Stock-constrained truss design exploration through Combinatorial Equilibrium Modelling

Jan BRÜTTING^{a*}, Patrick Ole OHLBROCK^b, Pierluigi D'ACUNTO^b, Julian HOFER^b

Affiliations:

^a Structural Xploration Lab, Swiss Federal Institute of Technology Lausanne (EPFL),
Passage du Cardinal 13b, 1700 Fribourg, Switzerland

*Corresponding author: jan.bruetting@epfl.ch (J. Brütting)

^b Chair of Structural Design, Swiss Federal Institute of Technology Zürich (ETH), Stefano-Franscini-Platz 1,
8093 Zürich, Switzerland

Abstract

Reusing structural components has potential to reduce environmental impacts of building structures because it reduces new material use, energy consumption, and waste. When designing structures through reuse, available element characteristics become a design input. This paper presents a new computational workflow to design structures made of reused and new elements. The workflow combines Combinatorial Equilibrium Modelling, efficient Best-Fit heuristics, and Life Cycle Assessment to explore different design options in a user-interactive way and with almost real-time feedback. The method applicability is demonstrated by a realistic case study. Results show that structures combining reused and new elements have a significantly lower environmental impact than solutions made of new material only.

Keywords: Reuse, structural design, circular economy, cutting-stock problem, best-fit algorithm, Combinatorial Equilibrium Modelling, graphic statics

ORCID

Jan BRÜTTING: <https://orcid.org/0000-0002-8947-0126>

1 Introduction

The construction of load-bearing systems involves resource- and energy-intensive processes that contribute to a significant share of adverse environmental impacts worldwide¹⁻³. Conventionally, structural engineers follow two strategies to reduce the environmental impact of new structures: 1) minimizing material input through designing better-performing systems, and 2) employing low-impact materials such as timber. However, focusing only on these two approaches is not sufficient and improved design methods and construction techniques that help reduce environmental impacts must be developed⁴. An alternative circular economy⁵ strategy has recently reemerged in practice and research: the *reuse* of structural components over multiple service cycles. Reuse reduces resource use, energy consumption for material reprocessing, and waste^{6,7}. Therefore, the adoption of reuse strategies in structural design has considerable potential to reduce the environmental impact of load-bearing systems significantly^{8,9}. Yet, this adoption implies an important paradigm shift: rather than manufacturing components after the structural system has been designed, the system must be synthesized from a stock of available components with predefined cross-sections and lengths¹⁰.

This paper presents a computational workflow to design structures made of a combination of reused components and new elements. The proposed workflow links Combinatorial Equilibrium Modelling (CEM), Finite Element Analysis (FEA), and Best-Fit heuristics (Figure 1). The CEM is a user-interactive form-finding engine that allows designing mixed tension-compression structures in static equilibrium. For a given topology and a dominant load case, the CEM is used to explore multiple equilibrium solutions by varying structure member lengths and the member internal force distribution. CEM solutions are then converted into statically (in-)determinate trusses and analyzed via FEA to obtain member forces for additional load cases. Lastly, a Best-Fit heuristic optimizes the assignment of available stock and new elements to member positions subject to element length and capacity constraints. An integrated Life Cycle Assessment (LCA) allows setting structure environmental impact minimization as the objective function of the Best-Fit heuristic. As a result, a new method for stock-constrained structural design is established that offers fast feedback and user-interactive exploration of different structural configurations. This way, it is possible to understand in early design stages when reusing available components is beneficial in environmental impact terms and in which case new element production should be preferred.

Figure 1: Stock-constrained truss design workflow.

The paper is outlined as follows. Section 2 presents a literature review and a discussion of methods for early structural design exploration and stock-constrained design. Section 3 describes the developed method to design structures made of reused and new elements. Section 4 highlights the relevance and applicability of the method through the design of a complex train station roof based on a realistic element stock. Section 5 discusses obtained results and gives an outlook for future research. Section 6 concludes the contributions of this paper.

2 Research context

2.1 Graphic statics and the Combinatorial Equilibrium Modelling

In recent years, different approaches have been proposed for the design of structures based on geometric variations¹¹. One of the earliest implementations of generative design applied to structural engineering has been shown by Shea et al.¹². More recently, Von Buelow¹³ and Brown and Mueller¹⁴ illustrated the use of top-down evolutionary optimization for structural design. Application of bottom-up grammar-based design rules for structural engineering has been shown by Mueller and Ochsendorf¹⁵ and Lee et al.¹⁶. As an alternative to the cited approaches, the Combinatorial Equilibrium Modelling (CEM)^{17–19} may overcome the difficulties associated with the unpredictability of structurally oriented bottom-up approaches and the use of typology-specific form-finding strategies.

Graphic statics is a geometry-based approach that enables to link the form of a loaded structure to the distribution of its internal forces and has been mainly developed at the end of the 19th century^{20–22}. The two fundamental constructions of graphic statics are the *form diagram* and the *force diagram*, which represent the geometry of a reticular structure and the equilibrium of the forces in the structure, respectively. The forces applied at each node of the structure in the form diagram define a dual closed polygon in the force diagram. The duality between form and force diagrams implies that every transformation of one of the two diagrams is followed by an associated transformation of the other diagram. By taking advantage of this duality and the lower-bound theorem of the theory of plasticity, the CEM can be used to find the geometry of different structure typologies^{17–19}. In addition to the form and force diagrams used in conventional graphic statics, the CEM introduces an operative *topological diagram* to control the connectivity of structure members and their combinatorial state, i.e. the distribution of tension and compression forces. As a result, it is possible to easily generate multiple design options (structure geometries) by simply varying the compressive and tensile forces or adapting the topology. In this way, the CEM enables the conception of structurally informed design solutions that overcome the fixation into conventional structural typologies. A detailed explanation of the CEM is given in Section 3.1.

2.2 Stock-constrained structural design

Combinatorial optimization subject to limited resources is common in operations research. Well-known examples are the *bin-packing problem* for the optimal packing of products into a minimum number of

containers^{23,24} and the one-dimensional *cutting-stock problem*^{24,25} where the objective is the minimization of trim losses²⁶. These combinatorial problems also arise when reticular structures are designed to fit given stock element characteristics, e.g. cross-sections and lengths. Fujitani and Fujii²⁷ employed evolutionary algorithms to minimize the weight of plane frames subject to a stock of one-time available cross-sections but without considering limited element lengths. Mollica and Self²⁸, Allner et al.²⁹, and Von Buelow et al.³⁰ presented the design of spatial reticular structures made of tree forks that were inventoried through 3D-scanning. Kovacs et al.³¹ showed an interactive tool for the design of space trusses made of reused plastic bottles. Clifford et al.³² showed the design of a dry-stacked “cyclopean masonry” wall made of stone and concrete panels originating from demolition debris. Similarly, Marshall et al.³³ combined genetic algorithms and bin-packing heuristics to design building envelopes composed of reclaimed concrete panels. Bukauskas et al.³⁴ presented “form-fitting” strategies for constructing trusses from a stock of irregular timber elements. The assignment of timber elements to member positions in the truss subject to element length and capacity constraints is carried out using bin-packing heuristics to minimize trim losses³⁴. An adapted version of the method³⁴ has been successfully applied to build a funicular timber gridshell³⁵. An extension to the method³⁴ allows consideration of displacement constraints and has been applied to small-scale examples³⁶.

In previous work, Brütting et al.^{37,38} showed discrete structural optimization techniques to design truss and frame structures that make best use of a stock of reclaimed elements whilst considering typical ultimate and serviceability limit states. The objective is the minimization of structure environmental impacts which are quantified through Life Cycle Assessment. The design process has been formulated as a Mixed-Integer Linear Programming (MILP) problem which produces global optima^{37,38}. It was shown that, depending on stock element availability, a combination of reused and new elements leads to structures with the least environmental impact³⁹.

2.3 Research gaps and new contribution

In recent years, the topic of reuse in structural design has gained interest in practice and research^{4,6}. As shown in Section 2.2, several methods to design structures based on an element stock are available. However, the existing methods^{28–30,34,35} are mainly concerned with stocks of timber elements and not with reclaimed elements from obsolete buildings. Some methods^{28,29,33,35} only focus on the geometric fitting of elements to pre-assigned target shapes, without considering structural limitations. Another crucial aspect, that so far has only been addressed by Brütting et al.^{37–39}, is the quantification of environmental impacts to show whether reuse is beneficial compared to new element production. The MILP-based optimization formulations^{37–39} allow such quantification. However, solving MILPs to global optimality through branch-and-bound methods is at worst-case exponentially complex⁴⁰ and hence it may require significant computation time (e.g. minutes to days, for the here concerned problem sizes^{38,39}).

This paper presents a new computational workflow that allows exploring different structural design solutions in a user-interactive way and with almost real-time feedback. The proposed workflow draws from the

127 interactive features of CEM and implements an efficient Best-Fit heuristic⁴¹ to design structures from reused
 128 and new elements. Instead of looking for global optima, the method allows generating and comparing diverse
 129 design solutions while providing relevant result metrics such as structure mass, cut-off waste, rate of reused
 130 stock elements, and structure environmental impact already in early, conceptual design stages where speed and
 131 interactivity are key. Different from the work by Bukauskas et al.^{34,36}, which is also based on Best-Fit
 132 heuristics, this work considers the quantification of environmental impacts, studies “hybrid” solutions that
 133 combine reused and new elements, and applies the proposed method to a realistic stock (Section 4).

134 3 Method

135 In the following, the term *edge* refers to a line in a topological, form, or force diagram. The term *member*
 136 denotes a bar in a truss structure. The term *element* denotes a component available for reuse from a stock or
 137 from new production. Elements are characterized by material properties, cross-section dimensions, length, and
 138 availability.

139 Figure 2 shows the workflow of the proposed method. Inputs are an initial topological diagram, a set of
 140 geometrical and structural parameters, and predefined boundary conditions (e.g. support locations). First, the
 141 equilibrium geometry is obtained via Combinatorial Equilibrium Modelling (CEM) for a dominant load case
 142 (Section 3.1). This form-active geometry is then transformed into a pin-jointed truss. Second, the maximum
 143 member tension and compression forces experienced among multiple load cases are computed via Finite
 144 Element Analysis (FEA, Section 3.2). Third, a Best-Fit heuristic is employed to optimize the cutting of
 145 members from stock elements and the assignment of new elements (Section 3.3). Details of the three steps are
 146 outlined in the following sections. The workflow has been implemented in the CAD software Rhinoceros⁴²
 147 and Grasshopper⁴³ that serve as a graphical user interface.

Figure 2: Method workflow.

3.1 CEM form-finding

Building on vector-based 3D graphic statics⁴⁴, the CEM finds the form of a structure in equilibrium following a sequential approach that breaks the global equilibrium problem down into smaller sub-problems¹⁹. This way, the CEM can be efficiently used for the rapid generation and manipulation of structures in equilibrium, where the distribution of internal compression and tension forces is a powerful generator of design variation.

The CEM process starts from a user-defined *topological diagram* (Figure 3a) and creates the *form diagram* (Figure 3b) and the associated *force diagram* (Figure 3c) of a structure in equilibrium. The CEM algorithm is initialized with a number of topological and metric parameters. Topological parameters include the connectivity of the structural members, taken from the topological diagram, and the combinatorial state of their internal forces, i.e. distribution of tension and compression. The metric parameters comprise the lengths of the *trail edges* and the magnitudes of the forces in the (*direct/indirect*) *deviation edges* (Figure 3a), which affect the form diagram and the force diagram, respectively. Further, the metric parameters include the applied loads, boundary conditions, and the position of the *origin nodes* in the form diagram, which correspond to the *starting vertices* of the topological diagram. The coordinates of all *generic nodes* are obtained by sequentially enforcing static equilibrium, from the *origin nodes* to the *support nodes* of the form diagram, the latter corresponding to the *support vertices* of the topological diagram¹⁹.

Figure 3: Relationship between (a) topological, (b) form and (c) force diagrams.

3.2 Finite Element Analysis

The input of the Finite Element Analysis (FEA) is the structure geometry (form diagram) obtained from the CEM (Section 3.1). This geometry is informed only by the dominant load-case and it is generally a kinematic structure. First, this kinematic structure is transformed into an (in-)determinate truss by adding diagonal members (Figure 4a). Next, the truss structure member forces are computed through a linear-elastic FEA. This

FEA is performed with the tool Karamba⁴⁵. Truss structures are modeled with pin-jointed members subjected to normal forces only. In addition to the dominant load case, other load cases are considered, e.g. horizontal or asymmetric loads. To run the FEA, generic member cross-sections are preliminarily defined based on the internal force distribution calculated in the CEM. The assignment of stock and new elements with discrete cross-sections is described in detail in Section 3.3.

Note that the presented method is meant for early design stages and focuses on the ultimate rather than on the serviceability limit state. Once a satisfying solution is obtained, the displacement of the structure should be evaluated carefully within the subsequent advanced design phase. The output of the FEA is a list of $i = 1 \dots m$ members, each described by its length l_i as well as its minimum compression force p_i^c (negative number, Figure 4b) and maximum tension force p_i^t (Figure 4c) experienced among all load cases.

Figure 4: Finite element analysis: (a) statically determinate truss subject to two different independent load-cases, (b) resulting minimum compression forces, and (c) maximum tension forces.

3.3 Best-Fit Heuristic

3.3.1 Concept

The “Best-Fit” algorithm⁴¹ is a heuristic for efficiently solving bin-packing and 1-dimensional cutting-stock problems (Section 2.2). Conceptually, the Best-Fit algorithm finds the best assignment of a “task” to an available “container” that meets the requirements⁴¹. This work adapts this concept to stock-constrained design to optimize the cutting of members from stock elements and the assignment of new elements. The method setup and output are first illustrated through an example; the algorithm details are given thereafter in Section 3.3.2.

Figure 5 shows the truss with $m = 27$ members that is output from the FEA. Figure 5(b) shows a set $\mathbf{R}$ of $g = 1 \dots 6$ stock element groups (grey bars), each containing n_g elements of the same type (i.e. identical cross-section and length). Elements available from new production constitute the set $\mathbf{N}$ (purple bars) which is appended to $\mathbf{R}$. The selection of cross-sections for new elements is constrained within a given finite inventory

198 (e.g. standard steel sections). Stock elements can be cut into one or more pieces (cutting-stock), but they cannot
 199 be extended. New elements are always adequately dimensioned for their position as a truss member without
 200 constraint on their length or availability. Stock element parts that are assigned to the structure are indicated by
 201 black bars in Figure 5(b). Similarly, dark purple bars indicate new elements assigned to the structure. The cut-
 202 off length of stock elements (waste) is represented by light grey bars above dark grey ones. When a member
 203 cannot be cut from any stock element (e.g. because of lack of force capacity or length) or if new element
 204 production is preferable in terms of the selected optimization objective (Section 3.3.3), then an adequately
 205 sized element is newly produced.

206
 207 Figure 5: Stock-constrained structural design: (a) structural system, (b) element stock and new elements. Grey bars represent stock elements, purple
 208 bars represent newly produced elements. Black bars overlaid on grey bars indicate element assignments.

209 3.3.2 Algorithm

210 The pseudocode of the developed Best-Fit heuristic is given in Algorithm 1. Stock elements (R) and new
 211 sections (N) are concatenated in a stacked array of $g = 1 \dots G$ groups. Each group constitutes an array of n_g
 212 elements (Figure 5b). The algorithm is initialized with an empty list of *OptimalAssignments*. The stacked array
 213 *RemainLengths* tracks whether a stock element is cut into one or more members. *RemainLengths* is initialized
 214 with the input stock element lengths and positive infinity for new elements. Then, **for each** member i , the
 215 optimal assignment to a stock or a new element is obtained (Algorithm 1, line 3). Per member, the variables
 216 gj_{opt} and *BestObjective* temporarily store the best assignment and objective value. Next, the algorithm loops
 217 through all element groups $g = 1 \dots G$ and the $j = 1 \dots n_g$ elements contained in each group (lines 9 and 10). The
 218 first **if**-condition checks whether a stock element is longer or equal than the member length. For new elements,
 219 this check is omitted. The second **if**-condition checks whether the stock or new element capacity (f_g^t and f_{ig}^c)
 220 is sufficient to resist the member forces (p_i^t and p_i^c) in tension and in compression. The tension capacity f_g^t is
 221 computed as the product of element cross-section area and material yield strength. The compression capacity
 222 f_{ig}^c considers the buckling resistance with respect to member length l_i and element cross-section properties.

223 f_{ig}^c can be computed according to national code regulations. If element length and capacity are sufficient, a
 224 feasible assignment is obtained (line 18). In this case, an objective value obj is computed through the function
 225 *GetObjectiveValue*. For instance, obj could represent the member mass that this assignment adds to the total
 226 structure mass or the associated environmental impact. Details of the function *GetObjectiveValue* are given in
 227 Section 3.3.3. If the current assignment has a smaller objective value obj than any previous assignment (line
 228 19), *BestObjective* and $g_{j_{opt}}$ are updated. If this assignment uses a previously unassigned element (line 22), the
 229 j -loop can be broken, because every element in the same group (higher j indices) would necessarily render an
 230 identical objective value.

231 If a feasible and optimal assignment has been obtained for member i , the assignment information $g_{j_{opt}}$ is stored
 232 in *OptimalAssignments*. For stock elements, the *RemainLength* is then reduced by the member length l_i (line
 233 32). In other words, for the following members, only the remaining stock element length is available. If for
 234 any member no feasible assignment can be obtained, the entire problem is infeasible. When each member is
 235 assigned, the optimal cutting pattern and use of new elements is output.

236 As the Best-Fit heuristic iteratively assigns members to elements, the method can only generate locally optimal
 237 solutions. However, at worst-case, the Best-Fit algorithm requires a polynomial number of $m \cdot s$ iterations,
 238 where $s = \sum_{g=1}^G n_g$ is the total number of elements (reuse and new). Therefore, the Best-Fit heuristic allows
 239 significantly faster computations compared to other approaches such as solving the same problem through
 240 MILP (Section 2.3).

241 In this work, structure members are sorted in descending order first by absolute member force and then by
 242 member length. Further, stock elements are sorted in ascending order first by tension capacity and then by
 243 length. This way, most demanding assignments (large member force and length) are obtained first. This aspect
 244 is relevant because processing members one after the other (for each loop, Algorithm 1) means that a previous
 245 assignment influences those that follow (reduced *RemainLength*).

Algorithm 1: Best Fit Algorithm

```
input : Structure with members, Stock and new elements
output: Optimal cutting of stock elements and new element assignment
1 OptimalAssignments  $\leftarrow$  new empty list
2 RemainLenghts  $\leftarrow$  Elements.Lengths
3 foreach Member in the Structure do
4   BestObjective  $\leftarrow +\infty$ 
5    $gj_{opt} = [0][0]$ 
6    $p_i^c = \text{Member.TensionForce}$ 
7    $p_i^t = \text{Member.CompressionForce}$ 
8    $l_i = \text{Member.Length}$ 
9   for  $g \leftarrow 1$  to  $G$  do Loop through all stock element groups
10    for  $j \leftarrow 1$  to  $n_g$  do Loop through all elements in group  $g$ 
11      Element = Elements[ $g$ ][ $j$ ]
12      type = Element.Type
13       $l_{gj} = \text{RemainLenghts}[g][j]$ 
14      if ( $l_i \leq l_{gj}$  or type == New) then
15         $f_g^t = \text{Element.TensionCapacity}$ 
16         $f_{ig}^c = \text{BucklingCapacity}(\text{Element}, l_i)$ 
17        if ( $f_{ig}^c \leq p_i^c$  and  $p_i^t \leq f_g^t$ ) then
18           $obj = \text{GetObjectiveValue}(\text{Member}, \text{Element}, l_{gj})$ 
19          if ( $obj < \text{BestObjective}$ ) then
20            BestObjective  $\leftarrow obj$ 
21             $gj_{opt} \leftarrow [g][j]$ 
22            if ( $\text{RemainLenghts}[g][j] == \text{Elements}[g][j].\text{Length}$ ) then
23              break
24            end
25          end
26        end
27      end
28    end
29  end
30  if  $gj_{opt} \neq [0][0]$  then
31    if ( $\text{Elements}[gj_{opt}].\text{Type} == \text{Reuse}$ ) then
32       $\text{RemainLenghts}[gj_{opt}] \leftarrow \text{RemainLenghts}[gj_{opt}] - l_i$ 
33    end
34    OptimalAssignments.Add( $gj_{opt}$ )
35  else
36    Problem is infeasible
37  end
38 end
```

Algorithm 1: Pseudocode of the Best-Fit algorithm for stock-constrained structural design.

3.3.3 Optimization objective

In Algorithm 1 - line 18, the scalar objective value obj is computed through the function *GetObjectiveValue* (Algorithm 2). In this work, the aim is to minimize the structure environmental impact, expressed in terms of embodied greenhouse gas (GHG) emission, measured in kgCO₂eq. Embodied GHG emissions are computed from process impacts occurring in the life cycle of a stock or new element (Figure 5). Process impacts are taken from Brütting et al.³⁸ where they have been quantified through Life Cycle Assessment (LCA) based on the Swiss federal LCA database KBOB⁴⁶. A description of each process and the corresponding GHG emissions coefficients are given in Table 1, for more details the reader is referred to Brütting et al.³⁸. In general, the Best-Fit method is not restricted to GHG minimization. Different objectives, such as structure mass or cut-off waste minimization, could be equivalently implemented.

Algorithm 2: $obj = \text{GetObjectiveValue}(\text{Member}, \text{Element}, l_{gj})$

input : Structure Member, Stock Element, RemainLength l_{gj}
output: Objective value for the assignment of a Member to an Element

```

1  $obj \leftarrow 0$ 
2  $m_s = l_{gj} \cdot \text{Element.CrossSectionArea} \cdot \text{Element.Density}$ 
3  $m_i = \text{Member.Length} \cdot \text{Element.CrossSectionArea} \cdot \text{Element.Density}$ 
4 if Element.Type == Reuse then Element is from stock
5   if  $l_{gj} == \text{Element.Length}$  then This is the first member to be cut from this element
6      $obj = obj + m_s \cdot (GHG^{DC} + GHG^{TF})$ 
7   end
8    $obj = obj + m_i \cdot (GHG^{TS} + GHG^A)$ 
9 else
10   $obj = m_i \cdot (GHG^{DM} + GHG^{TR} + GHG^P + GHG^{TP} + GHG^{TS} + GHG^A)$ 
11 end
12 return  $obj$ 

```

Algorithm 2: Pseudocode for function *GetObjectiveValue*.

Figure 6: LCA system boundary, adapted from Brütting et al.³⁸.

In the Best-Fit heuristic (Algorithm 1), the objective value obj is computed for each member assignment (Algorithm 2). After termination of Algorithm 1, all member assignments are obtained and the embodied GHG emissions of the final truss structure can be computed as:

$$GHG = (GHG^{DC} + GHG^{TF})M_S + (GHG^{DM} + GHG^{TR} + GHG^P + GHG^{TP})M_{New} + (GHG^{TS} + GHG^A)M$$

where M_S is the total mass of reused stock elements, M_{New} is the total mass of newly produced elements, and M is the total structure mass (combining reused and new elements). Cut-off waste can be computed as $W = M_S - (M - M_{New}) = M_S - M_{Reuse}$. The obtained optimal ratio between reused and new elements in the structure is denoted as “reuse rate” $RR = M_{Reuse}/M$.

Table 1: Process GHG emission factors, based on Brütting et al.³⁸.

	Process code	Description	Embodied GHG emissions [kgCO ₂ eq/kg]
<i>Stock elements</i>	GHG^{DC}	Selective deconstruction of an obsolete steel structure with worker crews and a diesel-powered mobile crane	0.337
	GHG^{TF}	Transport of stock elements from the place of disassembly to the steel fabrication workshop by truck (200 km)	0.022
<i>New elements</i>	GHG^{DM}	Demolition of an obsolete steel structure with excavators	0.050
	GHG^{TR}	Transport of demolition steel scrap to a recycling plant by truck (20 km)	0.002
	GHG^P	Production of new steel elements made of recycled steel	0.734
	GHG^{TP}	Transport of new steel elements to the steel fabrication workshop by truck (20 km)	0.002
<i>Final truss</i>	GHG^{TS}	Transport of truss parts from the steel fabrication workshop to the building site by truck (20 km)	0.002
	GHG^A	Assembly of the new structure with a diesel-powered mobile crane	0.110

272

273 4 Application

274 The workflow presented in Section 3 is applied to design a new roof for the train station in Lausanne
 275 (Switzerland) based on an element stock originating from disassembled electric pylons. The system boundary
 276 conditions, load cases, and element stock are taken from Brütting et al.³⁷ and briefly illustrated in Sections 4.1
 277 and 4.2. Sections 4.3 – 4.5 show parametric studies that explores several design alternatives.

278 4.1 Design context

279 Figure 7 shows the track and platform layout of Lausanne train station together with a roof truss design that
 280 has been previously shown by Brütting et al.³⁷. Equivalently, this work considers the roof to be constituted by
 281 21 identical transversal sections along the in total 200 m long train station. Transversal sections span 76 m
 282 over the four double tracks. The transversal section geometry is designed through the proposed workflow
 283 (Section 3) and based on a stock of individual elements. As assumed by Brütting et al.³⁷, secondary trusses that
 284 are taken from the electric pylons as complete modules span 10 m between transverse sections (Figure 7).

285 The boundary conditions set for the CEM form-finding consider that the structure can only be supported at the
 286 platforms and that clearance from the trains has to be kept. The dominant load-case is assumed to be the
 287 combination of superimposed dead load (roof cladding, 0.5 kN/m²) and snow load (1.0 kN/m²) acting on the
 288 whole roof surface. These loads are combined with design load factors of 1.35 and 1.50, respectively³⁷ and
 289 applied to structure top chord nodes based on tributary load areas. In the FEA, six additional ultimate limit
 290 state load-cases are considered: two symmetric and two asymmetric wind uplifts as well as two asymmetric
 291 snow distributions.

292 In the following case studies, structure members are subdivided into equal-length parts if they are longer than
 293 a user-defined length, here 3.0 m. This subdivision is made to respect available stock element lengths (Section

4.2). In other words, a structure member can also be a combination of multiple elements. Still, the buckling capacity is correctly computed based on the full (non-divided) member length.

Figure 7: Scheme of the train station design obtained by Brütting et al.³⁷.

4.2 Element stock and new sections

Stock elements originate from electric pylons which are planned to be disassembled in Switzerland³⁷. Archive plans have been used to characterize the element stock for reuse through manually extracting the element availability, length, material, and cross-section³⁷. The stock is made of steel L-profiles with cross-sections having equal leg angles of sizes between 40x4 and 120x8 (width x thickness in mm). Element lengths are between 27 cm and 3.30 m. One single power line comprises about 50 pylons, totaling up to 19'000 bars. Elements with same lengths and cross-sections were collated into 332 different groups³⁷. Considering elements available from five power lines and the construction of 21 identical transversal sections, for the design of one transversal section in total 4450 individual stock elements are available. New elements can be produced as equal leg angles of all 201 standard sizes reported in EN 10056-1⁴⁷ (size 20x3 to 300x35). Pylon and new elements are grade S235 steel with a yield strength of 235 MPa, a Young's modulus of 210 GPa, and a density of 7850 kg/m³. Critical buckling loads for L-sections are computed in accordance with Swiss standard SIA 263⁴⁸.

Depending on member forces, employing a single L-section element might be inefficient or infeasible because of limited capacity. For this case study, the Best-Fit algorithm (Section 3.3) is extended to consider the possibility of employing compound cross-sections made of two or four profiles (Figure 8). For each member position, the algorithm selects the solution with the least objective value, whether it is a single or a compound section. For compound sections, two respectively four, assignments with the same member length must be obtained. Compound cross-sections can be made of only reused or new elements, or of a combination of both.

Figure 8: Single and compound cross-sections made of equal leg angle profiles.

4.3 Design Concept 1: Arch-Cantilever

4.3.1 Setup and parameters

Figure 9 shows the setup for a first design concept which considers one arch spanning over the two central double tracks and two cantilevers that cover the double tracks and platforms at the perimeter. The horizontal tension forces in both cantilever top-chords are short-cut. The horizontal compression forces at the base are partially equilibrated and result in inclined legs, depending on the funicular geometry and the internal force distribution.

Figure 9: Setup design concept 1: arch-cantilever.

The topological diagram (Figure 9) for the CEM form-finding is composed of four compressive trails. The two trails that start in the center form the arch, while the two trails that start at the extremes define the cantilevers. Their origin nodes (Figure 9, o_1 and o_4) are connected with one deviation edge in tension. While the total width of the structure is kept constant, the number N of trail edges within each trail is a design parameter. Through this setup, the following parameters can be varied within the form domain: z -coordinates (z_1 and z_2) of the four origin nodes o_1 to o_4 and the number of *trail edges* N . Within the force domain, the following parameters can be varied: the force magnitude of the tensile *deviation edge* (d_1 , responsible for the rise of the two cantilevers) as well as the force magnitude of the compressive *deviation edge* (d_2 , responsible for the rise of the arch). This simple parametric setup in combination with the downstream FEA and Best-Fit steps allows to quickly and manually explore very different, structurally informed geometries and their related performance. For this case study, the average computation times on an Intel i7-7820HQ CPU were 20, 10, and 220 milliseconds for CEM, FEA, and Best-Fit algorithm, respectively (cf. 140 members, 4450 stock elements, 201 new elements with standard L-sections).

341 4.3.2 Design exploration

342 Figure 10 shows four different design options (a) – (d) that have been interactively generated through
343 parameter variations. For each option, the bar chart in Figure 10(e) gives obtained result metrics in terms of
344 total structure mass M , mass of reclaimed stock elements M_S , mass of reused elements in the structure M_{Reuse} ,
345 mass of newly produced elements M_{New} , cut-off waste W , and embodied greenhouse gas emissions GHG.

346 The generated form diagrams (Figure 10, left) of options (a) and (b) show cantilevers at $z_I = 10.0$ m above
347 ground. While (a) is characterized by a shallow arch (z_2 set to 8 m) and higher compressive forces, (b) has a
348 deep arch (z_2 set to 14 m) with lower compressive forces (c.f. force diagrams Figure 10, center). Options (c)
349 and (d) are variations of (a) and (b), respectively. For (c) and (d) the aim was to reduce the force magnitude of
350 the tensile chord. This is achieved by setting the height of the cantilever tips to $z_I = 14.0$ m. This in turn leads
351 to structures with longer members. Option (c) differs from (d) mainly in the rise and corresponding internal
352 force of the main arch.

353 Option (a) has a structure mass of $M = 9700$ kg, of which $M_R = 5700$ kg are reused elements and $M_N = 4000$ kg
354 are new elements. Hence, the reuse rate for option (a) is $RR = M_R/M = 59\%$. For option (b) the highest reuse
355 rate of 64% is obtained. This means that the member force and length distribution in option (b) correlate well
356 to the available element characteristics. With respect to embodied GHG emissions, options (c) and (d) perform
357 slightly better than (b), despite a lower RR . This is a result of the smaller force magnitudes in (c) and (d)
358 compared to (b) (cf. force diagrams). Generally, smaller forces allow assigning lower-impact elements. Option
359 (c) has a higher structure and stock mass than (d), yet requires fewer energy-intensive new production,
360 eventually resulting in the least embodied GHG emissions of the four options. Starting from options (c) and
361 (d), an additional sensitivity study is conducted to understand the relationship between structure form, internal
362 forces, and embodied GHG emissions.

Figure 10: Design concept 1 – Arch-cantilever: (a-d) design options based on CEM parameter variation, (e) result metrics.

4.3.3 Detailed sensitivity study

In this sensitivity study, the force magnitude of deviation edge d_2 (Section 4.3.1) in the main arch is regarded as the design variable. The force in d_2 is varied between -507 and -270 kN, which are the values taken for options (c) and (d), respectively. Changing d_2 has a direct influence on the rise of the central arch. The plots

369 in Figure 11 show the influence of d_2 on RR as well as M and GHG. Obtained RR s are between 51% and 61%.
 370 M and GHG are correlated and an option (c*) with minimum GHG emissions is obtained at $d_2 = -400$ kN,
 371 corresponding to the maximum RR of 61%. The structure layout for option (c*) is show at the bottom of Figure
 372 11. The embodied GHG of option (c*) is 4'820 kgCO₂eq and 12% less than option (c).

Figure 11: Sensitivity study: variation of the rise of the arch through changing d_2 . (c*) design with least GHG emissions.

375 Design (c*) is compared to a solution with the same geometry but made of only new elements, serving as a
 376 “best-practice” benchmark. This new element solution is equally obtained through the proposed method
 377 workflow, yet stock elements are omitted. The bar charts in Figure 12 give results for both the “hybrid” option
 378 (c*) and the new element solution. The structure made of only new elements has a lower mass because all
 379 standard L-sections are available in unlimited quantity. Note that, for the LCA assumptions made in Section
 380 3.3.3, GHG and mass minimization produce identical results for structures made of new elements only. Despite
 381 this oversizing, the hybrid option (c*) embodies 16% less GHG emissions.

Figure 12: Comparison between a hybrid structure (c*) combining reused and new elements and a structure made of new elements only.

4.4 Design Concept 2: Symmetric Cantilevers

4.4.1 Setup and parameters

The second design concept considers a structure comprising two symmetrical cantilevers, each spanning over two double-tracks. The cantilevers are connected and supported at the central platform. In addition, a vertical prestress force (tension) is applied at the cantilever tips.

Figure 13: Setup design concept 2: symmetric cantilevers.

The topological diagram of each cantilever is composed of two trails that are connected with deviation edges. Based on this setup, the following topological and metric parameters can be varied: the number of trail edges N and the related number of deviation edges, the z -coordinates z_1 and z_2 of the two origin nodes o_1 and o_2 , the prestressing force d_1 , the initial force magnitude d_2 in the top chord, and the force magnitudes of the deviation edges d_N . For this setup, the average computation times were 70, 10, and 220 milliseconds for CEM, FEA, and Best-Fit algorithm, respectively.

4.4.2 Design exploration

Figure 14(a) – (d) shows different design options that have been interactively generated and the bar chart in Figure 14(e) gives the corresponding performance metrics. The force magnitude d_2 is adjusted so that the horizontal components of the top and bottom chord forces remain equal among all options (cf. force diagrams).

401 Through manually varying the force magnitudes of the vertical deviation members d_N , the applied vertical
402 loads can be distributed differently between top and bottom chord. In (a), the applied vertical loads are evenly
403 distributed among top and bottom chord. In (b), the applied vertical loads are mainly transferred through the
404 bottom chord, while in option (c) the top chord is bearing the vertical loads. In option (d), the z -coordinate of
405 o_i is manually elevated. Through an additional increase of d_N , it is possible to avoid very long support columns
406 at the central platform. In general, the design options proposed in this second concept are structurally less
407 efficient and hence have a larger mass than those presented in the previous Section 4.3. This effect can be
408 explained by the increased span of the structure resulting in absolute larger member forces. Among the options,
409 (c) performs the worst as the diagonal located at the symmetry axis has to transfer by far the highest
410 compression force. As the element stock does not consist of elements with high enough capacity, many new
411 elements must be produced, eventually resulting in high embodied GHG emissions. For option (a), an
412 additional sensitivity study is conducted on the number of trail edges N .

Figure 14: Design concept 2 – symmetric cantilevers: (a-d) design options based on CEM parameter variation, (e) result metrics.

4.4.3 Sensitivity study - topology

In this sensitivity study, the number of trail edges N is varied between 9 and 29. All other parameters correspond to those set for previous option (a). The plots in Figure 15 show the influence of N on RR as well as M and GHG. The option (a2) with minimum embodied GHG emissions of 13'088 kgCO2eq is obtained at

419 N = 15. Figure 15 shows on the right the structural systems for (a2) as well as for the two extremes of the
 420 sensitivity study. For values of $N > 15$, M and GHG increase together with N as the total number of structure
 421 members also increases. Option (a2) embodies 13'100 kgCO2eq which is 3% less than option (a) at $N = 17$
 422 (Section 4.2).

Figure 15: Sensitivity study: variation of the number of trail edges N .

425 Option (a2) is equivalently compared to a benchmark solution made of new elements only. The bar charts in
 426 Figure 16 show that the hybrid option (a2) is slightly heavier than the new element structure. However, (a2)
 427 embodies 12% less GHG emissions.

Figure 16: Comparison between (a2) combining reused and new elements and a structure made of new elements only.

430 Figure 17 shows the use of compound cross-sections for option (a3) with $N = 29$. Figure 17(a) shows a
 431 perspective view where four and two L-sections are combined for the top and bottom chords, respectively.
 432 Figure 17(b) shows the distribution of compound cross-sections in the truss. Note that compound cross-
 433 sections are assigned for both reused and new elements (cf. Figure 15a3).

Figure 17: Compound cross-sections. (a) detailed view, (b) distribution of compound cross-sections in the truss.

4.5 3D structure

The presented computational workflow is equally applicable to the design of 3D structures. Figure 18 shows the design of spatial trusses spanning in the longitudinal direction of the train station. The structure is composed of two different modules (end and central part), each repeated six times. One module consists of 1200 members and is designed subject to 7500 stock elements. Computation times are 18, 1, and 4 seconds for CEM, FEA, and Best-Fit algorithm respectively. The complete roof shown in Figure 18 is the outcome of an interactive improvement of the structure form with the aim to obtain a high reuse rate and low embodied GHG emission. The structure has a mass of $M = 345$ tons at a reuse rate of $RR = 79\%$. The total embodied GHG emissions are 209 tCO₂eq. In comparison to a benchmark solution made of new elements only, this hybrid structure is 23% heavier but embodies 17% less GHG emissions.

Figure 18: Schematic views of a 3D structure concept.

448 **5 Discussion**

449 The presented case studies show that the proposed design workflow applies to realistic design scenarios. Since
450 solutions are computed in almost real-time (< 1 second for 2D case studies on a standard PC), a user-interactive
451 exploration of design options within the early design stage is feasible. The fast computation also allows using
452 global optimization tools in future work.

453 In this work, the FEA considers a limited number of load cases that are regarded to be most relevant for the
454 conceptual stage. In subsequent, advanced design phases, the selected solution should be checked for more
455 load cases. Such refined analysis might reveal that some members are overstressed or that displacement limits
456 are not met, and hence some members must be replaced with stronger and stiffer ones. For stock-constrained
457 cases, such elements might not be available in the stock. In this case, new elements must be produced, at a cost
458 of an increased structural environmental impact. Because of the fast computation times permitted by the
459 proposed workflow, one strategy to control this problem could be to start the process with a reduced allowable
460 member capacity utilization of for example 80% and to then iteratively refine the design in successive steps.

461 For the presented 2D case studies, it is expected that custom connection plates can be manufactured to connect
462 L-section elements, independent of whether reused or new elements are employed. For 3D cases, joining of
463 elements requires the design of more complex spatial connections. A detailed structural design and Life Cycle
464 Assessment of connections was out of the scope of this work. Future work could study the effect of connections
465 on the design and structure environmental impact in more detail.

466 **6 Conclusion**

467 This paper presents a computational workflow for the design of spatial reticular structures made of reused and
468 new components. Through the combination of the Combinatorial Equilibrium Modelling and efficient Best-
469 Fit heuristics, it is possible to explore a large variety of diverse structure layouts in a user-interactive way. An
470 integrated Life Cycle Assessment provides fast-feedback of the structure environmental impact and hence
471 gives an understanding for which design and stock configuration reuse of structural elements is beneficial
472 compared to new element production. Conducted sensitivity studies show that the method can not only be used
473 in a manual and interactive design mode, but also to analyze the effects of specific design parameters on the
474 results. Eventually, it is shown that even though structures made of reused and new elements have a higher
475 mass, they embody significantly less greenhouse gas emissions than structures made of new elements only.

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479 Declaration of conflicting interests

480 The authors declare that there is no conflict of interest.

481 References

- 482 1. International Energy Agency. *Material efficiency in clean energy transitions*. Paris,
483 www.iea.org/publications/reports/MaterialEfficiencyinCleanEnergyTransitions (2019, accessed 8 May
484 2019).
- 485 2. De Wolf C, Hoxha E, Hollberg A, et al. Database of Embodied Quantity Outputs: Lowering Material
486 Impacts Through Engineering. *J Archit Eng* 2020; 26: 04020016: DOI: 10.1061/(ASCE)AE.1943-
487 5568.0000408.
- 488 3. World Green Building Council. *Bringing embodied carbon upfront*. London, 2019.
- 489 4. Fivet C, Brütting J. Nothing is lost, nothing is created, everything is reused: structural design for a circular
490 economy. *Struct Eng* 2020; 98: 74–81.
- 491 5. Stahel WR. *The Circular Economy: A User's Guide*. 1st ed. London: Routledge, 2019.
- 492 6. Gorgolewski M. *Resource Salvation: The Architecture of Reuse*. John Wiley & Sons, 2017.
- 493 7. Addis B. *Building with reclaimed components and materials: a design handbook for reuse and recycling*.
494 London: Earthscan, 2006.
- 495 8. Pongiglione M, Calderini C. Sustainable Structural Design: Comprehensive Literature Review. *J Struct*
496 *Eng* 2016; 142: 04016139: DOI: 10.1061/(ASCE)ST.1943-541X.0001621.
- 497 9. Iacovidou E, Purnell P. Mining the physical infrastructure: Opportunities, barriers and interventions in
498 promoting structural components reuse. *Sci Total Environ* 2016; 557–558: 791–807: DOI:
499 10.1016/j.scitotenv.2016.03.098.
- 500 10. Gorgolewski M. Designing with reused building components: some challenges. *Build Res Inf* 2008; 36:
501 175–188.
- 502 11. Harding J. Dimensionality Reduction for Parametric Design Exploration. In: Adriaenssens S, Gramazio
503 F, Kohler M, et al. (eds) *Advances in Architectural Geometry 2016*. 2016. DOI: 10.3218/3778-4_19.
- 504 12. Shea K, Cagan J, Fenves SJ. A Shape Annealing Approach to Optimal Truss Design With Dynamic
505 Grouping of Members. *J Mech Des* 1997; 119: 388–394: DOI: 10.1115/1.2826360.
- 506 13. von Buelow P. Paragen: Performative Exploration of Generative Systems. *J Int Assoc Shell Spat Struct*
507 2012; 53: 271–284.
- 508 14. Brown NC, Mueller CT. Design for structural and energy performance of long span buildings using
509 geometric multi-objective optimization. *Energy Build* 2016; 127: 748–761: DOI:
510 10.1016/j.enbuild.2016.05.090.
- 511 15. Mueller CT, Ochsendorf JA. Combining structural performance and designer preferences in evolutionary
512 design space exploration. *Autom Constr* 2015; 52: 70–82: DOI: 10.1016/j.autcon.2015.02.011.
- 513 16. Lee J, Fivet C, Mueller C. Modelling with Forces: Grammar-Based Graphic Statics for Diverse
514 Architectural Structures. In: Thomsen MR, Tamke M, Gengnagel C, et al. (eds) *Modelling Behaviour:
515 Design Modelling Symposium 2015*. Cham: Springer International Publishing, pp. 491–504: DOI:
516 10.1007/978-3-319-24208-8_41.
- 517 17. Ohlbrock PO, Schwartz J. Combinatorial equilibrium modeling. *Int J Space Struct* 2016; 31: 177–189:
518 DOI: 10.1177/0266351116660799.
- 519 18. Ohlbrock PO, D'Acunto P, Jasienski J-P, et al. Constraint-Driven Design with Combinatorial
520 Equilibrium Modelling. In: *Proceedings of the IASS Annual Symposium 2017*. Hamburg, 2017.

- 521 19. Ohlbrock PO, D'Acunto P. A Computer-Aided Approach to Equilibrium Design Based on Graphic
522 Statics and Combinatorial Variations. *Comput-Aided Des* 2020; 121: 102802: DOI:
523 10.1016/j.cad.2019.102802.
- 524 20. Maxwell JC. On reciprocal figures, frames and diagrams of forces. *Philos Mag* 1864; 27: 250–261.
- 525 21. Culmann C. *Die Graphische Statik*. Zurich: Von Meyer & Zeller, 1866.
- 526 22. Cremona L. *Le figure reciproche nella statica grafica*. Milan: Tipografia Bernardoni, 1872.
- 527 23. Johnson DS. Fast allocation algorithms. In: *13th Annual Symposium on Switching and Automata Theory*
528 *(swat 1972)*. 1972, pp. 144–154: DOI: 10.1109/SWAT.1972.4.
- 529 24. Delorme M, Iori M, Martello S. Bin packing and cutting stock problems: Mathematical models and exact
530 algorithms. *Eur J Oper Res* 2016; 255: 1–20: DOI: 10.1016/j.ejor.2016.04.030.
- 531 25. Gilmore PC, Gomory RE. A Linear Programming Approach to the Cutting-Stock Problem. *Oper Res*
532 1961; 9: 849–859: DOI: 10.1287/opre.9.6.849.
- 533 26. Benjaoran V, Bhokha S. Three-step solutions for cutting stock problem of construction steel bars. *KSCE*
534 *J Civ Eng* 2014; 18: 1239–1247: DOI: 10.1007/s12205-014-0238-3.
- 535 27. Fujitani Y, Fujii D. Optimum structural design of steel plane frame under the limited stocks of members.
536 In: *Proceedings of the RILEM/CIB/ISO International Symposium, Integrated Life-Cycle Design of*
537 *Materials and Structures*. Helsinki, 2000, pp. 198–202.
- 538 28. Mollica Z, Self M. Tree Fork Truss. *Adv Archit Geom* 2016; 2016: 138–153.
- 539 29. Allner L, Kroehnert D, Rossi A. Mediating Irregularity: Towards a Design Method for Spatial Structures
540 Utilizing Naturally Grown Forked Branches. In: Gengnagel C, Baverel O, Burry J, et al. (eds) *Impact:*
541 *Design With All Senses*. Cham: Springer International Publishing, 2020, pp. 433–445: DOI: 10.1007/978-
542 3-030-29829-6_34.
- 543 30. Von Buelow P, Oliyan Torghabehi O, Mankouche S, et al. Combining parametric form generation and
544 design exploration to produce a wooden reticulated shell using natural tree crotches. *Proc IASS Annu*
545 *Symp* 2018; 2018: 1–8.
- 546 31. Kovacs R, Seufert A, Wall L, et al. TrussFab: Fabricating Sturdy Large-Scale Structures on Desktop 3D
547 Printers. In: *Proceedings of the 2017 CHI Conference on Human Factors in Computing Systems*. Denver,
548 Colorado, USA: Association for Computing Machinery, pp. 2606–2616: DOI:
549 10.1145/3025453.3026016.
- 550 32. Clifford B, McGee W, Muhonen M. Recovering Cannibalism in Architecture with a Return to Cyclopean
551 Masonry. *Nexus Netw J* 2018; 20: 583–604: DOI: 10.1007/s00004-018-0392-x.
- 552 33. Marshall D, Mueller CT, Clifford B, et al. Computational arrangement of demolition debris. *Detritus*
553 2020; 11: 3–18: DOI: 10.31025/2611-4135/2020.13967.
- 554 34. Bukauskas A, Shepherd P, Walker P, et al. Form-Fitting strategies for diversity-tolerant design. In:
555 *Proceedings of the IASS Annual Symposium*. Hamburg, 2017.
- 556 35. Baber KR, Burry JR, Chen C, et al. Inventory constrained design of a timber funicular structure. *SN Appl*
557 *Sci* 2020; 2: 1538: DOI: 10.1007/s42452-020-03314-9.
- 558 36. Bukauskas A. *Inventory-Constrained Structural Design*. PhD Dissertation, University of Bath,
559 <https://researchportal.bath.ac.uk/en/studentTheses/inventory-constrained-structural-design> (2020,
560 accessed 24 September 2020).
- 561 37. Brütting J, Desruelle J, Senatore G, et al. Design of Truss Structures Through Reuse. *Structures* 2019;
562 18: 128–137: DOI: 10.1016/j.istruc.2018.11.006.
- 563 38. Brütting J, Senatore G, Schevenels M, et al. Optimum Design of Frame Structures From a Stock of
564 Reclaimed Elements. *Front Built Environ*; 6. DOI: 10.3389/fbuil.2020.00057.

- 565 39. Brütting J, Vandervaeren C, Senatore G, et al. Environmental impact minimization of reticular structures
566 made of reused and new elements through Life Cycle Assessment and Mixed-Integer Linear
567 Programming. *Energy Build* 2020; 215: 109827: DOI: 10.1016/j.enbuild.2020.109827.
- 568 40. Nemhauser GL, Wolsey LA. *Integer and combinatorial optimization*. New York: John Wiley & Sons,
569 1999.
- 570 41. Coffman EG, Garey MR, Johnson DS. Approximation algorithms for bin packing: A survey. In:
571 Hochbaum DS (ed) *Approximation algorithms for NP-hard problems*. PWS Publishing Company, 1996,
572 pp. 46–93.
- 573 42. McNeel & Associates. Rhino 6, <https://www.rhino3d.com/> (2018, accessed 21 April 2020).
- 574 43. McNeel & Associates. Grasshopper - algorithmic modeling for Rhino, <https://www.grasshopper3d.com/>
575 (2018, accessed 21 April 2020).
- 576 44. D’Acunto P, Jasienski J-P, Ohlbrock PO, et al. Vector-based 3D graphic statics: A framework for the
577 design of spatial structures based on the relation between form and forces. *Int J Solids Struct* 2019; 167:
578 58–70: DOI: 10.1016/j.ijsolstr.2019.02.008.
- 579 45. Preisinger C. Linking Structure and Parametric Geometry. *Archit Des* 2013; 83: 110–113: DOI:
580 10.1002/ad.1564.
- 581 46. Koordinationskonferenz der Bau- und Liegenschaftsorgane der öffentlichen Bauherren (KBOB).
582 Ökobilanzdaten im Baubereich 2009/1:2016.
- 583 47. European Committee for Standardization. EN 10056-1 Structural steel equal and unequal leg angles.
- 584 48. SIA263. Stahlbau (Steel structures).
- 585