APS physics MARCH MEETING 2021
March 15-19 ONLINE

Numerical and experimental comparisons of the coplanar coalescence of two fluid-driven fractures

C. Peruzzo[1], B. Lecampion[1], N. O'Keeffe[2], P. F. Linden[2]

[1] Geo-Energy Lab - Gaznat Chair on Geo-Energy, Ecole Polytechnique Federale de Lausanne

[2] Department of Applied Mathematics and Theoretical Physics, University of Cambridge

Motivations

Spatio–temporal evolution of fluid driven fractures (FDF)

Motivations

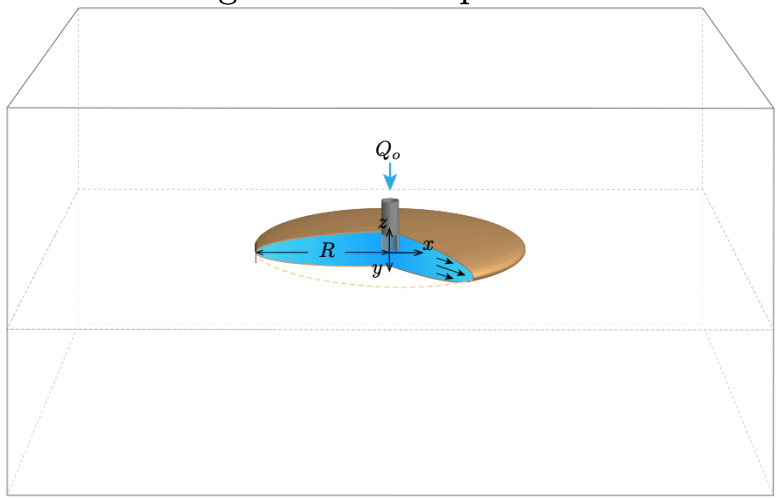
Spatio–temporal evolution of fluid driven fractures (FDF)

- Only few lab experiments involving FDF have been made

[Bunger, Detournay, 2008]

[Lecampion et al, 2017]

Single fracture experiment



Motivations

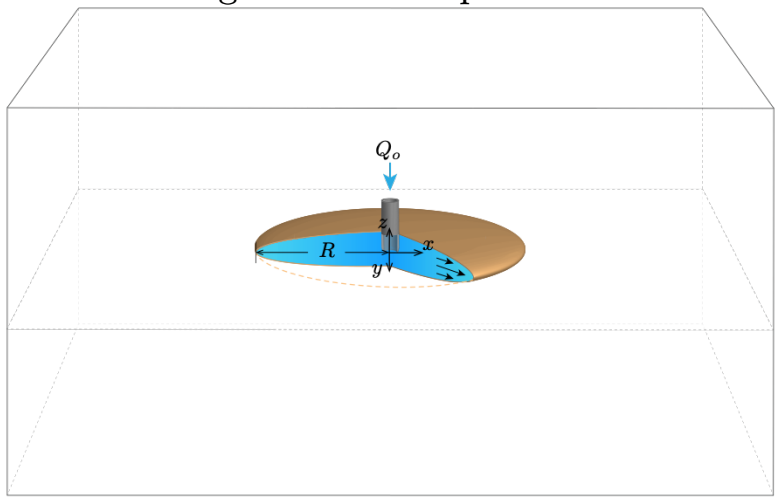
Spatio–temporal evolution of fluid driven fractures (FDF)

- Only few lab experiments involving FDF have been made

[Bunger, Detournay, 2008]

[Lecampion et al, 2017]

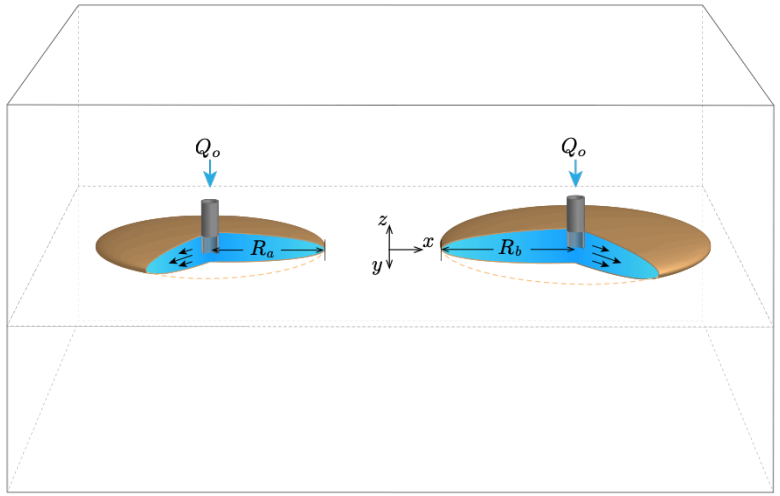
Single fracture experiment



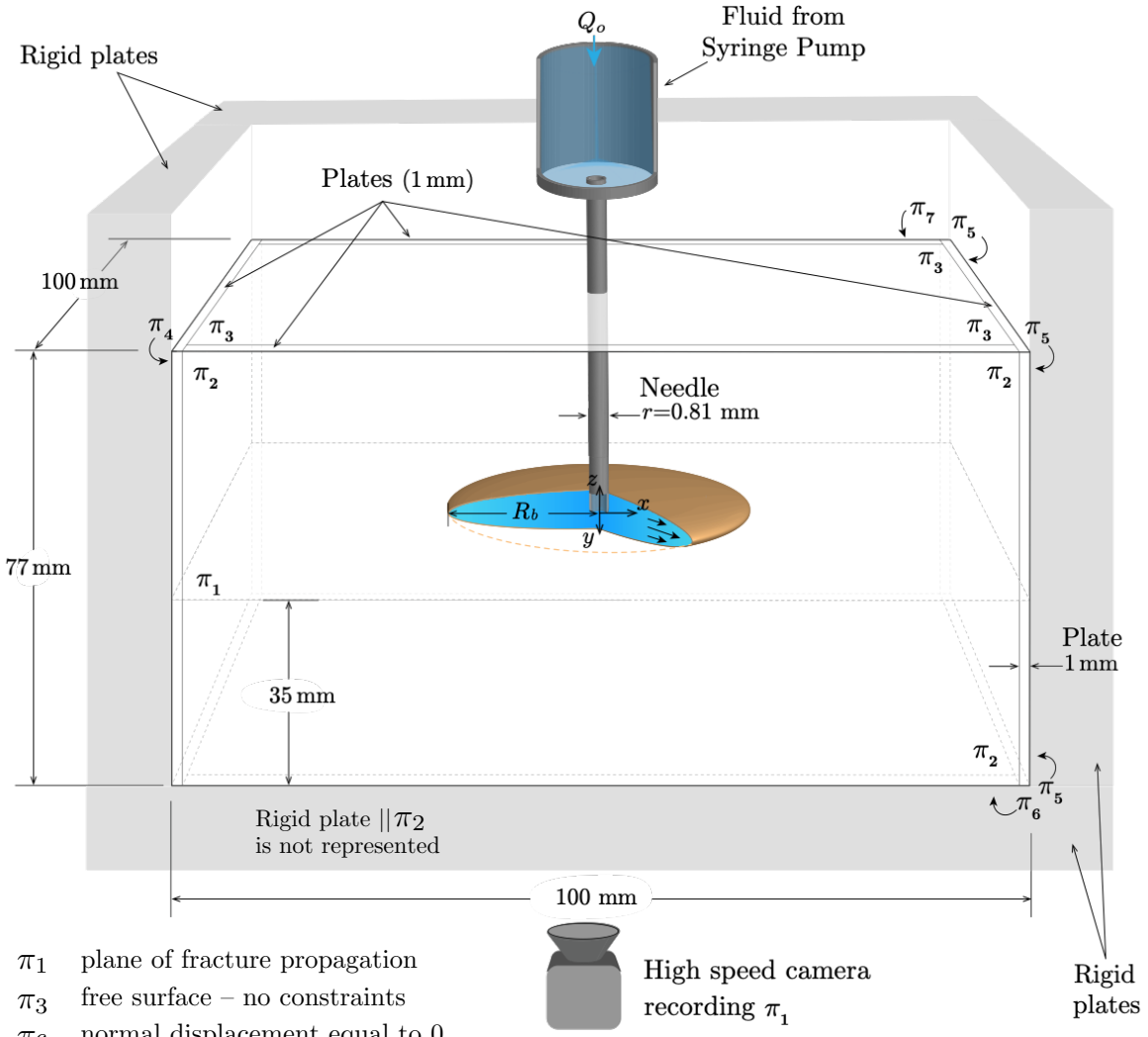
- No other experiments involving fluid velocity measurement and coalescence of two FDF have been published in the literature

[O'Keeffe et al, 2017; O'Keeffe, 2019]

Double fracture experiment



Experimental set up: single fracture



π_1 plane of fracture propagation

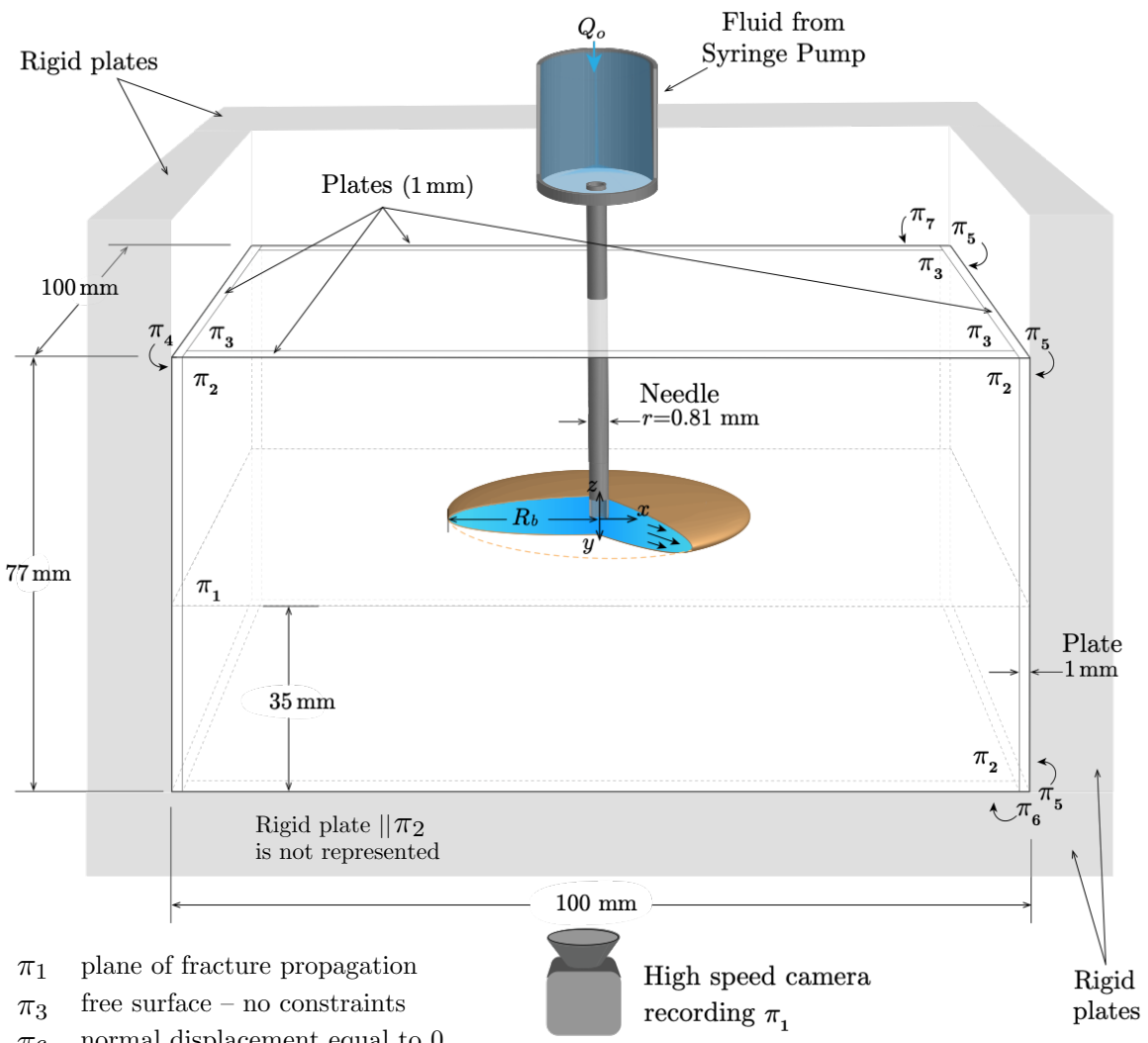
π_3 free surface – no constraints

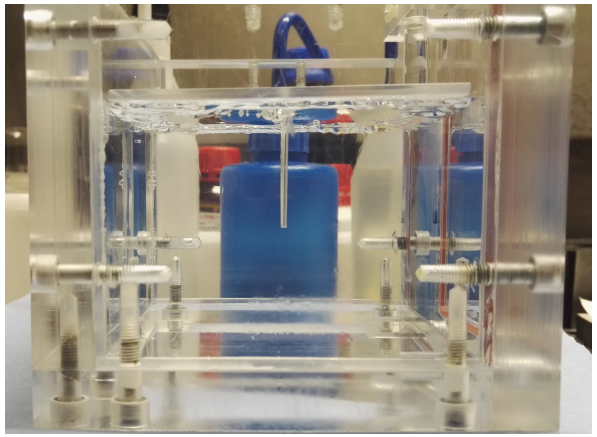
π_6 normal displacement equal to 0

$\pi_2, \pi_4, \pi_5, \pi_7$ applied $\perp$ displacement (1 mm)

Experimental set up: single fracture

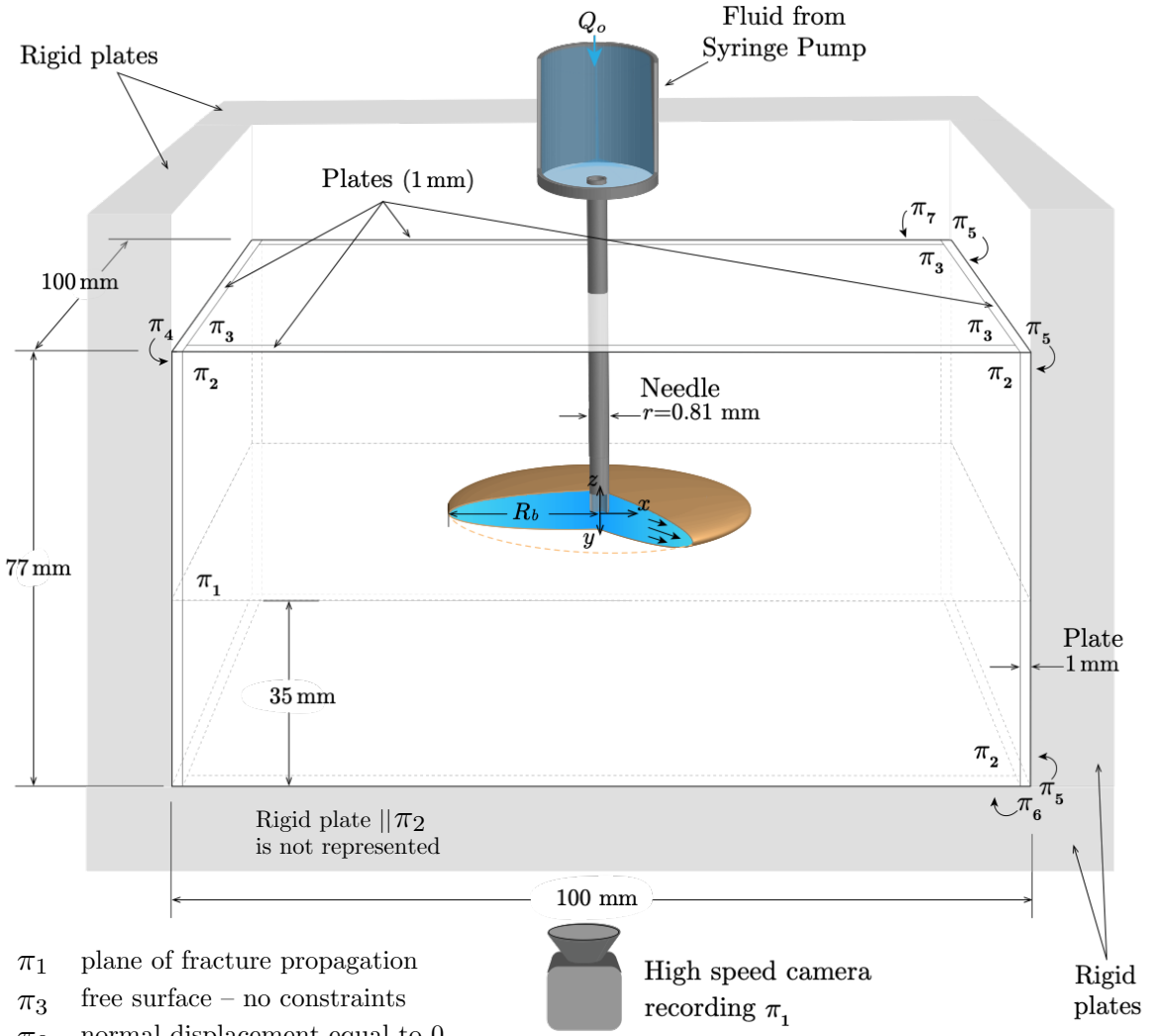
[O'Keeffe et al, 2017; O'Keeffe, 2019]





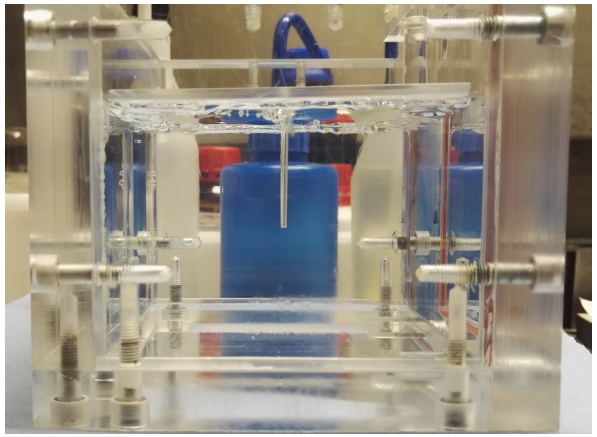
π_1 plane of fracture propagation
π_3 free surface – no constraints
π_6 normal displacement equal to 0
$\pi_2, \pi_4, \pi_5, \pi_7$ applied $\perp$ displacement (1 mm)

Experimental set up: single fracture



π_1 plane of fracture propagation
π_3 free surface – no constraints
π_6 normal displacement equal to 0
$\pi_2, \pi_4, \pi_5, \pi_7$ applied $\perp$ displacement (1 mm)

[O'Keeffe et al, 2017; O'Keeffe, 2019]

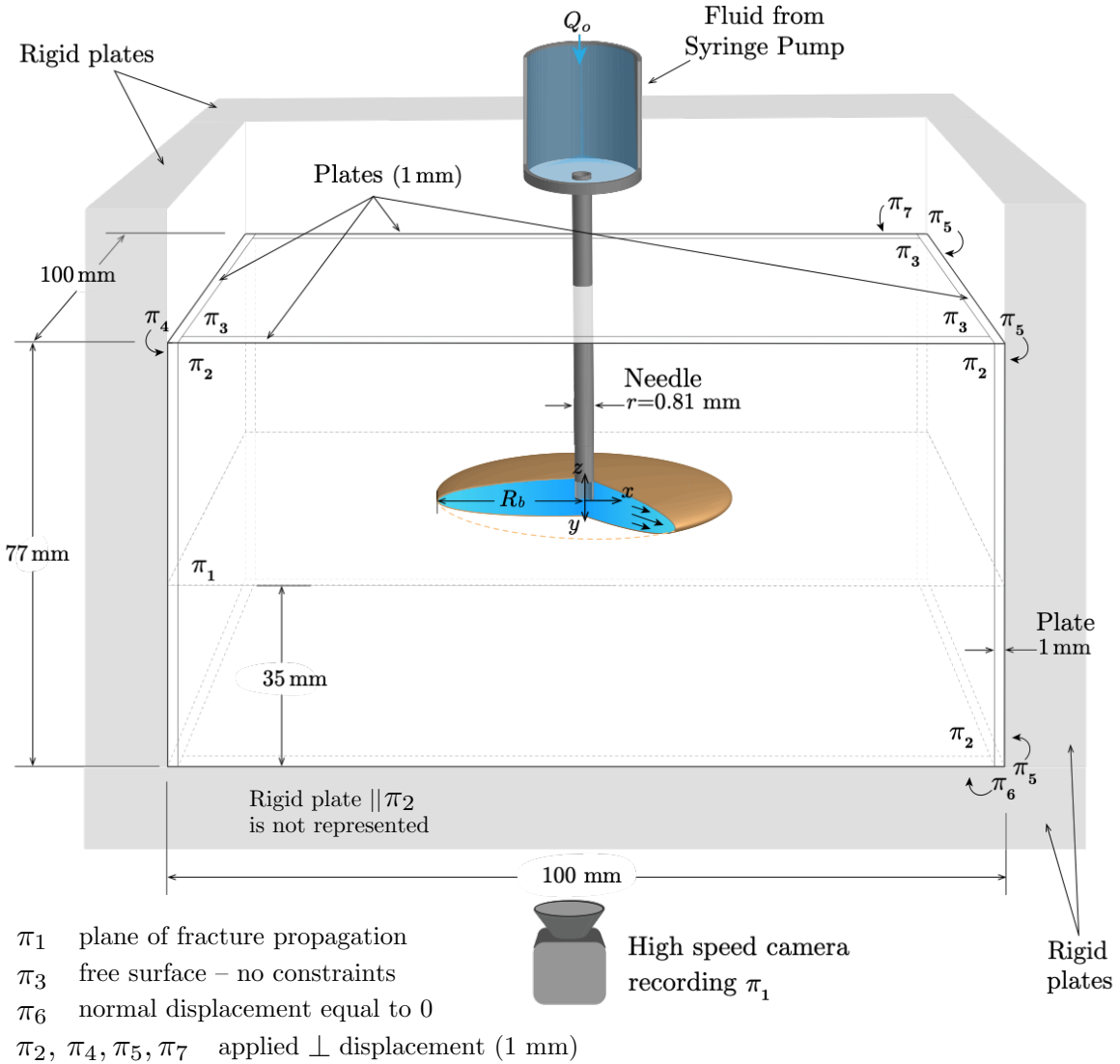


- **Solid medium**

  Polyacrylamide hydrogel
  - Linear elastic
  - Brittle

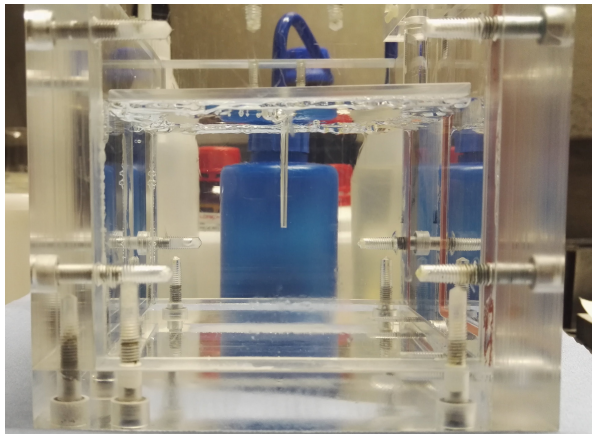
Poisson's ratio:	$\nu \sim 0.5$
Young's mod.:	$E \sim 10^2 - 10^3$ kPa
Fracture toughness:	$K_{Ic} \sim 10^3 - 10^4$ Pa m$^{1/2}$
Rayleigh wave speed:	$v_R \sim 5 - 20$ m/s

Experimental set up: single fracture



π_1 plane of fracture propagation
π_3 free surface – no constraints
π_6 normal displacement equal to 0
$\pi_2, \pi_4, \pi_5, \pi_7$ applied $\perp$ displacement (1 mm)

[O'Keeffe et al, 2017; O'Keeffe, 2019]



- **Solid medium**

  Polyacrylamide hydrogel
  - Linear elastic
  - Brittle

Poisson's ratio: $\nu \sim 0.5$

Young's mod.: $E \sim 10^2 - 10^3$ kPa

Fracture toughness: $K_{Ic} \sim 10^3 - 10^4$ Pa m$^{1/2}$

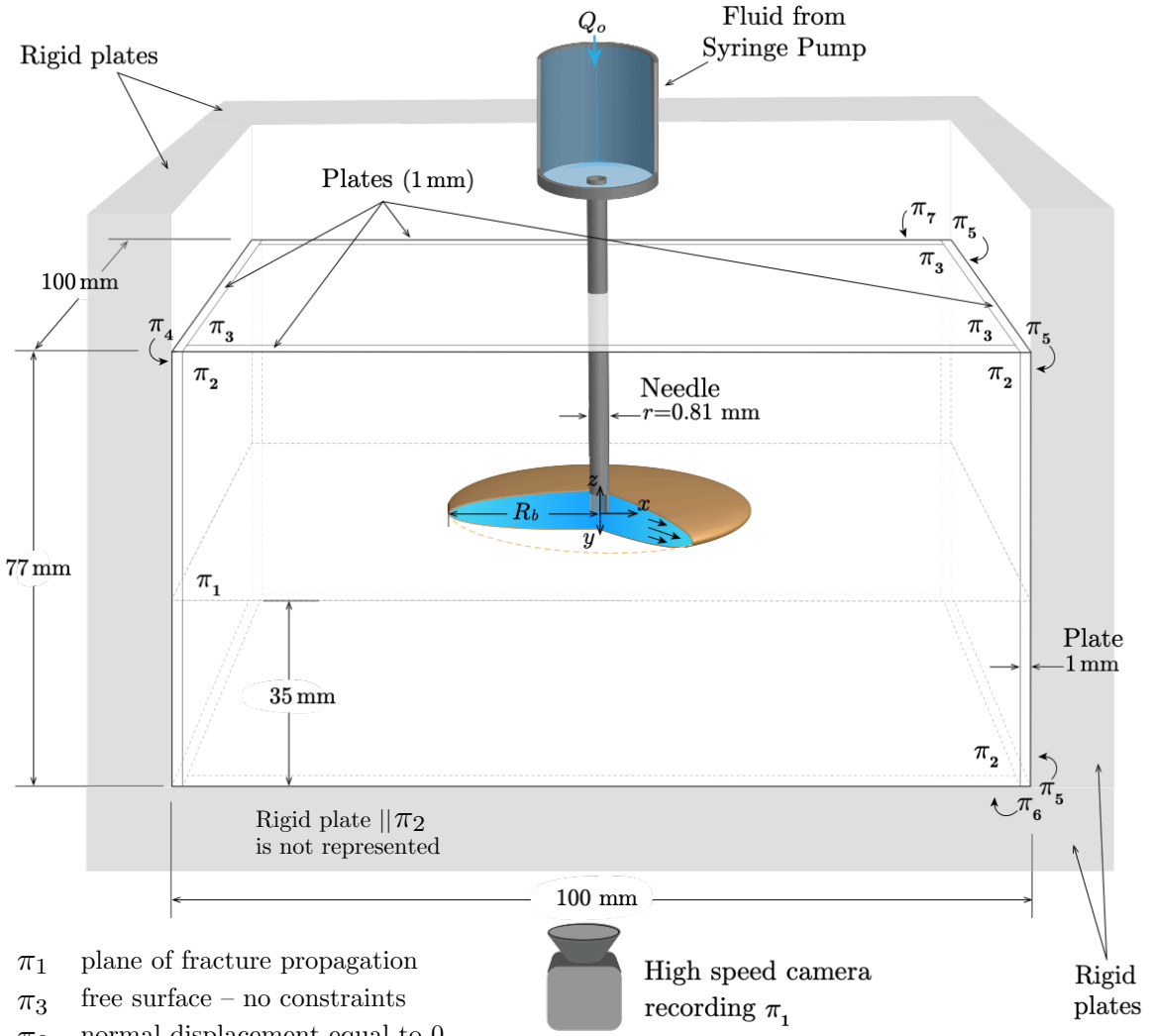
Rayleigh wave speed: $v_R \sim 5 - 20$ m/s

- **Injected Fluids**

  Glycerin, silicone, syrup, water

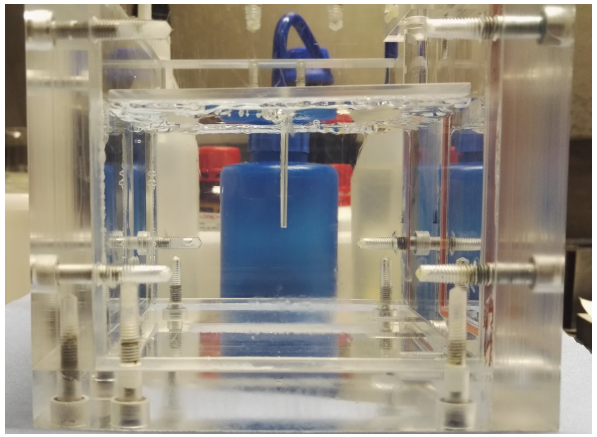
Dynamic viscosity: $\mu \sim 10^{-3} - 10^{-1}$ Pa $\cdot$ s

Experimental set up: single fracture

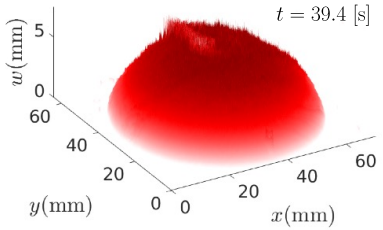


π_1 plane of fracture propagation
π_3 free surface – no constraints
π_6 normal displacement equal to 0
$\pi_2, \pi_4, \pi_5, \pi_7$ applied $\perp$ displacement (1 mm)

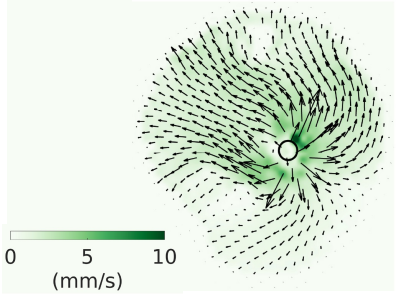
[O'Keeffe et al, 2017; O'Keeffe, 2019]



Light attenuation measurements



Particle image velocimetry



- **Solid medium**

  Polyacrylamide hydrogel
  - Linear elastic
  - Brittle

Poisson's ratio: $\nu \sim 0.5$

Young's mod.: $E \sim 10^2 - 10^3$ kPa

Fracture toughness: $K_{Ic} \sim 10^3 - 10^4$ Pa m$^{1/2}$

Rayleigh wave speed: $v_R \sim 5 - 20$ m/s

- **Injected Fluids**

  Glycerin, silicone, syrup, water

Dynamic viscosity: $\mu \sim 10^{-3} - 10^{-1}$ Pa $\cdot$ s

Mathematical model – solid medium

- Solid medium
  - homogeneous, isotropic
  - linear elastic
  - brittle
  - impermeable
  - infinite medium
  - negligible inertial effects

Mathematical model – solid medium

- Solid medium
  - homogeneous, isotropic
  - linear elastic
  - brittle
  - impermeable
  - infinite medium
  - negligible inertial effects

Linear momentum balance for a 3D-planar crack:

Young's mod. Fracture opening

$$p(x,y) - \sigma_0(x,y) = -\frac{E}{8\pi(1-\nu^2)} \int_{A(t)} \frac{w(x',y')\mathrm{d}A(x',y')}{[(x'-x)^2+(y'-y)^2]^{3/2}}$$

Fluid pressure Confining stress Poisson's ratio Fracture area at a given time

Mathematical model – solid medium

- Solid medium
  - homogeneous, isotropic
  - linear elastic
  - brittle
  - impermeable
  - infinite medium
  - negligible inertial effects

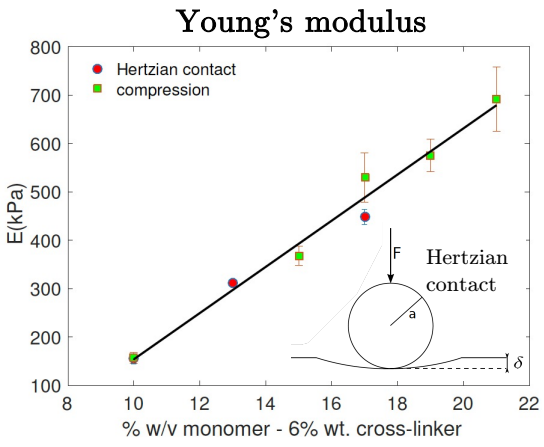
Linear momentum balance for a 3D-planar crack:

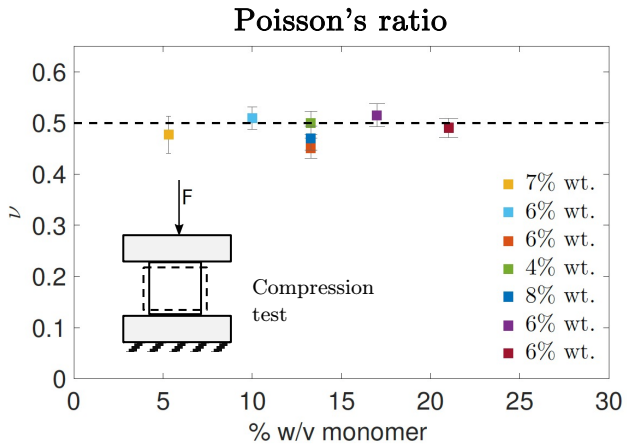
Young's mod. Fracture opening

$$p(x,y) - \sigma_0(x,y) = -\frac{E}{8\pi(1-\nu^2)} \int_{A(t)} \frac{w(x',y')\mathrm{d}A(x',y')}{[(x'-x)^2+(y'-y)^2]^{3/2}}$$

Fluid pressure Confining stress Poisson's ratio Fracture area at a given time

Material characterization taken from [O'Keeffe, 2019]





Mathematical model – solid medium

- Solid medium
  - homogeneous, isotropic
  - linear elastic
  - brittle
  - impermeable
  - infinite medium
  - negligible inertial effects

Linear momentum balance for a 3D-planar crack:

Young's mod. Fracture opening

$$p(x,y) - \sigma_0(x,y) = -\frac{E}{8\pi(1-\nu^2)} \int_{A(t)} \frac{w(x',y')\mathrm{d}A(x',y')}{[(x'-x)^2+(y'-y)^2]^{3/2}}$$

Fluid pressure Confining stress Poisson's ratio Fracture area at a given time

Propagation condition:

$$(K_I - K_{Ic}) \leq 0$$

$$(K_I - K_{Ic}) \times V = 0$$

$$K_{Ic} \propto \gamma_s$$

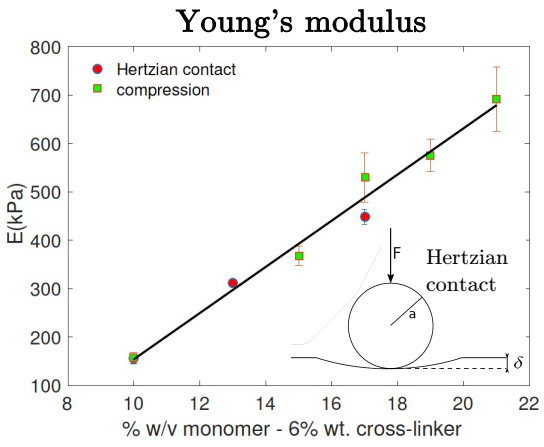
Fracture energy

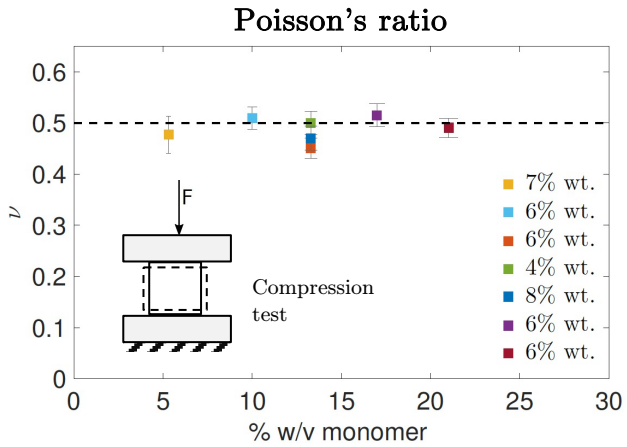
Local fracture front velocity

Critical, mode I, stress intensity factor

Local mode I, stress intensity factor

Material characterization taken from [O'Keeffe, 2019]





Mathematical model – solid medium

- Solid medium
  - homogeneous, isotropic
  - linear elastic
  - brittle
  - impermeable
  - infinite medium
  - negligible inertial effects

Linear momentum balance for a 3D-planar crack:

Young's mod. Fracture opening

$$p(x,y) - \sigma_0(x,y) = -\frac{E}{8\pi(1-\nu^2)} \int_{A(t)} \frac{w(x',y')\mathrm{d}A(x',y')}{[(x'-x)^2+(y'-y)^2]^{3/2}}$$

Fluid pressure Confining stress Poisson's ratio Fracture area at a given time

Propagation condition:

$$(K_I - K_{Ic}) \leq 0$$

$$(K_I - K_{Ic}) \times V = 0$$

$$K_{Ic} \propto \gamma_s$$

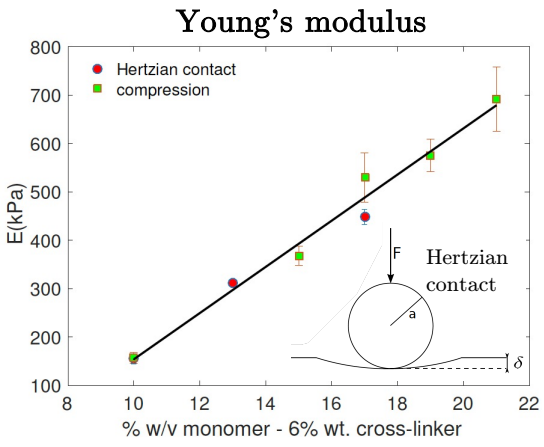
Fracture energy

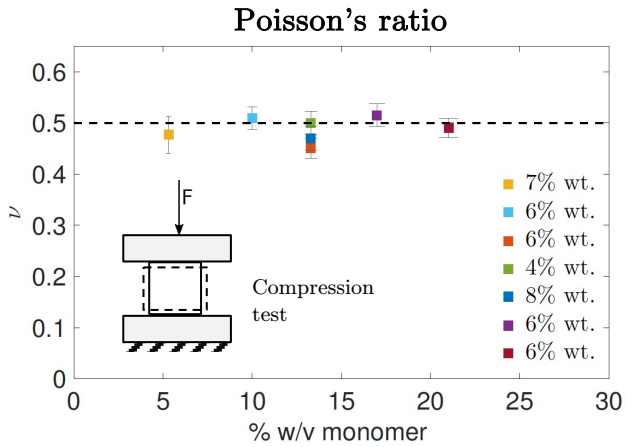
Local fracture front velocity

Critical, mode I, stress intensity factor

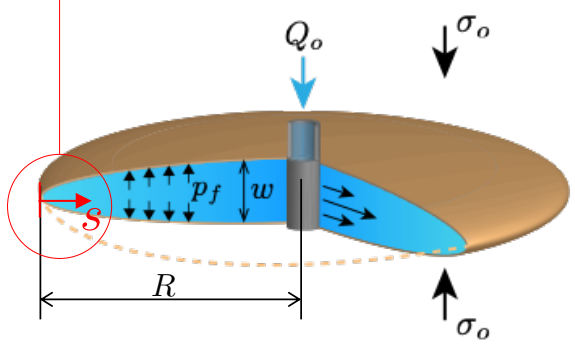
Local mode I, stress intensity factor

Material characterization taken from [O'Keeffe, 2019]





Crack tip opening: $\lim_{s\to 0} \frac{w}{\sqrt{s}} = \sqrt{\frac{32}{\pi}} \frac{K_{Ic}}{E}(1-\nu^2)$



Mathematical model – solid medium

- Solid medium
  - homogeneous, isotropic
  - linear elastic
  - brittle
  - impermeable
  - infinite medium
  - negligible inertial effects

Linear momentum balance for a 3D-planar crack:

Young's mod. — Fracture opening

$$p(x,y) - \sigma_0(x,y) = -\frac{E}{8\pi(1-\nu^2)} \int_{A(t)} \frac{w(x',y')\mathrm{d}A(x',y')}{[(x'-x)^2+(y'-y)^2]^{3/2}}$$

Fluid pressure — Confining stress — Poisson's ratio — Fracture area at a given time

Propagation condition:

$$(K_I - K_{Ic}) \leq 0$$

$$(K_I - K_{Ic}) \times V = 0$$

$$K_{Ic} \propto \gamma_s$$

Fracture energy

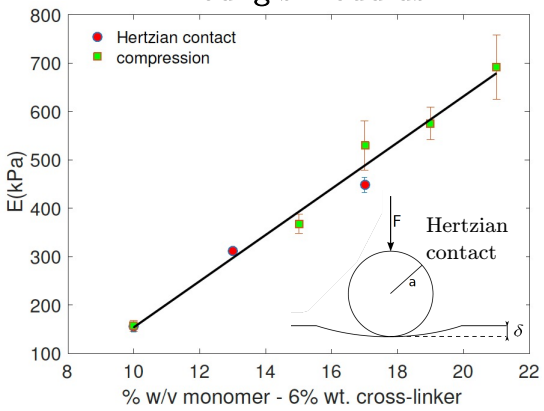
Local fracture front velocity

Critical, mode I, stress intensity factor

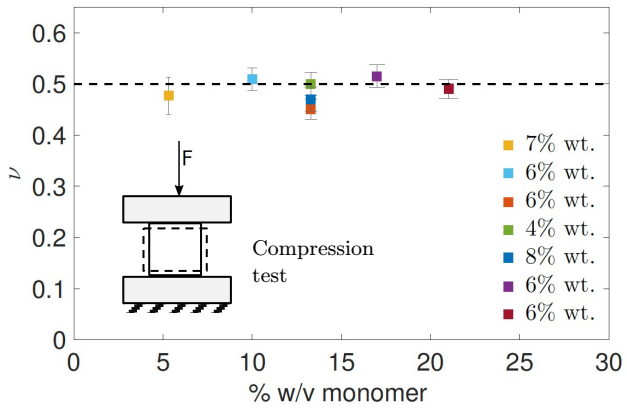
Local mode I, stress intensity factor

Material characterization taken from [O'Keeffe, 2019]

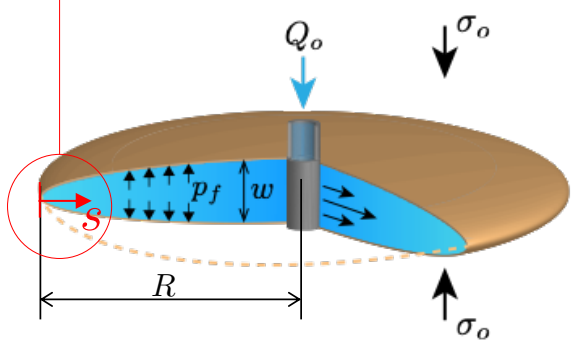
Young's modulus



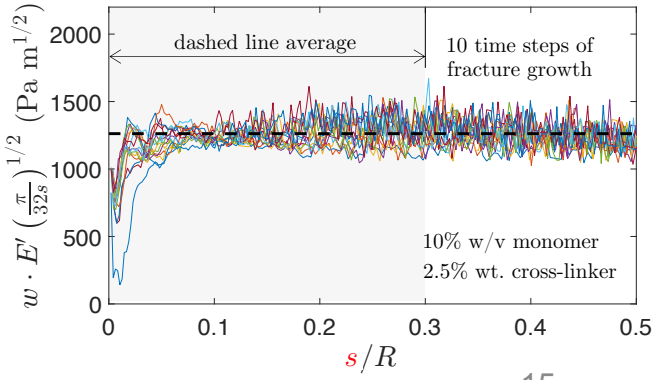
Poisson's ratio



Crack tip opening: $\lim_{s\to 0} \frac{w}{\sqrt{s}} = \sqrt{\frac{32}{\pi}} \frac{K_{Ic}}{E}(1-\nu^2)$

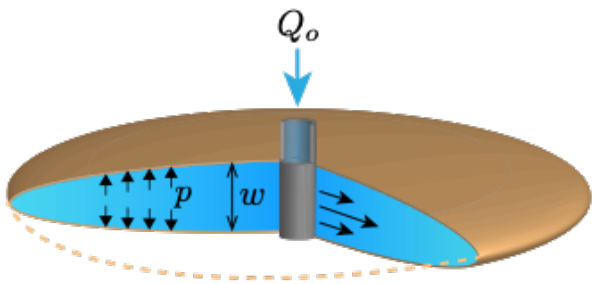


Fracture toughness



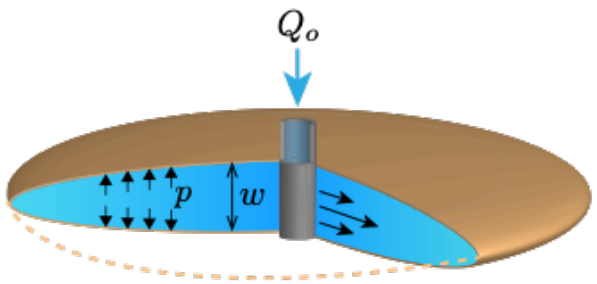
Mathematical model – fluid phase

- Injected Fluid
  - Newtonian, incompressible
  - Lubrication approximation [Batchelor, 1967]
    - Small Reynolds number
    - Transverse length scale of the flow << length scale in the flow direction
  - Negligible fluid lag [Lecampion et al, 2017]
  - Constant injection rate



Mathematical model – fluid phase

- Injected Fluid
  - Newtonian, incompressible
  - Lubrication approximation [Batchelor, 1967]
    - Small Reynolds number
    - Transverse length scale of the flow << length scale in the flow direction
  - Negligible fluid lag [Lecampion et al, 2017]
  - Constant injection rate



Width average mass conservation:

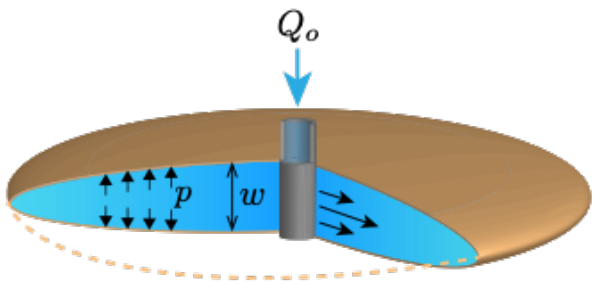
Fracture opening Inlet flux

$$\frac{\partial w}{\partial t} = -\nabla \cdot \mathbf{q} + Q_o(x, y)$$

Fluid flux

Mathematical model – fluid phase

- Injected Fluid
  - Newtonian, incompressible
  - Lubrication approximation [Batchelor, 1967]
    - Small Reynolds number
    - Transverse length scale of the flow << length scale in the flow direction
  - Negligible fluid lag [Lecampion et al, 2017]
  - Constant injection rate



Width average mass conservation:

Fracture opening Inlet flux

$$\frac{\partial w}{\partial t} = -\nabla \cdot \mathbf{q} + Q_o(x, y)$$

Fluid flux

Width average balance of momentum:

Fluid pressure

$$\mathbf{q} = \frac{-w^3}{12\mu} \nabla p$$

Dynamic viscosity

Mathematical model – fluid phase

- Injected Fluid
  - Newtonian, incompressible
  - Lubrication approximation [Batchelor, 1967]
    - Small Reynolds number
    - Transverse length scale of the flow << length scale in the flow direction
  - Negligible fluid lag [Lecampion et al, 2017]
  - Constant injection rate

Width average mass conservation:

Fracture opening Inlet flux

$$\frac{\partial w}{\partial t} = -\nabla \cdot \mathbf{q} + Q_o(x, y)$$

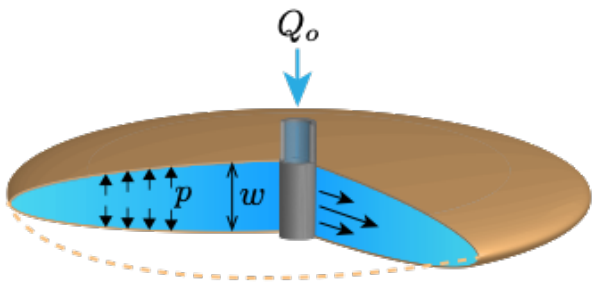
Fluid flux

Width average balance of momentum:

Fluid pressure

$$\mathbf{q} = \frac{-w^3}{12\mu} \nabla p$$

Dynamic viscosity



Dynamic viscosity

This parameter has been measured using a U-tube viscometer [O'Keeffe, 2019]

Mathematical model – fluid phase

- Injected Fluid
  - Newtonian, incompressible
  - Lubrication approximation [Batchelor, 1967]
    - Small Reynolds number
    - Transverse length scale of the flow << length scale in the flow direction
  - Negligible fluid lag [Lecampion et al, 2017]
  - Constant injection rate

Width average mass conservation:

Fracture opening Inlet flux

$$\frac{\partial w}{\partial t} = -\nabla \cdot \mathbf{q} + Q_o(x,y)$$

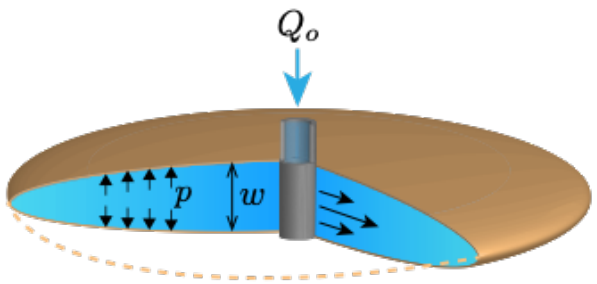
Fluid flux

Width average balance of momentum:

Fluid pressure

$$\mathbf{q} = \frac{-w^3}{12\mu}\nabla p$$

Dynamic viscosity



Dynamic viscosity

This parameter has been measured using a U-tube viscometer [O'Keeffe, 2019]

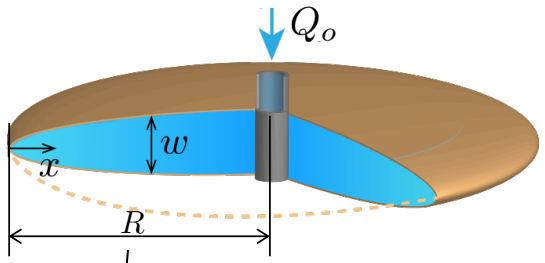
Experimental errors

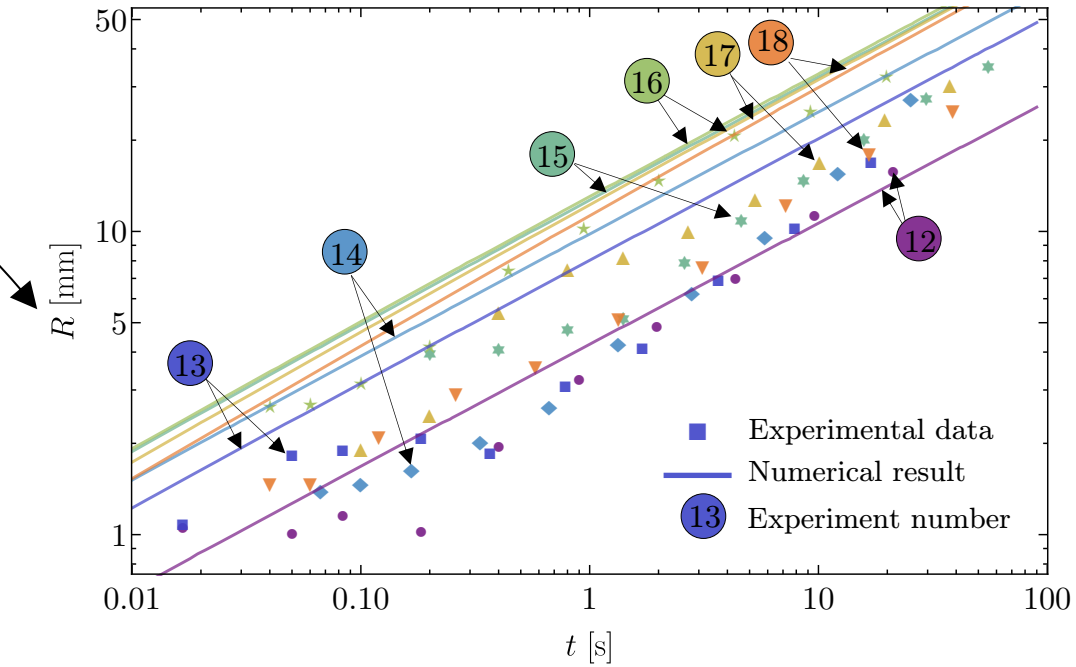
The orders of magnitude of the experimental errors have been estimated [O'Keeffe, 2019]

Young's modulus	E	$\approx \pm 10\%$
Fracture energy	γ_s	$\approx \pm 10\%$
Dynamic viscosity	μ	$\approx \pm 5\%$
Inlet flux	Q_o	$\approx \pm 10\%$

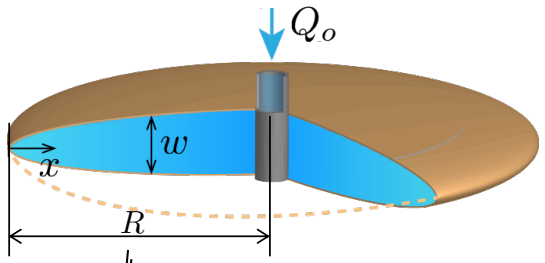
See [Zia & Lecampion 2020] for further details and code verifications against semi-analytical solutions.

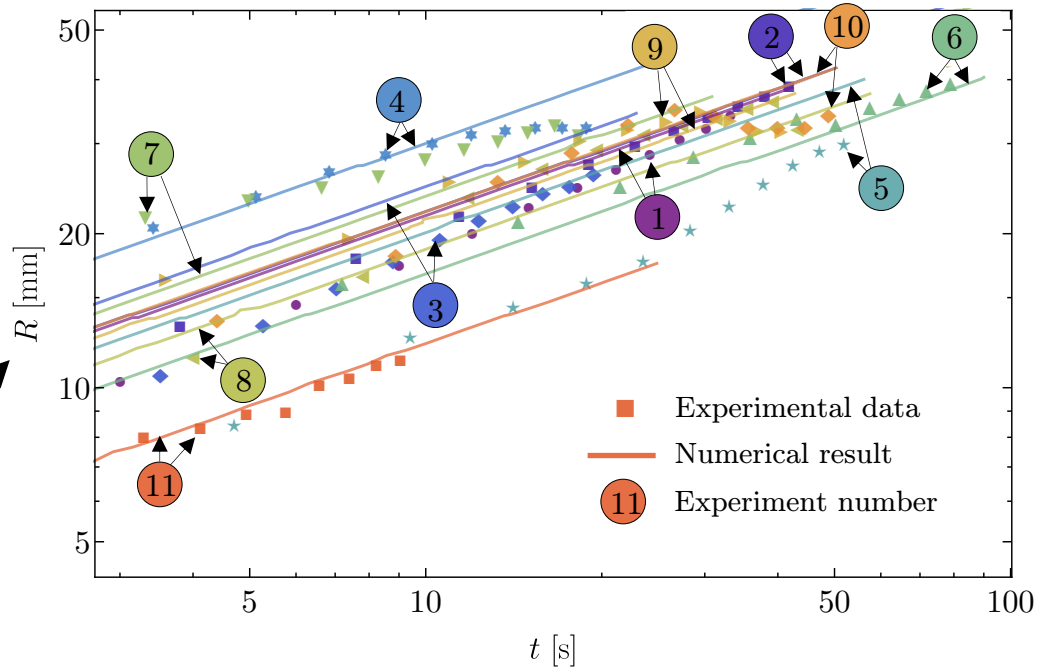
Single fracture experiment

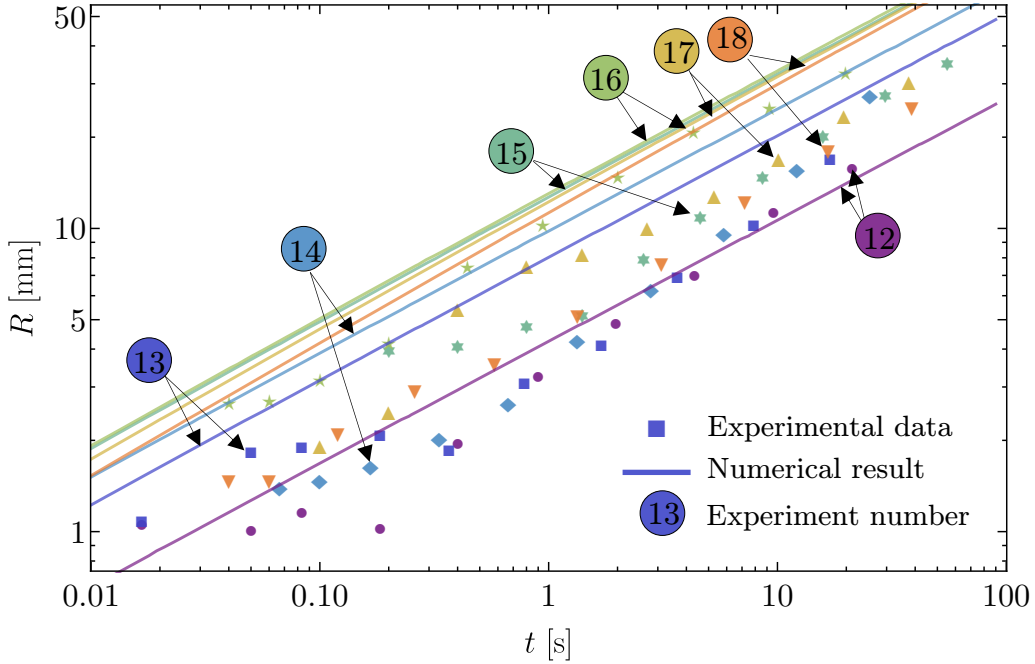




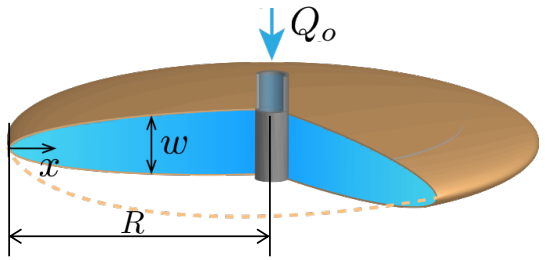
Single fracture experiment

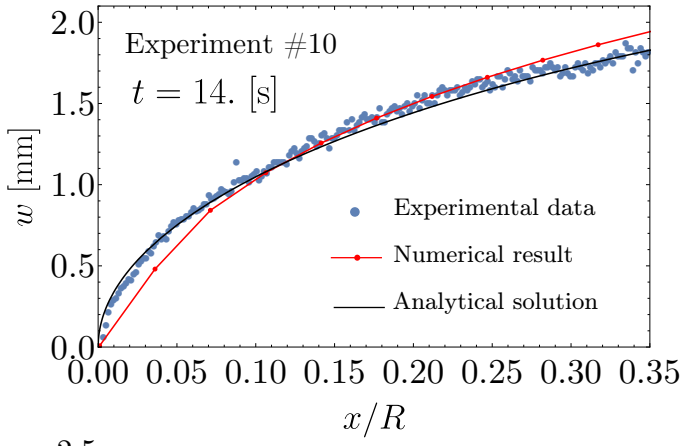


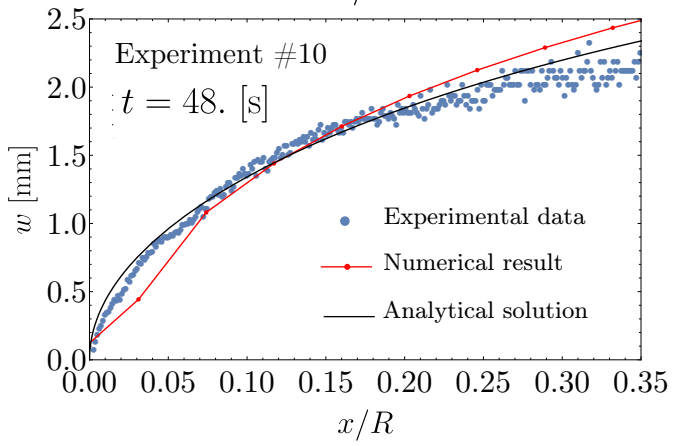


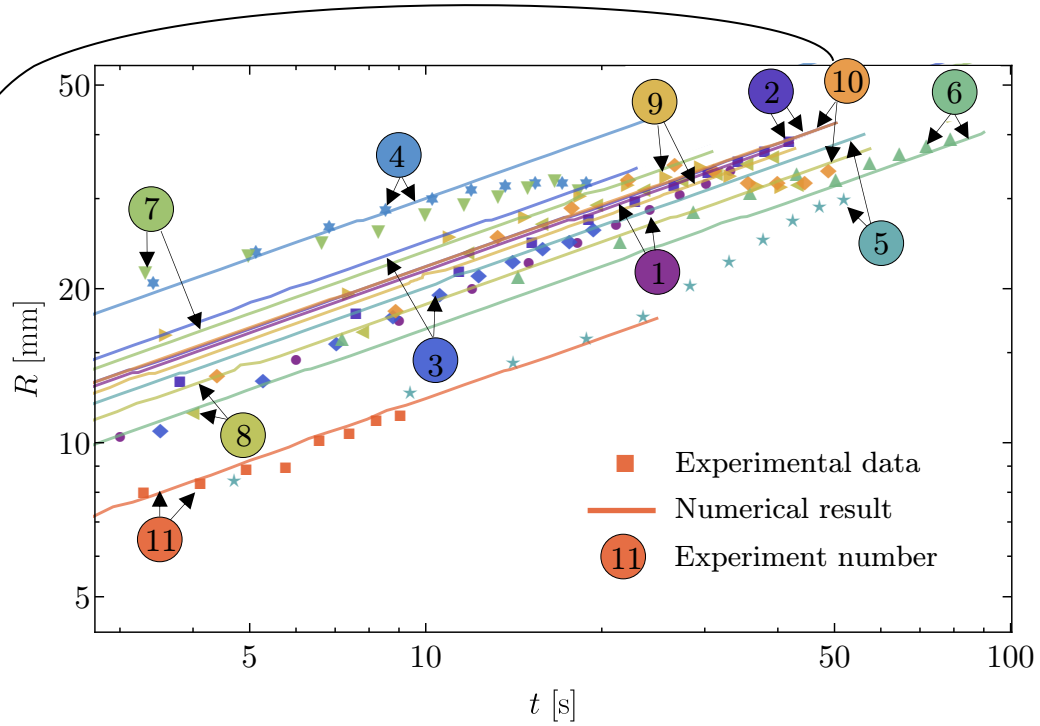


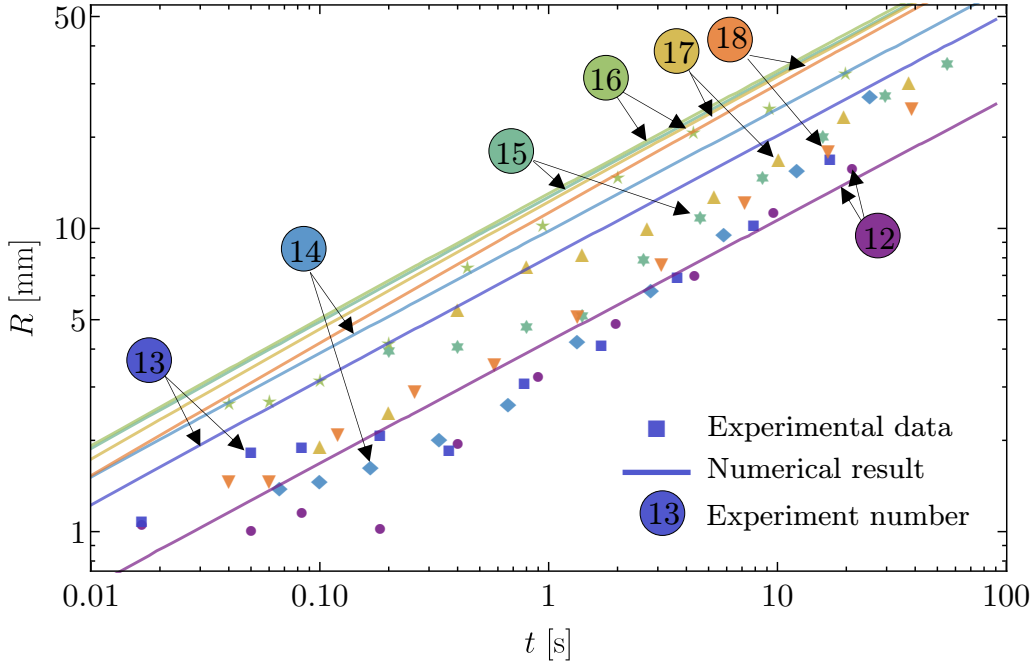
Single fracture experiment



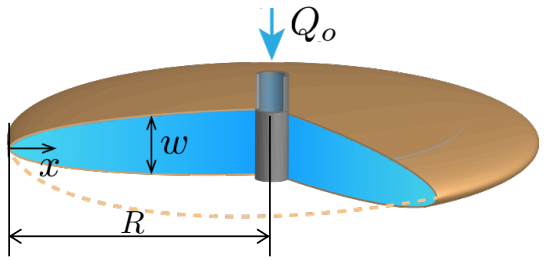


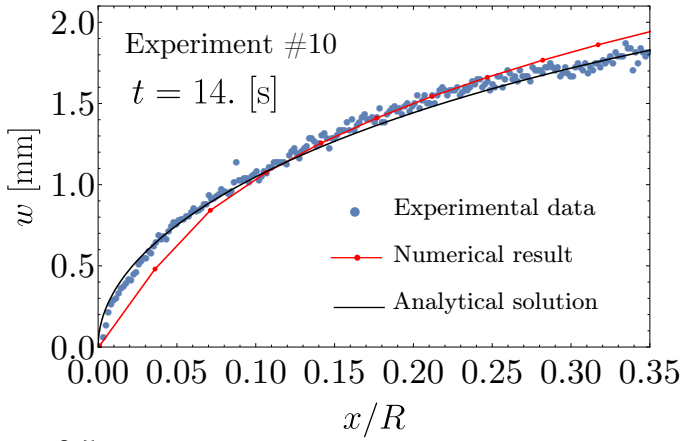


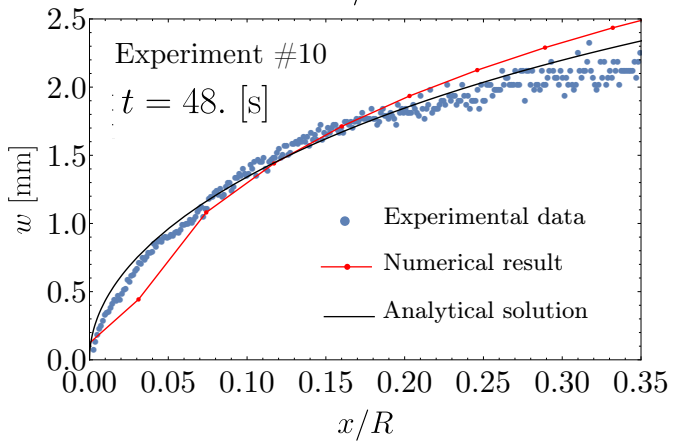


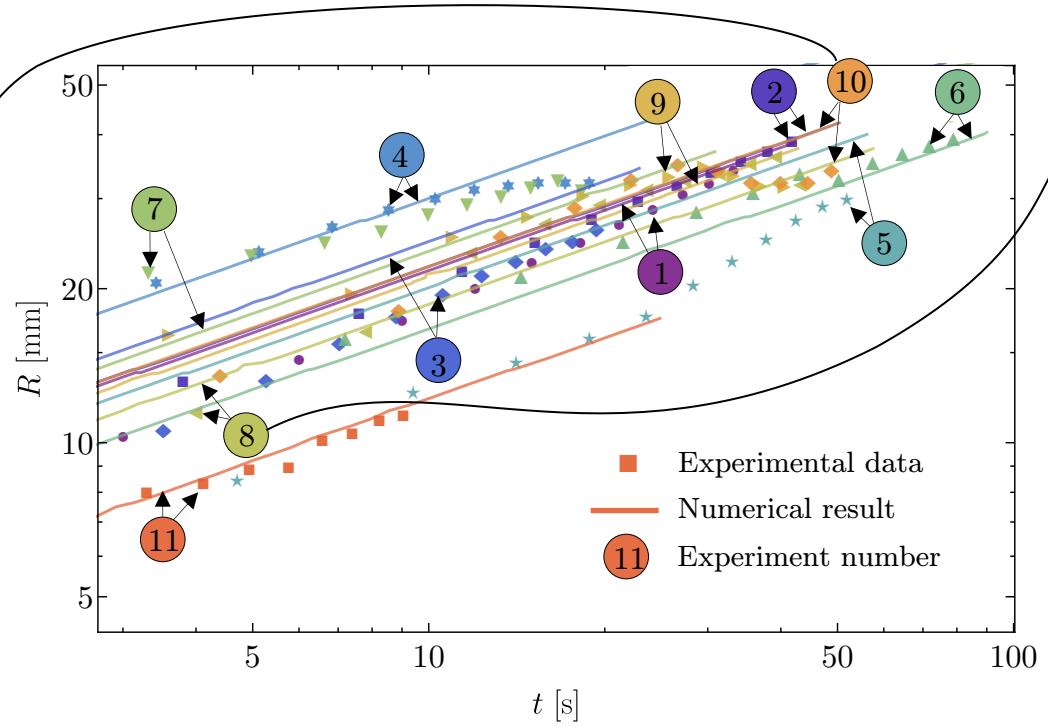


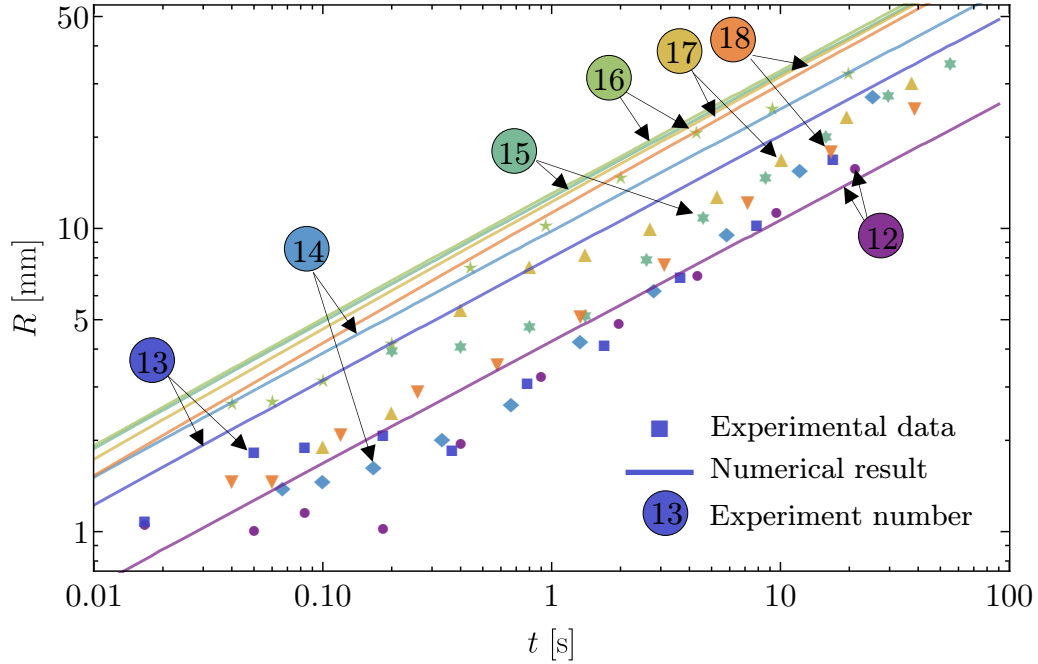
Single fracture experiment

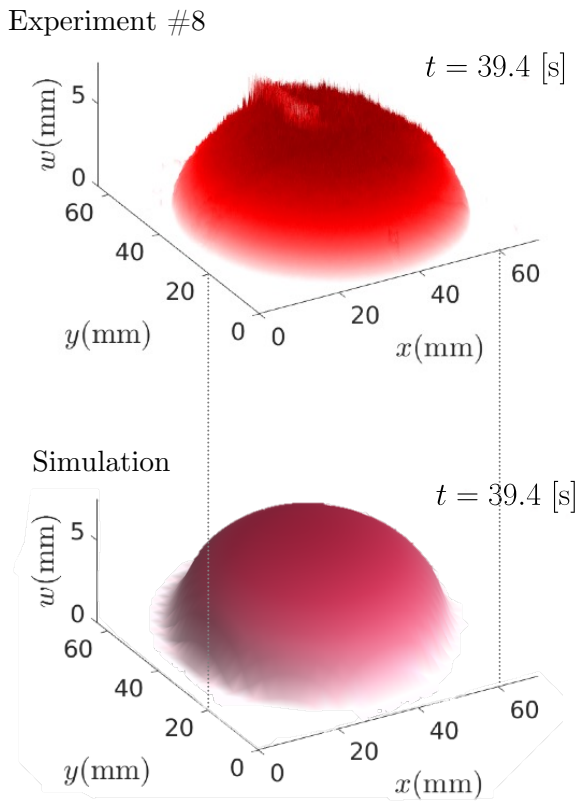


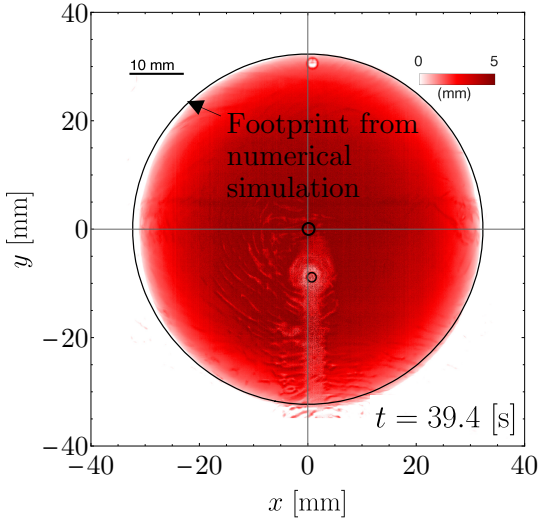




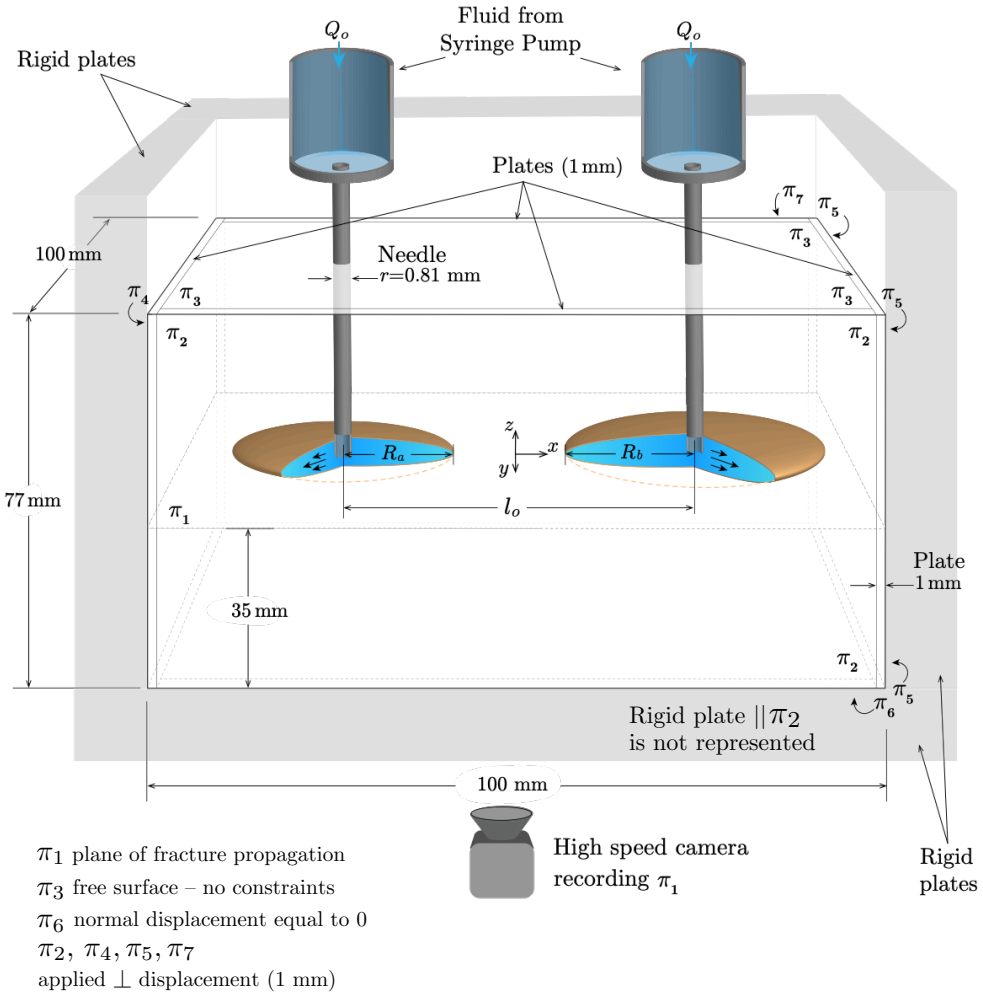




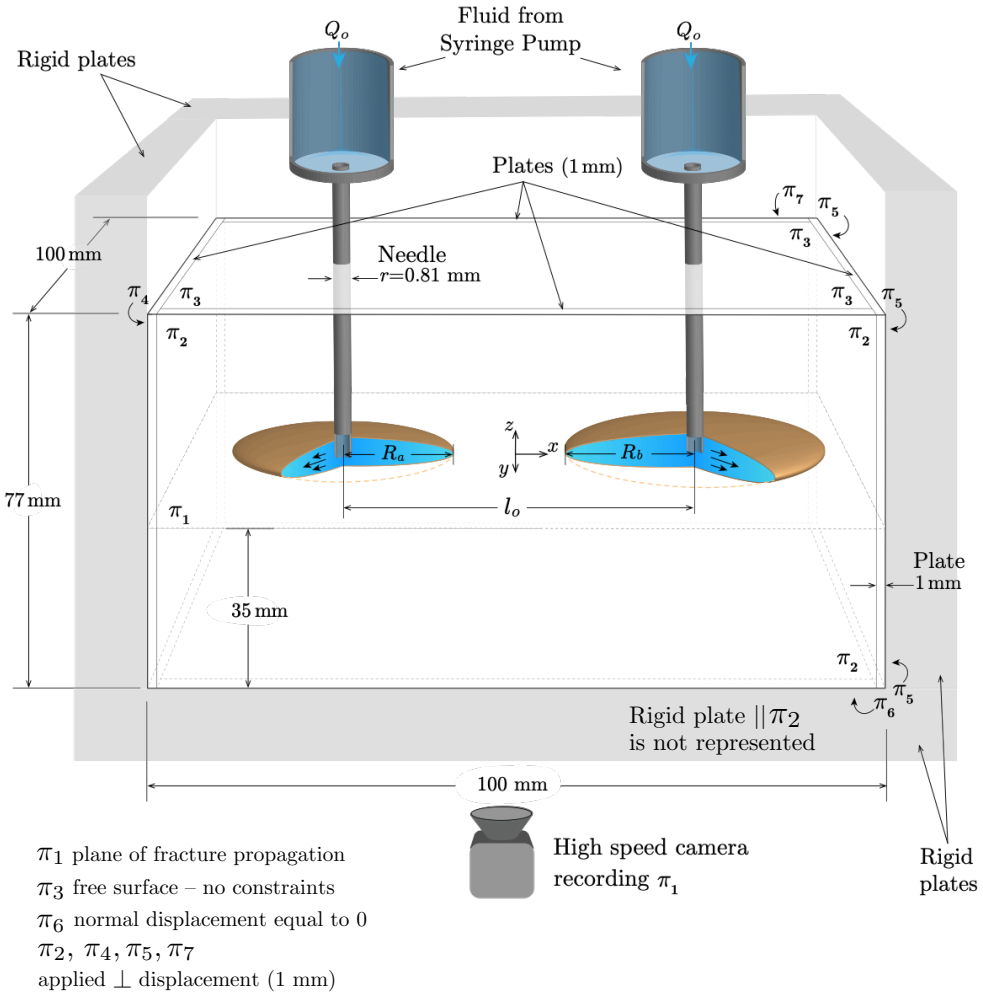


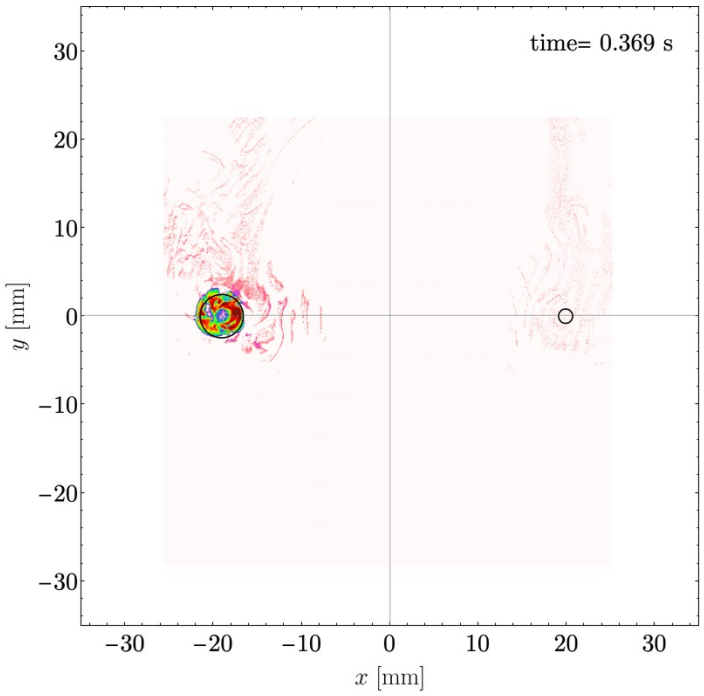


Experiment 1

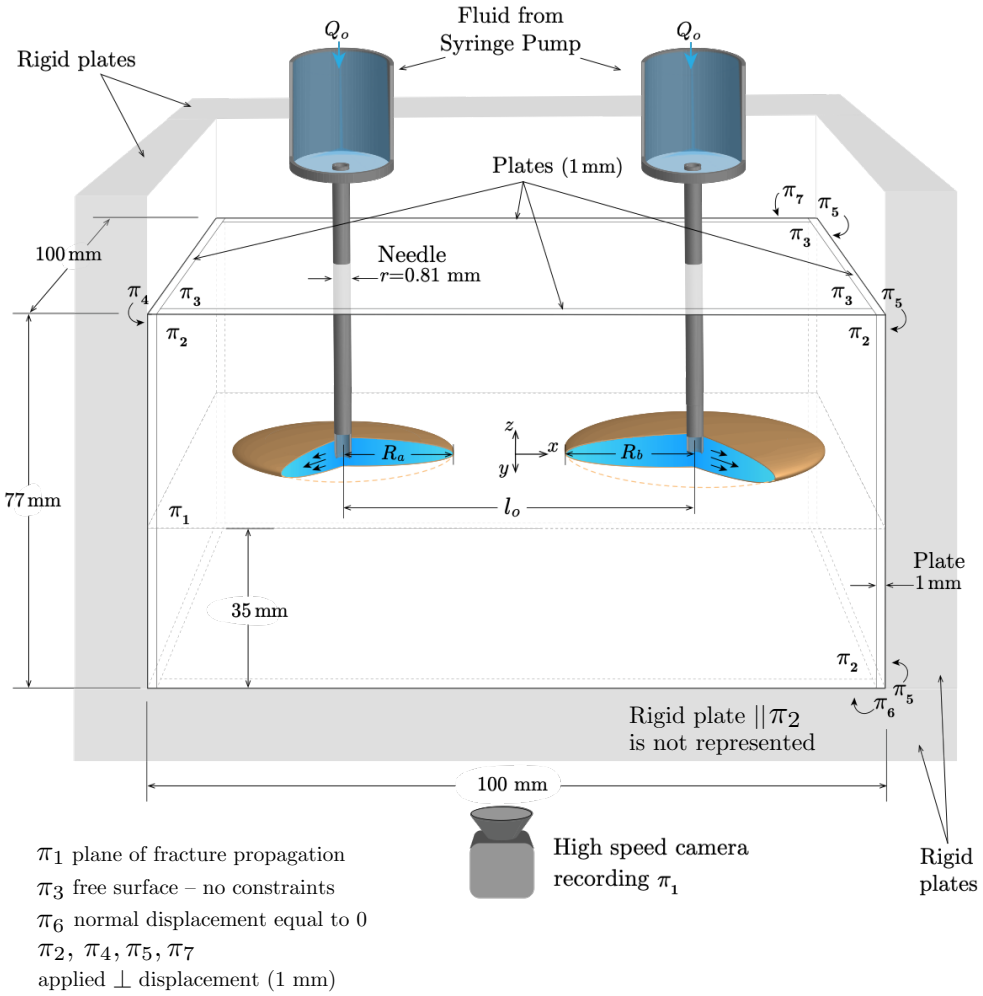


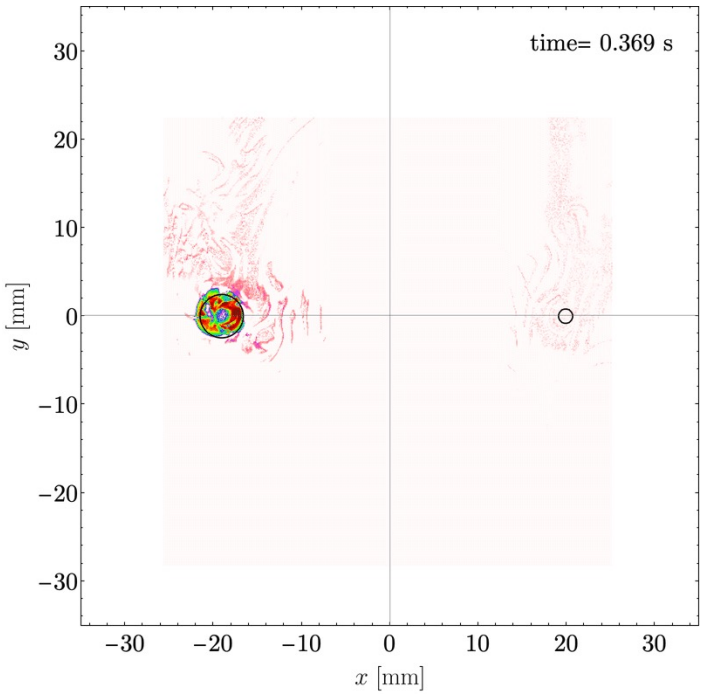
Experiment 1

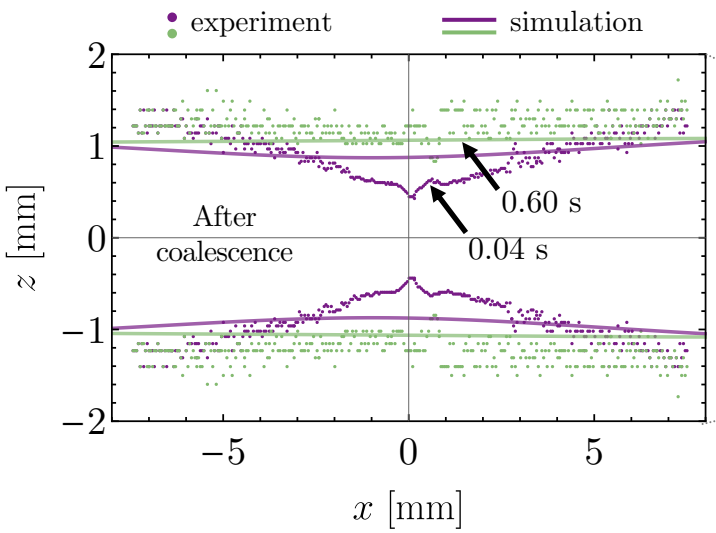




Experiment 1

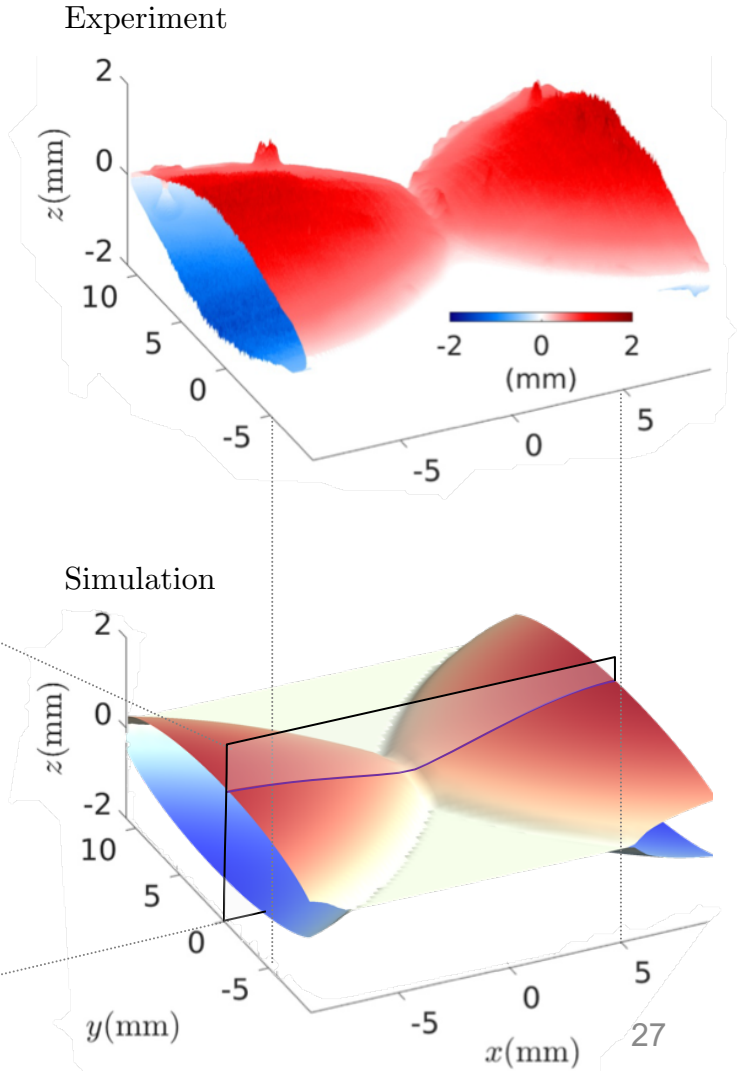




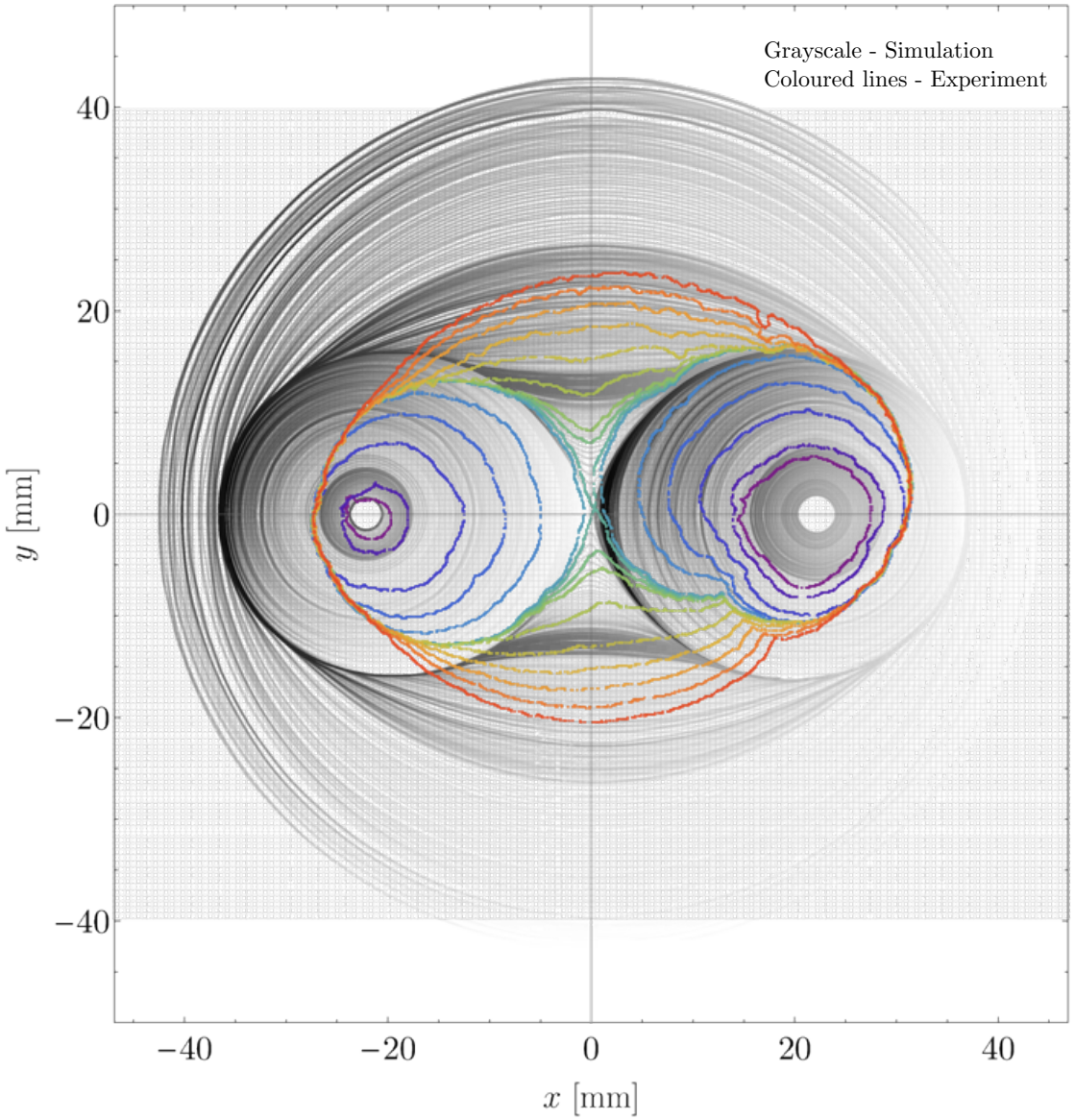


3D fracture profile

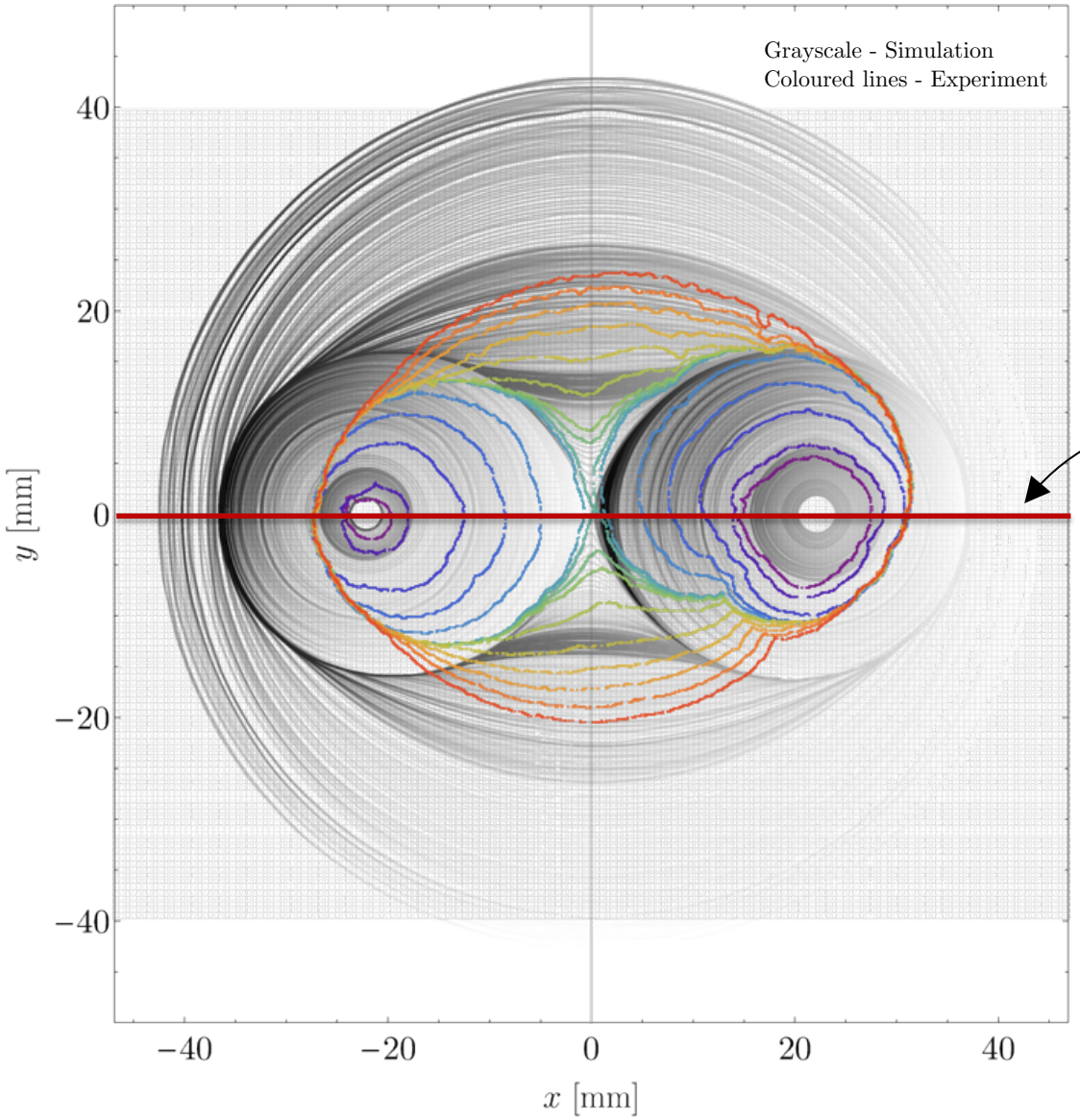
$t = 0.04$ s after coalescence

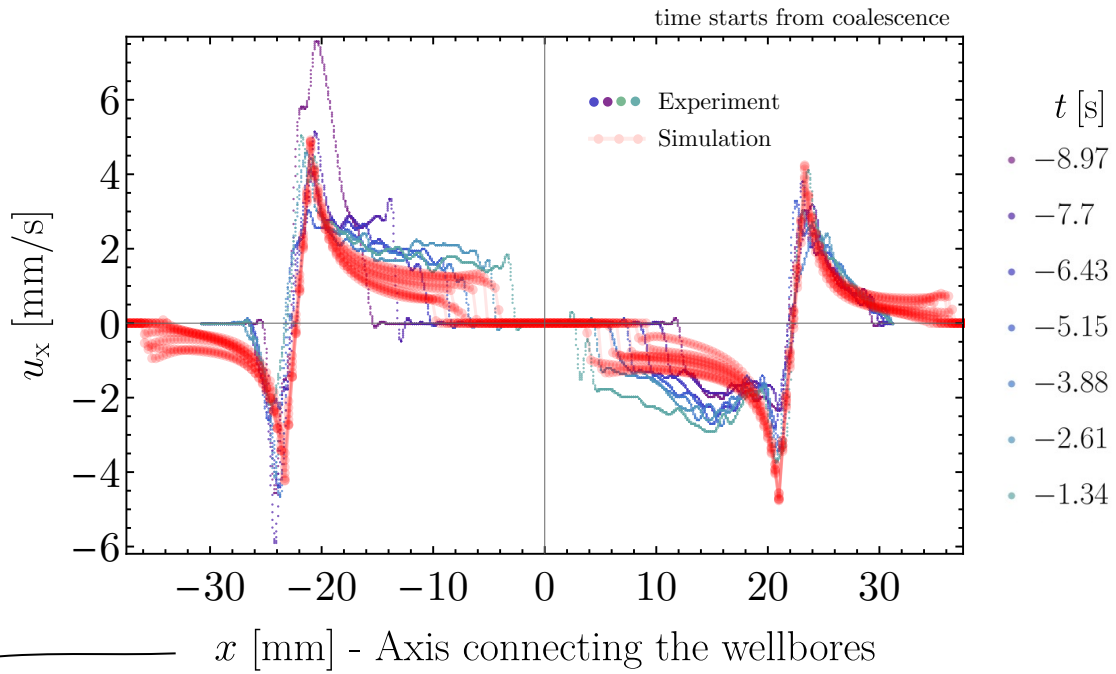


Experiment 2

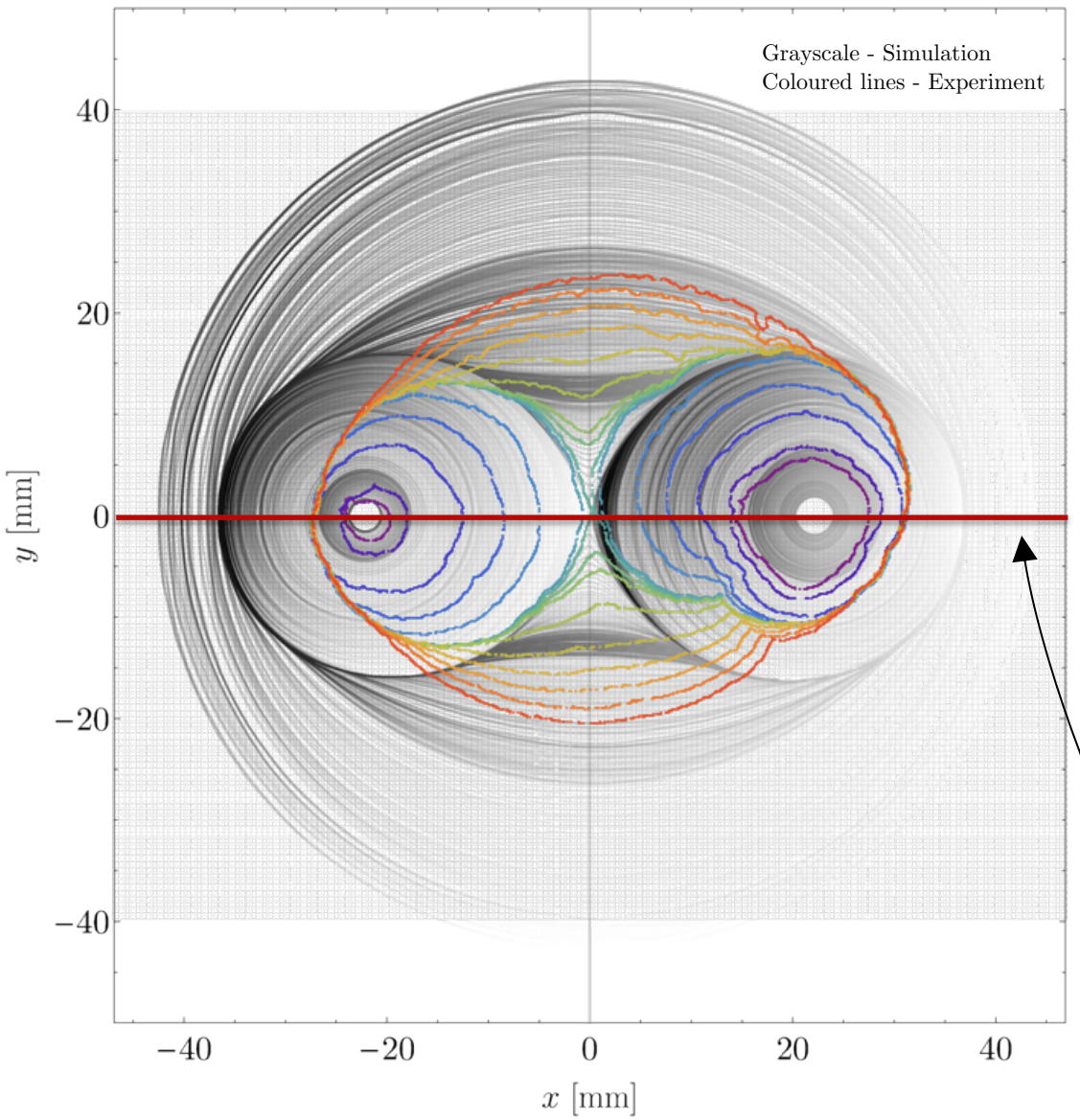


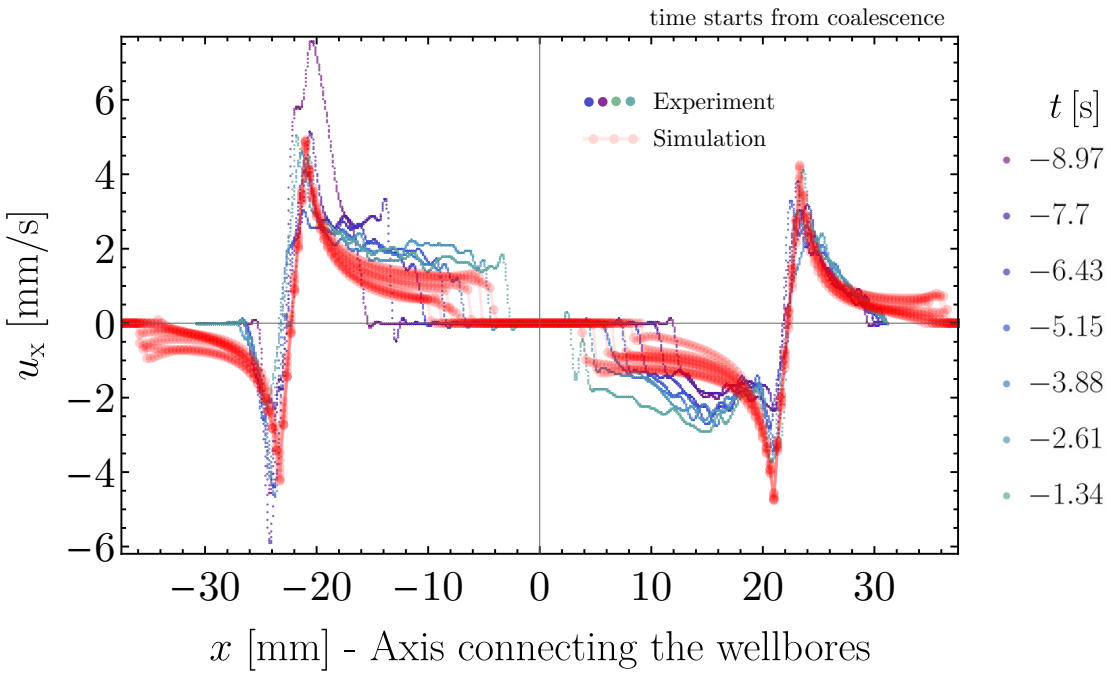
Experiment 2

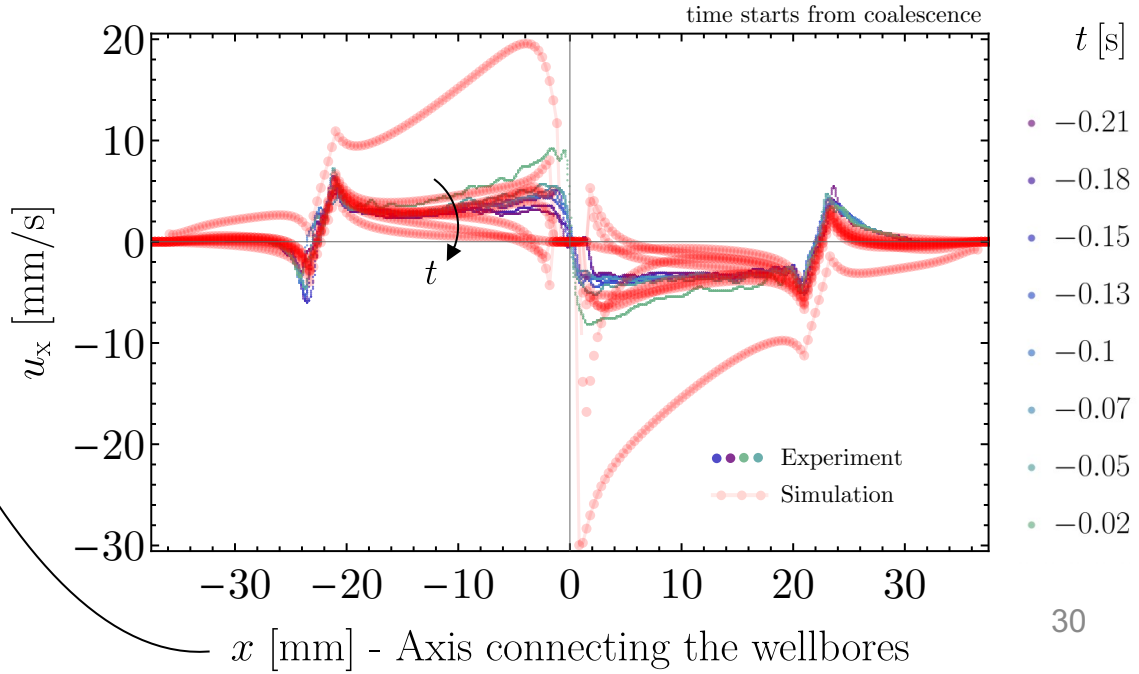




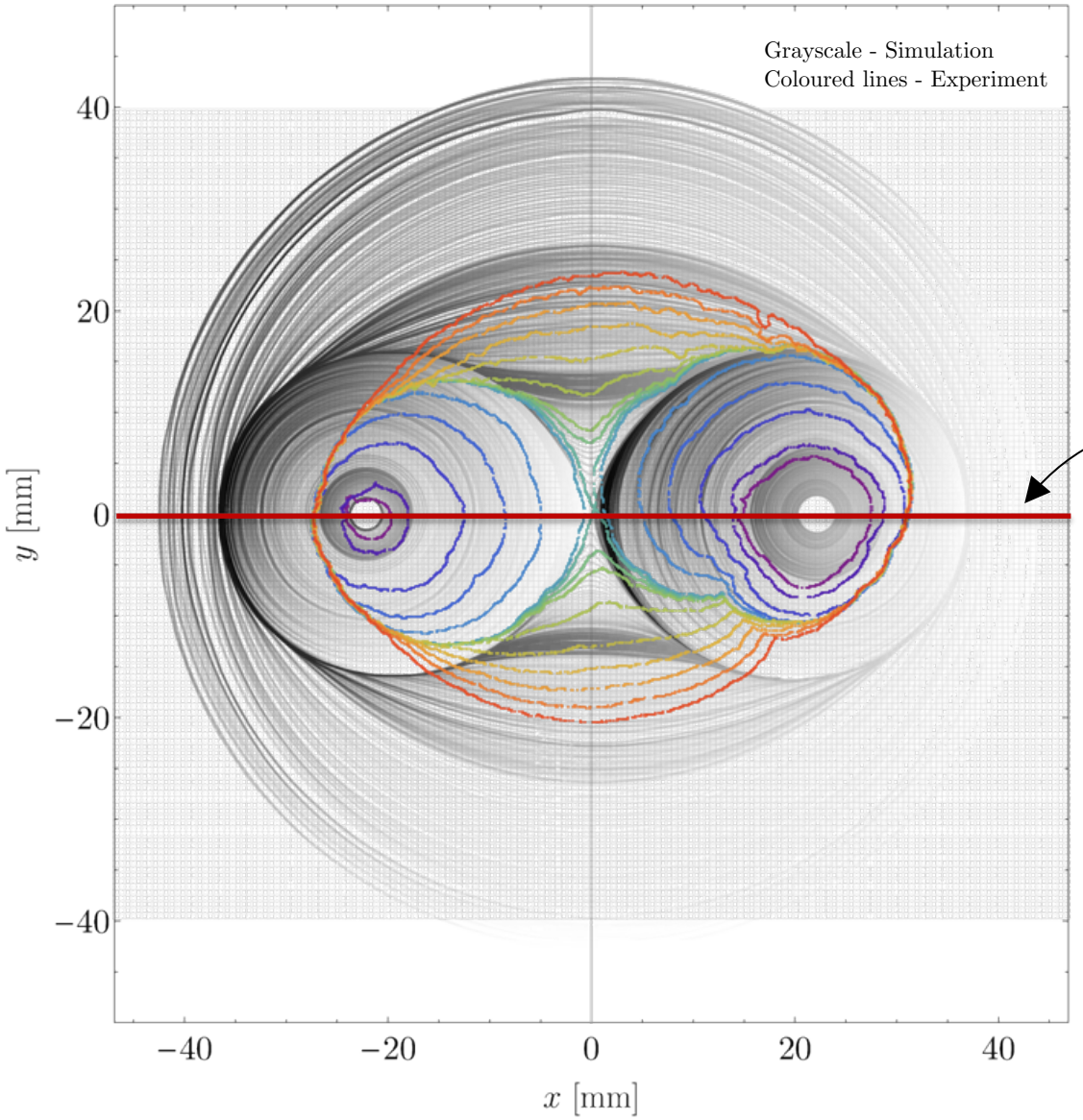
Experiment 2

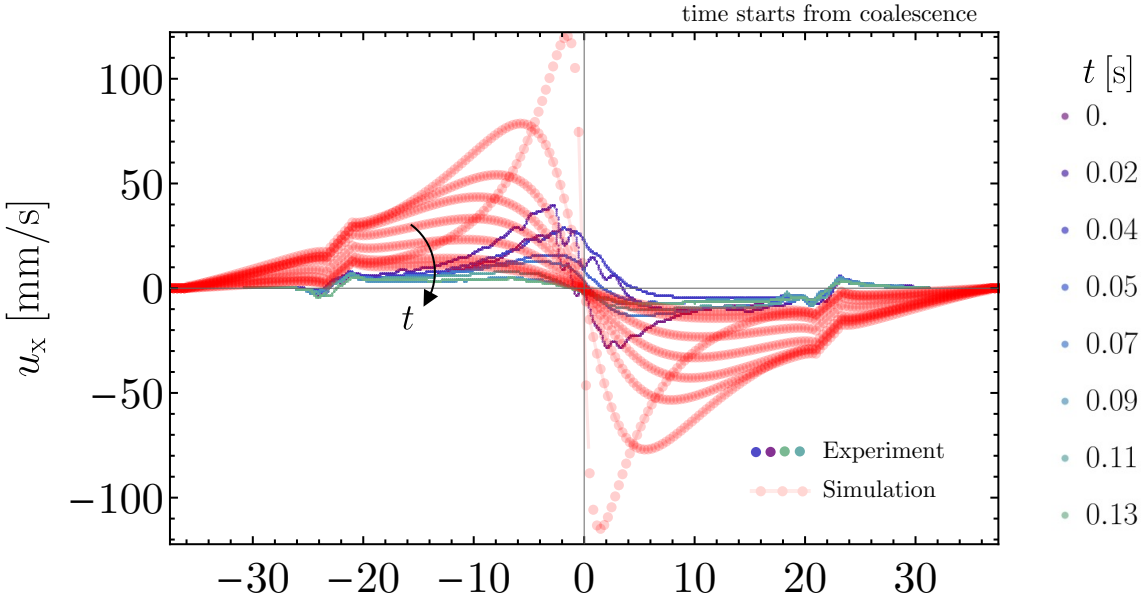






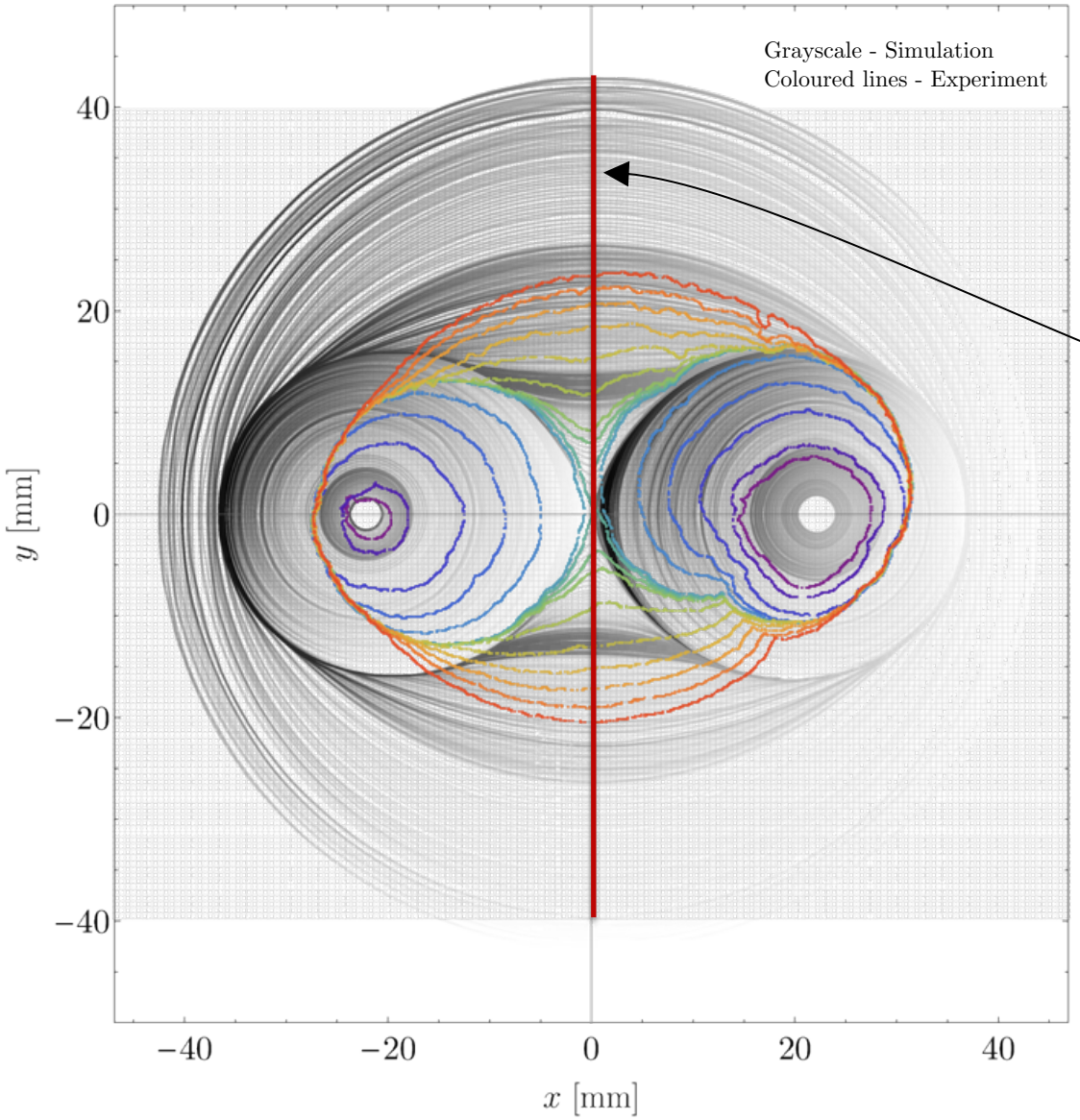
Experiment 2

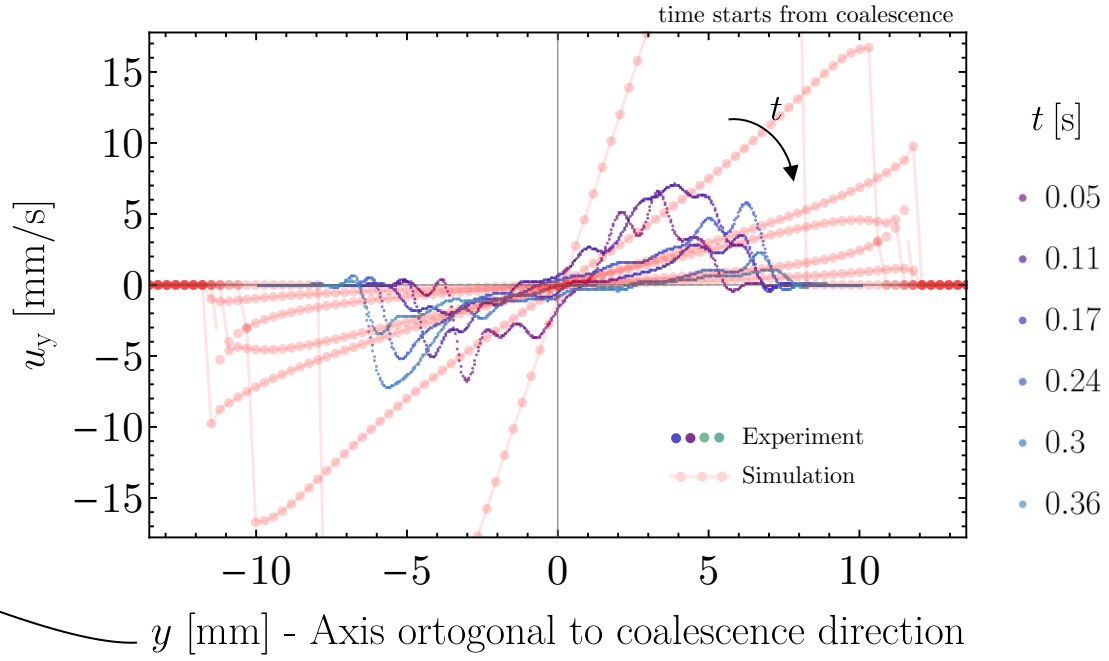




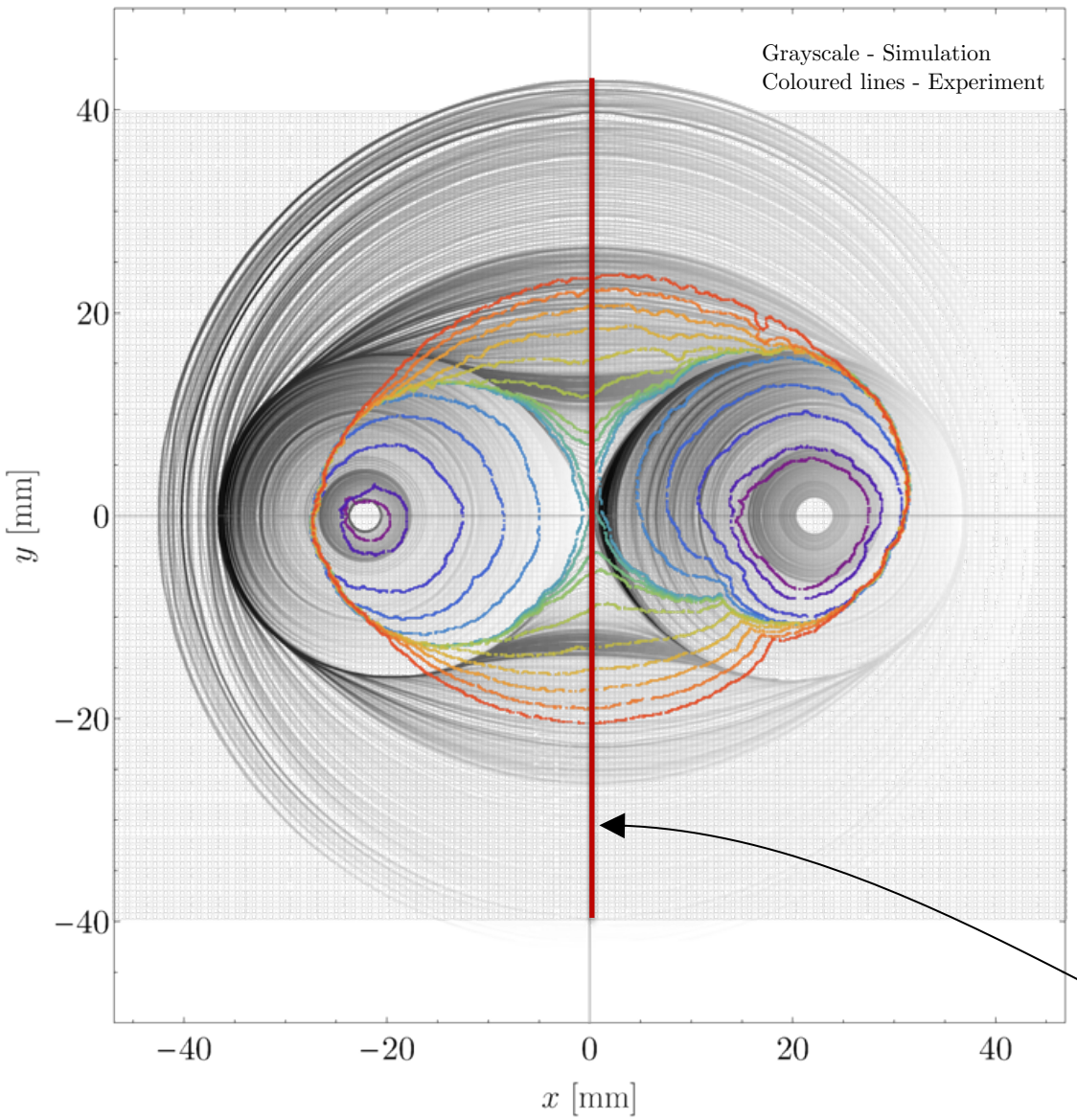
x [mm] - Axis connecting the wellbores

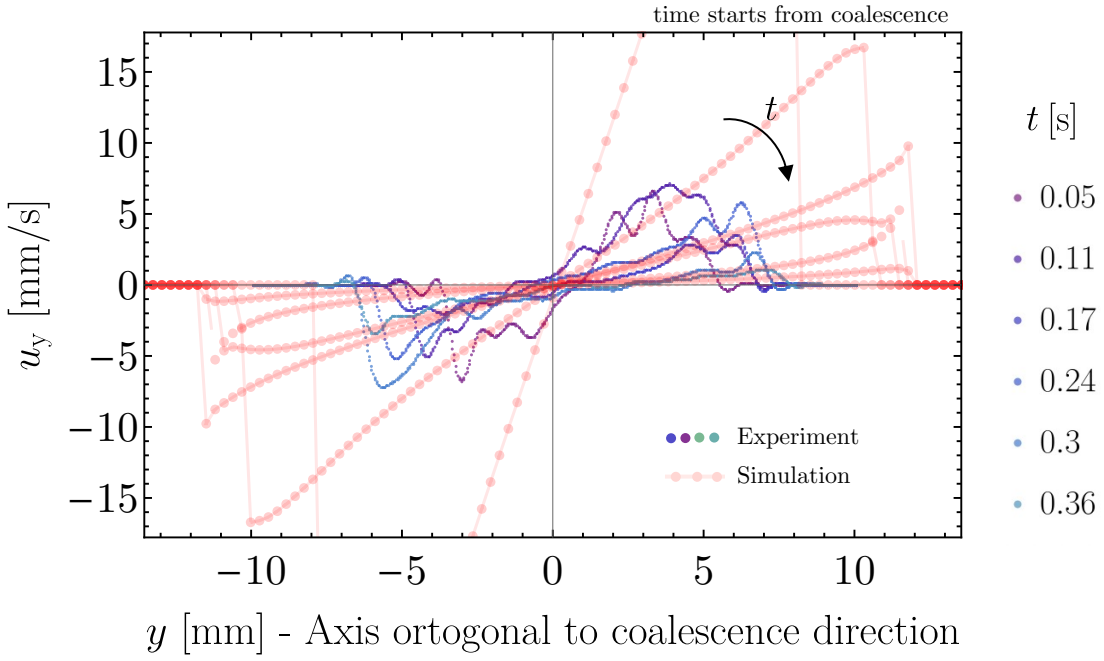
Experiment 2

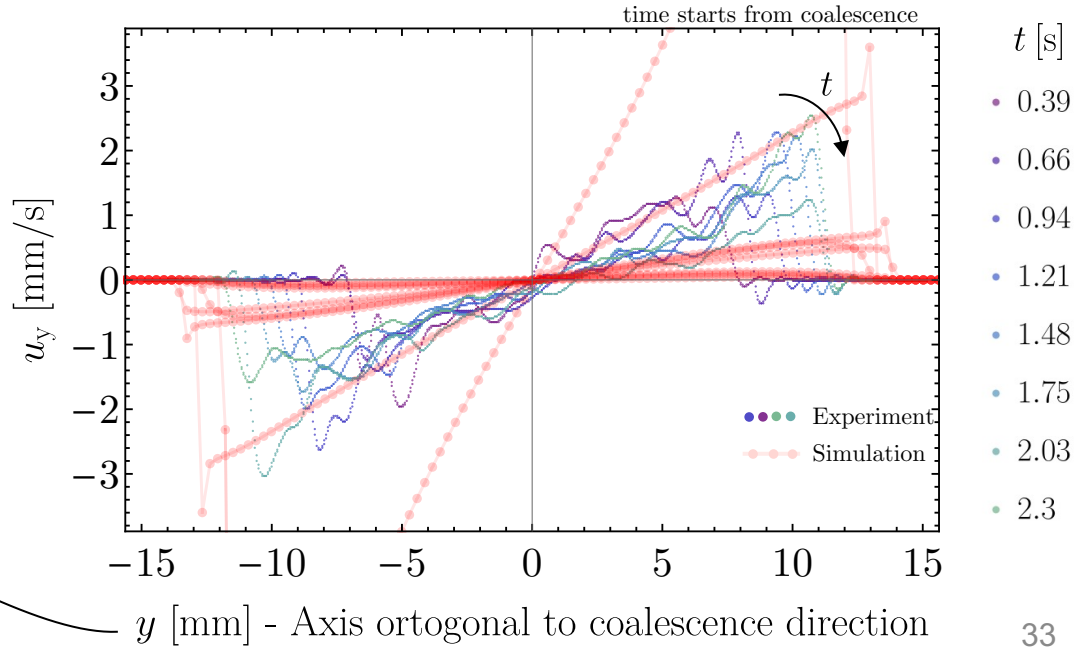




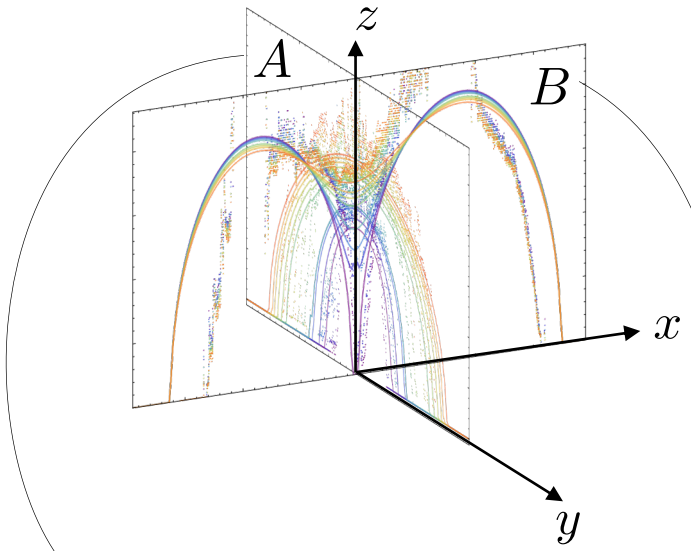
Experiment 2

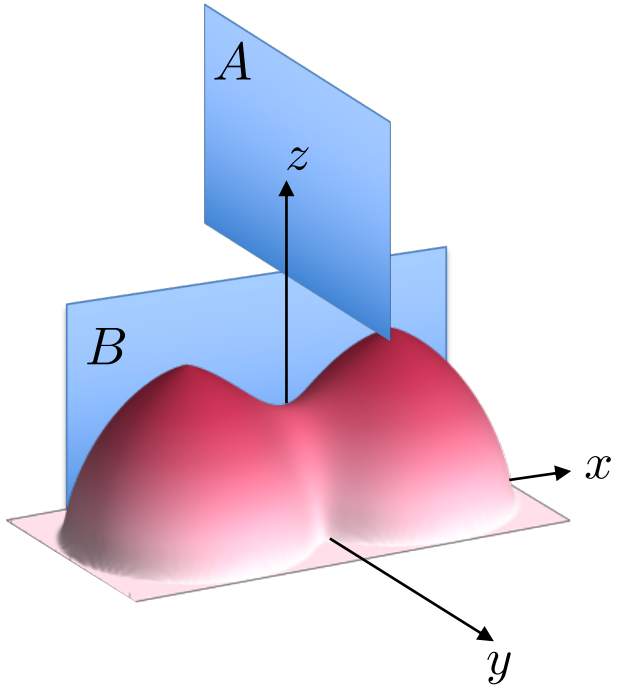


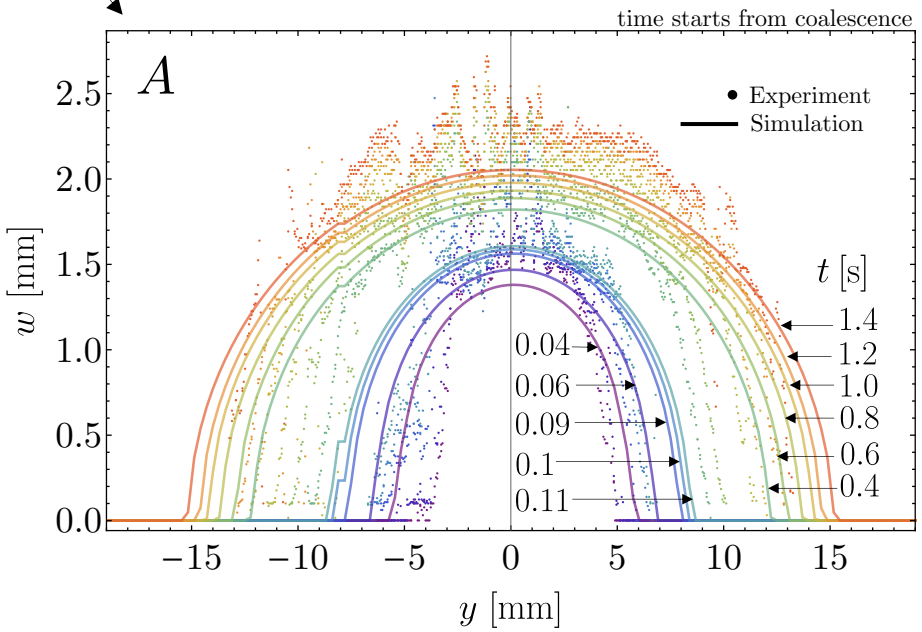


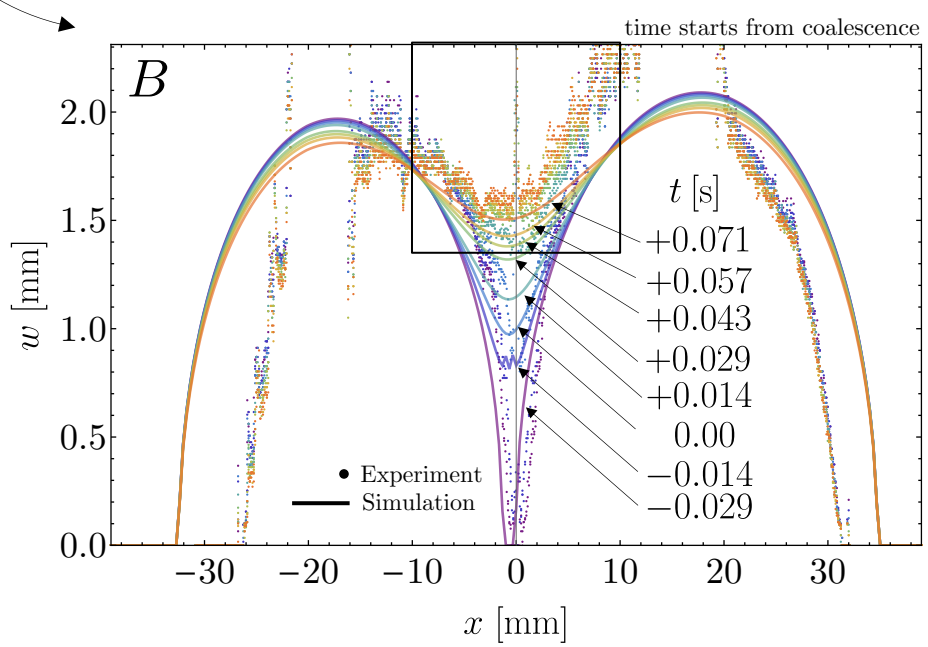


Experiment 3

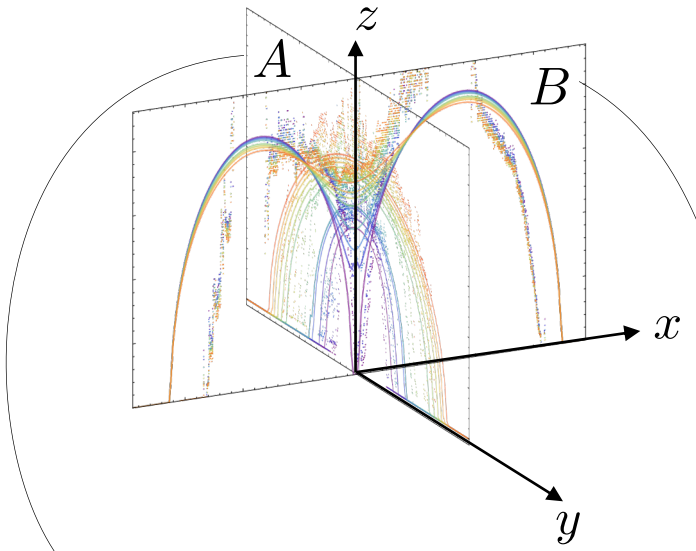


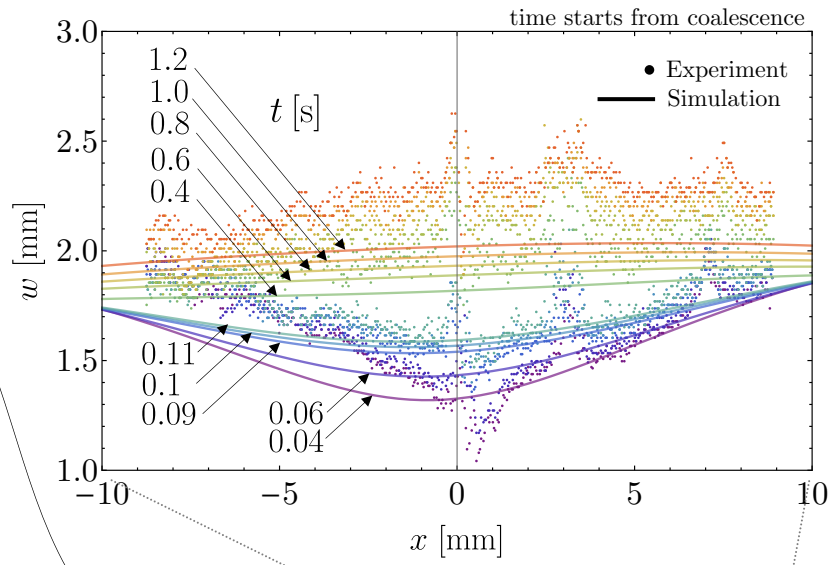


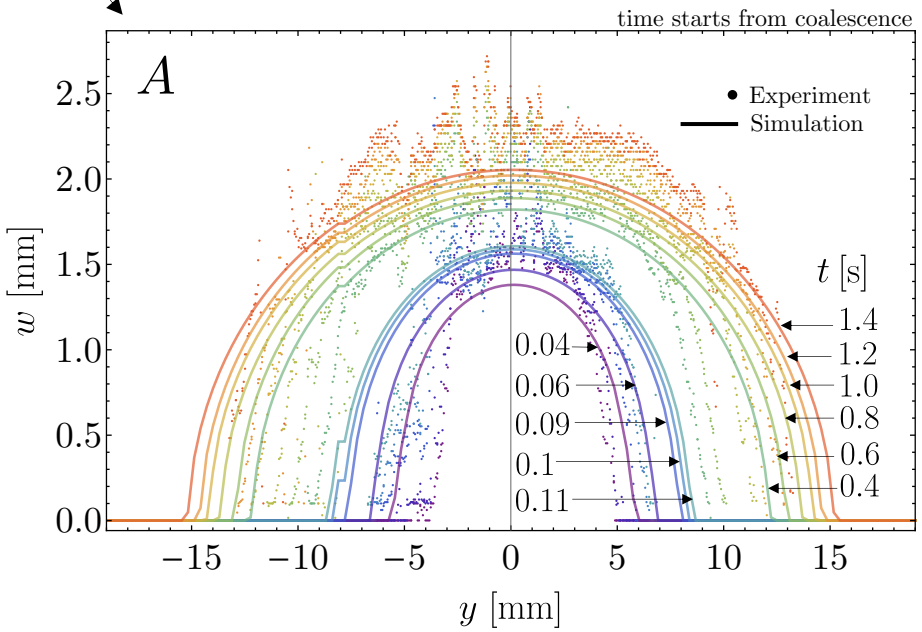


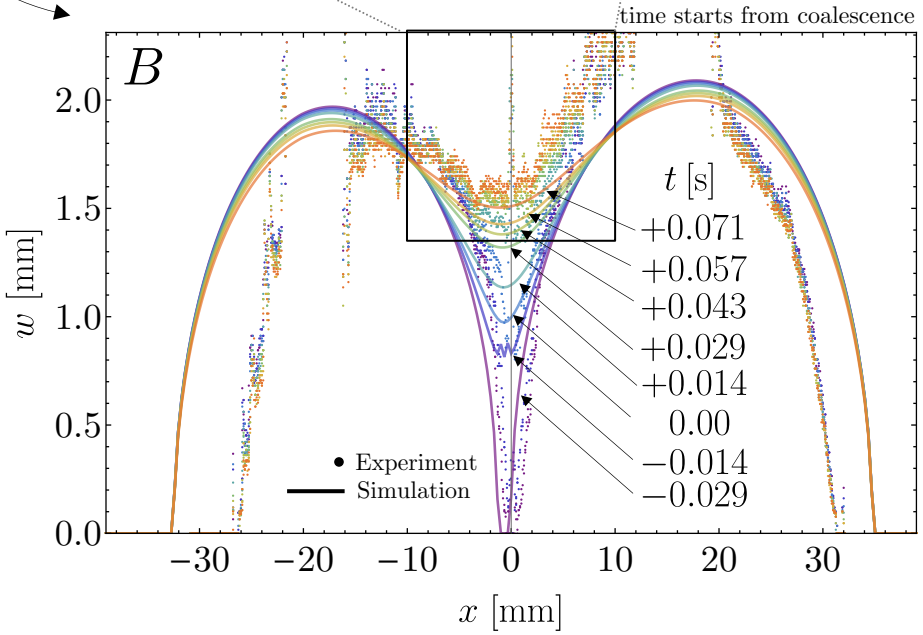


Experiment 3

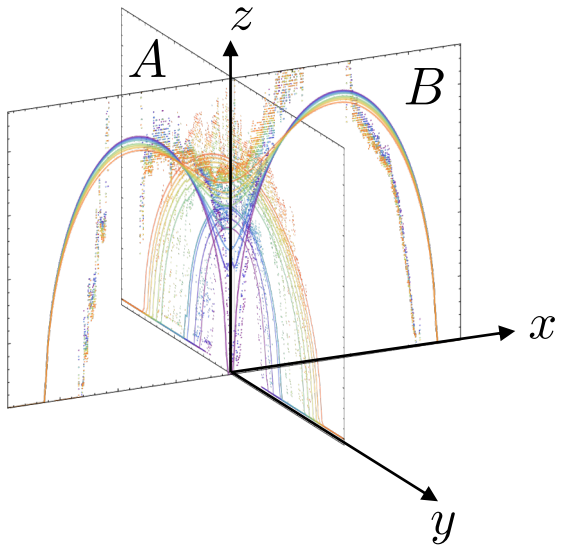


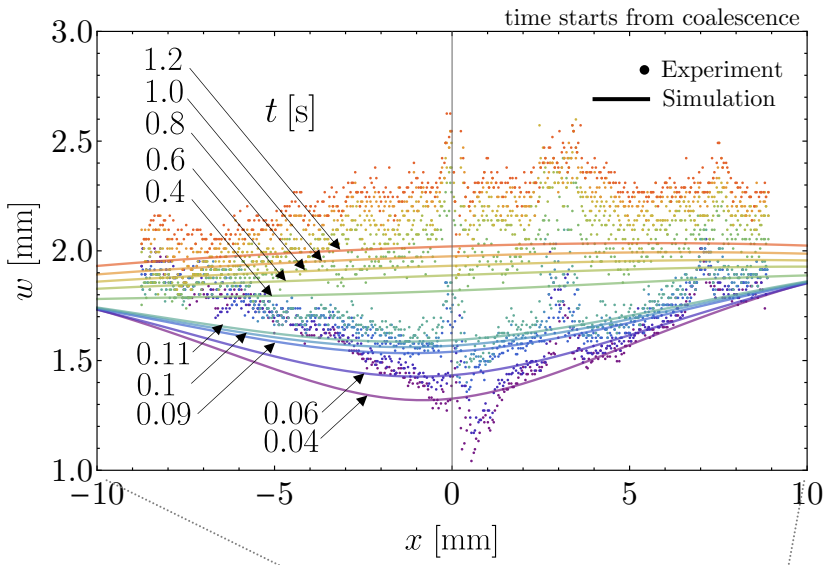




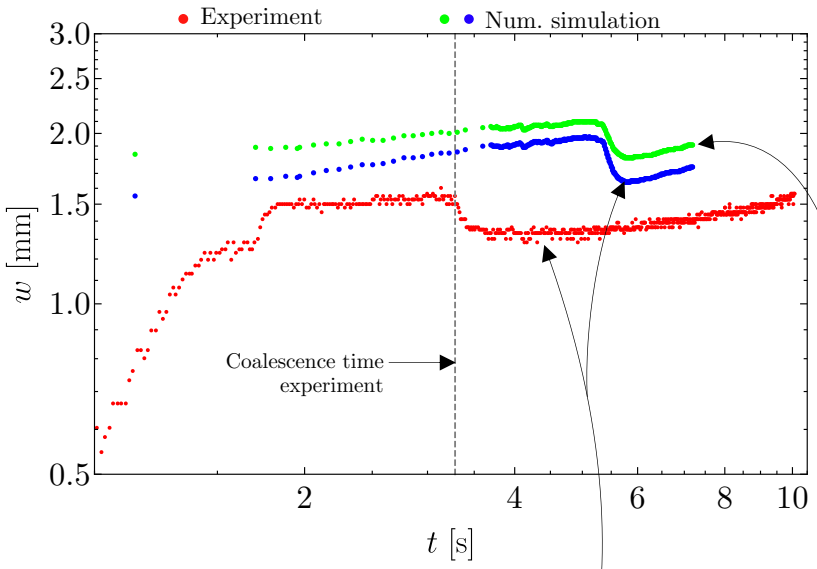


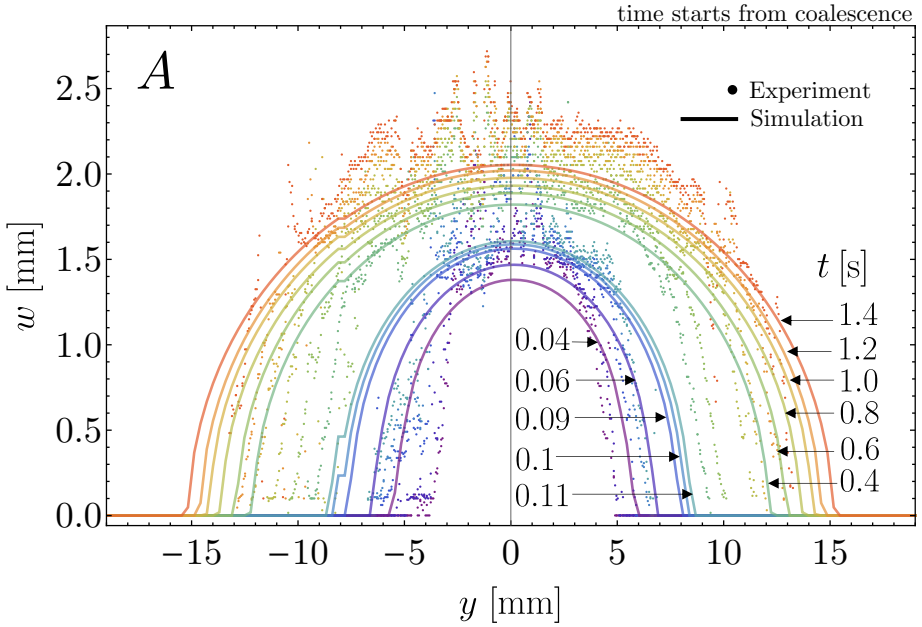
Experiment 3

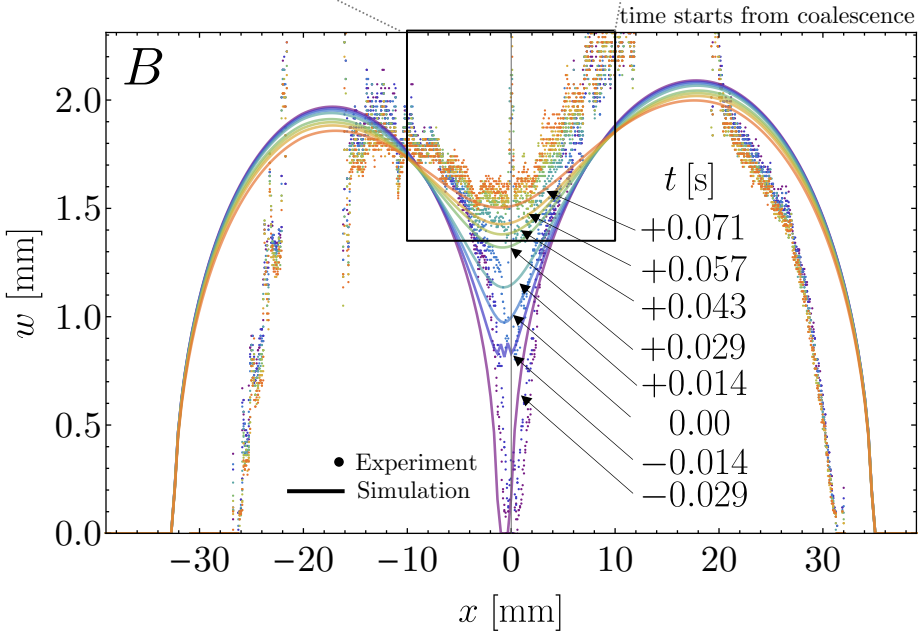


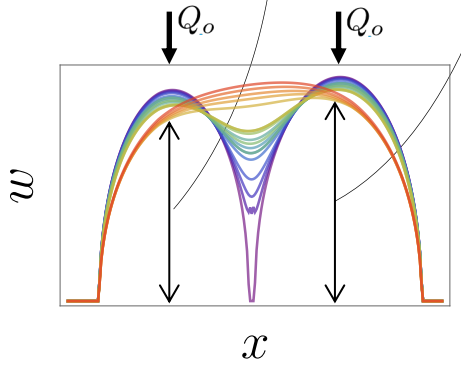


Fracture opening at the injection points:









Conclusions

- The coalescence of two fluid driven fracture can be reproduced by the mathematical model, pending the elastic effect of the specimen boundaries.

- The order of magnitude of the fluid velocity is captured.