

Computational tool for stock-constrained design of structures

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Abstract

Designing structures from reused elements is becoming an increasingly important design task for structural engineers as it has potential to significantly reduce adverse environmental impacts of building structures. To allow for a broad application of this design approach, this paper presents an interactive computational tool to design structures from a stock of reclaimed components as well as with new components. The tool provides a user-friendly data input, visualizes results, and comprises two methods for stock-constrained design: 1) discrete optimization based on Mixed-Integer Linear Programming, and 2) a newly developed heuristic. Both methods are combined with Life Cycle Assessment to design structures with least environmental impact. The applicability of the tool is demonstrated through spatial structure case studies. Results show that employing the Mixed-Integer Linear Programming methods – which produce globally optimal solutions in terms of environmental impact – are useful in detailed design stages. Instead, applying the heuristic produces solutions with slightly higher impact but requires significantly less computation time, thus enabling an interactive exploration of solutions in early conceptual design stages. The case studies show that often a combination of reused and new elements leads to structures with least environmental impact.

Keywords: Reuse, conceptual design, structural optimization, interactive design

1. Introduction

Reducing the adverse environmental impacts (EI) of buildings is a prevailing challenge since today's construction industry is responsible for a vast amount of global material use and greenhouse gas emissions (GHG) [1]. One approach to reduce EI is the reuse of structural elements over multiple service cycles [2,3]. For example, it has been shown that reusing reclaimed steel elements from obsolete buildings can significantly reduce the embodied EI of building structures [4,5]. In conventional structural design, the structural layout determines the required elements to be manufactured. Instead, the design of structures through reuse describes the inverse: structure topology and geometry must be designed to make best use of available stock elements. Depending on stock element characteristics, reused elements might also be combined with new elements to obtain structures with least EI (Fig. 1).

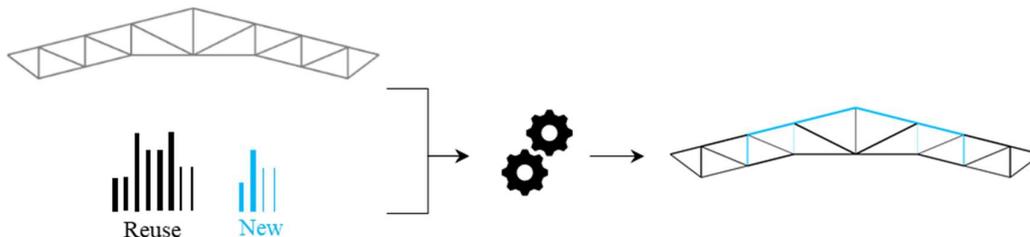


Figure 1. Stock-constrained design workflow.

Today, there is an increasing interest in these circular design approaches by practitioners and researchers. However, only few computational methods that support the stock-constrained design of structures exist. Bukauskas et al. [6] presented “form-fitting” strategies for constructing trusses from a stock of irregular timber elements. The assignment of timber elements to member positions in the truss subject to element length and capacity constraints is carried out using bin-packing heuristics to minimize trim losses. Kim and Kim [7] employed genetic algorithms to optimize the weight, embodied carbon, and cost of frame structures made from reclaimed steel elements. In previous work, Brütting et al. [8,9] presented methods to design truss and frame structures from reused and new elements. In their work, Mixed-Integer Linear Programming (MILP) and Life Cycle Assessment (LCA) have been combined to obtain a globally optimal usage of stock and new elements with least EI.

To make stock-constrained design methods available to a broader audience, this paper presents the new computational tool **Phoenix3D** [10] implemented for the CAD and visual programming environment Rhino/Grasshopper [11]. The tool enables a user-interactive and parametric stock-constrained design workflow as well as the visualization of solutions and results (Section 2). In Section 3, the potential of the proposed computational tool is demonstrated via its application to spatial structure case studies, including the benchmarking of the computational performance of both methods.

2. Method

In the following the term *element* denotes a stock component available for reuse or from new production. The term *member* denotes a bar in a truss structure. The workflow of the proposed tool is illustrated in Fig. 2. First, a structural layout, i.e. its geometry and topology, is manually defined through linear members (lines) in the CAD environment. Then, the support conditions and external loads are defined. The second input consists of the stock comprising the available reused and new elements. Reused elements are characterized by their material, cross-section, length and number of available elements. New elements are characterized by material and candidate cross-section. It is assumed that new elements are produced with standard section shapes, in adequate element length, and in unlimited quantity.

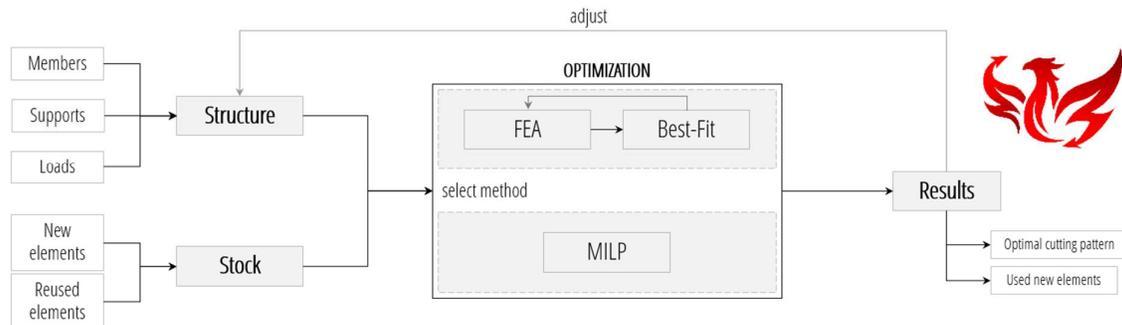


Figure 2. Phoenix3D workflow.

The key of the tool is finding the optimal assignment of either reused or new elements to member positions in the structure. Hereby, the optimization is constrained by element capacities and lengths. In detail, the normal force of the truss member must be smaller than the force capacity of the assigned element (tension and member buckling). At the same time, reused stock elements can be partitioned (cut) into one or more structure members. To solve this stock-constrained design problem, two methods are made available: 1) the MILP-based optimization formulations from [5] (Section 2.1), and 2) a newly developed “Best-Fit” heuristics (Section 2.2). Further, the two methods are combined with Life Cycle Assessment (Section 2.3) to enable setting minimization of structure environmental impacts as the objective function. The output of the tool is a visualization of the optimal assignment of elements to each member position in the structure, the optimal cutting of reclaimed elements into one or more

members, and result metrics on structure mass, cut-off, ratio of reused elements in the structure, and total EI. To close the design workflow, the starting structure layout can then be manually adjusted and improved.

2.1. Mixed-Integer Linear Programming

Brütting et al. [8,9] presented a discrete structural topology optimization method to trusses and frames with reused and new components and subject to ultimate (ULS) and serviceability limit state (SLS) constraints. This method is based on a simultaneous analysis and design (SAND) approach [12] where both design and state variables are simultaneously treated in the optimization problem. This allows treating structure equilibrium, compatibility, and member length and capacity constraints in one Mixed-Integer Linear Programming problem formulations. Such a formulation has the advantage that it can be solved to global optimality via well-established branch-and-bound techniques [13]. Combining these structural optimization methods with Life Cycle Assessment also allowed for the minimization of structure environmental impacts. For the proposed tool, the MILP-method is integrated via the C# API of the commercial branch-and-bound solver Gurobi 9.1.1 [14]. Nevertheless, solving a MILP is at worst-case exponentially complex and hence it may require significant computation time for larger-scale problems.

2.2. Best-Fit heuristic

A *heuristic* is an algorithmic process that aims to produce a solution based on an iterative application of practical rules. Such iterative procedure can be carried out very fast in computation time terms. As a tradeoff, a heuristic cannot provide any information on solution optimality. A well-known heuristic to minimize trim losses in one-dimensional cutting-stock problems is the Best-Fit algorithm [15]. In this work this algorithm is adapted to obtain an optimal assignment of either reclaimed or new elements to member positions of the structure subject to element length and capacity constraints. The concept of the Best-Fit heuristic is described through the following pseudo code:

```
foreach member  $i$  in the structure
   $obj_i = \text{Inf}$ 
  foreach reused and new element
    if element capacity  $\geq$  member force
      if element == New or reused element length  $\geq$  member length
         $obj_{temp} = \text{ComputeObjectiveValue}(\text{member}, \text{element})$ 
        if  $obj_{temp} < obj_i$ 
           $obj_i = obj_{temp}$ 
          update member/element assignment information
    if assigned element == Reuse
      reduce assigned element's length by member length
```

Pseudocode: Concept of the stock-constrained Best-Fit Heuristic

For each member i in the structure the best assignment of a reused or a new element is searched. Therefore, for each available reused or newly produced element it is first checked, whether the element capacity is larger or equal than the member normal forces. These normal forces are obtained through a linear-elastic Finite Element Analysis (FEA). Note that, to obtain member forces prior to the first Best-Fit application, a generic cross-section is set for each member. In successive FEA/Best-Fit iterations, the member cross-sections are updated to the assigned ones accordingly (Fig. 2). For reused elements it is then checked, whether the element length is longer or equal to the member length. For new elements this check is omitted as new elements can be adequately dimensioned. If an assignment is feasible with respect to element capacity and length, an objective value is computed. In this work, the objective value is computed through a Life Cycle Assessment (Section 2.3) and represents the environmental impact (EI) that the member assignment contributes to the total structure EI. If successive element iterations produce assignments with lower objective value, the member assignment information is updated to the

respectively better solution. The heuristic ends when an assignment has been obtained for each member. Note that this setup also permits to cut multiple members from one reused element. Hence, if a reused element is assigned to a member position, its remaining length is accordingly reduced by the member length (see last part in the pseudo code).

In contrast to the MILP formulation (Section 2.1), the Best-Fit heuristic cannot consider topology optimization. In addition, the heuristic only considers member capacities but does not account for serviceability limit states (SLS) and corresponding deformation limits, i.e. it is preferably applicable to strength-governed problems. Different from the work by Bukauskas et al. [6], which is also based on Best-Fit heuristics, this work considers the quantification of EI and studies solutions that combine reused and new elements.

2.3. Life Cycle Assessment

To minimize the environmental impact (EI) of a structure made of reused and new elements, Life Cycle Assessment is combined with the MILP and Best-Fit methods. The LCA methodology, system boundary, and processes are those presented in [8]. The total EI of a structure is the sum of all EIs from sourcing over manufacturing to assembling the structure components, including transports. The LCA considers that reclaimed elements are obtained through a selective deconstruction of a building structure. For new elements, the LCA considers their production from recycling steel. In this work, EI is expressed in terms of greenhouse gas emissions measured in units of equivalent carbon [kgCO₂eq]. Based on the numeric values given in [8], the total GHG emissions embodied in a structure can be computed as:

$$GHG_{Total} = \frac{0.3546 \text{ kgCO}_2\text{eq}}{\text{kg}} \cdot M_{Stock} + \frac{0.11 \text{ kgCO}_2\text{eq}}{\text{kg}} \cdot M_{Reuse} + \frac{0.8973 \text{ kgCO}_2\text{eq}}{\text{kg}} \cdot M_{New} \quad (1)$$

where M_{Stock} is the mass of the reused stock elements before cutting, M_{Reuse} is the mass of the reused elements eventually used in the structure, and M_{New} is the mass of all newly produced elements. Note that Eq. (1) expresses GHG_{Total} as one summation. Only the MILP (Section 2.1) allows the global minimization of Eq. (1). Instead, the Best-Fit heuristic (Section 2.2) iteratively computes the corresponding GHG of each member assignment individually (obj_i). After all member assignments are obtained, Eq. (1) is then employed to quantify GHG_{Total} of the entire structure.

3. Case Studies

This section presents the application of the presented stock-constrained design tool Phoenix3D to three roof structures. The first two case studies are the design of a determinate and an indeterminate 2D truss. For both truss designs the result metrics and the computational performance of the MILP and Best-Fit heuristic are benchmarked. The third case study shows the design of a 3D space-frame. This case study applies only the heuristic.

3.1 Statically determinate 2D truss

Fig. 3(a) shows the layout, support conditions, and loading of a 2D determinate truss. The applied loads consider a superimposed dead load and snow load on the roof. Since the structure is statically determinate, the member forces can be computed through external equilibrium equations and independently of member stiffness. Thus, the FEA and Best-Fit heuristic must be applied only once. Fig. 3(b) shows three element stocks of reclaimed elements that are considered. The three stocks differ in the element lengths, which are represented by black bars. All three stocks comprise elements with standard circular-hollow sections of sizes 21.3×3.2, 33.7×4, 42.4×4, 48.3×5, 60.3×5, 76.1×2.9, 88.9×6.3, 101.6×10, 114.3×10, and 139.7×10 and all elements are 10-times available. In addition, new elements with circular-hollow sections of standard sizes between 21.3×3.2 and 193.7×12.5 reported in EN 10210 [16] are available.

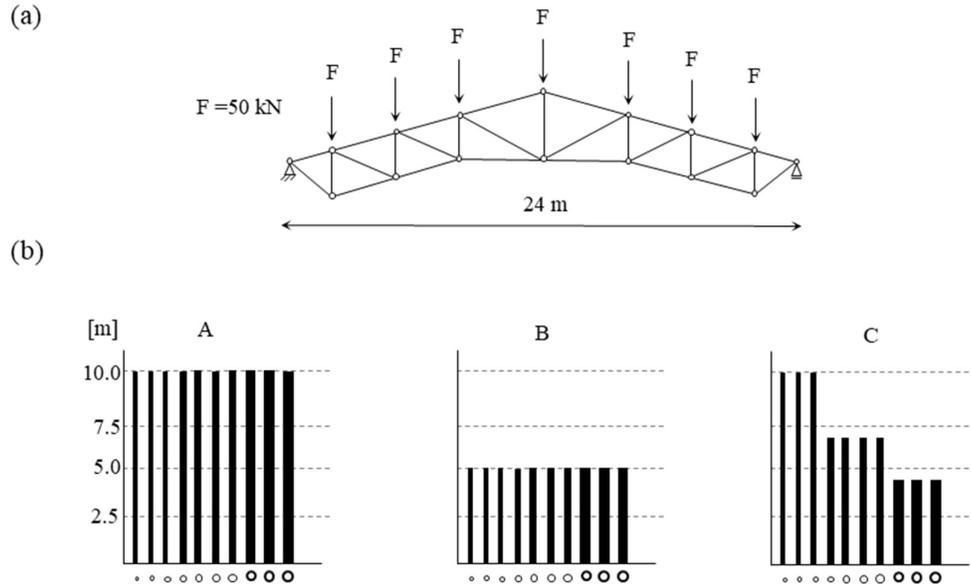


Figure 3. (a) Structure system, loads, and support conditions, (b) available stocks A, B, and C.

Fig. 4(a) shows the optimal structure designs and element usage for stocks A, B, and C when applying the MILP. Fig. 4(b) shows the obtained results when applying the Best-Fit heuristic. For each case, the assigned reused (black) and new (blue) elements are shown below the structure.

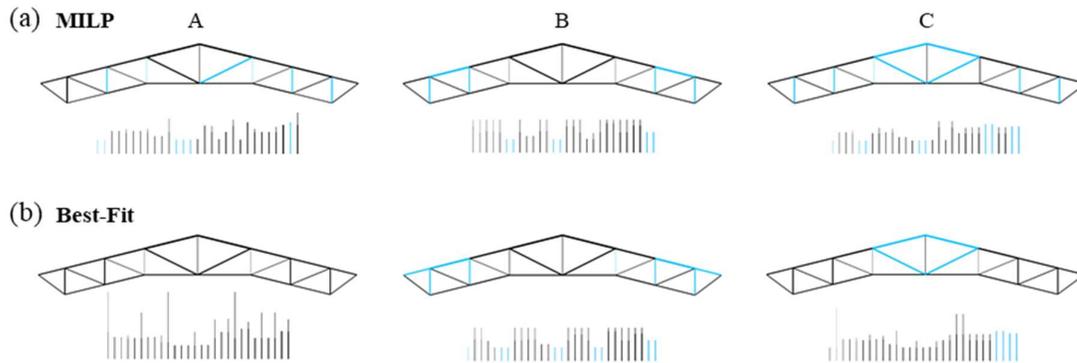


Figure 4. Case Study 1: (a) optimization results for MILP, (b) optimization results for Best-Fit.

Table 1 gives result metrics for MILP and Best-Fit. All computations have been carried out on an Intel Core i7-9850 2.60 GHz CPU. In the following, the term reuse rate (RR) denotes the ratio of the mass of reused elements in the structure over the total structure mass. The globally optimal structure designs obtained through MILP have an embodied GHG of 652 to 725 kgCO₂eq at RRs between 67% and 100%. For Stocks A and C, the Best-Fit heuristic produces solutions with 5% and 18% higher GHG, respectively. For Stock B the heuristic equally provides the globally optimal solution. Computation time of Best-Fit is 12 to 190 times faster than MILP. However, results for both methods are computed in almost real time, i.e. a few seconds, for this example of a statically determinate structure. The significantly better global optimum obtained through MILP for Stock A might not be negligible in practice. Yet, depending on stock characteristics, the Best-Fit produces acceptable results.

Table 1. Optimization results for determinate structure

Stock	MILP					Best-Fit				
	Time [ms]	Mass [kg]	Cut-off [kg]	RR [%]	GHG [kgCO ₂ eq]	Time [ms]	Mass [kg]	Cut-off [kg]	RR [%]	GHG [kgCO ₂ eq]
A	3040 (x190)	1213	124	92	652	16	1255	528	100	770 (+18%)
B	260 (x12)	1165	330	87	725	21	1149	310	84	725 (+0%)
C	272 (x18)	1039	132	67	678	15	1080	231	73	710 (+5%)

3.2 Statically indeterminate 2D truss

Fig. 5 shows the layout, support conditions, and loading of a 2D indeterminate truss. Since the structure is statically indeterminate, assigned member stiffnesses have an effect on the member force distribution. Therefore, FEA and Best-Fit heuristic are successively applied for eight times. Reported is the obtained solution after convergence. Instead, the MILP formulation can consider indeterminate structures directly. The same three stocks as in the previous case study (Section 3.1) are employed.

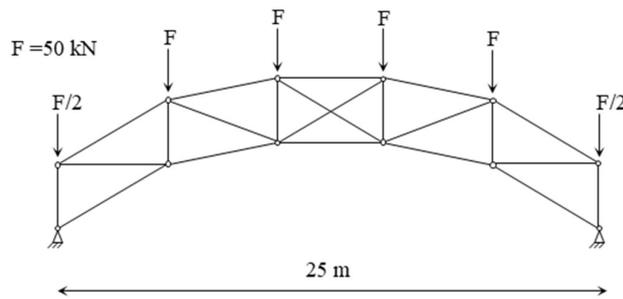


Figure 5. Structure system, loads, and support conditions.

Fig. 6(a) shows the optimal structure designs and element usage for stocks A, B, and C when applying the MILP. Fig. 6(b) shows the obtained results when applying the Best-Fit heuristic. Further, Fig. 6 indicates that the RR for all structures is relatively low which means that the structure characteristics (dimensions and loading) does not fit well with the characteristics of the available reused elements (cross-section capacities and lengths). Thanks to the tool, such incoherence can be directly captured. Subsequently, the structure can then be adjusted or another stock can be found.

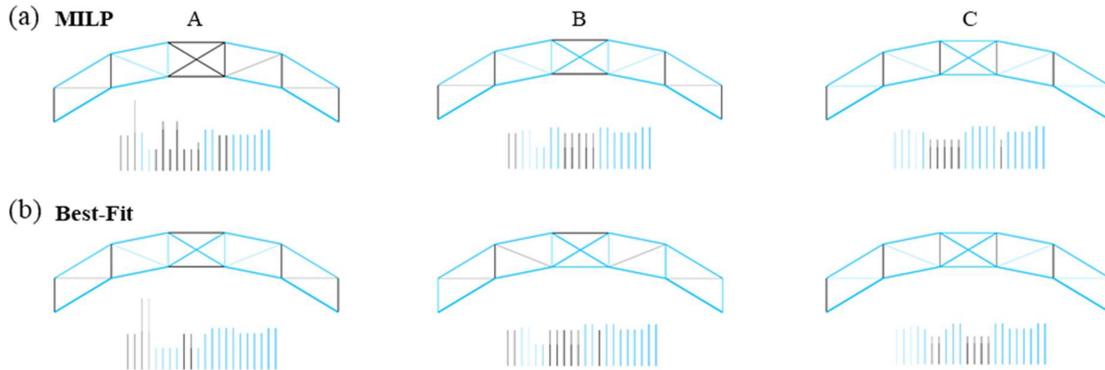


Figure 6. Case study 2: (a) Optimization results for MILP, (b) optimization results for Best-Fit.

Table 2 gives metrics for the indeterminate truss structure. Obtained through MILP, the globally optimal designs have an embodied GHG of 572, 601, and 614 kgCO₂eq for stocks A, B, and C, respectively. Best-Fit only diverges up to +9% from these global optima. However, the heuristic solutions are obtained in significantly less computation time (minutes vs milliseconds). It follows that for this case study Best-Fit is preferable to MILP. This applies in particular when fast results are crucial, as it is the case in the early conceptual design phase. However, if a more advanced planning phase requires to reduce EI as much as possible, it is worthwhile to resort to MILP.

Table 2. Optimization results for indeterminate structure

Stock	MILP					Best-Fit				
	Time [ms]	Mass [kg]	Cut-off [kg]	RR [%]	GHG [kgCO ₂ eq]	Time [ms]	Mass [kg]	Cut-off [kg]	RR [%]	GHG [kgCO ₂ eq]
A	2·10 ⁶ (33 min)	768	24	38	572	73	723	24	10	626 (+9%)
B	32·10 ⁴ (5 min)	734	42	23	601	72	734	42	23	601 (+0%)
C	19·10 ⁴ (3 min)	717	32	13	614	69	729	29	12	627 (+2%)

3.3 3D space-frame structure

In this case study a 3D space-frame structure is evaluated. In order to simulate a real-world problem as close as possible, a realistic stock with more than 3000 elements is defined. The structure comprises 250 members. Such design problem cannot be efficiently handled through the exponentially complex MILP-based approach. Therefore, this case study only employs the Best-Fit heuristic.

Fig. 7(a) shows the space-frame system, applied loads, and support conditions. Fig. 7(b) shows the optimized structure and the assignment of reused and new elements with a total mass of 1436 kg and a RR of 56%. This solution is computed through the Best-Fit method in 7.6 seconds.

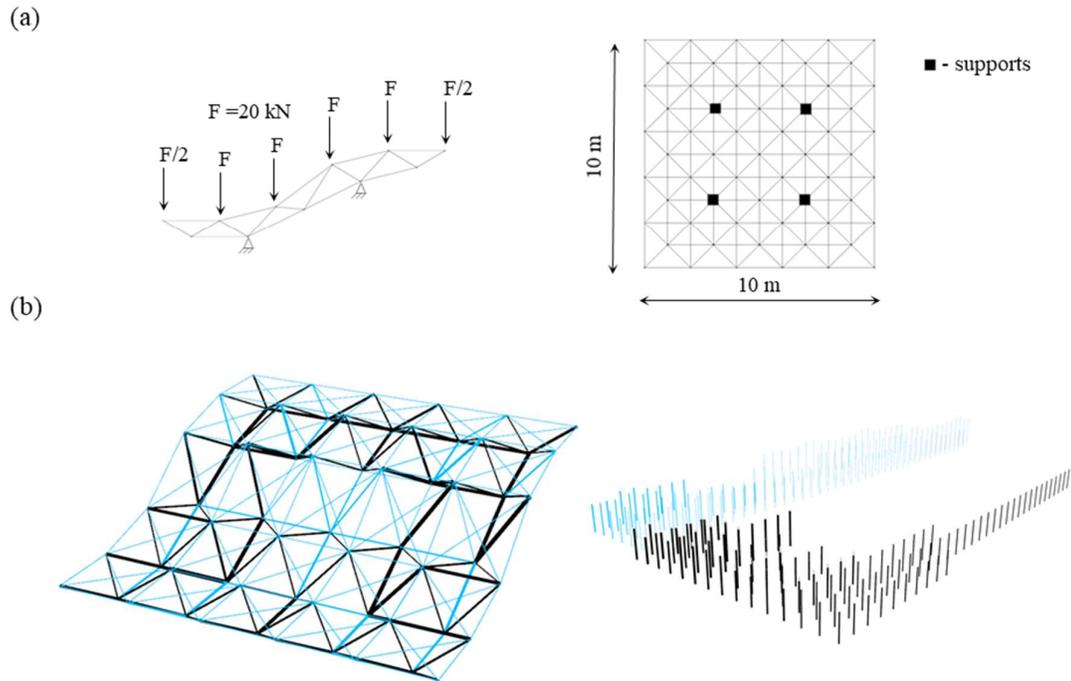


Figure 7. (a) System, loads, and support conditions of the 3D space-frame, (b) Best-Fit solution structure design and reused (black) and new (blue) elements.

4. Discussion

The case study results show that employing the Mixed-Integer Linear Programming (MILP) method produces globally optimal solutions in terms of environmental impact, making this approach very useful in detailed design stages. Instead, applying the heuristic produces solutions with slightly higher impact but requires significantly less computation time, thus enabling an interactive exploration of solutions in early conceptual design stages. The MILP can account for serviceability limit states (SLS) and corresponding deformation limits [9], which is not possible with the current implementation of the Best-Fit heuristic. In future work, the heuristic will be extended to equivalently consider deformation limits, e.g. by artificially assigning stronger elements to increase the structure stiffness.

5. Conclusion

This paper has presented a computational tool to design truss structures from a stock of reclaimed and new elements in an interactive and user-friendly way. This paper has also introduced the implementation of an algorithm to design these structures in almost real time making use of a Best-Fit heuristic. The two methods to solve the assignment and cutting-stock problem have been successfully applied to three case studies. The newly developed Best-Fit algorithm shows potential for application to large-scale real-world structural design tasks due to its fast computation time and interactive handling. In addition, the combination with Life Cycle Assessment to design structures with least environmental impact made of reused and new elements is a unique contribution of this work. From a broader perspective, the presented tool makes a strong contribution towards more circular structural design.

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