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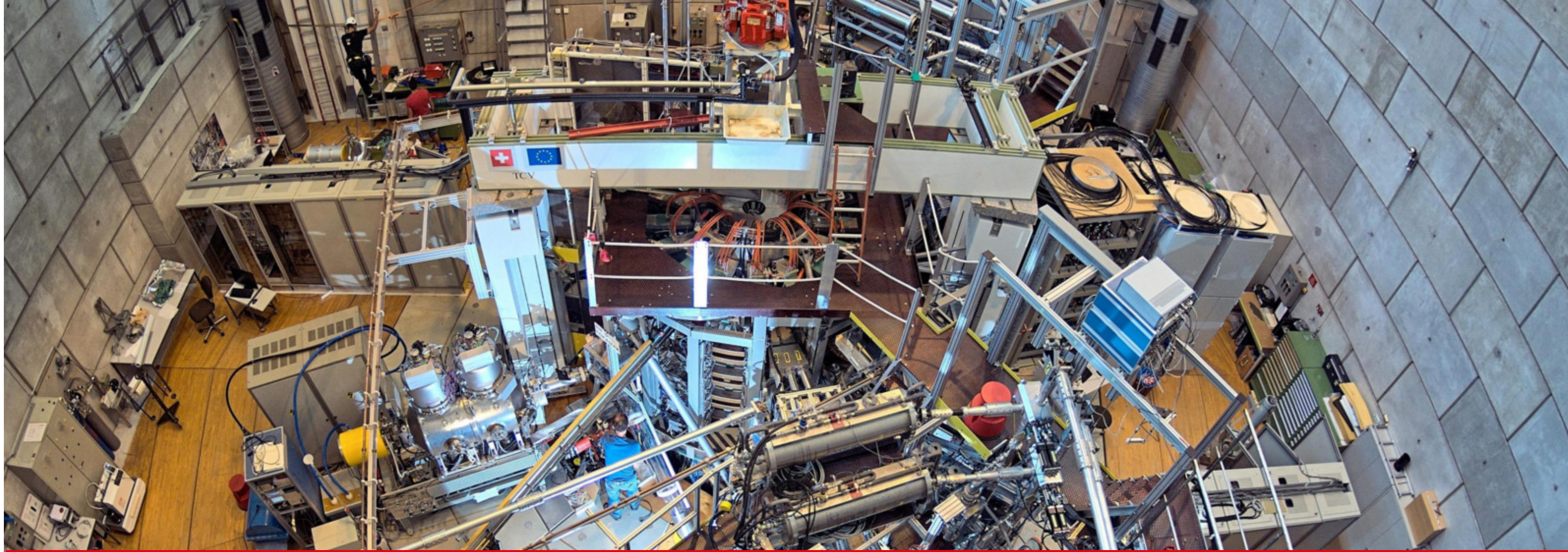

A Gyrokinetic Moment-Based Approach for the Simulation of the Boundary Plasmas in Fusion Devices

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Challenges of Simulating the Tokamak Boundary Plasmas

- **Boundary region** (Edge + Scrape-off-Layer) controls the performance of fusion devices:
  - H-L transition, pedestal, ELMs, …
  - heat and particle exhaust,
  - Impurity removal and plasma refuelling
  - ….
- **Collisions** (usually small in the core) are important and affect the turbulent transport and the energy confinement
- Boundary region characterised by **different plasma collisionality regimes**:

ITER: $T_e \sim 10 - 10^4$ eV and $n \sim 10^{18} - 10^{20}$ m^{-3} $\Rightarrow \lambda_{mpf}/R \sim \dfrac{CT_e^2}{n_e R} \sim 10^{-1} - 10^3$

- Two approaches to model the turbulent plasma dynamics in the boundary region:
  - **Drift-reduced (DK) fluid modelling** (lowest-order moments, less expensive, $k_\perp \rho_i \ll 1$ and $\lambda_{mfp}/L_\parallel \ll 1$, no kinetic effect)
  - **Gyrokinetic (GK) modelling** (expensive, $k_\perp \rho_i \sim 1$, kinetic effects)

Main Present approach to solve the GK Boundary Model

- The GK Boltzmann equation:

GK Collision Operator

$$\frac{\partial}{\partial t}F_a + \dot{\boldsymbol{R}} \cdot \frac{\partial}{\partial \boldsymbol{R}}F_a + \dot{v}_\parallel \frac{\partial}{\partial v_\parallel}F_a = \sum_b \langle C_{ab} \rangle \qquad \text{with } F_a = F_a(\mathbf{R}, \mu, v_\parallel, t)$$

- Finite difference schemes in $(\mu, v_\parallel)$ (e.g., GENE [Jenko, 2000], GT5D [Idomura, 2007], GKW [Peeters, 2009]) or in $(\lambda = v_\perp^2/v^2, v^2)$ (e.g., GS2 [Kotschenreuther, 1995] and GYRO [Candy, 2003]).
- PIC (Particle-in-cell) methods (e.g., ORB5 [Lanti, 2020], XGC1[Ku, 2016] and GTC [Lin, 2000])
- Semi-Lagrangian method (e.g., GYSELA [Grandgirard, 2006])
- Pseudo-Spectral method using velocity-space polynomials in $(\xi = v_\parallel/v, v)$ (e.g., CGYRO [Candy, 2016])
- Discontinuous Galerkin method (e.g., Gykell [Mandell, 2020])
- And others…

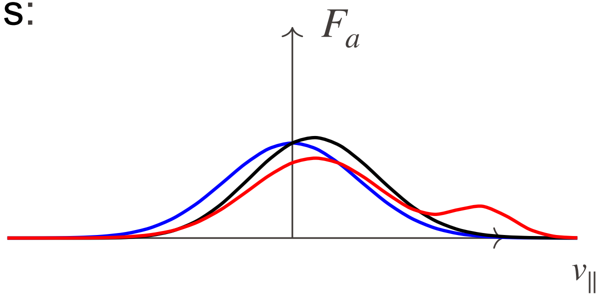
We aim to develop a **GK Model using a Moment Approach**

Outline

- The Moment Approach to the GK Boundary Model
- The GK Collision Operator
- Code Implementation/Validation
- Conclusions

The new Moment Approach to the GK Boundary Model

Expansion of F_a on a **Hermite-Laguerre polynomials** basis:



Gyro-Moment of F_a

$$F_a = \sum_p \sum_j \boxed{N_a^{pj}(\boldsymbol{R}, t)} \frac{H_p(v_\|) L_j(\mu)}{\sqrt{2^p p!}} F_{Ma}$$

Project of the GK Eq. on the same basis

$$\int dv_\| d\mu d\theta H_p(v_\|) L_j(\mu) \, (\text{GK Equ.})$$

Gyro-Moment Hierarchy Equation + self-consistent GK field equations

$$\frac{d}{dt} N_a^{pj} + \cdots = \sum_b C_{ab}^{pj}$$

B. J. Frei *et al.*, JPP **86**, 905860205 (2020)

Nonlinear GK Coulomb (Landau) Collision Operator

- In $(\mathbf{x}, \mathbf{v})$ phase-space, **Coulomb (Landau) collision** operator

$$C_{ab}[f_a, f_b] = C_{ab}(\boldsymbol{x}, \boldsymbol{v}) = \frac{\partial}{\partial \boldsymbol{v}} \cdot \left[\boldsymbol{A}_{ab} f_a + \frac{\partial}{\partial \boldsymbol{v}} \cdot (\boldsymbol{D}_{ab} f_b) \right]$$

$$\boldsymbol{A}_{ab} = -\left(1 + \frac{m_a}{m_b}\right) \frac{\partial}{\partial \boldsymbol{v}} \cdot \boldsymbol{D}_{ab}$$

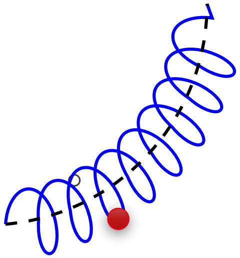
$$\boldsymbol{D}_{ab} = -\nu_{ab} \frac{\partial^2}{\partial \boldsymbol{v} \partial \boldsymbol{v}} G_b$$

$$\nabla_v^2 G_b = H_b$$

$$H_b = \int d\boldsymbol{v}' \frac{f_b(\boldsymbol{v}')}{|\boldsymbol{v}' - \boldsymbol{v}|}$$

$$G_b = \int d\boldsymbol{v}' f_b(\boldsymbol{v}') |\boldsymbol{v}' - \boldsymbol{v}|,$$

- From $(\mathbf{x}, \mathbf{v})$ to gyrocenter phase-space $(\mathbf{R}, \mu, v_\parallel, t)$



spatial and velocity space coordinate mixing

$$\boldsymbol{x} = \boldsymbol{R} + \boldsymbol{\rho}_a(\mu, \theta)$$

Gyro-average

$$\langle C_{ab} \rangle (\boldsymbol{R}, \mu, v_\parallel) = \frac{1}{2\pi} \int d\theta C_{ab}$$

Finite Larmor radius (FLR) in GK collisions (important for $k_\perp \rho_{a,b} \sim 1$ and for energetic particles)

Nonlinear GK Coulomb Operator in the Gyro-Moment Approach

$$C_{ab}^{pj} = \int d\boldsymbol{v} H_p(v_\parallel) L_j(\mu) \langle C_{ab} \rangle$$

- Perform the **gyro-average** of C_{ab} using a **spherical harmonic multipole** expansion
- Multiply by $H_p(v_\parallel)L_j(\mu)$, integration in (ξ, v) coordinates using a **basis transformation** $(T^{-1})_{pq}^{rsm}$

Full-F Nonlinear GK Coulomb Collision Operator

$$C_{ab}^{pj} = \sum_{\boldsymbol{k},\boldsymbol{k}'} \sum_{l=0}^{\infty} \sum_{m=-l}^{m=l} \sum_{r,s,p,q,\dots} \nu_{ab\dots}^{lpj\dots} \left(T^{-1}\right)_{pq}^{rsm} \dots \boxed{\mathcal{K}_s(k_\perp \rho_a)\mathcal{K}_q(k_\perp \rho_b) N_a^{rs}(\boldsymbol{k}) N_b^{pq}(\boldsymbol{k}')}$$

Numerical coefficients

Convolution between gyro-moments

- Valid for arbitrary $k_\perp$, and m_a/m_b , T_a/T_b ratios

Jorge R., B. J. Frei and P. Ricci, JPP **85**, 905850604 (2019)

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First Numerical Investigations: linearized GK Collision Operator

- Assuming $F_a = F_{aM} + f_a \quad (f_a \ll F_{aM})$

$$C_{ab} \simeq C_{ab}^T + C_{ab}^F$$

Test component

$$C_{ab}^T = C_{ab}[f_a, f_{bM}]$$

Field component

$$C_{ab}^F = C_{ab}[f_{aM}, f_b]$$

- Study the **effects of GK collisions** (i.e. FLR effects) and **collision operator models** (w.r.t to Coulomb) on linear modes

- Because numerically and analytically challenging, **simplified/*ad hoc*** linearized GK (or $k_\perp \rho_{a,b} \ll 1$) collision operator models (e.g. Sugama operator) are often considered

Catto and Tsang, Phys. of Fluids, **1977**

Abel I. G. *et al.,* Phys. Plasmas, **2008**

Sugama H. *et al.*, Phys. Plasmas, **2009**

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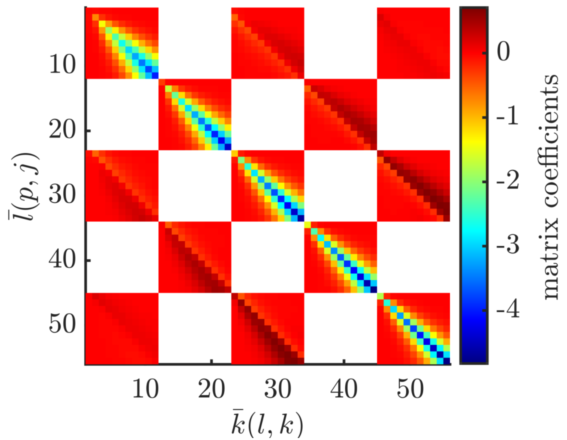
Numerical Implementation of Lin. GK Coulomb Operator

- Linearized GK **Coulomb** Collision Operator:

Numerical coefficients

$$C_{ab}^{Tpj} = \sum_{l=0}^{\infty} \sum_{m=-l}^{m=l} \sum_{r,s,p,q,\dots} \nu_{ab\dots}^{Tlpj} \dots \left(T^{-1}\right)_{pq}^{rsm} \boxed{K_s(b_a) N_a^{rs}}$$

- Similar expressions for **simplified/*ad hoc* operators** (e.g. Sugama operator)
- Large number of sums of large number to perform numerically (round-off error, i.e. $(T^{-1})_{50,50}^{50,50,50} \sim 10^{1000}$)
- Arbitrary-precision arithmetic library to avoid overflows and numerical loss precision



B. J. Frei *et al., arXiv:2104.11480 (2021)*

Linearized Gyro-Moment Hierarchy Equation

Landau Damping — Particle trapping — Magnetic Drifts

$$\frac{\partial}{\partial t}\mathbf{N}_a + \mathbf{H}_a \cdot \nabla_\parallel \mathbf{N}_a + (\mathbf{M}_a \cdot \mathbf{N}_a)\nabla_\parallel \ln B + \mathbf{D}_a \cdot \mathbf{N}_a$$

$$= \mathbf{S}_\phi \phi + \mathbf{S}_\psi \psi + \sum_b \left(\mathbf{C}_{ab}^T \cdot \mathbf{N}_a + \mathbf{C}_{ab}^F \cdot \mathbf{N}_b\right)$$

Background Gradient drive

Linearized GK Collision Operators

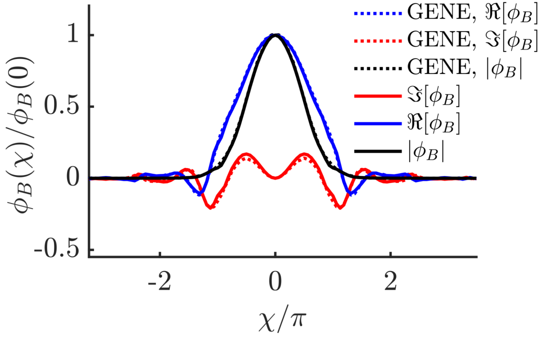
With $\delta \mathbf{E}_{es} = -\nabla\phi, \delta\mathbf{B}_\perp \simeq -\mathbf{b}\times\nabla\psi$ and $\mathbf{N}_a = [N_a^{00}, N_a^{01}, \dots N_a^{10}, N_a^{11} \dots N_a^{PJ}]^T$

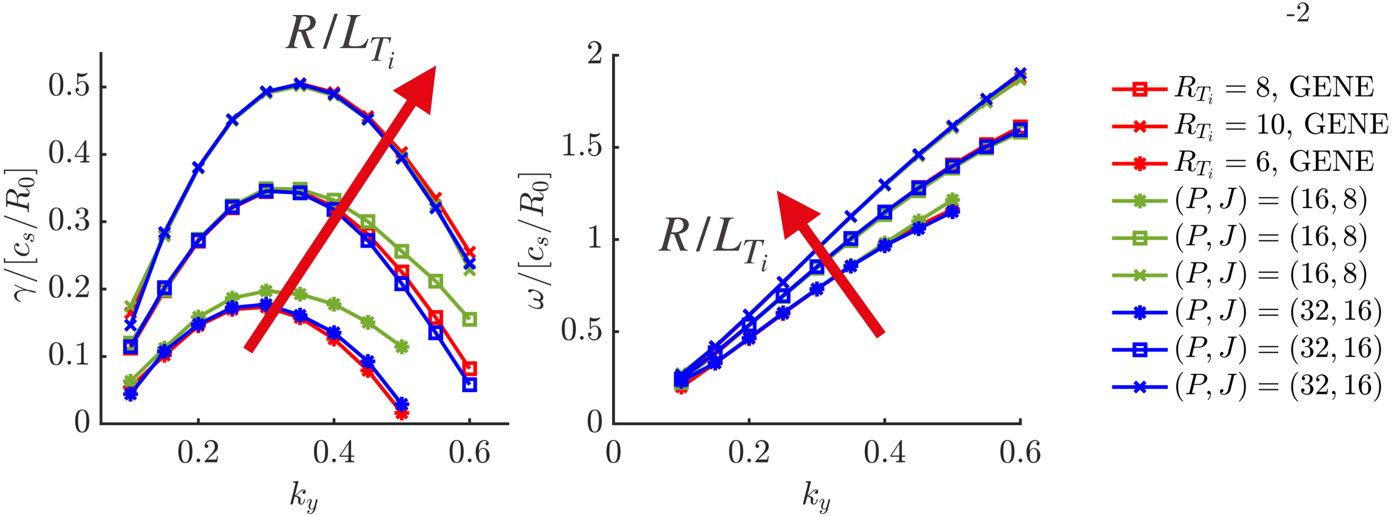
- Closure at arbitrary collisionality available: $P = P(\nu, k_\parallel), J = J(\nu, k_\perp)$
- Implemented in a flux-tube code **MOLI-X**

Cyclone Base Case in agreement with GENE

- **Collisionless** ITG with **adiabatic electron**:
  - $\mathbf{B} = B_0 \nabla x \times \nabla y$ and $\mathbf{k}_\perp = k_x \nabla x + k_y \nabla y$

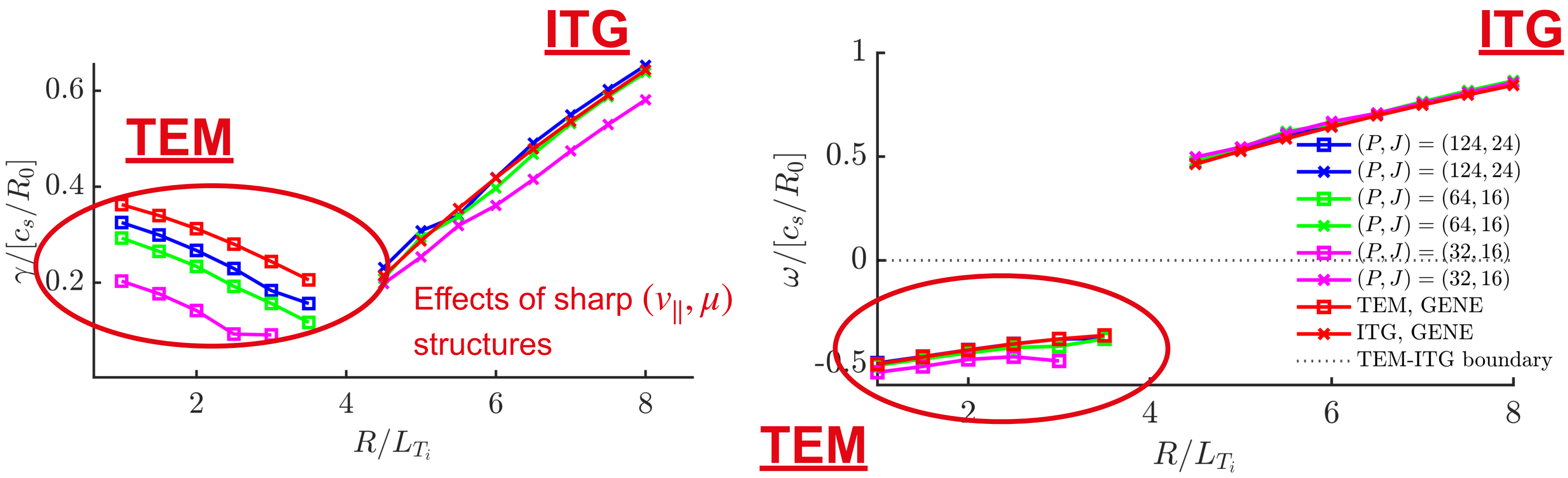
$$\frac{R}{L_{Ti}} \simeq -R\partial_x \ln T_i$$





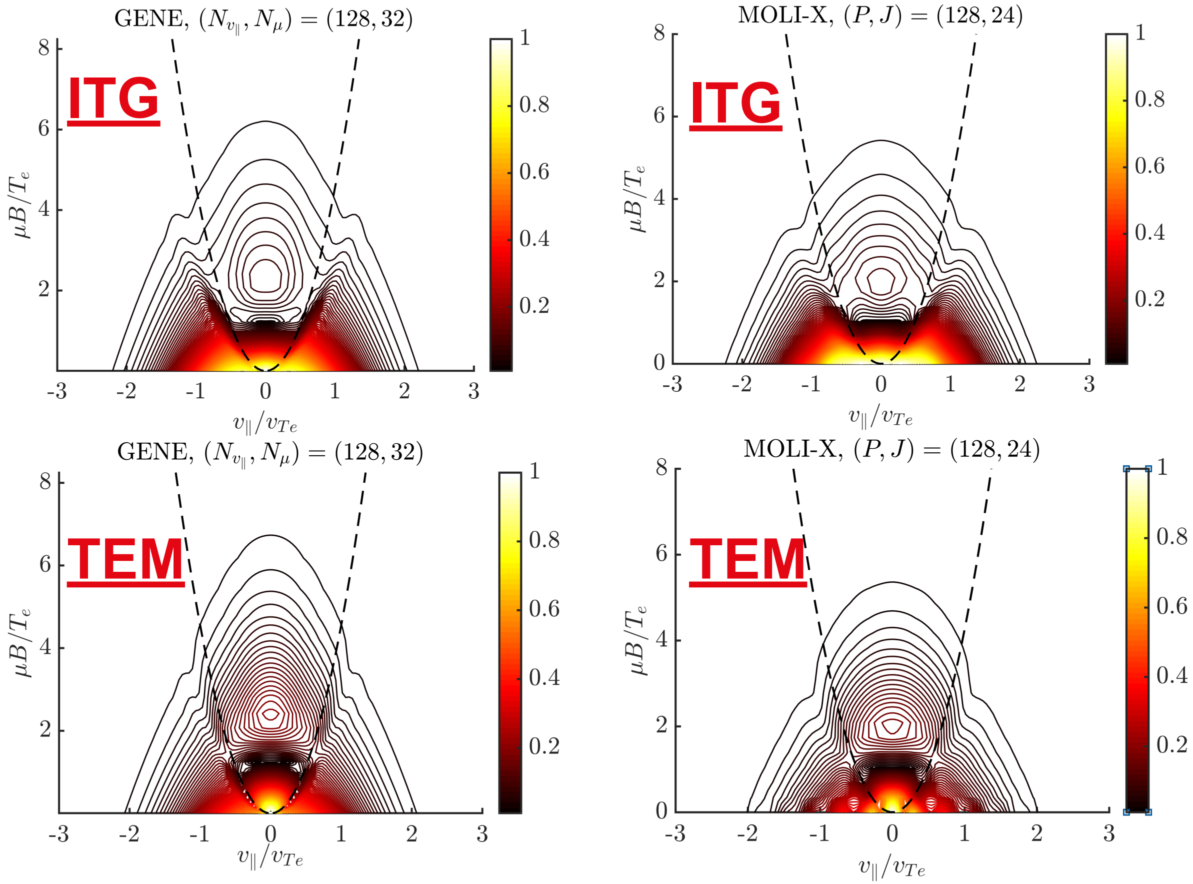
Collisionless Transition TEM-ITG in agreement with GENE

- **Trapped electron mode** (TEM) and **Ion temperature gradient** (ITG) are main candidate to explain anomalous turbulent transport; propagate in the electron ($\omega < 0$) and ion ($\omega > 0$) diamagnetic direction, respectively



TEM & ITG Velocity-Space Structures in agreement with GENE

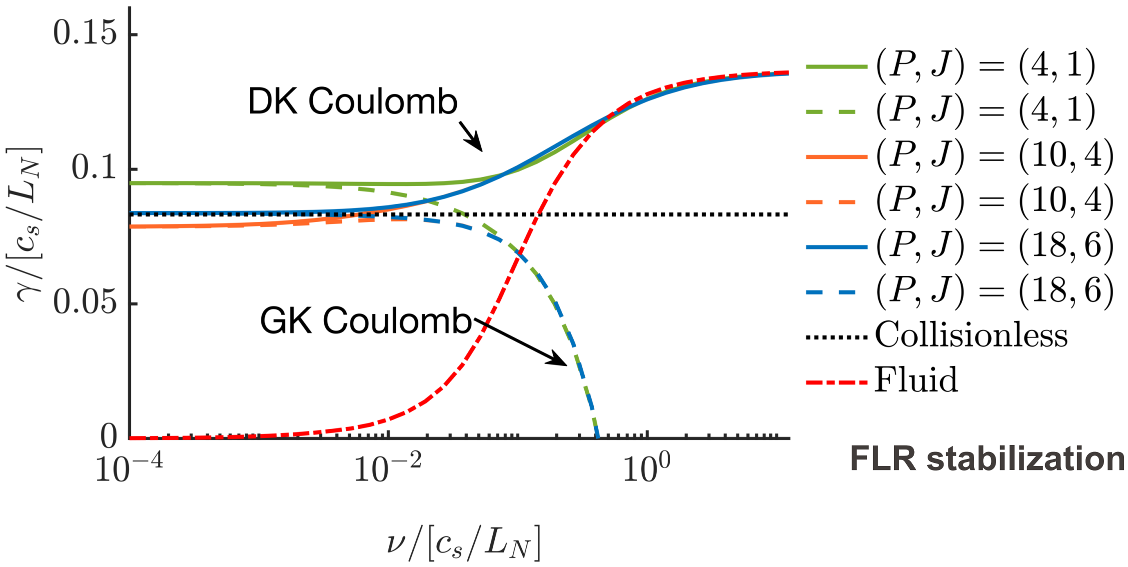
- Collisionless **normalized electron perturbed distribution function** g_e at the outer midplane ($k_x = 0$)
- Ultimately, **collisions smear out velocity-space structures** $\Rightarrow$ fewer gyro-moment required



B. J. Frei, PASC 2021

Collisional Slab Ion-Temperature Gradient (sITG)

- sITG with adiabatic electrons $k_\parallel = 0.1/L_N$ and $R/L_{T_i} = 3$

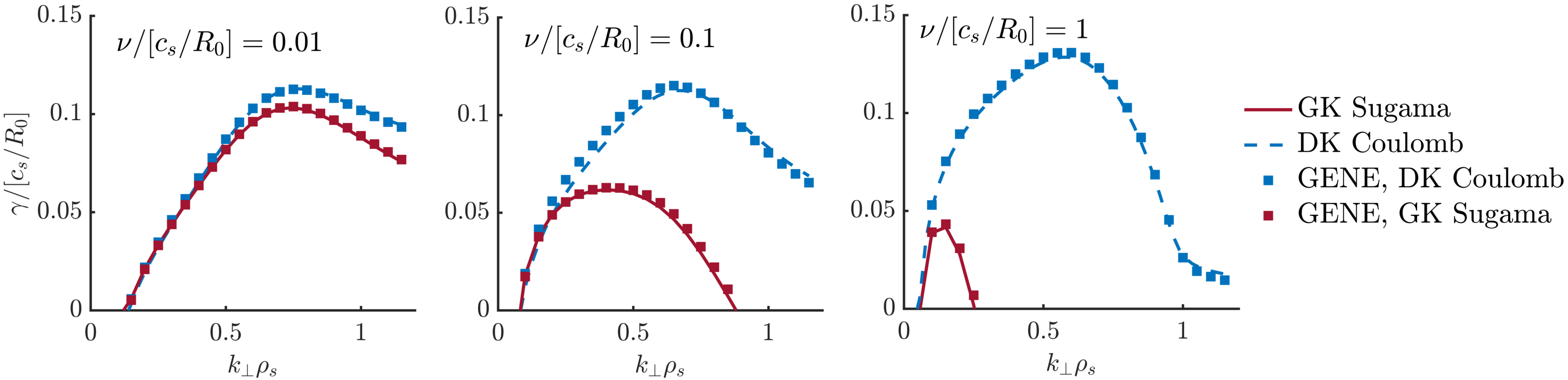


FLR stabilization by GK Coulomb

- Collisionless limit retrieved with $(P, J) \simeq (18{,}6)$ when $\nu \ll 1$
- <u>**Number of needed gyro-moments is reduced with increasing collisionality**</u>

sITG: Collision Operators in agreement with GENE

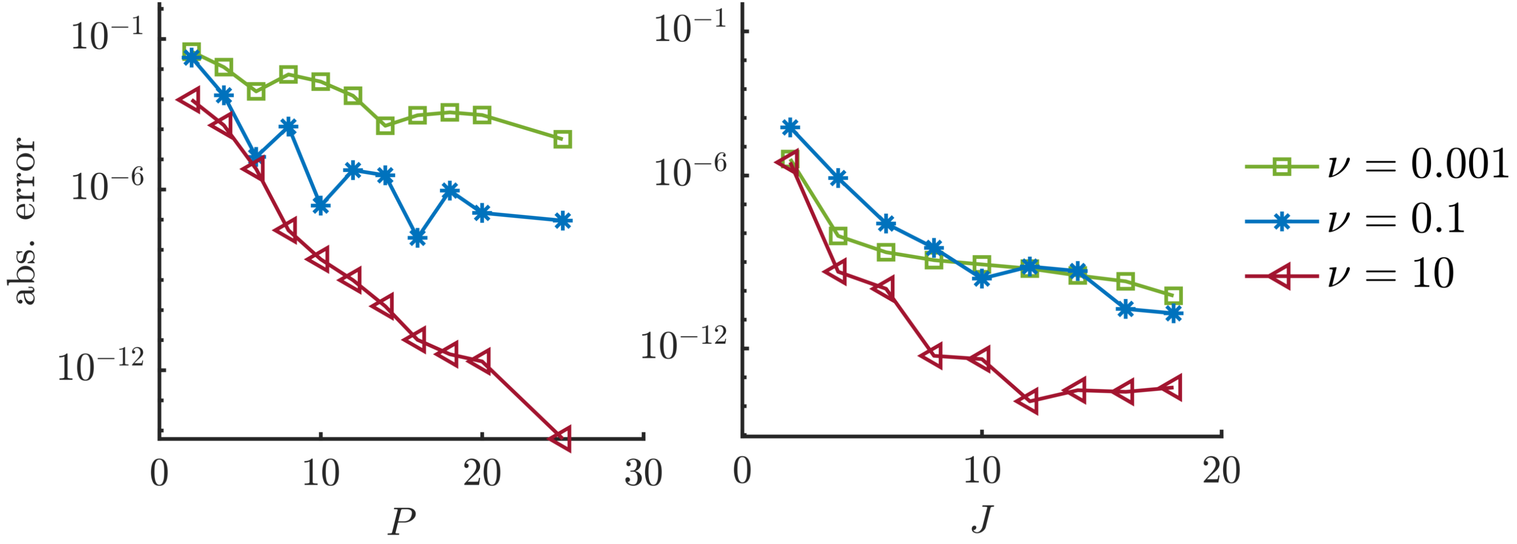
- Agreement between the **DK Coulomb** and **GK Sugama** collision operators with **GENE**



- Ability of the gyro-moment approach to **describe GK collisional effects**

sITG: Gyro-Moment Convergence rate

- Convergence rate **increases with collisionality** ν on sITG growth rate peak



- **Ideal** for tokamak boundary modelling since $\nu \sim nT^{-3/2}$

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First Nonlinear Simulation using the Gyro-Moment Approach

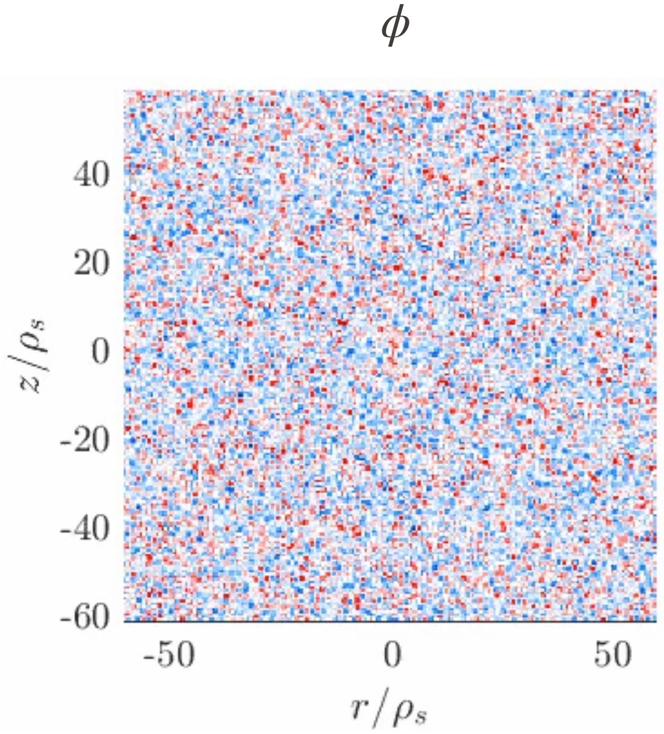
- **Z-Pinch** geometry **($k_{\parallel} = 0$)**, kinetic electrons

- Retain $\mathbf{E} \times \mathbf{B}$ nonlinearity

$$\sim \sum_{\boldsymbol{k},\boldsymbol{k}'} \frac{i\boldsymbol{E}(\boldsymbol{k}) \times \boldsymbol{B}}{B^2} \cdot \boldsymbol{k}' N_a^{pj}(\boldsymbol{k}') e^{i(\boldsymbol{k}+\boldsymbol{k}')\cdot \boldsymbol{R}}$$

- Collisional effects using **linearized GK collision operator**

- Cyclic, bursting behaviour of fluctuations, transport, and zonal flows



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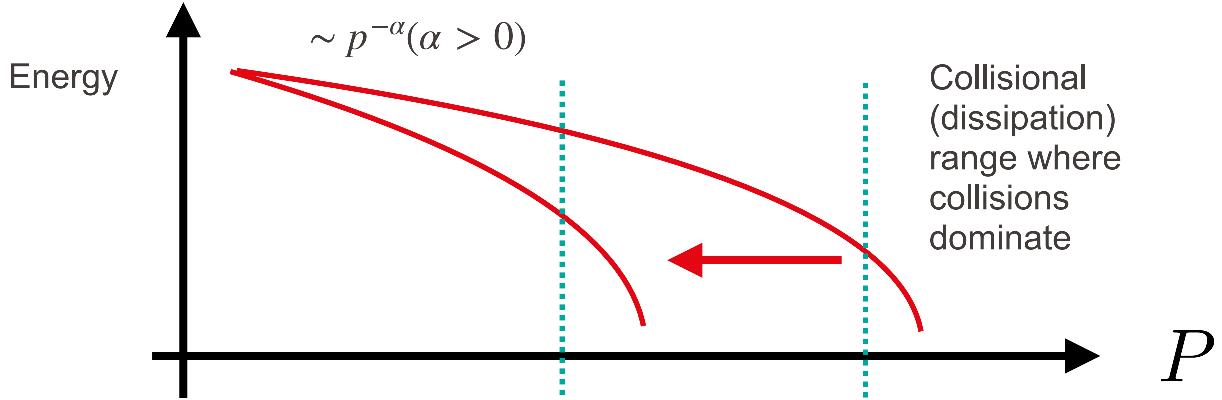
Conclusions

- A **new GK model** for the tokamak boundary is developed based on a **Hermite-Laguerre polynomials basis**
- A **nonlinear GK Coulomb collision operator** (valid for arbitrary mass and temperature ratios) is derived; a **linear version derived and numerically implemented**.
- The gyro-moment hierarchy expansion applied to different **simplified/*ad hoc*** collision operator models (e.g., Sugama)
- Numerical tests show the ability of the gyro-moment approach to describe key boundary linear modes (e.g., ITG, TEM, KBM and Zonal flow); systematic comparison with **GENE** (good agreement on complex mode frequency, mode structure and collision operators)
- Numerical efficiency as **the number of gyro-moments decreases with collisionality; ideal for the simulation of the boundary plasmas**
- Extend nonlinear simulation to 3D **realistic** geometry and **full-F**.

Thank you for your attention

Backup: Closures at Arbitrary Collisionality are Available

In practice, solve $\partial_t \mathbf{N}_a = \ldots$ up to $(P_c, J_c) \ll \infty$. How to properly choose (P_c, J_c) ?



$$P_c \simeq P_c(\nu, k_\parallel)$$

$$J_c \simeq J_c(\nu, k_\perp)$$

- At arbitrary collisionality, an asymptotic semi-collisional closure; $\mathbf{H}_a, \mathbf{M}_a, \mathbf{D}_a \sim O(\sqrt{p}, \sqrt{j})$ and $\partial_t \mathbf{N}_a \ll \mathbf{C}_{ab}^T \cdot \mathbf{N}_a \sim \nu p^\alpha \mathbf{N}_a$ assuming $(P_c, J_c) \gg 1$,
- In the high-collisional limit, Chapman-Enskog procedure:

$$\mathbf{N}_a \simeq \mathbf{N}_a^{(0)} + \epsilon \mathbf{N}_a^{(1)} \quad \epsilon \sim \lambda_{mfp}/L_\parallel \quad \Rightarrow \quad Q_\parallel \simeq -\chi_\parallel \nabla_\parallel T_\parallel$$

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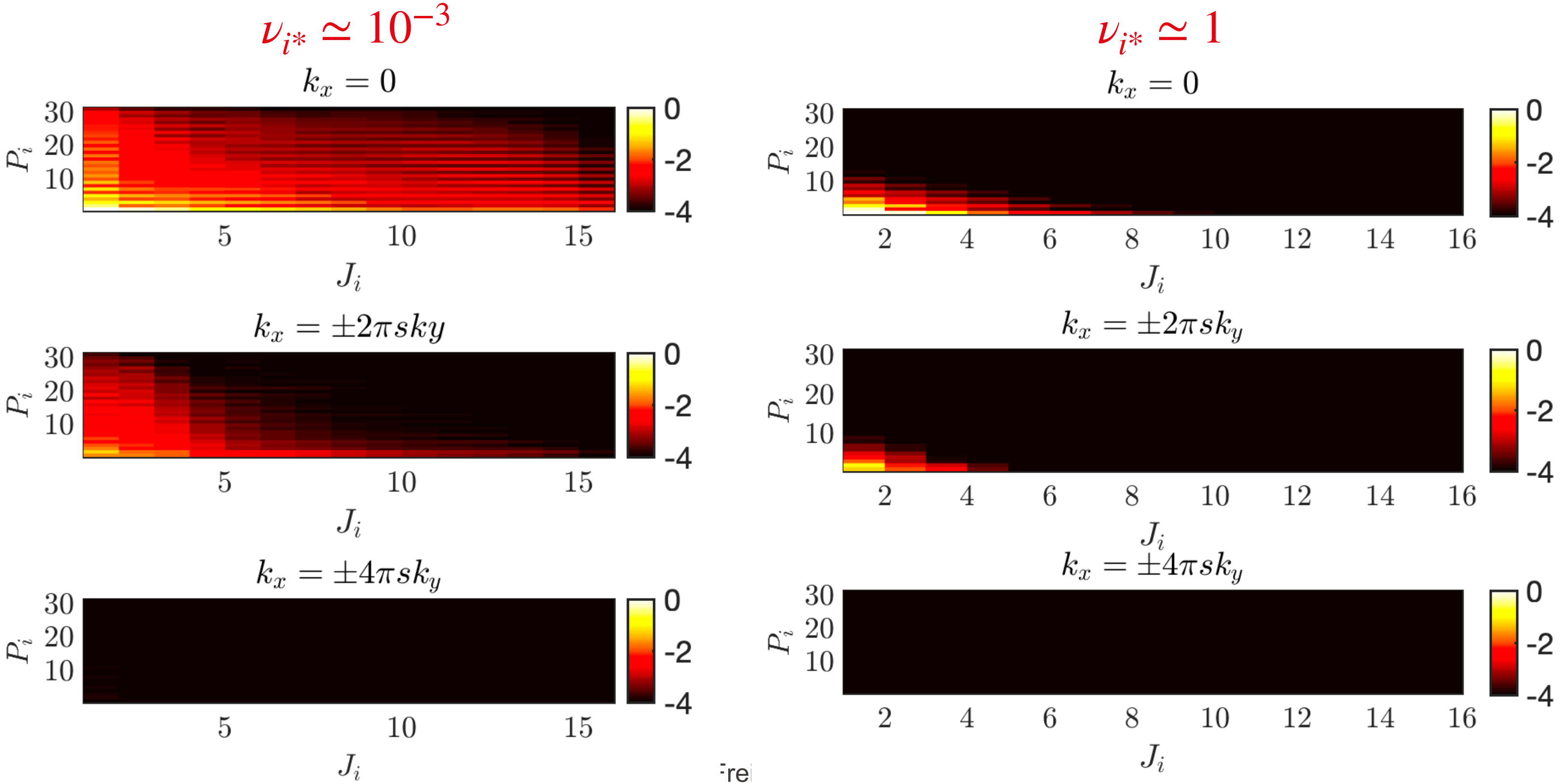
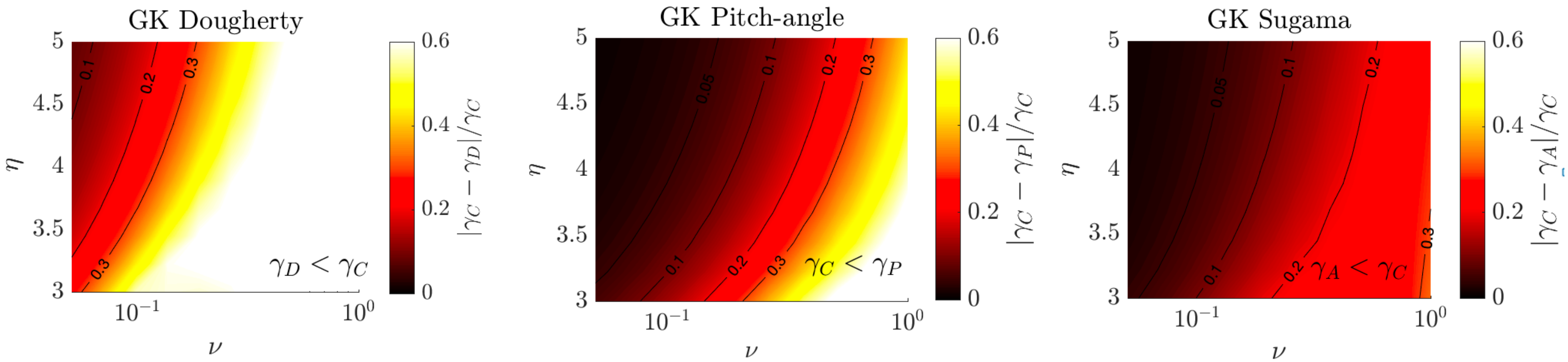
Backup: Velocity-space resolution

- Finite shear yields k_x modes coupling via the parallel to the magnetic field boundary condition



Backup: large differences between GK Coulomb and other models

- Deviation of ITG peak from GK Coulomb as a function of collisionality ν and temperature gradient η



- Enhanced stabilization by energy scatter (overestimation by the GK Pitch-angle)

- Deviation (up $20\ \%$) for GK Sugama compare to GK Coulomb

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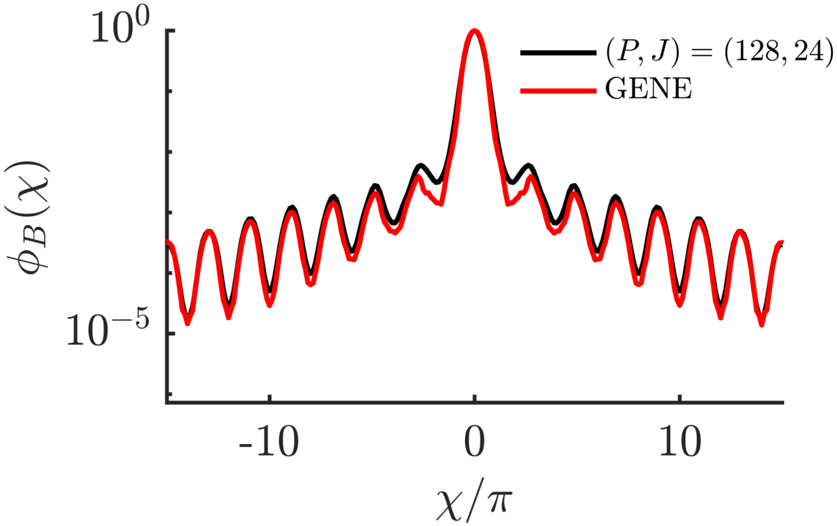
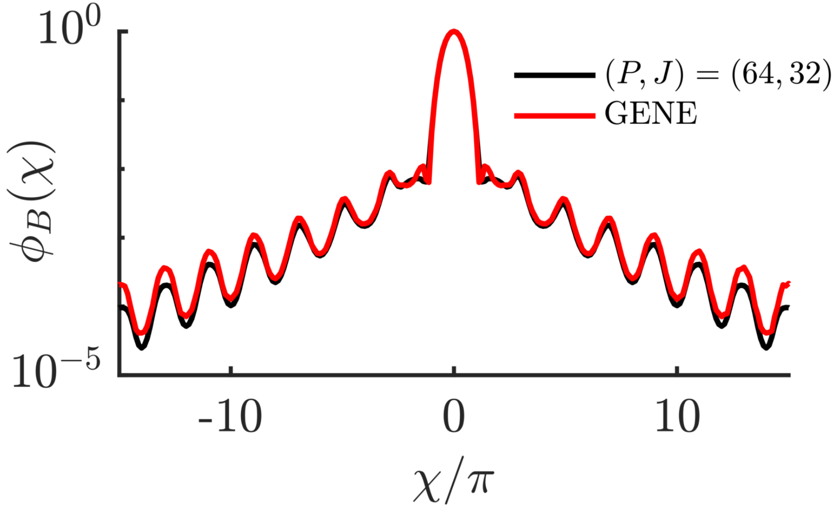
Backup: Ballooning Mode Structures in agreement with GENE

- **“Giant tails” in the ballooning mode** structure $\phi_B(\chi)$ because of kinetic electron response
- **Good agreement** with GENE

TEM

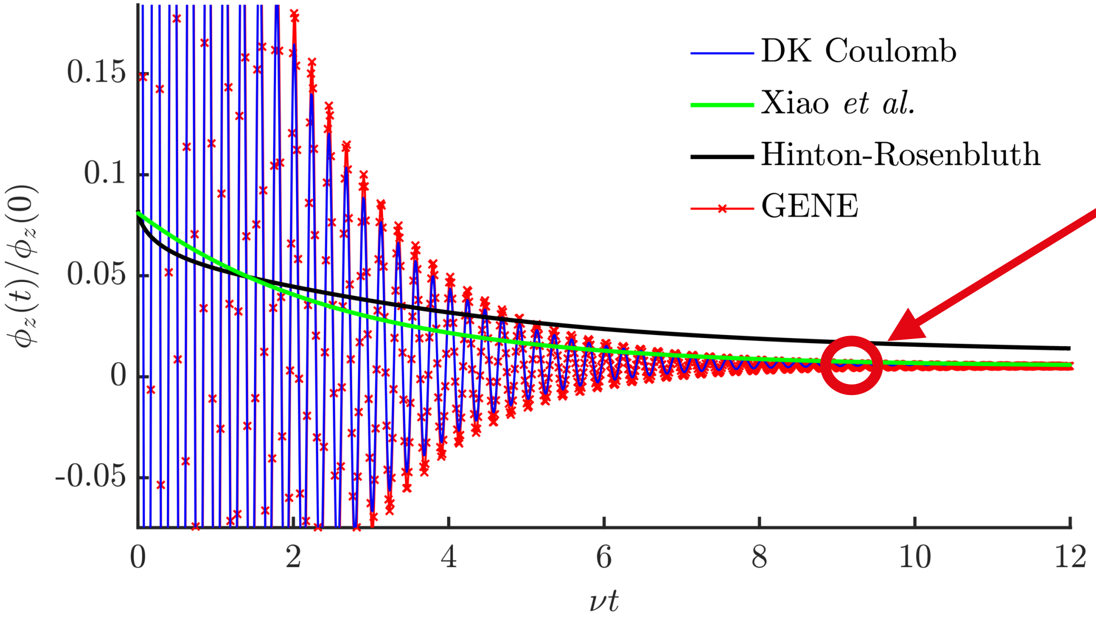


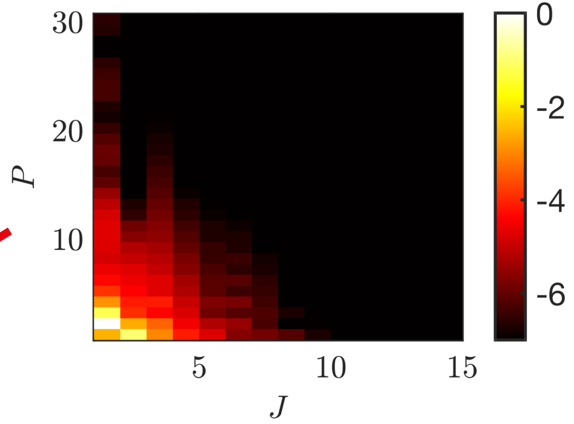
ITG



Backup: Collisional Zonal Flow (ZF) Residual Damping

- ZF residual is affect by collisions via particle detrapping
- Collisional damping of ZF with **DK Coulomb** ($k_x = 0.05$ and $\nu_{i*} \simeq 3.1$)
- **Good agreement with GENE, and agree with analytical predictions**





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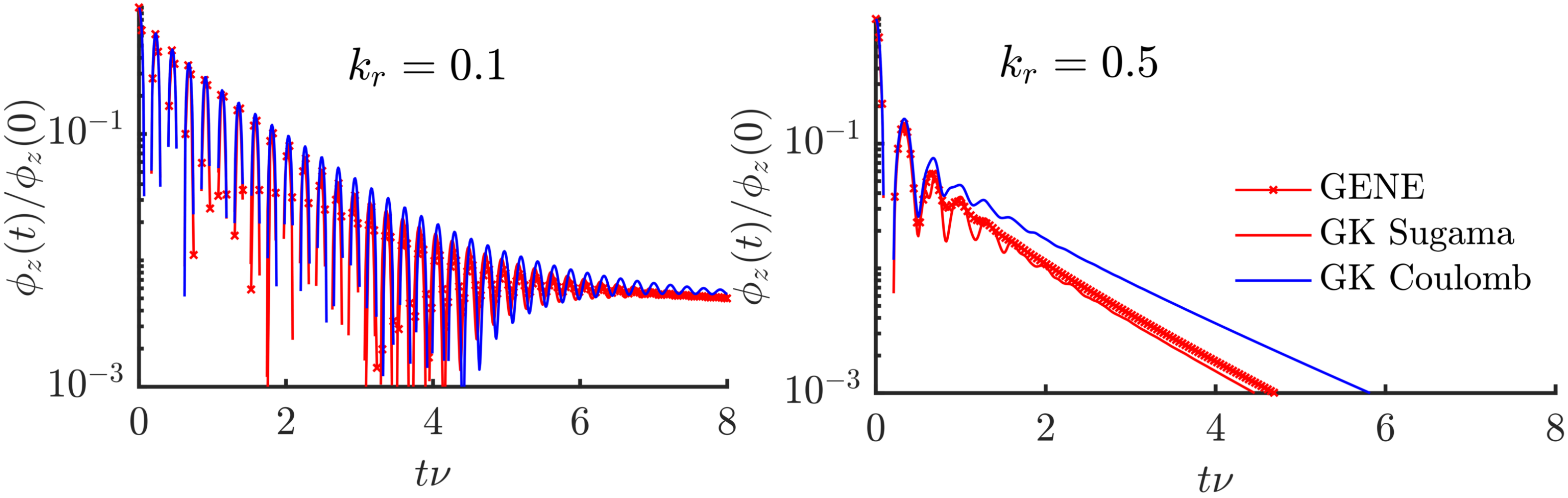
Backup: Collisional Zonal Flow (ZF) Residual Damping

- **FLR collisional ZF damping with GK Sugama and GK Coulomb when $k_x = 0.1$ and $k_x = 0.5$ at $\nu_{i*} \simeq 3.1$**
- **GK Sugama yields stronger ZF damping than GK Coulomb; good agreement with**



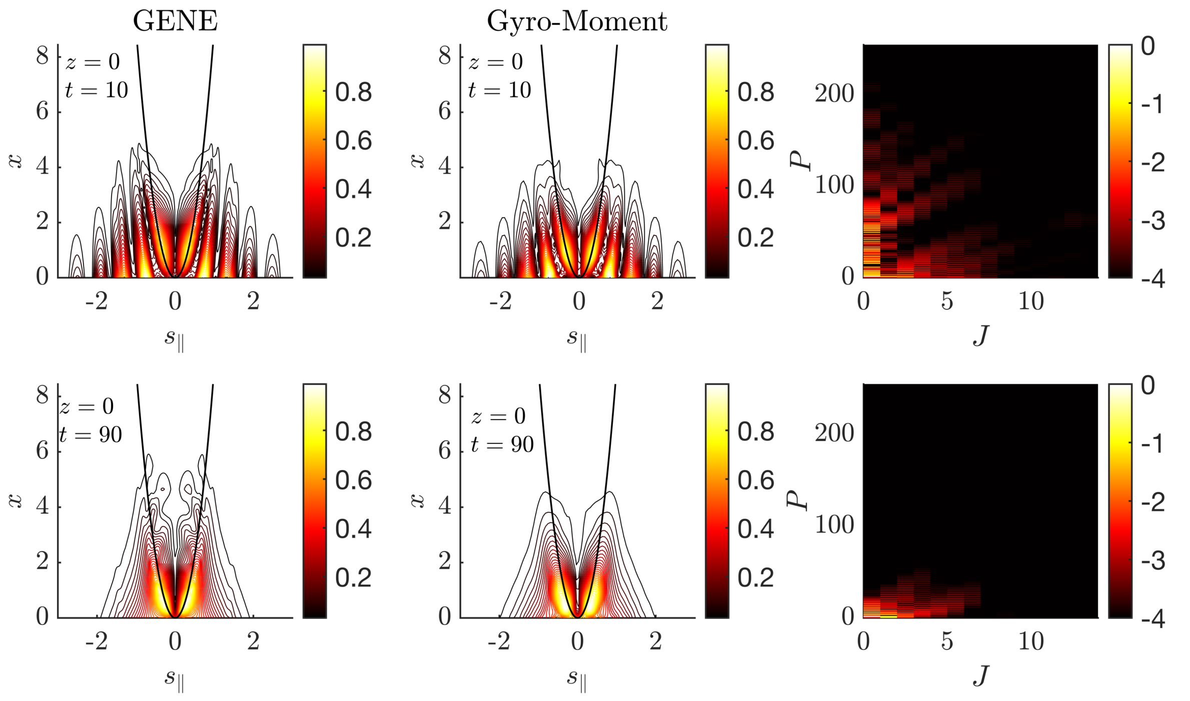
B. J. Frei *et al., arXiv:2104.11480 (2021) (accepted in JPP)*

Backup: Zonal Flow Velocity-Space Structure

- Zonal flow ($k_y = 0$) residual is important in regulation of turbulent transport
- Ballistic response of passing particle creates fine $v_\parallel$ structures ($k_{v_\parallel} \sim k_\parallel t$)

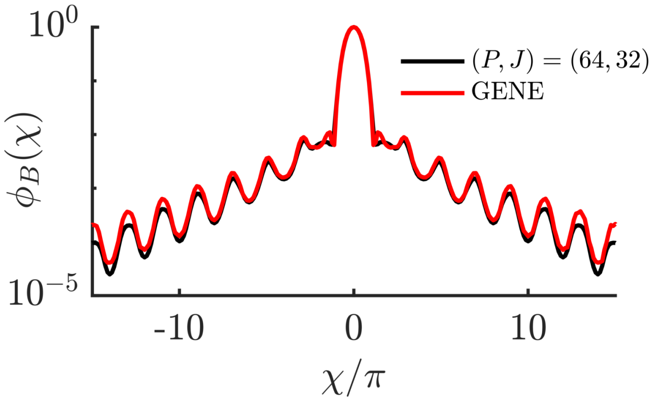
- **In GENE, $k_{v_\parallel} \sim N_{v_\parallel}/v_T$; in MOLIX-X $k_{v_\parallel} \sim \sqrt{p}/v_T$.**

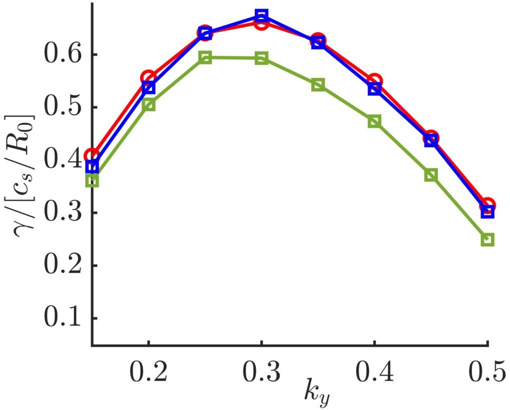
- **Velocity space structure comparison with GENE ($k_x = 0.05/\rho_s$ and $\nu_{i*} \simeq 0.03$)**

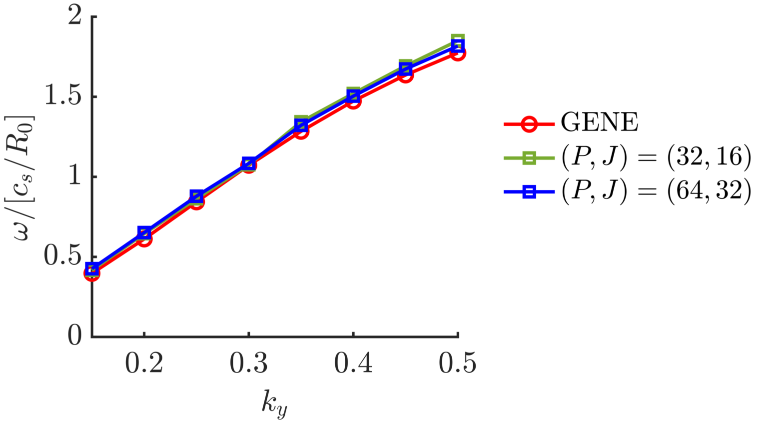


Backup: ITG with kinetic Electrons

- Adiabatic electron approximation fails near rational surface where $k_{\|} \simeq 0$
- Because of $m_e/m_i \ll 1$, larger number of gyro-moment to resolve **electron Landau channel**
- **Good agreement** with **collisionless** GENE runs







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