed for the solution of unstructured problems when deterministic methods are not applicable, most simulation-based algorithms (Genetic Algorithms, Particle Swarm Optimization, Ant Colony Optimization etc.) are biology-inspired and simulate life's main problem of evolution by natural selection, per Darwin who linked the evolving process of species to their adaptation to their environment. Evolutionary algorithms were first developed during the 1960s, but the implementations of Genetic Algorithms (GA) by Holland (1975) and Goldberg (1989) are dominant until nowadays and have gained ground in the field of structural engineering and optimization (Goldberg et al. 1986), computational morphogenesis (Pugnale et al. 2014) and architectural (Turrin et al. 2012) or structural design (von Buelow 2012) (Mueller et al. 2015).

Recently, Harding et al. enriched the arsenal of available design space explorers, upon recognizing that multiple simultaneous displays are crucial for effective design space exploration (Harding 2016). *Biomorpher* is a Grasshopper plug-in that allows any parametric definition constructed in the same environment to be explored and optimized using interactive genetic algorithms (Harding et al. 2018). The content of this paper is built on top of its open-source implementation, its user-friendliness, its power and efficiency to thoroughly explore the design space, following respective modifications.

3 CONCEPT

Early, schematic options of static equilibrium representations by means of networks of bars in tension or compression, spawn at the early (conceptual) stage of a project. The generation of bespoke bar networks in static equilibrium, is crucial for the design of unconventional structural forms. The presented structural design framework stems from the need of an equilibrium-aware design space explorer that goes beyond predetermined design supported by numerical inputs (Mirtsopoulos et al. 2022). Specifically, it constitutes a bottom-up design exploration approach:

- that incrementally grows and transforms bar networks within specified geometric domains;
- that maintains their static equilibrium at every intermediate transformation step;
- that is based on a parametric, equilibrium-aware policy, controlled by a choice of four low- or high-level rules;
- that is not constrained to precedent typologies or recursing topology patterns (e.g. triangles);
- that allows designers to steer the exploration through subjective criteria, when fused with IGA.

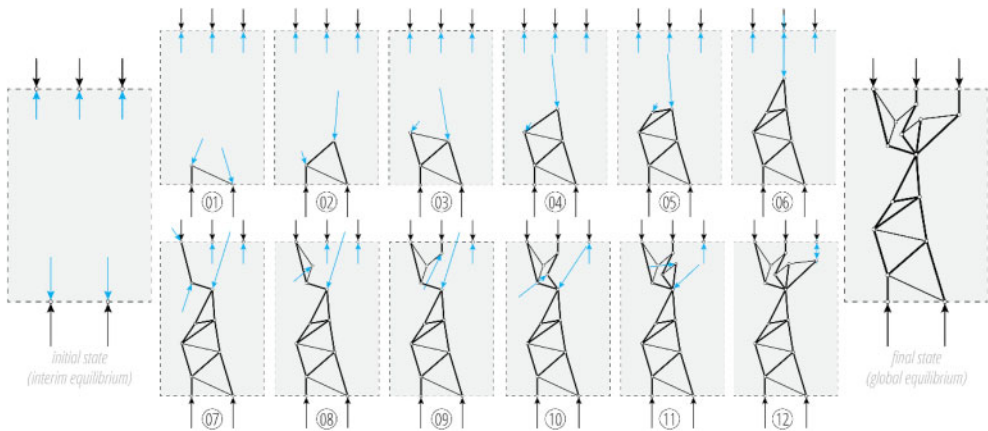


Figure 1. Incremental transformations for the generation of a pylon-like network of bars in static equilibrium.

In more depth, this transformative design approach aims at the transition from a disconnected network of force vectors in interim equilibrium (Figure 1 – left), to a connected network of bars and force vectors in global static equilibrium (Figure 1 – right). Applying the chosen policy results in network transformations that each consists in: the introduction of a new node (P); the replacement of selected interim forces by new bars in tension or compression; and the introduction of new interim forces to retain static equilibrium, if necessary. Transformations end when the pool of remaining interim forces is empty, meaning that all externally-applied forces are connected by a network of bars in static equilibrium. The replacement of a bar element by two force vectors, located at the antipodal points, of equal magnitude but opposite directions, can resume the transformations. The policy parameterization allows the designer to control the transformation without jeopardizing the loss of static equilibrium, interim or global. While the transformative process continues, the pool of interim design candidates shrinks, or expands respectively.

Overall, the incremental transformation of bar networks, gradually expands the size of the network, by increasing the number of bars (Figure 1), but also filters out structurally invalid forms. Under no circumstances it is expected to constrain its exploratory power. On the contrary, the hypothesis is that it unveils unprecedented typologies, dodges premature design fixation, provokes creativity and facilitates decision making. Additionally, its exploratory power is augmented by the possibility to backtrack on previous states when desired qualitative criteria are not met (e.g. aesthetics) (Figure 2).

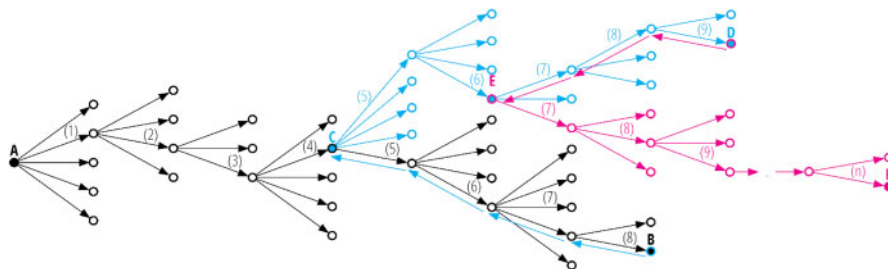


Figure 2. Design exploration; out of a vast range of options designers explore only a tiny fraction of the design space before choosing a unique design. Starting from state A, the designer browses through various aesthetics and different design branches while backtracking to previous states until completing the design process at state F.

4 EQUILIBRIUM-AWARE PARAMETRIC POLICY AND ITS APPLICATION

The entire process is fully based on a unique parametric policy, which is defined as a set of choices to control the growth of the bar networks. The rules defining the policy describe: (–) the change of entropy, (–) the selection of force vectors to replace, (–) the placement of the new node and (–) the force indeterminacies, if necessary. Operating on models in interim equilibrium, the transformations impact the number of the remaining interim forces in the model. The difference before and after the transformation is a balanced ternary rule called entropy rate and leads to the convergence (–1), stagnation (0) or divergence (1) of the entire process. Another rule selects a set of three interim forces, which depending on whether they are coincident with each other or not, lead to monomial, binomial or trinomial forces input. None of the selected forces remains in the model at the end of each incremental transformation. Each network transformation imposes the introduction of a set of bars to replace them with. The bars introduction is supported by introducing a new node P , which ensures the bars connectivity. The force magnitudes in the bars and the location of P are described by respective rules. Diverse network topologies occur when different connectivity configurations are tried out. Interim forces are introduced to impose the conservation of the network’s static equilibrium. Mirtsopoulos et al. (2022) provides in depth explanation of the parametric policy and its rules.

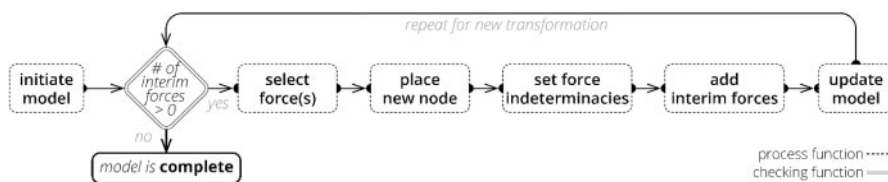


Figure 3. Algorithmic workflow of the presented methodology.

The algorithmic workflow of the presented methodology is illustrated on Figure 3 and consists in six stages. The first one (initiate model) is executed once at the beginning of the process and builds an incomplete network in interim static equilibrium. The remaining stages are repeated incrementally and transform the initiated network for as long as the pool of interim forces is not empty. At that moment the network of bars is closed and in global static equilibrium.

At the second, third and fourth stage (*select force(s)*, *place new node* and *set indeterminacies*) the designer is expected to provide the *interim forces*, the *coordinates of node P* and if necessary the *force indeterminacies* respectively. The definitions of these inputs are either explicit (low-level) or implicit through high level rules. The next stage (*add interim forces*) is controlled by the *entropy rate*. The number of added interim forces depends on the designer’s intention to converge,

stagnate, or diverge the exploration process. This stage also ensures that the network remains in static equilibrium throughout the process. At the last stage of the loop (*update model*), the added elements are integrated into the existing network of bars and the selected force vectors are removed from the pool of interim forces.

5 INTERACTIVE GENETIC ALGORITHMS (IGA)

According to Banzhaf (1997), interactive evolution involves human users in the variation-selection loop of evolutionary algorithms. Additionally, it does not only prevent his/her passive observation while the process evolves but also allows faster convergence onto the fitness landscape. In cases that fitness criteria cannot be defined explicitly, interactive evolution can be applied on a comparative case-by-case basis.

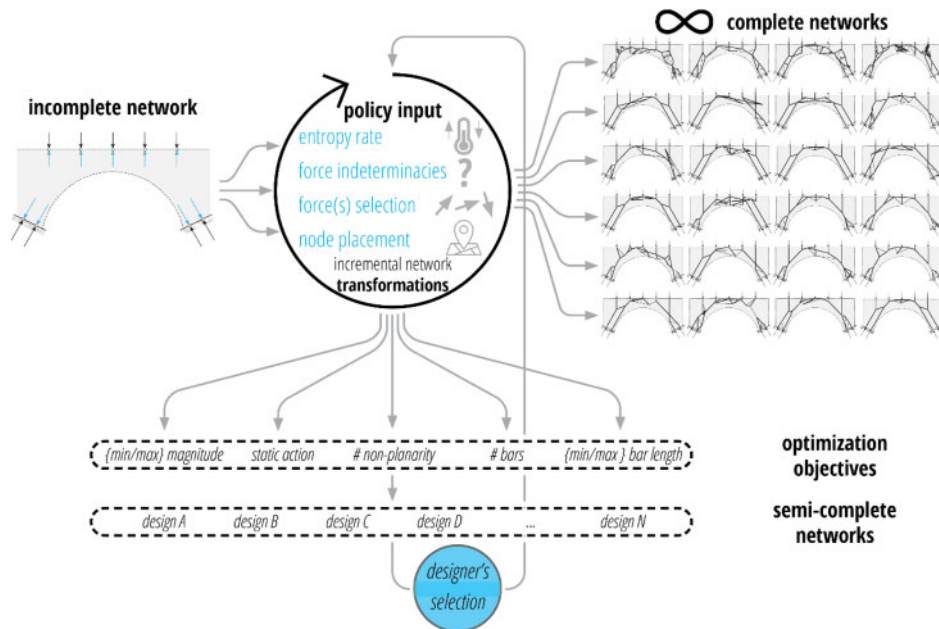


Figure 4. Integration of the proposed design space explorer with interactive genetic algorithms.

5.1 Necessity and benefits

Considering the above, equilibrium-based exploration of the design space combines both quantitative (e.g. static action) and qualitative (e.g. aesthetics) criteria and thus interactive genetic algorithms are tailored to this design space explorer. Like most of the simulation-based stochastic algorithms, GA satisfies two features: *exploration* and *exploitation*.

Within the context of this research, exploration allows the generation of “infinite” design candidates that are not resembling catalogued structural typologies and have diverse topology. This hypothesis features the design space explorer as a form-finding engine of alternative topologies which are not known a priori. The possibility to select, or discard, design options that are appealing, or not, and eventually crossover and mutate only the preferred ones indicates high level of control during the exploration and accelerates immensely the entire process. The evaluation of each network is feasible at every intermediate step, while the network grows. This way, exploration occurs node by node and not only statically invalid complete networks are filtered out, but also undesired incomplete networks are excluded from further transformations.

Exploitation allows the networks optimization with regards to engineering performance or graph theory related metrics. In the course of just a few generations, new, diverse bar networks, which outperform the earlier generations, are added to the list of design candidates. For the specific methodology, performance optimization is not the main exploitation goal. The policy's capacity to operate on any given force vectors and its freedom to build bespoke topologies unconstrained by specific typologies, requires the human interaction to steer the process towards human-approved results. Thanks to IGA, the process can continue in an automated way with human interaction limited to aesthetics criteria. In other words, exploitation results in more controlled design candidates in an automated way without being self-constraining.

5.2 Implementation

The design space explorer has been implemented as a plug-in for the parametric environment of Grasshopper in McNeel's Rhinoceros. The implementation of the IGA for the same platform is John Harding's and Cecile's Brandt-Olsen courtesy (<https://github.com/johnharding/Biomorpher>). The policy parameterization, along with their open-source implementation, followed by respective modifications, allowed the fusion of the equilibrium-based design exploration with IGA.

5.3 Integration

The integration of IGA into the presented design space explorer is illustrated in Figure 4. The designer selects the policy input to transform the network of bars for the course of a single, or multiple, iteration of the genetic algorithm, as well as the optimization objectives, if any. Evolutionary algorithms tune the supplementary parameters of the inputs, subject to single or multi-optimization. Each generation of the evolutionary algorithm plots a population of new incomplete bar networks in a specially designed interactive user interface. When the evolution has stopped, the designer chooses, if he/she wishes, the designs that satisfy subjective or other quantitative criteria. The design exploration process illustrated in Figure 3 through backtracking to multiple design branches is applicable to this integrated interactive workflow too.

6 APPLICATION STUDY

Evolutionary algorithms are exploited for the generation and optimization of bar networks through the presented methodology. The situation of three supports, an applied load at the top and equally distributed lateral loading that simulates wind loads is examined. This simple set up resembles the form-finding process for the Eiffel tower (Figure 5 - initial state). Indeed, after mirroring the generated networks, many of them resemble the form of the famous landmark.

Figure 5 presents 288 effortlessly generated, in an automated way, incomplete tower designs. For every model, 30 transformations are performed. The total number of transformations until the pool of interim forces gets empty cannot be predicted. Assigning, bounds to the bar lengths (Mirtsopoulos et al. 2021) makes such a prediction even harder. Thus, most of the networks still contain interim forces. Though not complete, their completion is only a few transformations far. The designer can either manually continue this process or select the most appealing design candidate and backtrack to previous states (Figure 2) or simply use the available list of design candidates as an inspiration for a brand-new design candidate.

Multi-objective optimization for the minimization of the *static action* and the *degree of non-planarity* of the networks, frame the setup of the exploration process. In the course of 8 generations, both objectives are achieved (Figure 5 – performance evolution). The former results in more efficient use of material and the latter reduces the bar intersections and ultimately the total number of nodes. Last but not least, it indirectly, assists the growth of networks in larger areas and leads to networks with sparsely-populated nodes.

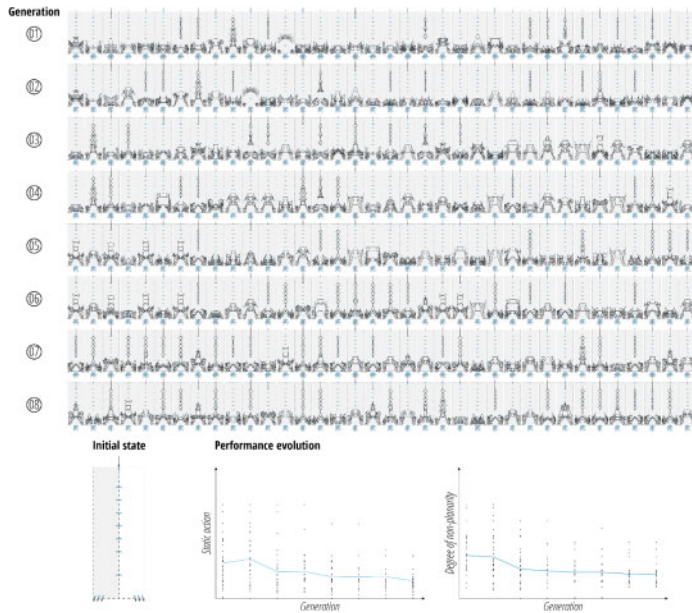


Figure 5. Evolutionary generation of diverse bar networks in static equilibrium; static action and number of intersections are used as optimization objectives.

7 CONCLUSIONS

This paper presented the definition of an equilibrium-based design space explorer that uses policies and interactive evolution for the generation of bar networks in static equilibrium. The approach considers incomplete networks that incrementally transform, for the sake of exploration, and, optionally, optimizes them. At the end of the process, diverse bar networks in static equilibrium are outputted, per the human-user interaction and the optimization objectives. The interactive evolution successfully enables a synergy between the human and the machine. It is believed that without this synergy many results could not be conceived. Overall, the methodology allows designers to generate valid and appealing networks of bars in static equilibrium without knowledge of structural mechanics.

Future developments include the improvement of the established synergy between human and machine and the evolution of the latter as a collaborative partner during the design process, that contributes with its own intelligence towards the final design.

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