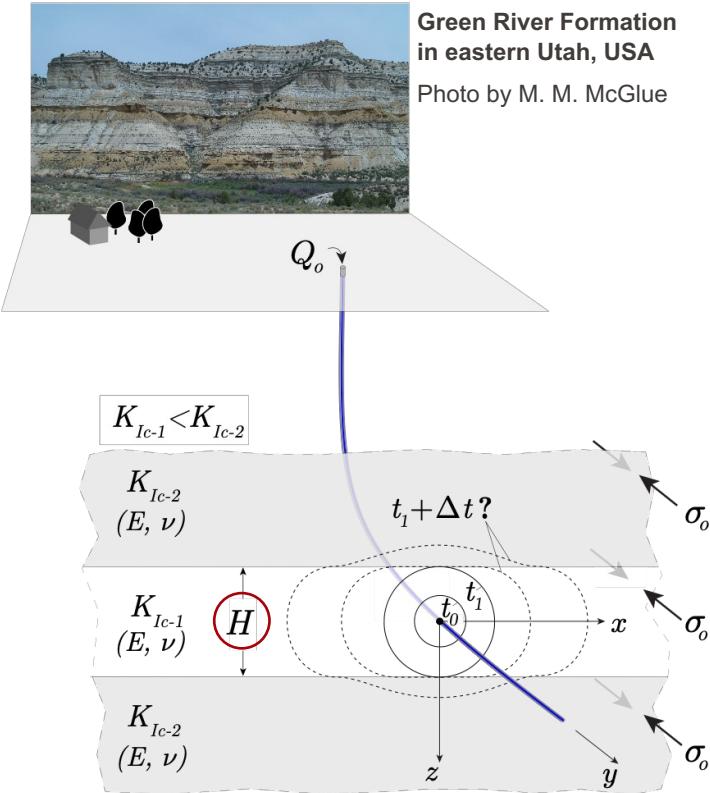


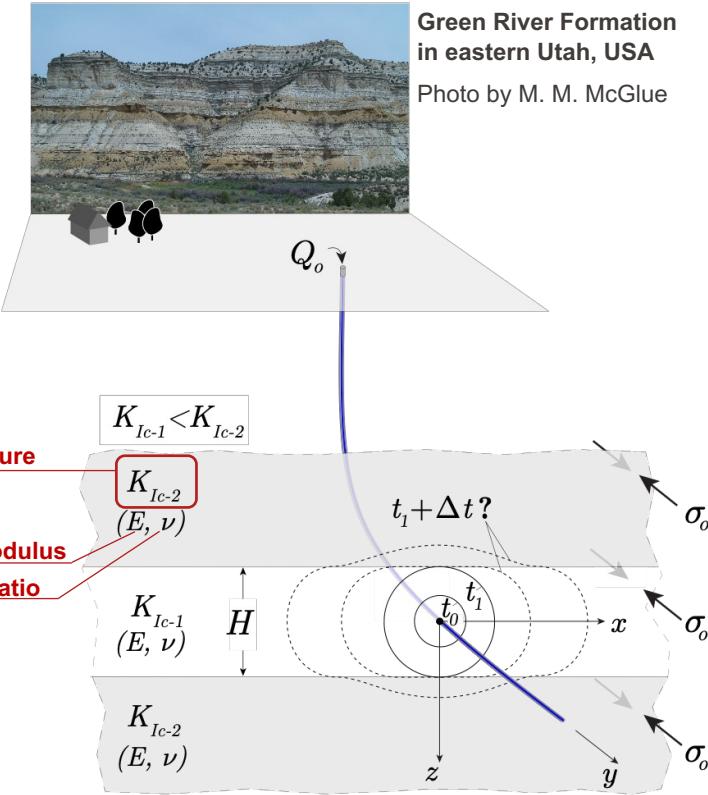
When and
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the propagation of a fracture
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Carlo Peruzzo, Judith Capron,
Brice Lecampion



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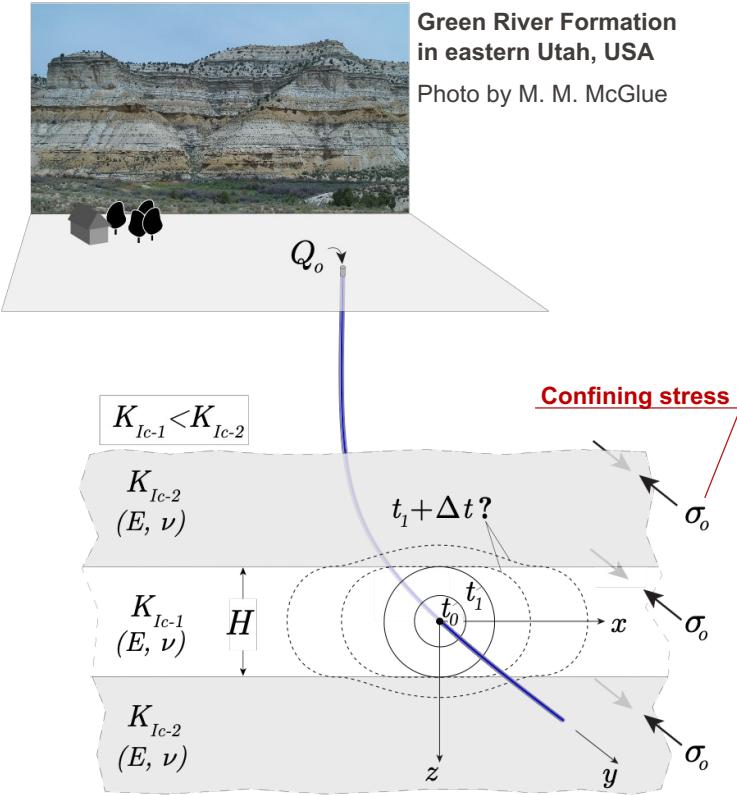
Mode I fracture
toughness

Young's modulus

Poisson's ratio

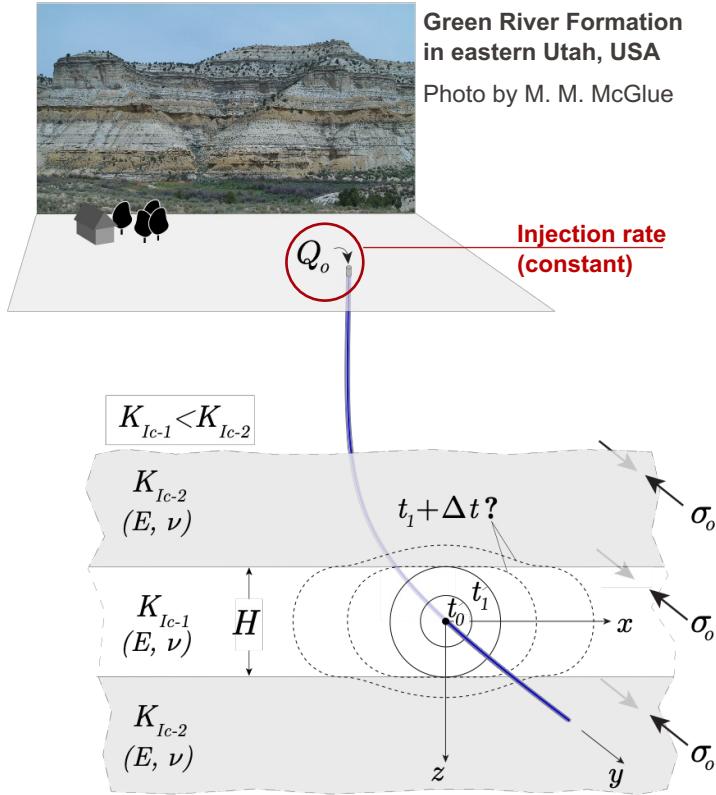
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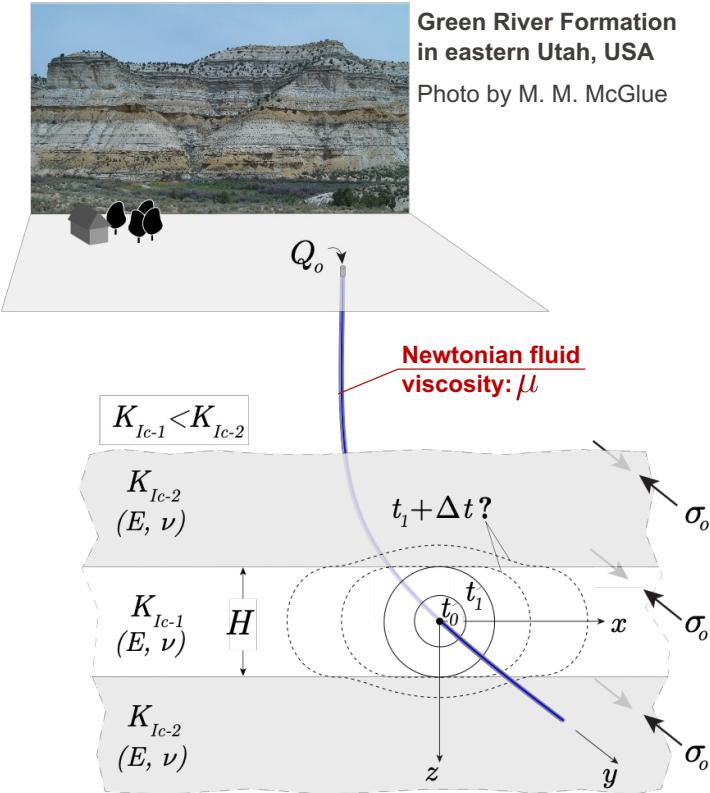
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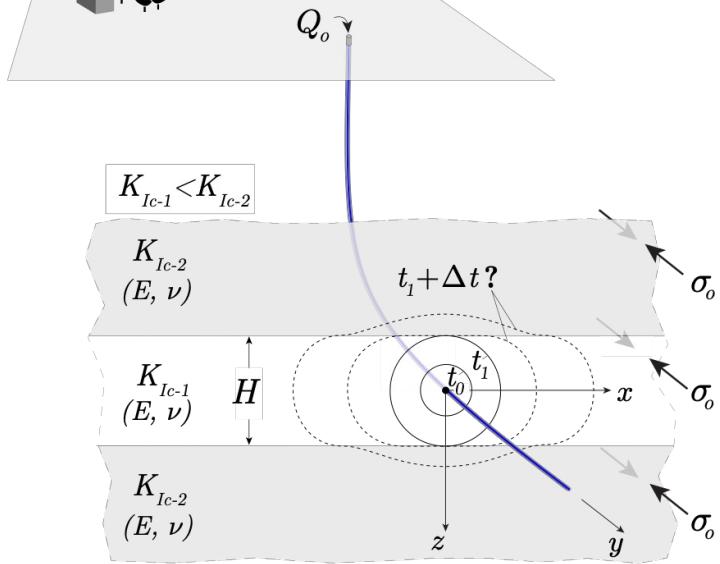
Carlo Peruzzo, Judith Capron,
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$$\frac{K_{Ic-2}}{K_{Ic-1}}$$

$$\bar{\mathcal{K}} = K_{Ic-1} \left(\frac{H}{E'^3 Q_o \mu'} \right)^{1/4}$$



Green River Formation
in eastern Utah, USA
Photo by M. M. McGlue



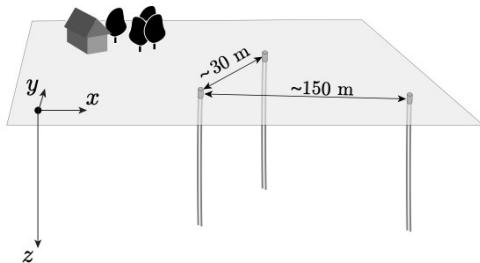
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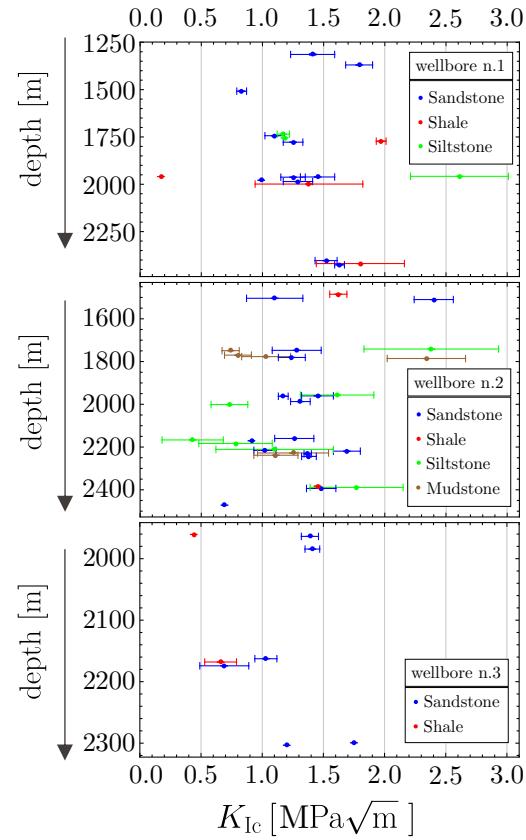
Fracture toughness ratios for different rocks

[Senseney et al., 1984]

Rulison Field – Piceance
Basin, Garfield Country,
Colorado, USA



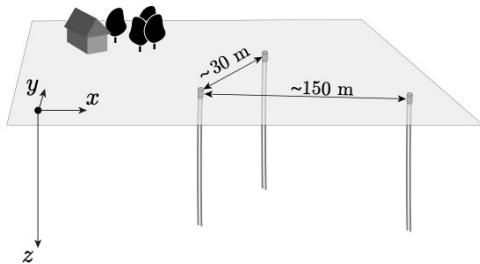
$$\frac{K_{Ic-2}}{K_{Ic-1}} \sim 1 - 5$$



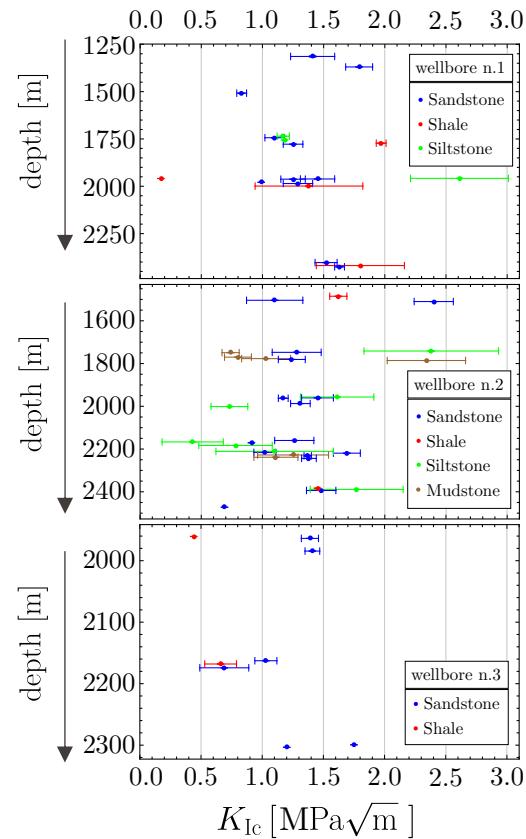
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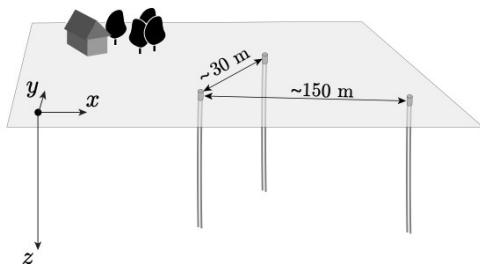


Lithology	n. of rock units	K_{Ic} [MPa m ^{1/2}]	$\frac{K_{Ic-max}}{K_{Ic-min}}$	Ref.
Sandstone	3	$0.36 < K_{Ic} < 0.89$	2.5	Nara et al., 2012
	3	$0.37 < K_{Ic} < 1.1$	2.9	Roy et al., 2017
	5	$0.49 < K_{Ic} < 1.60$	3.3	Noël et al., 2021
	3	$0.98 < K_{Ic} < 2.12$	2.2	Thiercelin, 1989
Limestone	3	$0.53 < K_{Ic} < 0.73$	1.38	Chandler et al., 2016
	2	$0.48 < K_{Ic} < 0.92$	1.9	Chandler et al., 2016
		$1.36 < K_{Ic} < 2.06$	1.5	Gumsallus et al., 1984

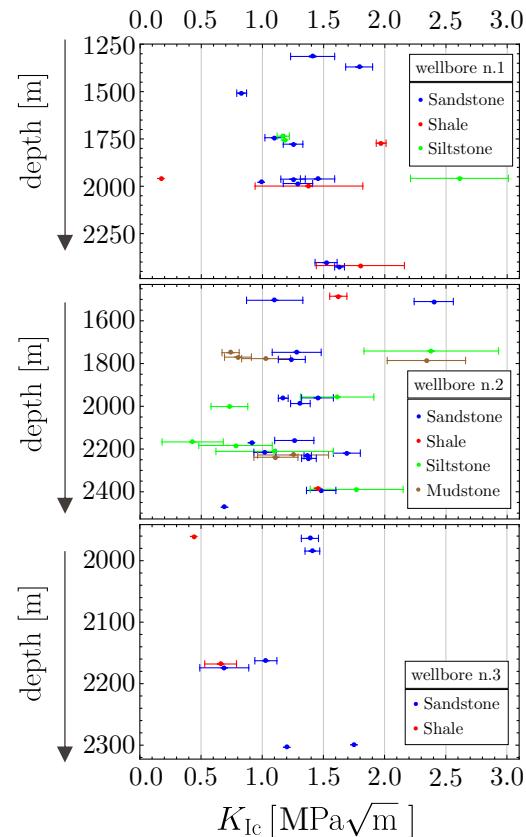
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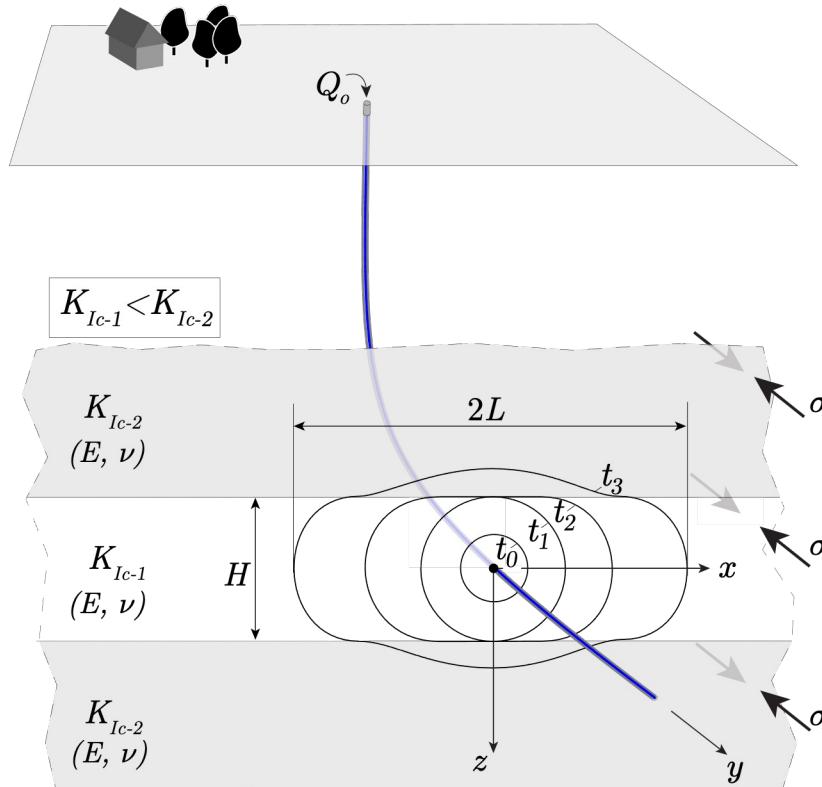
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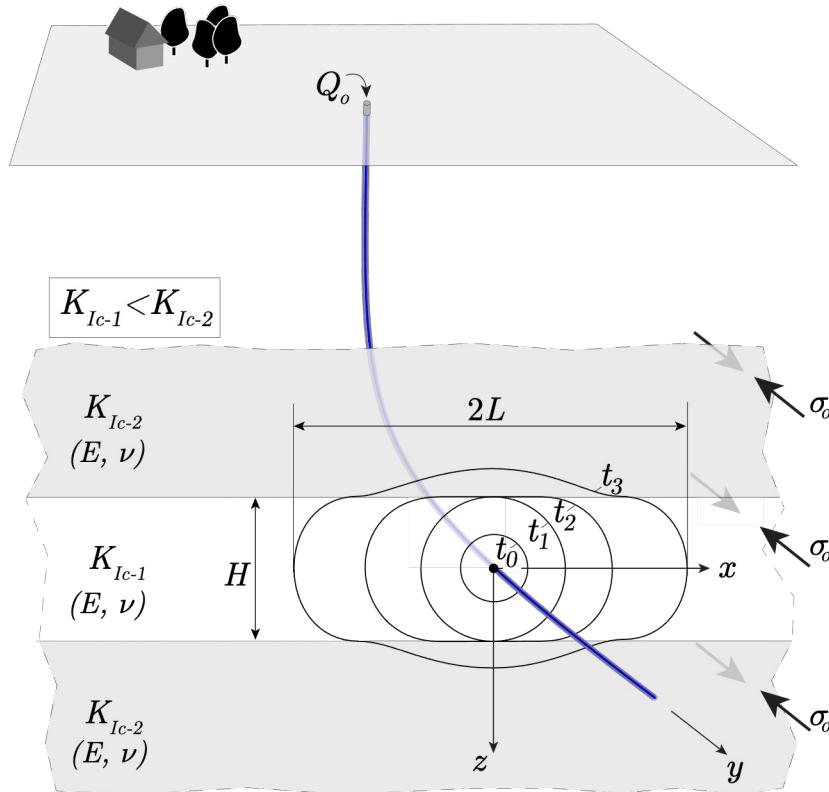
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	1	2.14	5.9	Nara et al., 2012
Granite ^a	3	$0.36 < K_{Ic} < 0.89$		
Shale ^b	1	0.31	3.4	Roy et al., 2017
Sandstone*	3	$0.37 < K_{Ic} < 1.1$		
Mudstone*	1	2.12	4.9	Thiercelin, 1989
Shale*	1	0.43		
Sandstone*	3	$0.98 < K_{Ic} < 2.12$		
Shale ^a	1	0.44		
Shale ^b	1	0.44	2.1	Chandler et al., 2016
Limestone*	3	$0.48 < K_{Ic} < 0.92$		
Sandstone*	3	$0.53 < K_{Ic} < 0.73$		
Dolostone*	5	$1.66 < K_{Ic} < 2.47$		
Sandstone*	1	1.47		
Limestone*	2	$1.36 < K_{Ic} < 2.06$	1.8	Gunsallus et al., 1984



Assumptions:

- infinite space
- LHF M (Linear Hydraulic Fracture Mechanics)
- no gravity
- impermeable media

Scenario n. 1

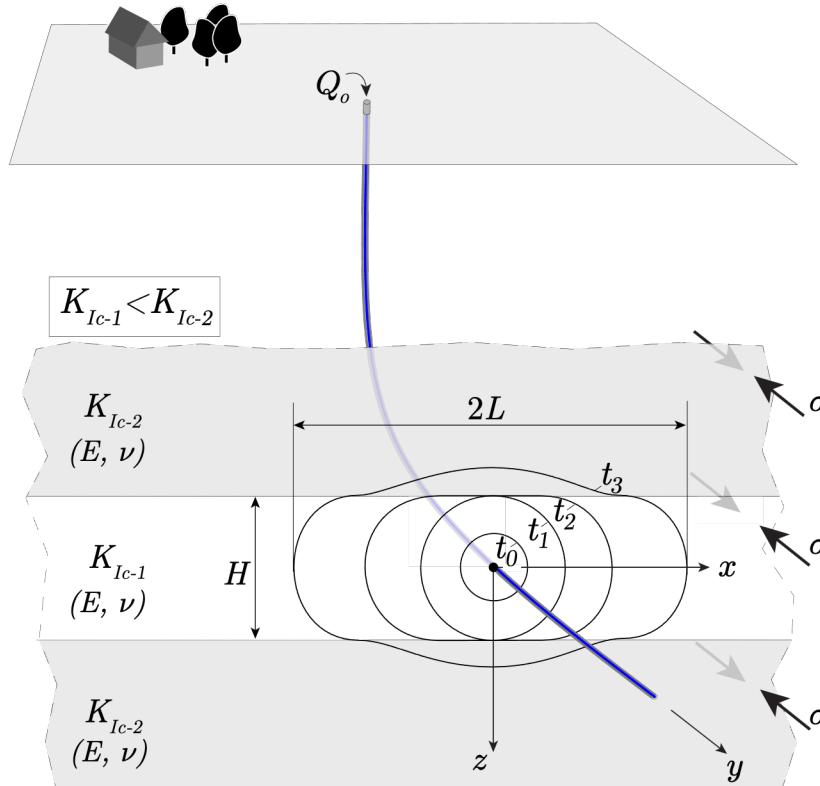


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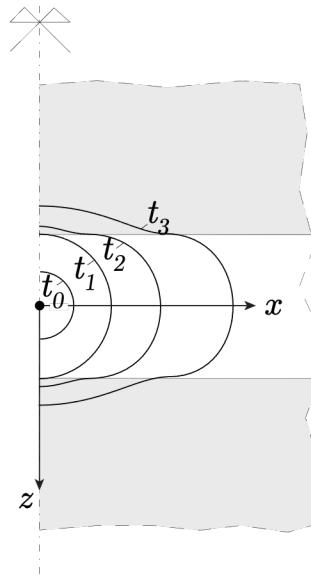
Possible scenarios



Scenario n. 1

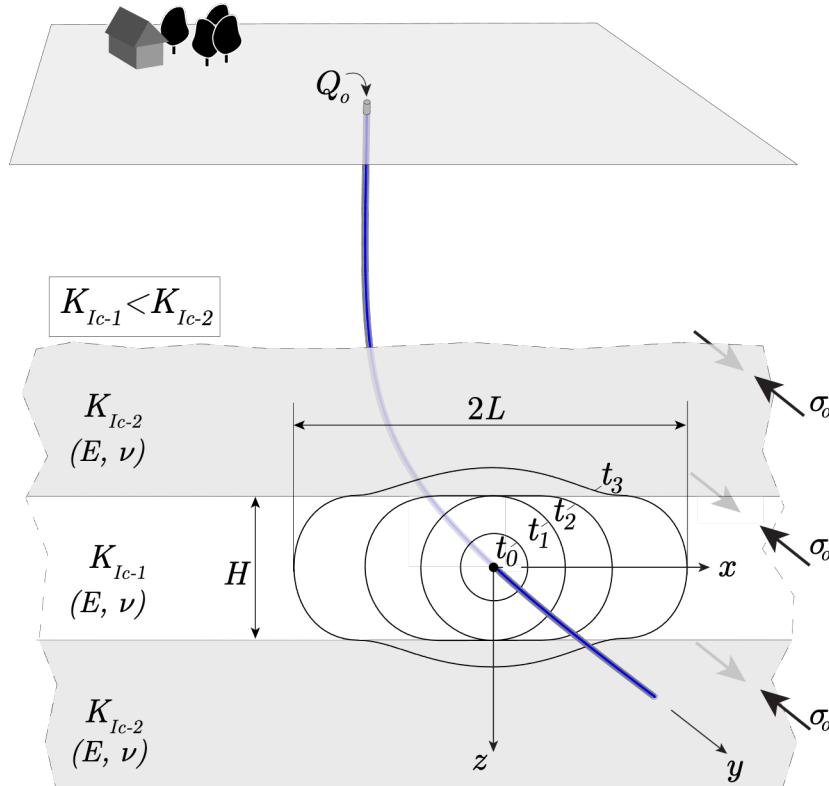
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Scenario n. 2

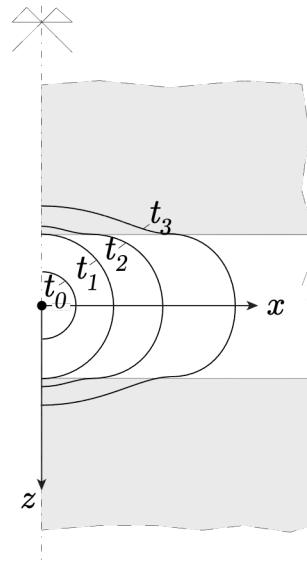
Possible scenarios



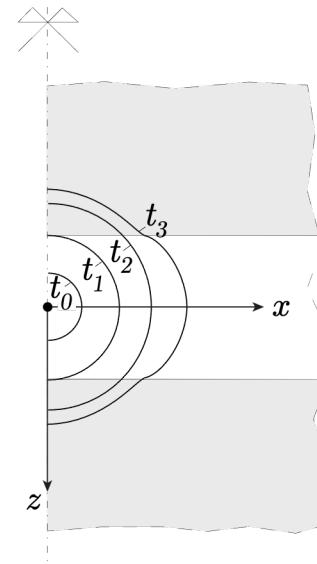
Scenario n. 1

Assumptions:

- infinite space
- LHF M (Linear Hydraulic Fracture Mechanics)
- no gravity
- impermeable media



Scenario n. 2



Scenario n. 3

Elasticity: [Hills et al, 2013][Crouch et al, 1983]

$$p = p_f - \sigma_o = -\frac{E'}{8\pi} \int_{A(t)} \frac{w(x', z', t) dA(x', z')}{[(x' - x)^2 + (z' - z)^2]^{3/2}}$$

Net pressure

Volume conservation:

[e.g.: Batchelor, 1967]

$$\frac{\partial w(x, z, t)}{\partial t} + \nabla \cdot \mathbf{q} - \delta(x)\delta(z)Q(x, z, t) = 0$$

Propagation conditions:

$$V \geq 0$$

$$K_I \leq K_{Ic}$$

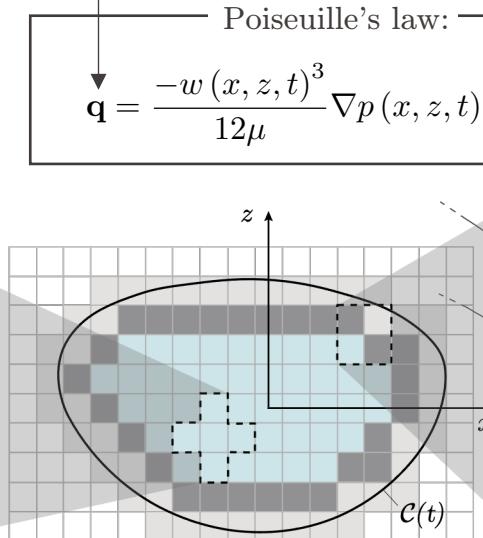
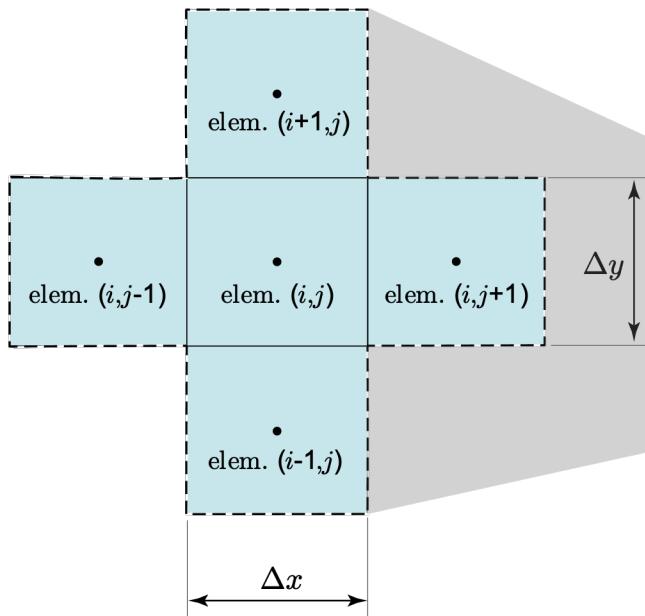
$$(K_I - K_{Ic})V = 0$$

} on $\mathcal{C}(t)$

Poiseuille's law:

$$\mathbf{q} = \frac{-w(x, z, t)^3}{12\mu} \nabla p(x, z, t)$$

Fracture velocity



- \mathcal{A}_t = tip elements
- \mathcal{A}_s = survey elements
- \mathcal{A}_c = channel elements

Elastohydrodynamic system

[Peirce et al., 2008] [Zia & Lecampion, 2020]

$\mathbf{A}(\mathbf{x}^n) \cdot \mathbf{x}^n = \mathbf{b}(\mathbf{x}^n)$ → Anderson acceleration of fixed point iterations [Anderson, 1965]
 [Walker & Ni, 2011]

$$\underbrace{\begin{bmatrix} \mathbf{I} + \Delta t \mathbf{L}^{cc} \mathbf{E}^{cc} & \Delta t \mathbf{L}^{ct} \\ \Delta t \mathbf{L}^{tc} \mathbf{E}^{cc} & \Delta t \mathbf{L}^{tt} \end{bmatrix}}_{\text{Elasticity operator}} \underbrace{\begin{bmatrix} \mathbf{x}^n \\ \Delta \mathbf{w}^c \\ \Delta \mathbf{p}^t \end{bmatrix}}_{\mathbf{x}^n} = \underbrace{\begin{bmatrix} \mathbf{b}(\mathbf{x}^n) \\ \mathbf{f}_L^c - \Delta t \mathbf{L}^{cc} \mathbf{E}^{ct} \Delta \mathbf{w}^t \\ \mathbf{f}_L^t - \Delta \mathbf{w}^t + \Delta t \mathbf{L}^{tc} \mathbf{E}^{ct} \Delta \mathbf{w}^t \end{bmatrix}}_{\text{Finite difference operator}}$$

- Positive definite
- Full matrix
- Positive definite
- 5-banded matrix

Elastohydrodynamic system

[Peirce et al., 2008] [Zia & Lecampion, 2020]

$\mathbf{A}(\mathbf{x}^n) \cdot \mathbf{x}^n = \mathbf{b}(\mathbf{x}^n)$ → Anderson acceleration of fixed point iterations [Anderson, 1965]
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- Positive definite
- Full matrix
- Positive definite
- 5-banded matrix
- 9-banded matrix

Solution of the linear system of equations

$$\mathbf{A}(\mathbf{x}_k^n) \cdot \mathbf{x}_{k+1}^n = \mathbf{b}(\mathbf{x}_k^n)$$

- Non symmetric
- Non diagonal dominant

BICGSTAB
 $\mathbf{A}\mathbf{P}^{-1}\mathbf{z} = \mathbf{b}$
 $\mathbf{x} = \mathbf{P}^{-1}\mathbf{z}$

- \mathbf{P}^{-1} reduces the spectral radius of \mathbf{A}
- Fast to compute
- Memory cheap
- Fast to apply

[Peirce, 2005]

Preconditioner
 $\mathbf{P}^{-1} \sim \mathbf{A}^{-1}$

Approximate \mathbf{A}
 $\tilde{\mathbf{A}}^{-1} \approx \mathbf{A}^{-1}$
 $\tilde{\mathbf{A}} = f(\tilde{\mathbf{E}})$

Calculate
 $\mathbf{P} = \text{ILUT}(\tilde{\mathbf{A}})$

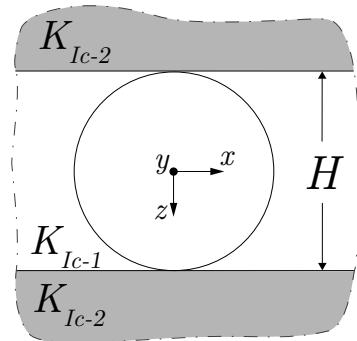
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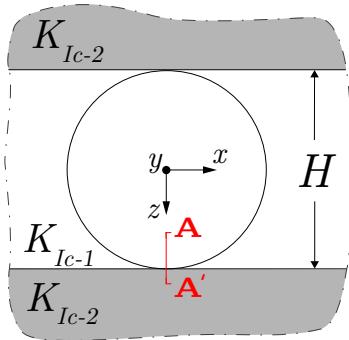
Finite difference operator

Toepplitz structure → Dot product in N^2 → Storage in N
 \mathcal{H} -matrix [Hackbusch, 1999] → Dot product in $N \log(N)$ → Storage in $N \log(N)$

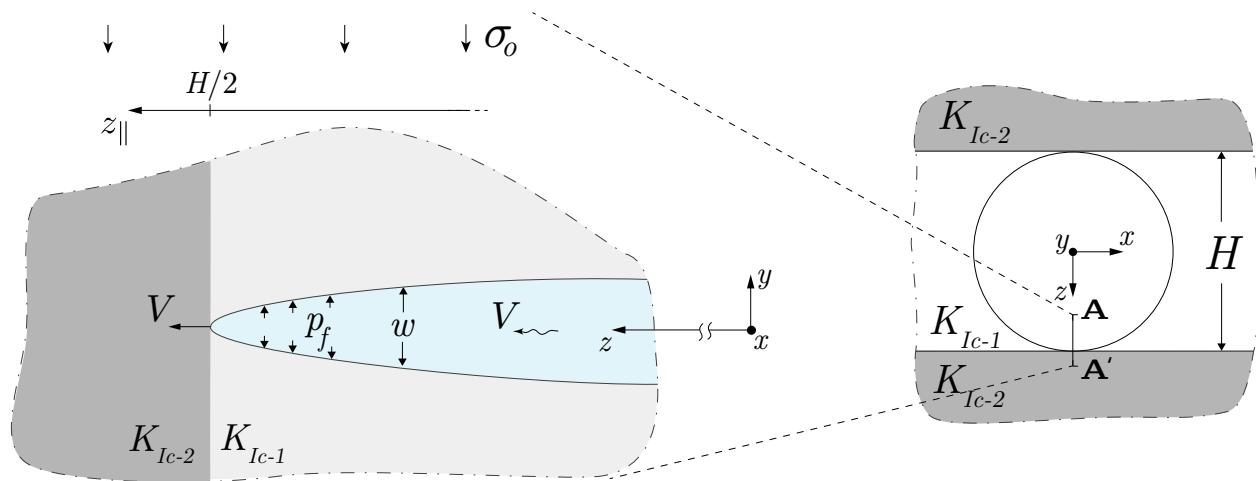
Minimum toughness ratio that stops the fracture



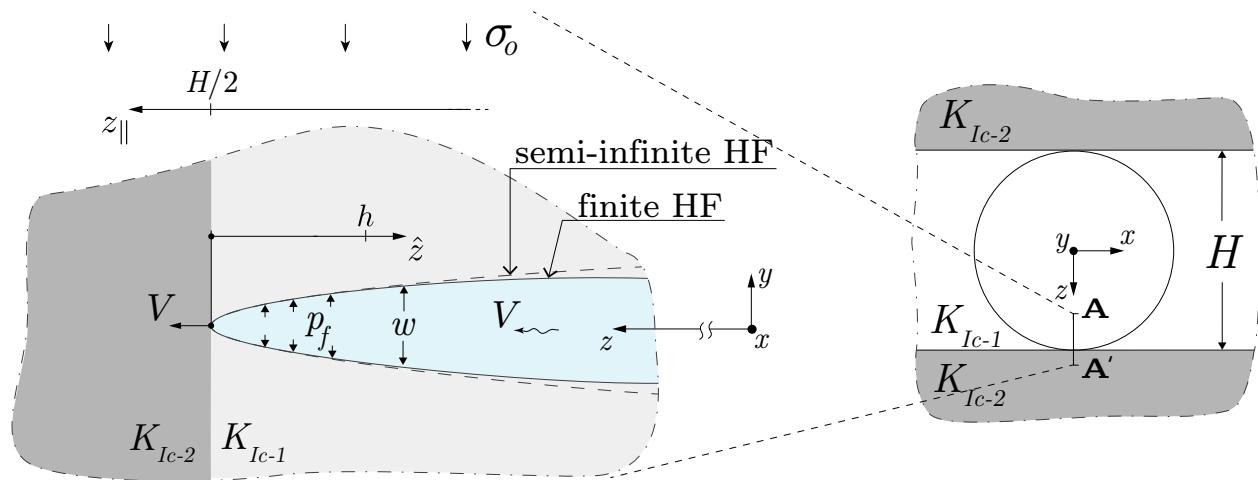
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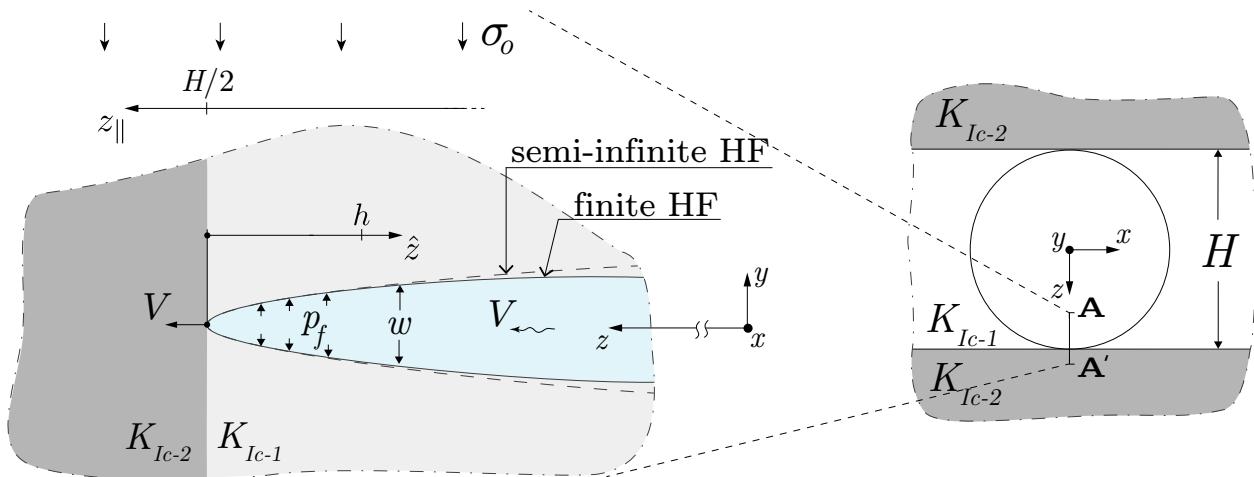
Minimum toughness ratio that stops the fracture



Minimum toughness ratio that stops the fracture



Minimum toughness ratio that stops the fracture



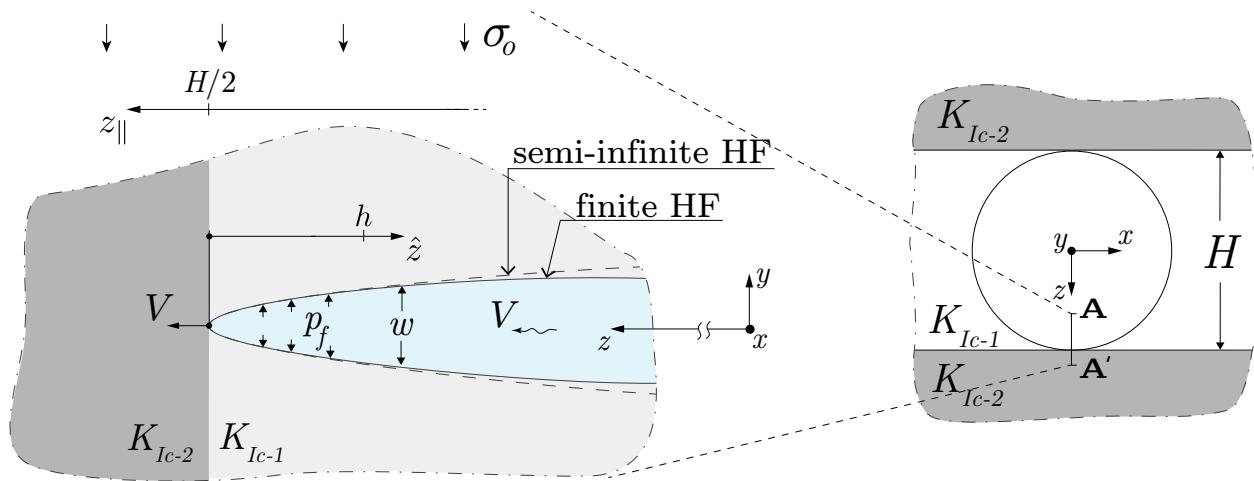
Power balance in a moving coordinate system:

$$\underbrace{12\mu V^2 \int_0^h \frac{1}{w(\hat{z})} d\hat{z}}_{\text{viscous dissipation rate}} + \underbrace{G \times V}_{\text{power rel. rate}} + \underbrace{\int_0^h \frac{1}{2} V \frac{d}{d\hat{z}} (w \times (p_f - \sigma_o)) d\hat{z}}_{\text{rate of elastic energy accretion}} = \underbrace{V w(h) p(h)}_{\text{external power}}$$

Energy balance:

$$\underbrace{12\mu V \int_0^h \frac{1}{w(\hat{z})} d\hat{z}}_{\text{viscous dissipation}} + \underbrace{G}_{\text{energy rel. rate}} = \underbrace{\frac{1}{2} w(h) p(h)}_{\text{external dissipated energy}}$$

Minimum toughness ratio that stops the fracture



Propagation conditions:

$$\begin{cases} V \geq 0 \\ G \leq G_c \\ (G - G_c) V = 0 \end{cases}$$

Energy balance:

$$\underbrace{12\mu V \int_0^h \frac{1}{w(\hat{z})} d\hat{z}}_{\text{viscous dissipation}} + \underbrace{\frac{G}{2} w(h) p(h)}_{\text{energy rel. rate}} = \underbrace{\frac{1}{2} w(h) p(h)}_{\text{external dissipated energy}}$$

$$\begin{cases} V \geq 0 \\ G \leq G_c \\ \left(\underbrace{\frac{1}{2} w(h) p(h)}_{\text{external dissipated energy}} - \underbrace{12\mu V \int_0^h \frac{1}{w(\hat{z})} d\hat{z}}_{\text{viscous dissipation}} - G_c \right) V = 0 \end{cases}$$

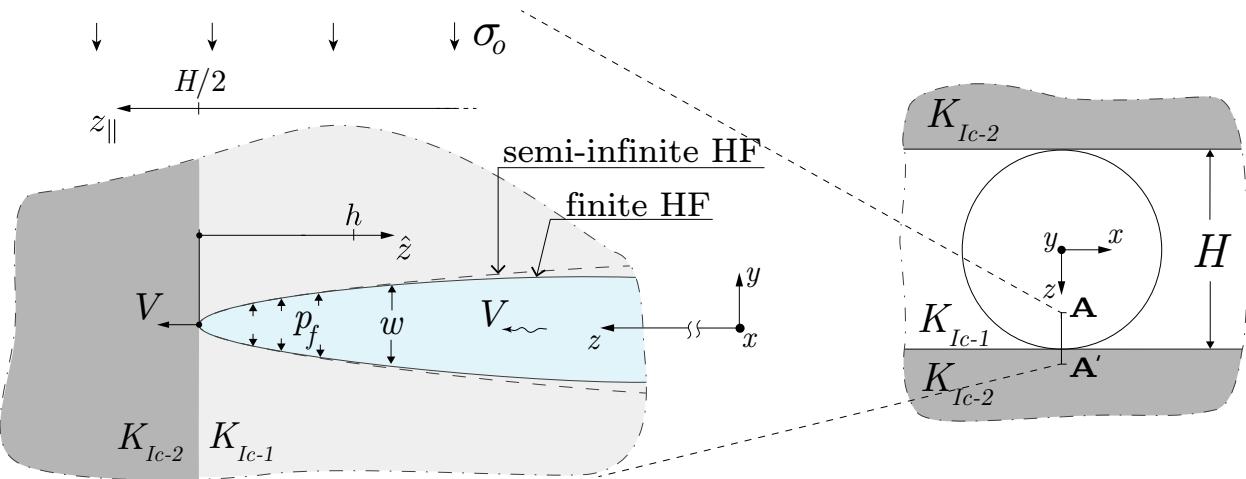
Minimum toughness ratio that stops the fracture

STATE 1:

$$\left\{ \begin{array}{l} V^- > 0 \\ G = G_{c1} \\ \underbrace{\frac{1}{2}w(h)p(h)}_{\text{external dissipated energy}} = \underbrace{12\mu V \int_0^h \frac{1}{w(\hat{z})} d\hat{z}}_{\text{viscous dissipation}} + G_{c1} \end{array} \right.$$

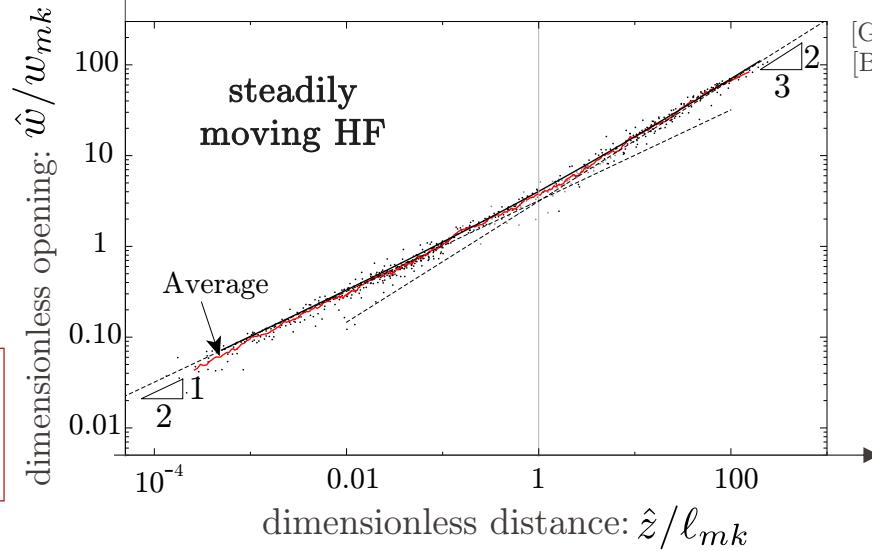
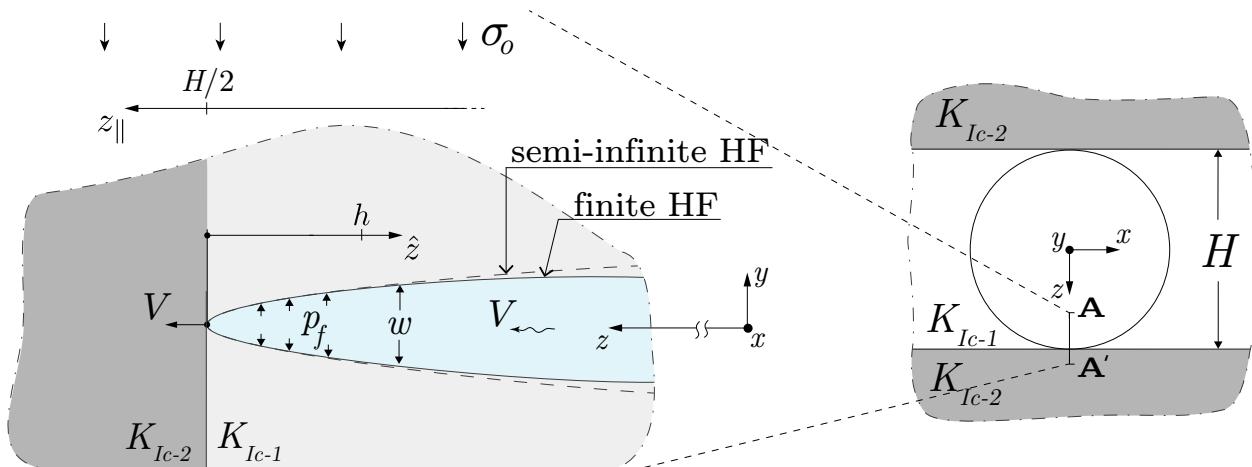
STATE 2:

$$\left\{ \begin{array}{l} V^+ = 0 \\ G = G_{c2} \\ \boxed{\underbrace{12\mu V^- \int_0^h \frac{1}{w(\hat{z})} d\hat{z}}_{\text{viscous dissipation}} + G_{c1} = G_{c2}} \end{array} \right.$$



$$\left\{ \begin{array}{l} V \geq 0 \\ G \leq G_c \\ \left(\underbrace{\frac{1}{2}w(h)p(h)}_{\text{external dissipated energy}} - \underbrace{12\mu V \int_0^h \frac{1}{w(\hat{z})} d\hat{z}}_{\text{viscous dissipation}} - G_c \right) V = 0 \end{array} \right.$$

Minimum toughness ratio that stops the fracture



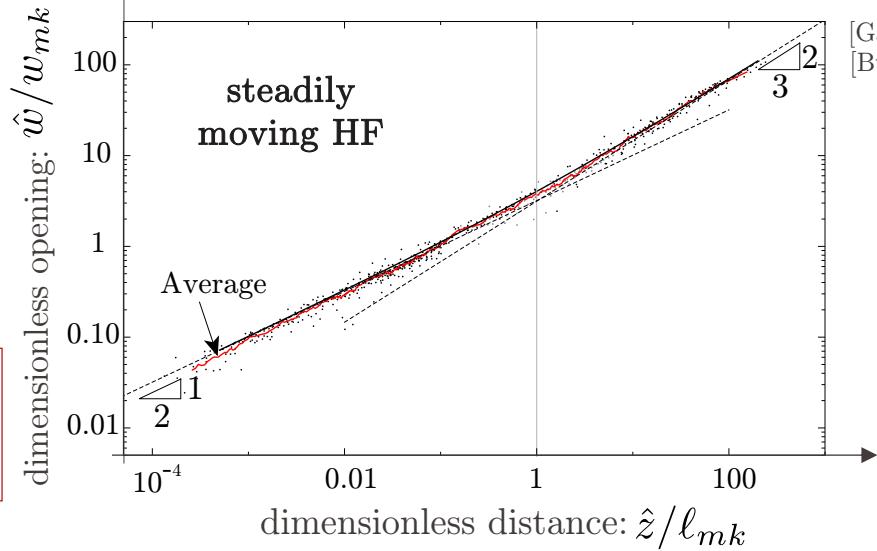
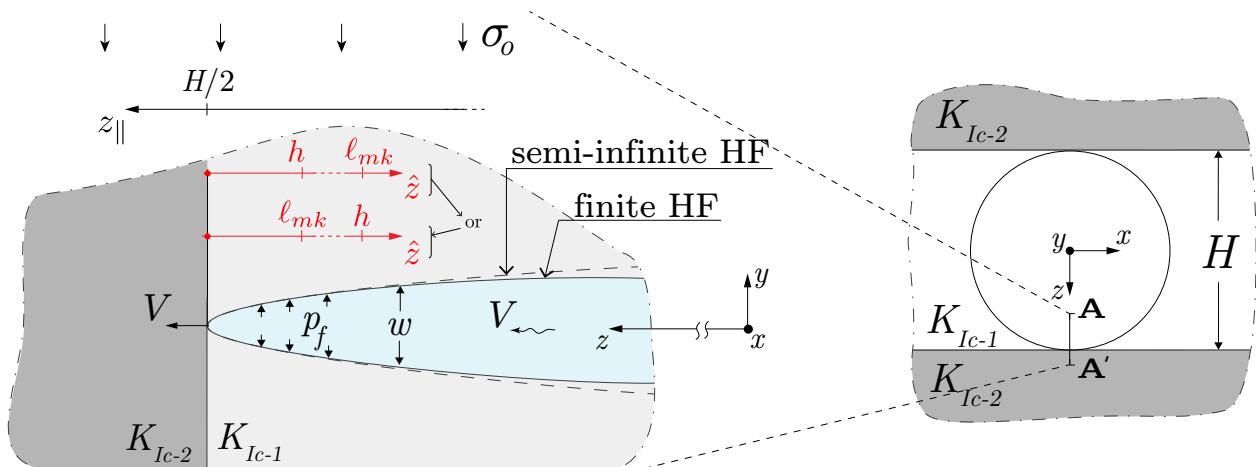
Minimum toughness ratio that stops the fracture

STATE 2:

$$V^+ = 0$$

ANSWER

$$\underbrace{12\mu V^- \int_0^h \frac{1}{w(\hat{z})} d\hat{z}}_{\text{viscous dissipation}} + G_{c1} = G_{c2}$$



[Garagash *et al.*, 2011]
 [Bunger *et al.*, 2008]

Minimum toughness ratio that stops the fracture

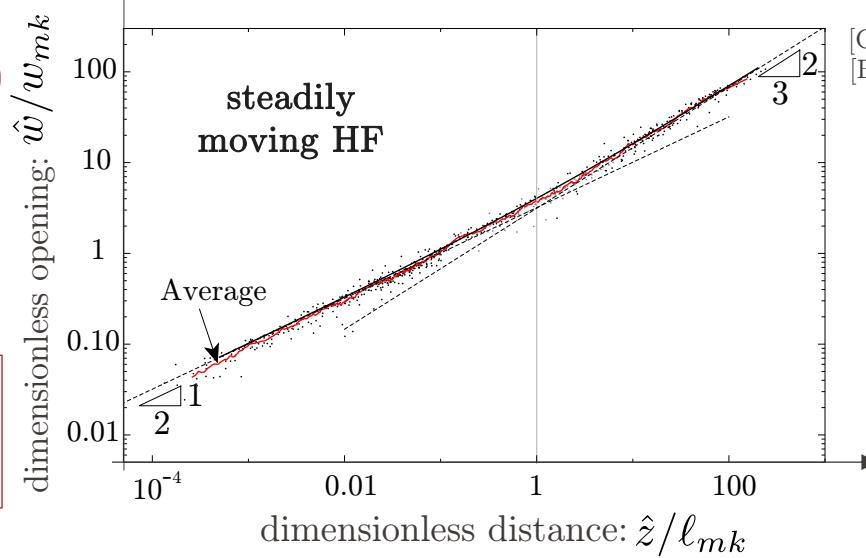
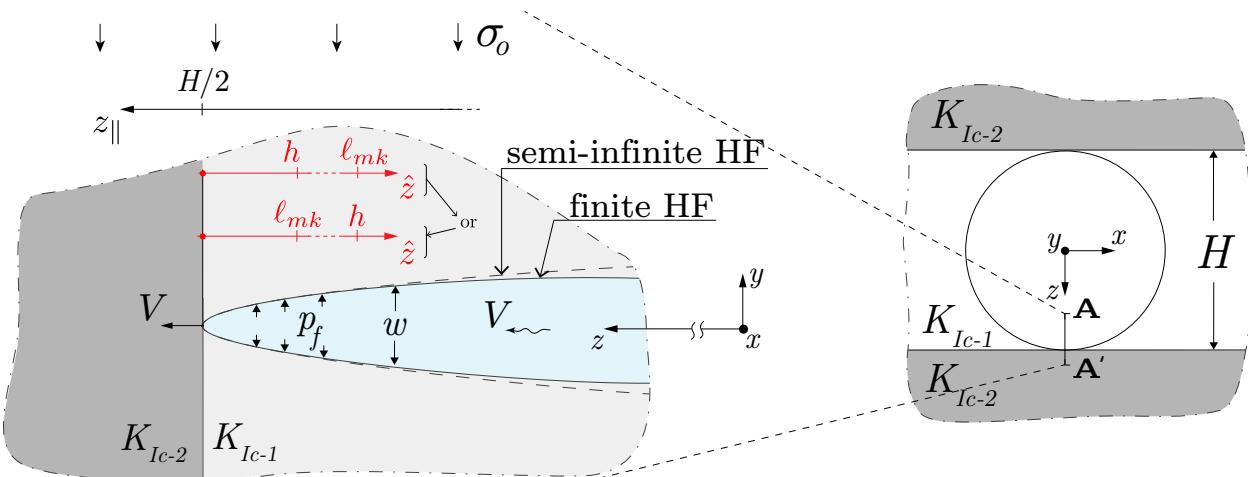
[Garagash, 2009]

$$\bar{\mathcal{K}} = K_{Ic-1} \left(\frac{H}{E'^3 Q_o \mu'} \right)^{1/4}$$

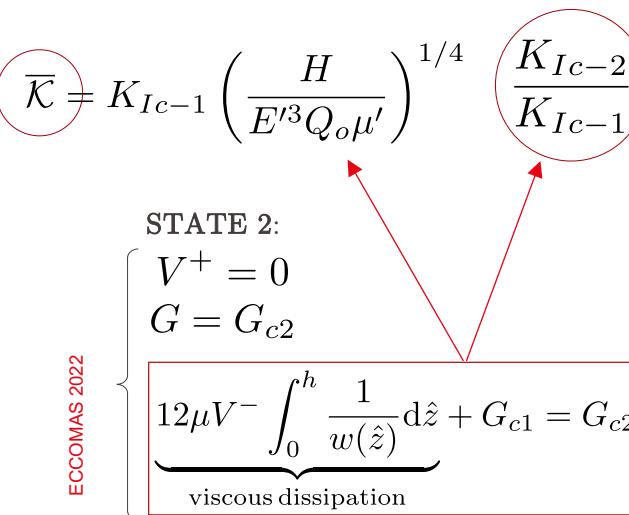
STATE 2:
 $V^+ = 0$
 $G = G_{c2}$

$$12\mu V^- \int_0^h \frac{1}{w(\hat{z})} d\hat{z} + G_{c1} = G_{c2}$$

viscous dissipation



Minimum toughness ratio that stops the fracture

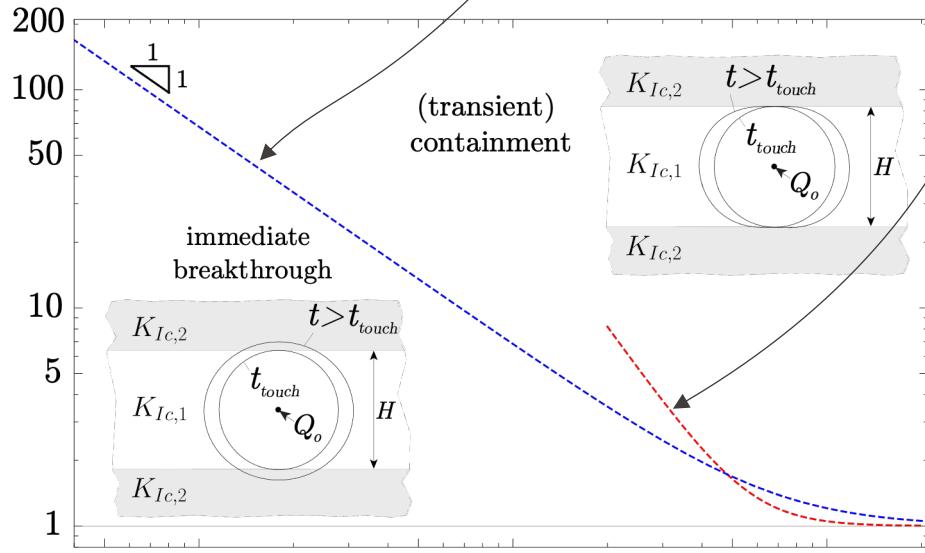


$$\frac{K_{Ic-2}}{K_{Ic-1}} (\bar{\mathcal{K}}) \text{ such that } V^+ = 0$$

$$\frac{K_{Ic-2}}{K_{Ic-1}}$$

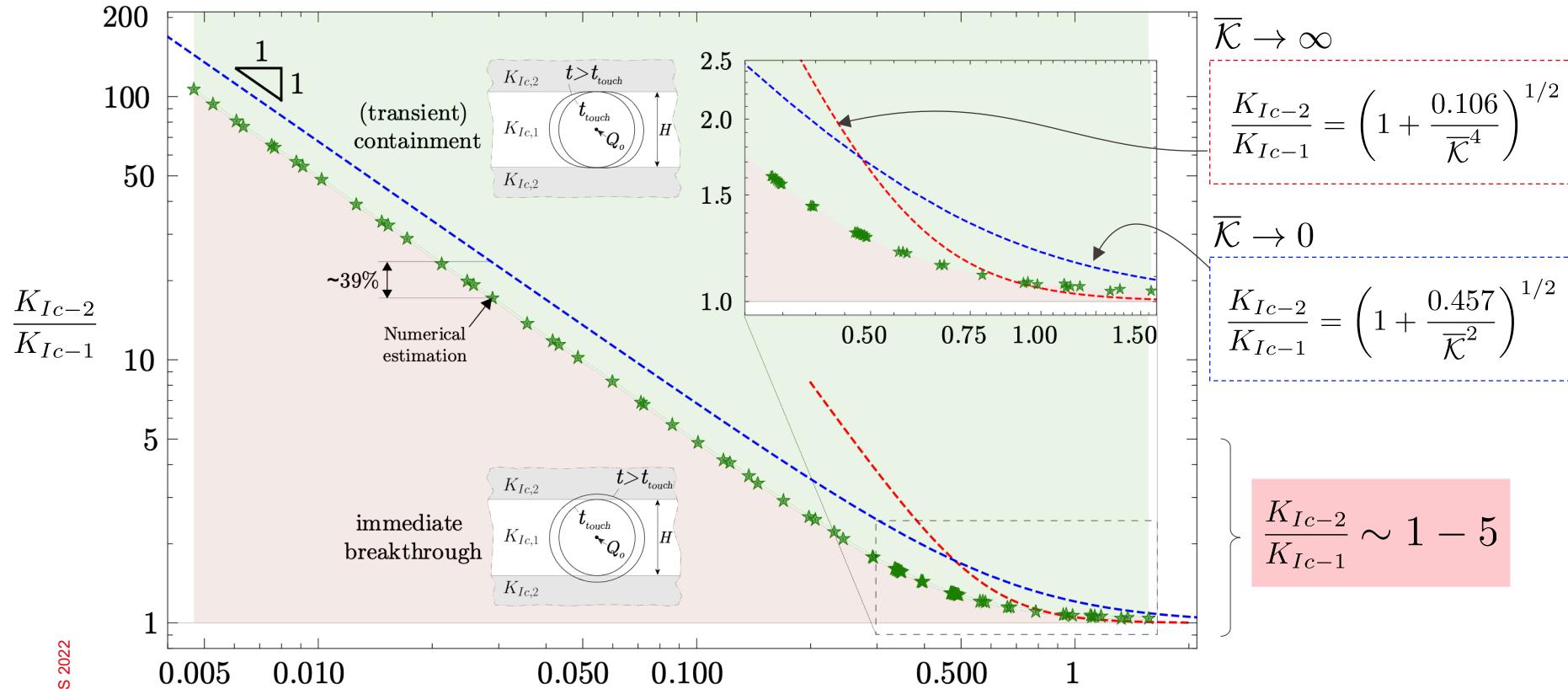
↑

$$\begin{aligned} \bar{\mathcal{K}} &\rightarrow \infty & \frac{K_{Ic-2}}{K_{Ic-1}} &= \left(1 + \frac{0.106}{\bar{\mathcal{K}}^4}\right)^{1/2} \\ \bar{\mathcal{K}} &\rightarrow 0 & \frac{K_{Ic-2}}{K_{Ic-1}} &= \left(1 + \frac{0.457}{\bar{\mathcal{K}}^2}\right)^{1/2} \end{aligned}$$



$$\bar{\mathcal{K}} \propto \frac{\text{fracture en. dissipation}}{\text{viscous en. dissipation}}$$

Numerical results using a full 3D-Planar hydraulic fracture simulator



$$\bar{\mathcal{K}} = K_{Ic-1} \left(\frac{H}{E'^3 Q_o \mu'} \right)^{1/4} \propto \frac{\text{fracture en. dissipation}}{\text{viscous en. dissipation}}$$

Conclusions

- Two dimensionless number defines if the fracture breakthroughs immediately:

$$\frac{K_{Ic-2}}{K_{Ic-1}} \quad \bar{\kappa} = K_{Ic-1} \left(\frac{H}{E'^3 Q_o \mu'} \right)^{1/4}$$

- The importance of fracture toughness contrast observed in nature is not sufficient per se to identify when, if immediate, breakthrough happen.
- The dominant dissipation mechanism determines whether an hydraulic fracture is immediately stopped by a toughness heterogeneity.

Acknowledgements:



Green River Formation
in eastern Utah, USA

Photo by M. M. McGlue

