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Incidence of Upward Lightning Triggered by Nearby Lightning: A Monte Carlo Simulation

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***Abstract*— Upward lightning is the dominant type of lightning discharge to tall structures. It has been observed that a significant number of these upward flashes are initiated by nearby lightning activity. The aim of this study is to estimate the incidence of upward lightning flashes from tall structures caused by nearby lightning based on Monte Carlo simulations and using a simplified electrostatic and corona model. We present the spatial distribution of different nearby lightning events that can potentially trigger upward lightning from a given structure. Our results suggest that Eriksson's empirical formulas might significantly underestimate the total number of flashes to a tall structure.**

Keywords—lightning, upward, triggered, other-triggered, Monte Carlo, incidence

I. INTRODUCTION

Upward lightning initiation has been observed either due to the slowly rising background electric field caused by cloud charges (the so-called 'self-triggered' or 'self-initiated'), or due to the faster electric field changes caused by nearby lightning ('other-triggered') [1], superimposed on the slow background electric field. The estimation of the number of upward flashes from a given structure is of high importance for determining the risk level for vulnerable structures such as tall wind turbines built from nonconducting materials.

Simple empirical equations exist to estimate the number of downward and upward lightning flashes to a structure of a given height, such as the formula derived by Eriksson [2] or the one defined in the IEC standard 61400-24:2010 for lightning protection of wind turbines [3]. However, field observations suggest that these formulas underestimate the number of flashes [4-6]. Becerra et al. [7] estimated that both downward and self-initiated upward lightning can be responsible for only a limited number of flashes, 8 to 20% of observed events.

Saba et al. [8] and Schumann et al. [9] observed four different scenarios leading to the triggering of an upward flash from a tower (see Figure 6-8 in [9]). One of them is due to intracloud discharges, and three are due to different phases in a positive cloud to ground (CG) flash. All of them are characterized by a horizontal leader propagation over the tower.

In their study, Becerra et al. [7] estimated the proportion of positive CG flashes that are capable to trigger upward lightning, by interpolating the number of triggering events as a function of distance from [8]. Further, they assumed different probabilities for those events to produce a critical electric field resulting in a triggered upward lightning. It is worth noting that Becerra et al. [7] only considered relatively slow electric field changes associated with leader propagation, as observed in [1]. However, faster field changes of return strokes can also trigger upward lightning with much smaller field magnitudes [10].

In this paper, we present a method to estimate the number of upward flashes from a tower triggered by nearby lightning. Based on the analytical model presented in [10,11], we employ Monte Carlo simulations considering a possible range of values for the electrical and geometrical parameters of positive cloud to ground (CG) lightning. We take into account both, relatively slow processes due to the leader propagation and faster return stroke processes. The adopted geometrical parameters are based on the scenarios observed by Schuman et al. [9]. Similar to [7], one of the scenarios in which cloud discharges are the triggering mechanism was not considered because of the low efficiency of lightning location systems to detect these events and lack of available statistical data. This omission would, however, not significantly impact the overall prediction since this scenario represents only 13% of the observed cases [8,9].

II. METHOD

A. Electrostatic Model

The full derivation of the electrostatic field due to horizontal and vertical line charge densities involved in triggering processes as observed in [8,9] can be found in the appendix of [10]. Here, for the sake of conciseness, we only present the general ideas. The electric potential of any charge distribution at an arbitrary observation point (*x*,*y*,*z*) in free space can be calculated as:

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r} \tag{1}$$

Integrating along the leader, one can obtain the electric potential at each point in space. The components of electric field are given by:

$$\overrightarrow{E_x}(x,y,z) = -\frac{\partial V(x,y,z)}{\partial x}\,\widehat{e_x} \tag{2}$$

$$\overrightarrow{E_y}(x,y,z) = -\frac{\partial V(x,y,z)}{\partial y}\,\widehat{e_y} \tag{3}$$

$$\overrightarrow{E_z}(x,y,z) = -\frac{\partial V(x,y,z)}{\partial x}\,\widehat{e_z} \tag{4}$$

The influence of the flat ground can be taken into account with image theory. The expressions for the electric fields were derived for the scenarios observed by Schuman et al. [9] shown in Fig. 1 associated with a nearby CG positive flash. For more details see the Appendix of [10].

B. *Sustained Leader Criteria*

The electrostatic model is capable of predicting the electric field (*E*) due to a nearby lightning event in the case of a flat ground. These fields can later be used in simplified corona models to evaluate whether the conditions for a sustained leader initiation are satisfied [10]. In case of lightning protected objects with sharp lightning rods, the following condition has to be satisfied to initiate a sustained leader [11]:

$$U_m \geq K * 3.54\,\tau^{\frac{5}{16}}\,[MV] \tag{5}$$

where K is a coefficient that takes into account the complexity of the structure geometry, τ is the 10-90% risetime of the electric field *E* and U_m is 80% of the peak voltage induced on a structure of height *h*. Assuming that the electric field is constant along its height, U_m can be evaluated as:

$$U_m = 0.8E_m h \tag{6}$$

Note that Eq. (5) is obtained using a simple geometry of a sphere located at an altitude *h* and connected to ground with a wire, neglecting the charge distribution along the wire. It was estimated that a more realistic structure consisting of a rod with hemispheric top would differ by a factor of about two [11]. Therefore, we choose K = 2. Further for the chosen coefficient critical electric field for sustained leader initiation is similar to one obtained using numerical modeling in n Fig. 6 of [7] for the case of zero background field. Note that the simplified approach used here aims to provide a rough estimate of the total number of other-triggered flashes, rather than an accurate representation of the complex mechanisms involved. For more details, see Section 4 of [10].

C. *Monte Carlo Model*

In order to estimate the incidence of upward lightning triggered by nearby events, we will use Monte Carlo simulations.

The general geometry applied to the three different scenarios is shown in Fig. 2. The structure is located at the origin of the coordinate system S and it is along the y axis. The location of the nearby lightning flash ground termination point is at coordinates (xl,zl) in S and at the origin of the coordinate system S'. The red line denotes a horizontal leader with an arbitrary angle Φ with respect to the x axis of the coordinate system S.

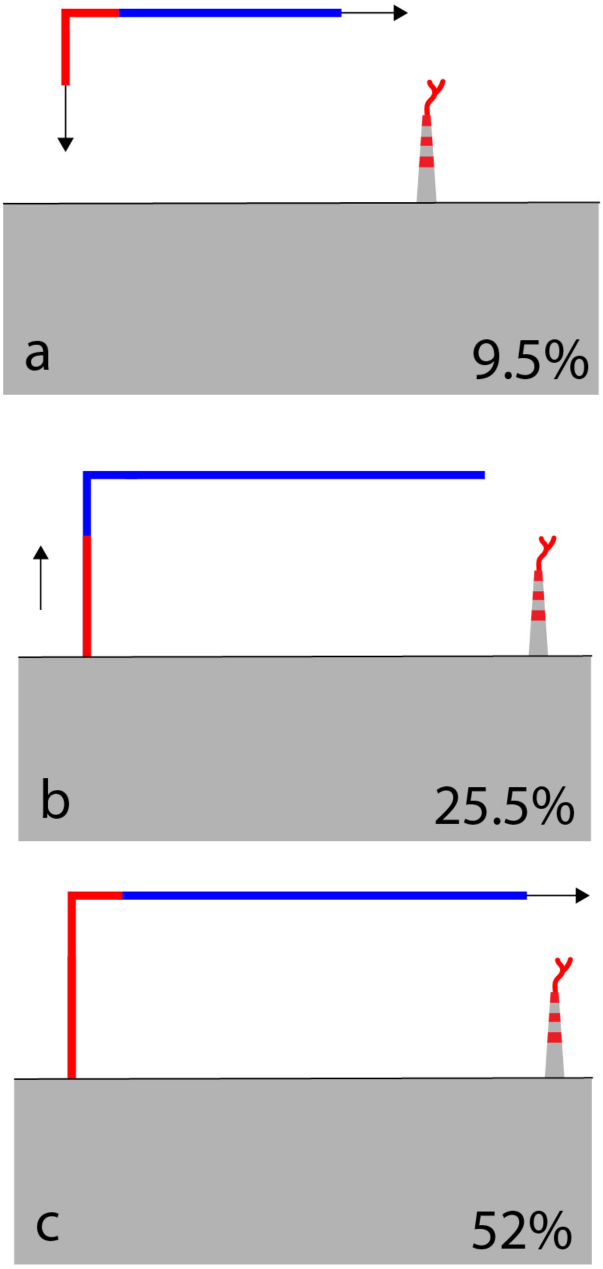

Fig. 1. Upward lightning triggered by different subsequent phases of a nearby CG positive flash. Positive discharges are represented in red and negative in blue. a) Positive leader approaching the ground, b) positive RS, c) during the CC phase.

Note that the formulas for the electric fields derived in [10] are defined for a coordinate system in which the x axis is parallel to the leader direction. In order to apply the same formulas in [10] to horizontal leaders with an arbitrary orientation, we will first transform location of structure to one from the coordinate system S to S' with its x axis parallel to the negative leader and with the ground termination point of the positive CG at its origin:

$$\begin{bmatrix} x_S' \\ y_S' \end{bmatrix} = \begin{bmatrix} cos\phi & sin\phi \\ -sin\phi & cos\phi \end{bmatrix} \begin{bmatrix} x_S - x_L \\ y_S - y_L \end{bmatrix} \qquad (7)$$

Now we can calculate the vertical electric field by directly using the equations from [10].

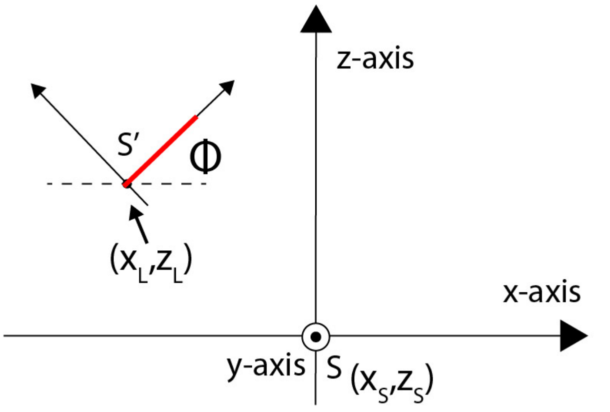

Fig. 2. Geometry of problem. Location of the structure (x_S,y_S) and ground termination point of the positive lightning flash (x_L,y_L).

The range of values considered for our Monte Carlo simulations is presented in Table 1. These values are chosen based on some typical values observed experimentally [e.g., 12-16]. The input parameters of our model are the leader angle, the initiation altitude above the ground, the speed of the positive leader end, the speed of the negative leader end, the return stroke velocity, the duration of the horizontal propagation before the leader veers down to ground, the duration of the continuous current phase, and the line charge density of the negative leader end. Note that the line charge density of the positive end is obtained by assuming a zero net charge along the leader (see [10] for more details). For the sake of simplicity and due to lack of experimental data, we use a uniform distribution for each random experiment. More advanced models can be built by using more representative distributions for each parameter.

Table 1 – Input parameters and considered ranges of variation.

	MIN	MAX
Φ [°]	0	360
altitude [km]	1.5	5
$v_{Positive}$ [10^4 m/s]	1	4
$v_{Negative}$ [10^5 m/s]	0.5	2
$v_{return\ stroke}$ [10^8 m/s]	0.7	1.2
Horizontal duration [ms]	30	200
CC duration [ms]	200	700
$\lambda_{negative}$ [C/km]	0.5	2

The aim of each random experiment is to evaluate the criterion given by Eq. (5) for each successive scenario related to the process of a positive lightning flash, as shown in Fig. 3. For the scenario of a positive leader approaching the ground (Fig 1.a), Eq. (5) is evaluated just prior to the attachment to the ground. The criterion for the positive return stroke (Fig. 1b) is evaluated when the whole positive charge is neutralized. Finally, the criterion for the continuous current (CC), which has a given duration, is evaluated at five equally distant points in time since, in some cases, the criterion might be satisfied at earlier times rather than late times.

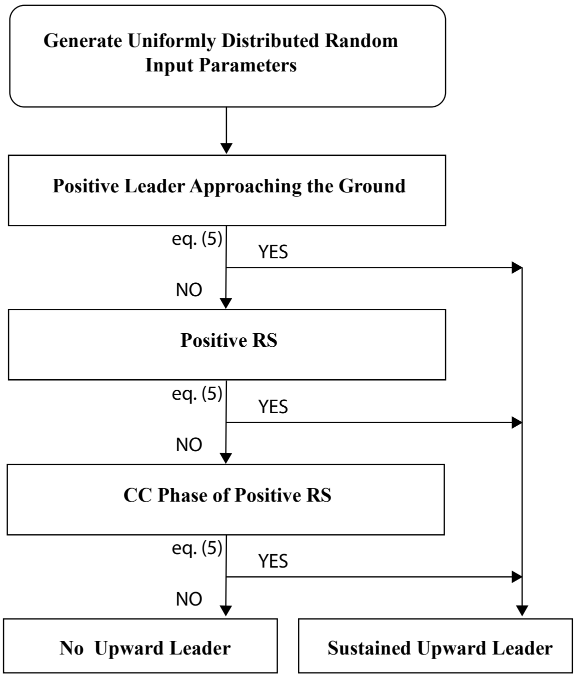

Fig. 3. Flowchart for each random experiment.

III. RESULTS

In this section, we present the results of Monte Carlo simulations for structures of different height. We analyzed an area of 60 x 60 km^2 around the structure. The x and y coordinates of each flash are randomly generated. We assumed a flash density N_g = 2 flashes/km^2 year. Furthermore, we assume that only 7.5 % of flashes are positive. This results in 540 positive flashes per year in the considered 60 x 60 km^2 area.

Fig. 4 and Fig. 5 present the spatial distribution of ground termination points for positive flashes with respectively a 100-m and a 200-m tall structure at the origin for a period of 100 years. The adopted analytical approach enables us to simulate 54000 events in less than two minutes on a typical modern personal computer without any specific optimization or parallelization. Note that in figures we presented 60 x 60 km^2 area centered around the structure since the majority of events that result in sustained leader initiation are within it, but the statistical results are obtained for a 120 x 120 km^2 area to take into account less likely distant events. Grey colored markers denote positive flashes that did not cause the initiation of upward lightning from the tall structure.

The red colored dots represent ground termination points of positive flashes that initiated upward lightning during the initial phase of the positive leader approaching the ground (Fig. 1a). About 90% of these events are within a radius of about 25 km around the 100-m structure and about 28 km in the case of the 200-m tall structure.

Blue color denotes ground termination points of positive return strokes initiating an upward flash (Fig. 1b). About 90% of them are located within a radius of 9 km around the 100-m tall structure and a radius of 10 km around the 200-m tall structure.

CC phase events shown in green can have the most distant ground termination points since, if their horizontal propagation is in the direction of the structure, they can reach its proximity. About 90% of these events are located within a radius of 53 km around the 100-m tall structure and 55 km around the 200-m tall structure. Note that there is no CC events in very close proximity to the tower since these would have already triggered upward lightning by their preceding processes.

Note finally that in our modelling we did not take into account the fact that a small fraction of the positive flashes in the immediate proximity of the structure might connect directly to it.

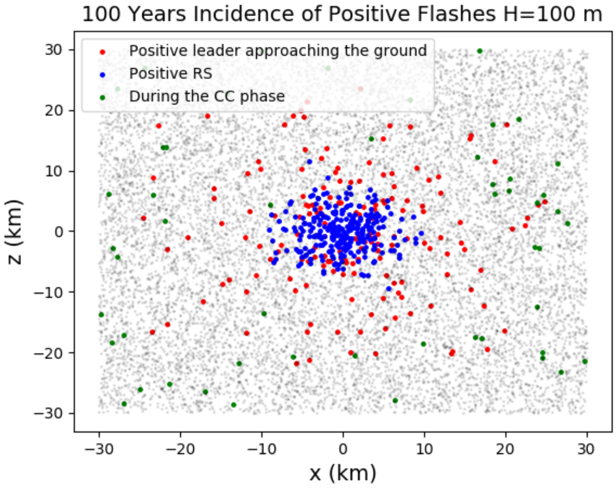

Fig. 4. Distribution of positive lightning flashes in a 60 x 60 km^2 flat ground area with a 100-m tall structure at the origin. Grey markers denote events that did not cause a sustained upward leader. The red, blue and green dots correspond to the scenarios in Fig. 1a, 1b and 1c, respectively.

Fig. 6 presents the reverse cumulative distribution of CC phase events that triggered an upward lightning versus the distance from the structure. We can observe that there are no events with a distance less than 10 km. Furthermore, a similar distribution is observed for both heights.

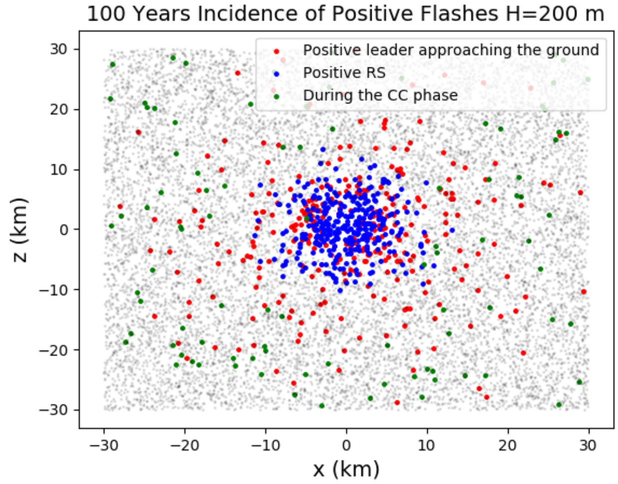

Fig. 5. Distribution of positive lightning flashes in a 60 x 60 km^2 flat ground area with a 200-m tall structure at the origin. Grey markers denote event that did not cause a sustained upward leader. The red, blue and green dots correspond to the scenarios in Fig. 1a, 1b and 1c, respectively.

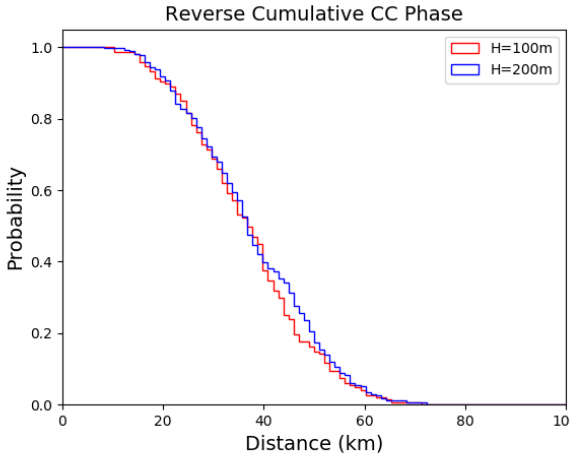

Fig. 6. Reverse cumulative distribution of CC phase events that triggered an upward lightning versus the distance from the structure.

The total number of both downward and upward flashes to a structure of height h can be estimated using the well-known Eriksson's empirical formula [2]:

$$N_T = 24 * N_g * h^{2.05} * 10^{-6} \tag{8}$$

The percentage of upward lightning can be obtained using [17]:

$$P_U = 24 * N_g * h^{2.05} * 10^{-6} \tag{9}$$

Note that in both equations, the structure is assumed to be located on flat ground; in case of an elevated terrain, the physical height has to be replaced by the effective height of the object (e.g. [18]).

Table 2 presents the total number of expected flashes to structures of different heights using eqs. (8) and (9). We also present the total estimated number of other-triggered (OT) flashes obtained by using the Monte Carlo model averaged over a period of one year. We can observe that the number of estimated OT flashes is 3 to 10 times (depending on the structure height) higher than the number of flashes predicted using Equation (8), and the percentage of upward flashes is about 4 to 80 times higher than the number predicted by Equation (9). In line with [7], we observe a significant underestimation of Eriksson's empirical formulas, just by considering upward flashes caused by nearby lightning without contribution of downward lightning and self-initiated upward lightning.

Table 2 – Statistics for a 60 x 60 km^2 observation area, and comparison with equations (8) and (9). N_g= 2 flashes / km^2

Height [m]	100	125	150	175	200	250
N_T from (8)	0.6	0.95	1.39	1.9	2.5	3.95
P_U [%] from (9)	13.2	25	34.6	42.7	49.8	61.5
Scenario a [%]	33	33	33	34	34	35
Scenario b [%]	43	42	40	39	36	35
Scenario c [%]	24	25	27	27	30	30
Total number OT flashes per year	6.2	6.66	7.75	8.33	9.46	11.42

Note that the estimated number of OT flashes does not take into account the contribution of intracloud processes to which 13% of OT flashes are attributed [8,9]. Furthermore, in order to obtain more accurate predictions, more exact statistics of the parameters in Table 1 should be used.

Note also that the percentage of occurrence of each of the three scenarios is somehow different to those observed in [8,9]. This might be explained by the fact that our model does not include the electric field change due to the preceding events (see [10] for more details), which could increase the occurrence of scenarios (b) and (c) (Fig. 1), so that they might become more similar to the observations reported in [8,9].

IV. CONCLUSION

Using a simplified electrostatic model coupled with a simplified corona model, Monte Carlo simulations were carried out to estimate the incidence of upward lightning flashes from a tall tower caused by nearby positive cloud to ground flashes.

The study allowed for the first time to obtain spatial distributions of triggering events based on their nature. Our results suggest that Eriksson's empirical formulas could significantly underestimate the total number of flashes to tall structures. Based on our analysis, the number of upward flashes triggered by nearby lightning could be, depending on the height of the structure, as much as three to ten times as high as the total number of both upward and downward lightning flashes estimated using conventional empirical formulas.

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