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Predictive user-based relocation through incentives in one-way car-sharing systems

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ABSTRACT

Car-sharing systems are an attractive alternative to private vehicles due to their benefits in terms of mobility and sustainability. However, the distribution of vehicles throughout the network in one-way systems is disturbed due to asymmetry and stochasticity in demand. As a consequence, vehicles need to be relocated to maintain an adequate service level.

In this paper, we develop a user-based vehicle relocation approach through the incentivization of customers and a predictive model for the state of the system based on Markov chains. Our methods determine the optimal incentive as a trade-off between the cost of an incentive and the expected omitted demand loss while taking into account the value of time of customers. We introduce a learning algorithm that allows the operator to estimate unknown customer preferences to find the optimal incentive.

Experimental results in an event-based simulation of a real system show that the use of incentives can significantly increase the service level and profitability of a car-sharing system and decrease the number of staff members needed to balance the vehicles in the system. Thereby, incentives are a more sustainable alternative to staff-based relocations. Extensive sensitivity analyses show the prospective benefits in terms of customer flexibility and the robustness of our results to varying customer preferences.

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1. Introduction

Sharing economy and collaborative consumption concepts have influenced mobility. The classical public transportation with fixed routes and schedules cannot always have high utilization due to limited accessibility or higher waiting times. Instead, various types of on-demand services offer alternatives that can decrease car ownership and private car trips. Due to their benefits in terms of mobility and sustainability, Car-Sharing Systems (CSSs) have become an interesting alternative to private vehicles. Benefits for the individual users include reduced transportation cost and mobility enhancement, while society as a whole benefits from reduced congestion and emissions (see for example [Martin and Shaheen \(2011\)](#) and [Baptista et al. \(2014\)](#)). Over the last years, the number of car-sharing users has increased rapidly. A recent study by [Frost & Sullivan \(2016\)](#) has shown that the increase in the users of CSSs is likely to continue over the coming years.

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A problem that is generally encountered in CSSs is asymmetry and uncertainty in demand, causing an imbalance of vehicles in the spatial and temporal distribution of vehicles. This imbalance refers to the shortage of vehicles at some locations and the abundance of vehicles at others. Due to the limited number of parking spaces in urban areas, the latter leads to a shortage of available parking spaces as well. The shortage of cars or parking spaces can, in turn, lead to the loss of demand.

The vehicle imbalances can be reduced by relocating the vehicles from locations where there is an abundance of vehicles to locations where there exists a shortage of vehicles. This problem has been studied by, among others, [Jorge et al. \(2014\)](#) and [Boyacı et al. \(2015\)](#). The operational management of car-sharing systems is complex due to the stochastic and dynamic nature of demand in time and space, and the limited availability of information as people intend to usually do last minute reservations. The most common type of relocation is staff-based relocation, where staff members relocate vehicles. A disadvantage of this type of relocation is that it is inconvenient due to the size and lack of portability of cars and the significant personnel cost. User-based relocations can potentially overcome these disadvantages. By offering the users incentives (i.e. discounts) for undertaking a trip which is slightly less convenient in terms of access time, user-based relocations are encouraged. These relocations can help the operator to reduce vehicle imbalances and thereby avoid future demand losses. Nevertheless, for such a system to be successful, it requires the availability of resources (in terms of available vehicles or available parking spots) in the proximity of desired origin and destination of a trip to keep the service attractive.

In this paper, we introduce a new predictive user-based relocation strategy that determines the optimal incentive based on both the current state of the system (i.e. distribution of vehicles throughout the network) and expected future demand. In doing so, we aim to anticipate future demand and therefore avoid expected future demand losses. Our user-based relocation strategy builds on unknown customer preferences. These preferences can be approximated by learning from previously offered incentives. The obtained estimates can, in turn, be used to dynamically determine the optimal value of an incentive, as well as the optimal pickup and delivery location of the vehicle. Our method is adaptive, in the sense that the value of the incentive is adjusted to the value of time of customers, as well as the current and expected future states of the car-sharing system.

By offering incentives, the operator stimulates customers to relocate vehicles from over-saturated to under-saturated locations. We evaluate our strategy using an event based simulation model with synthetic data from a real experiment, which allows us to compare our methods to existing relocation policies. Our results show that incentives can increase the service level of car-sharing systems and decrease the number of staff members needed to achieve this level. Furthermore, our results indicate that incentives are a more profitable and sustainable way of relocating vehicles, compared to staff-based relocations. By using a hybrid operator-user-based relocation strategy, profit and service level can be maximized.

The remainder of this paper is organized as follows. [Section 2](#) presents a review of the relevant literature. [Section 3](#) provides a description of the problem we consider. [Section 4](#) describes the methodology. Experimental results are provided in [Section 5](#). In [Section 6](#), the paper is concluded and possible directions for further research are suggested.

2. Literature review

Over the last decades, car-sharing systems, as well as other vehicle-sharing systems such as bike- and ebike-sharing, have received increasingly more attention. In many large cities, vehicle-sharing systems emerge for various modes of transportation. The systems can be classified as either free-floating or station-based. The first refers to the case where vehicles can be dropped off at any location where parking is permitted within the specified operating area. This type of system has been considered by, among others, [Weikl and Bogenberger \(2013\)](#) and [Herrmann et al. \(2014\)](#). The environmental effects of such a system are significant, as described by [Firkorn and Müller \(2011\)](#). However, they are often also harder to handle. [Li et al. \(2018\)](#) incorporate free-floating car-sharing in a dynamic user equilibrium model and thereby illustrate the supply-demand interaction for shared cars. Station-based CSSs on the other hand, require vehicles to be picked up and dropped off at a limited number of stations. A major advantage of this type of system is that electric vehicles can be charged at these stations. As described by [Li et al. \(2016\)](#), this innovative mobility service has benefits in terms of sustainability and the environment.

Station-based CSSs can be either one-way or two-way systems. Two-way systems require the customers to drop-off the vehicle at the same station as where they picked it up. One-way systems allow the customer to drop their vehicle off at any other station of their choice. Due to the increased level of flexibility of these systems, they are commonly viewed as a more attractive alternative for customers compared to two-way systems. As described by [Boyacı et al. \(2015\)](#), the attractiveness of a CSS is not only determined by its flexibility, but also by its level of service. The level of service consists of two important factors: accessibility and availability. Accessibility refers to the distance of the origin and destination of a customer to the available vehicle. Availability refers to the availability of a vehicle at the right time and the right place.

System performance (i.e. profit and service level) is optimized on a strategic, tactical and operational level. On a strategic and tactical level, long and midterm decisions are made. These decisions include the locations of stations ([Kumar and Bierlaire, 2012](#); [Brandstätter et al., 2017](#)), the size of the fleet of vehicles ([George and Xia, 2011](#); [Nair and Miller-Hooks, 2011](#)) and the number of staff members ([Kek et al., 2009](#)). In this paper, we focus on the operational decisions, which regard the redistribution of vehicles in the network to guarantee a minimum service level.

In one-way vehicle-sharing systems, the availability of vehicles is often problematic. Uncertain and asymmetric demand are the main causes of the existence of balancing problems. At a location where the demand for vehicles is high, the number of available vehicles declines rapidly. On the other hand, at a location where the supply for vehicles is high, the number

of available parking places declines. Due to the limited availability of both vehicles and parking spaces the service level of CSSs decreases. Non-reserved vehicles should be relocated either to create available vehicles for stations with high density of origins or to create available slots for stations with high density of destinations. Nevertheless, due to limited resources for relocation and the fact that vehicles are unavailable during this movement, an optimization framework should be integrated.

To solve this balancing problem, vehicles should be relocated. For an extensive review of relocation strategies in one-way car-sharing systems, the reader is referred to [Illgen and Höck \(2019\)](#). In the literature, both static and dynamic relocation policies are considered. The static relocation policy assumes that no demand occurs during the relocation of vehicles, suggesting the vehicles are relocated at night. [Chemla et al. \(2013\)](#) consider a capacitated pickup and delivery problem to describe the static bike relocation problem. Static relocation problems are easier to solve as they are less prone to uncertainty. However, they are also less effective as temporary imbalances during the day can not be resolved.

A dynamic relocation policy considers the relocation of vehicles during the day. This is discussed by among others [Caggiani and Ottomanelli \(2013\)](#) and [Boyacı et al. \(2015\)](#). Dynamic relocation policies are more effective as vehicles can be relocated throughout the day. However, as customers arrive dynamically, uncertainty forms a major burden. Due to this uncertainty, most relocation policies rely on simple benchmarks such as a minimum number of vehicles at each station. As these policies do not incorporate expected future demand, they are classified as non-predictive. Predictive methods incorporate expected future demand and thereby expected future states of the system. Such a predictive relocation policy was developed by [Repoux et al. \(2019\)](#). They use a Markovian model to describe the state of the system and optimize their staff-based relocation policy based on this.

In practice, vehicle relocations are mostly performed by staff members. Staff members pick up vehicles at over-saturated locations and deliver them to under-saturated locations. In bike-sharing systems, a truck can be used to relocate multiple bikes at the same time by a single staff member ([Caggiani and Ottomanelli, 2013](#)). However, in car-sharing systems this procedure is less efficient as only a single car can be moved at the same time by one staff member. User-based relocation offers a more sustainable and less costly alternative to staff-based relocation. User-based relocation refers to the case where users are stimulated to relocate the vehicles themselves, thereby contributing to a more balanced system. An example of such a method is paid relocation, as described by [Schulte and Voß \(2015\)](#), where users are paid free minutes or other bonuses. [Jorge et al. \(2015\)](#) use dynamic trip pricing to reduce imbalances. They offer higher prices to trips that increase imbalances and lower prices to those trips that improve the state of the system.

The most common type of user-based relocation is customer incentivization. In this case, customers are stimulated to change their pickup or delivery location by offering them a discount. By doing this, a less favorable location in terms of access time may be chosen by the customer, which aims to reduce the balancing problem. [Correia et al. \(2014\)](#) investigate that if customers are more flexible in their choice for pickup and delivery locations, a significant increase in profit can be obtained by incentivizing customers. [Angelopoulos et al. \(2016\)](#) provide discounts to customers if they contribute to the balancing process. Their decision is based on priorities that are assigned based on the capacity and occupancy of the stations. Similarly, [Brendel et al. \(2016\)](#) assume that the price of a ride is a function of the extra time that is required to perform a relocation. They assume the same value of time applies to all customers, thereby disregarding customer heterogeneity. Most of the literature considers policies where incentivization decisions are made based on threshold values ([Clemente et al., 2017](#)) or problematic scenarios at stations such as being completely full or completely empty ([Singla et al., 2015](#)). [Di Febbraro et al. \(2018\)](#) determines the best incentive stations and discount in a sequential manner. They determine the best station based on the relative demand for vehicles at all stations and the best discount value is determined to maximize the systems' profit. These approaches can be classified as non-predictive, in the sense that they do not incorporate expected future demand loss caused by insufficient vehicles or parking spaces. Future demand is integrated by [Pfrommer et al. \(2014\)](#), who incorporate the difference between supply and demand rates of bikes in their decisions. They use truck routing and dynamic incentives to relocate bikes in a bike-sharing system.

In on-demand transportation systems, incentives or dynamic (surge) pricing are often used to balance demand and supply. For example, [Yang et al. \(2020\)](#) design a reward scheme integrated with surge pricing for the ride-sourcing market. Similarly, [Zha et al. \(2018\)](#) propose equilibrium models for supply in ride-sourcing and investigate the effect of surge pricing. [Xiong et al. \(2019\)](#) design an incentive scheme to create energy efficient mobility systems using personalized traveler information. [Ma et al. \(2017\)](#) propose an emission pricing model for dynamic traffic networks. They determine the optimal first-best emission pricing by solving an optimal control problem.

In this paper, we consider a station-based one-way car-sharing system, where a user-based relocation policy is implemented through customer incentivization. Contrary to what has been done in the literature, our approach uses information on the current state of the system, as well as expected future demand through a Markovian model. This model allows us to better predict future demand through vehicle and parking space reservation information. The incorporation of expected future demand makes our policy predictive. Using a predictive policy allows the operator to better anticipate expected future demand and thereby increase the service level. Our policy is adaptive in the sense that the discount value depends on the value of time of customers, as well as the current and expected future states of the car-sharing system. In addition to this, we assume that customer preferences are generally unknown. As our policy builds on the value of time of customers, we develop a machine learning approach to estimate these unknown customer preferences. Using this, we solve an optimization problem to choose the best set of pickup and delivery locations as well as the optimal discount value.

3. Problem description

The characteristics of the designed system are similar to those considered by Repoux et al. (2019) and occur in many real cities such as Toyota City in Japan and Grenoble in France. We consider a one-way car-sharing system where once customers arrive to the system, they select their preferred origin and destination stations. This type of reservation policy is referred to as a complete journey reservation policy. The customer is allowed to reserve the vehicle a short time in advance and a parking space is reserved at the destination until the vehicle is returned. We consider that customers are possibly offered an alternative and less convenient trip after revealing their preferences, in return for a small discount. Due to a shortage of either vehicles or parking spaces, a customer's first choice trip may be unavailable. In that case, the customer can choose to accept the incentive or decline and choose a different mode of transportation. We should keep in mind that the short time for reservations does not allow for proactive relocations based on real information. Nevertheless, a predictive model utilizing historical data and the current state of the system can be proved beneficial compared to standard threshold based strategies that are among the most established in the state-of-practice.

We assume that customers make reservations using their customer ID, which is for example linked to their driving license. For this reason, we can collect customer-specific data, which contributes to the learning process. Such a system is commonly used in practice to ensure that only registered people with a valid driving license can reserve a vehicle.

Besides user-based relocations, we consider that vehicles can be relocated by staff members. Staff members can pick up vehicles at locations with a shortage of parking spaces and deliver them to locations with a shortage of vehicles. We note that using staff members in car-sharing systems is not necessarily efficient, as only one car can be moved by a staff member at a specific time. On the other hand, user-based relocations are less flexible as users typically do not want to spend too much effort to reach their destination. Therefore, user-based relocations are mainly short-distance relocations. Thus, even if customers are willing to change their origin or destination following the recommendations of the system (through some incentives) this cannot guarantee that it can lead to a proper rebalancing of the system.

4. Methodology

In this section, we first elaborate on the relocation policy of the operator in Section 4.1. We provide an alternative welfare-maximization objective in Section 4.2. The customer decision process is described in Section 4.3. Thereafter, we describe a method that allows the operator to estimate customers' value of time by learning from their previous decisions and incorporate this into their relocation policy in Section 4.4. Results of the different policies will follow in Section 5.

4.1. Relocation policy

Car-sharing operators usually focus on simultaneously maximizing their profit and the level of service they offer to their customers, which is reflected in their relocation policy. In our approach, the operator can offer each arriving customer an incentive. Upon the arrival of a customer, the operator determines i) whether to offer an incentive, ii) what the discount value of the optimal incentive is and iii) between which stations the vehicle should be relocated. Therefore, an optimization problem is solved upon the arrival of every customer. In this section, we provide a detailed description of this optimization problem.

We define I as the set of feasible incentives. An incentive is feasible if there are sufficient available vehicles at the pickup location and sufficient available parking spaces at the drop-off station. Sufficiency suggests that at least one vehicle is available at the origin and at least one parking space is available at the destination. Thereby, we limit this set to only contain incentives for which the customer can reach the stations within a given time interval (see Section 4.3). For every incentive i we define $\Delta_{time}(i)$, the increase in access time the customer experiences when accepting the incentive, and $\Delta_{cost}(i)$, the discount value that is offered. The estimated probability that a customer accepts an incentive, \hat{P}_{acc} , is based both on $\Delta_{time}(i)$ and $\Delta_{cost}(i)$. The shape of the estimated probability function $\hat{P}_{acc}(\Delta_{time}, \Delta_{cost})$ is described in more detail in Section 4.3.

The aim of offering incentives is to relocate the vehicles to omit expected future losses in demand due to vehicle imbalances. We refer to this as the expected omitted demand loss, $ODL(i)$, for the system when the customer accepts incentive $i \in I$. Thereby, we define w as the importance of demand loss relative to the cost of incentives. That is, the higher the value of w , the higher the relative importance of demand loss. Given that demand is known in the system operator with a short notice, a predictive framework has to be integrated. The idea is that based on the current state of each station (which is measured in real time), and the historical demand between origins and destinations, the operator can estimate the probability that in a given future window a station will run out of vehicles or slots. If this probability is multiplied by the expected demand for origins (related to available vehicles) or demand for destinations (related to available parking spots) an expected loss can be estimated. We refer to the original pickup and delivery stations as o and d respectively. The pickup and delivery stations that are chosen as a consequence of the acceptance of the incentive are referred to as o^* and d^* . As depicted in Fig. 1, the use of one incentive implicitly replaces at most two staff-based relocations. The operator can determine for every possible incentive $i \in I$ what the optimal discount $\Delta_{cost}(i)$ is. For the operator, this decision is based on a trade-off between cost and the probability that the incentive is accepted. Note that for a given incentive i , the corresponding values of $\Delta_{time}(i)$

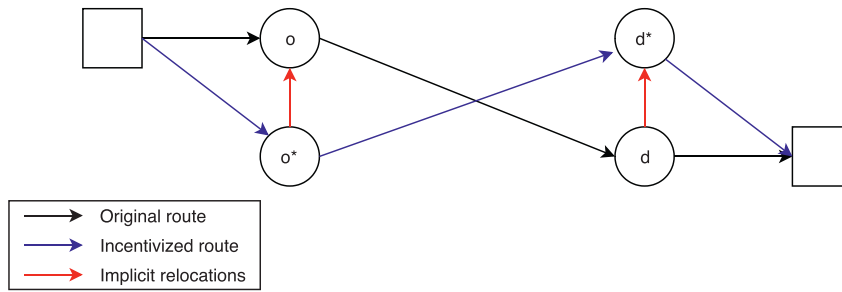


Fig. 1. Implicit relocations experienced due to incentive.

and $ODL(i)$ are fixed. This can be formulated as follows:

$$f_i(\Delta_{cost}(i)) = \max_{\Delta_{cost}(i) \geq 0} \hat{P}_{acc}(\Delta_{time}(i), \Delta_{cost}(i)) \cdot (wODL(i) - \Delta_{cost}(i)) \tag{1}$$

The objective is to maximize the expected additional profit of offering the incentive. The additional profit is defined as the extra profit obtained compared to the base case when the incentive is not offered. It consists of the weighted omitted demand loss, minus cost incurred by offering the discount. This discount, of course, needs to be non-negative. An additional constraint may be imposed which says that the discount cannot be higher than the price of the trip. The expected values of w and $ODL(i)$ are inserted in the objective function in a predict-then-optimize fashion. As the objective is linear in these uncertain parameters and given their likely independence, this does not effect optimality (Elmachtoub and Grigas, 2017). The value of $ODL(i)$ is estimated using Markov chains as explained in the remainder of this section. The function P_{acc} is estimated using a logistic regression model, using independent data to avoid bias caused by the optimization. This is explained in detail in Section 4.4. The optimal value of each incentive can be found by solving the optimization problem in (1), which can be done efficiently as the function has a single stationary point, as stated in Theorem 1. A proof of this theorem is included in the Appendix.

Theorem 1. *If for a given incentive i a profitable discount value $\Delta_{cost}(i)$ exists, there exists a unique most profitable (optimal) discount value $\Delta_{cost}^*(i)$ for which the derivative of the subproblem is equal to.*

Following from this theorem and using the fact that discounts are in whole cents and therefore integer, we initialize $\Delta_{cost}^0 = w \cdot ODL$ and iteratively reduce the discount until the objective function starts to decrease or if it is equal to zero. If the function starts to decrease, the optimal incentive is found. If the objective value is maximal at zero, no discount exists for which this incentive is profitable. The optimal discount $\Delta_{cost}^*(i)$ is non-decreasing in the value of $ODL(i)$ if all other variables remain constant. This is shown analytically through Theorem 2, of which a proof is included in the Appendix. This implies that incentives that yield a higher expected omitted demand loss in general receive higher discounts.

Theorem 2. *The optimal discount $\Delta_{cost}^*(i)$ is non-decreasing in the value of $ODL(i)$*

The best incentive can be chosen by optimizing over the set of possible incentives. We refer to the optimal incentive as i^* and to the corresponding optimal value of the incentive as $\Delta_{cost}^*(i^*)$ (which is the argument of the sub-problem). The total objective can be formulated as follows:

$$i^* = \arg \max_{i \in I} f_i(\Delta_{cost}^*(i)) \tag{2}$$

To determine the value of $ODL(i)$ we construct a Markovian model expanding the model proposed by Repoux et al. (2019). We consider a separate Markov chain for every station, which allows us to define the expected loss of future demand given the current state of the system. Through this Markov chain, we incorporate trip reservation information to better predict future states of the system. Every parking spot at a station can have one of the following five states: occupied by an available vehicle (x_{av}), occupied by a reserved vehicle for either a one-way (x_{rv}) or a two-way trip ($x_{rv'}$), not occupied but reserved for a vehicle (x_{rp}) or not occupied and available (x_{ra}). Therefore, the state of a station is defined by the number of parking spots that are in any of the first four states. As the capacity C of a station is fixed and known, the number of parking spots in the fifth state can be deduced from the first four.

The state of a station changes because of arrivals of vehicles or reservations made by customers. For this, arrival rates can be determined based on historic data. We determine arrival rates of customers looking to rent a vehicle and arrival rates of customers returning a vehicle to a reserved parking spot. Hourly arrival rates are used to capture the dynamic demand pattern of the historic data. Using these states and arrival rates we can estimate the expected loss of customers. Loss of customer demand is encountered if either the desired pickup location has no available vehicles or the desired drop-off location has no available parking spaces. The expected demand loss is numerically obtained in the same way as in

Repoux et al. (2019) using the approximation method described by Raviv and Kolka (2013). We denote the expected loss given the state of the station as $EL(x_{av}, x_{rv}, x_{rv'}, x_{rp})$.

The omitted demand loss can then be calculated as the difference between the expected demand loss in the original situation and the expected demand loss after the relocation has been performed. Following Repoux et al. (2019), we first determine the omitted demand loss for every station separately, given the implicit relocations in Fig. 1. The *ODL* for every station is given in Eqs. (3)–(6). For ease of notation, the variables x_{av} , x_{rv} , $x_{rv'}$ and x_{rp} belong to the station of the corresponding *ODL*. Intuitively, without an incentive, a vehicle is reserved at station o and no change is observed at station o^* . If the incentive is accepted, a vehicle is reserved at station o^* and no change is observed at station o . A similar intuition applies to the destination stations.

$$ODL_o = EL(x_{av} - 1, x_{rv} + 1, x_{rv'}, x_{rp}) - EL(x_{av}, x_{rv}, x_{rv'}, x_{rp}) \tag{3}$$

$$ODL_{o^*} = EL(x_{av}, x_{rv}, x_{rv'}, x_{rp}) - EL(x_{av} - 1, x_{rv} + 1, x_{rv'}, x_{rp}) \tag{4}$$

$$ODL_d = EL(x_{av}, x_{rv}, x_{rv'}, x_{rp} + 1) - EL(x_{av}, x_{rv}, x_{rv'}, x_{rp}) \tag{5}$$

$$ODL_{d^*} = EL(x_{av}, x_{rv}, x_{rv'}, x_{rp}) - EL(x_{av}, x_{rv}, x_{rv'}, x_{rp} + 1) \tag{6}$$

Using this, we can compute the total omitted demand loss as a consequence of the incentive. Other than in Repoux et al. (2019) where the relocation is always between a unique origin and a unique destination, user-based relocations depend on the relation between the four stations that may be included in the relocation. If the origin station is not changed because of the incentive, $o = o^*$ and the first two components (i.e. ODL_o and ODL_{o^*}) cancel out. Similarly, the last two components cancel out if the destination station is not changed. We consider other special cases for which two or more stations are equal in a similar manner. For example, in case the original trip is a two-way trip (i.e. $o = d$), we consider the reservation of a round-trip vehicle at that station. If the incentive changes one of the stations, the trip becomes a one-way trip instead. Therefore, we verify for every incentive the exact reservations that were made in the old and the new situation, to accurately estimate the omitted demand loss for every involved station. For example, consider a customer travelling from A to B and an incentive being offered to change the destination station from B to A . In the old situation, a vehicle was reserved for a one-way trip at A , whereas in the new situation a vehicle is reserved for a two-way trip. At B , a parking space was reserved in the old situation, but remains unused in the new situation. This yields the following calculation of the omitted demand loss with respect to this incentive:

$$ODL_A = EL(x_{av} - 1, x_{rv} + 1, x_{rv'}, x_{rp}) - EL(x_{av} - 1, x_{rv}, x_{rv'} + 1, x_{rp}) \tag{7}$$

$$ODL_B = EL(x_{av}, x_{rv}, x_{rv'}, x_{rp} + 1) - EL(x_{av}, x_{rv}, x_{rv'}, x_{rp}) \tag{8}$$

If the original trip is unavailable, the composition of the omitted demand loss, to which we will refer as ODL' , is also slightly different. Stations o and d are ignored because no change is observed here. If the incentive is accepted, vehicles and parking spaces are reserved at the incentivized location and if the incentive is not accepted, the customer is lost and therefore no reservations are made. In case the original trip is not available, the corresponding customer is not lost if the incentive is accepted, but is lost if it is not accepted. This customer is therefore included in the omitted demand loss.

$$ODL(i) = ODL_o + ODL_{o^*} + ODL_d + ODL_{d^*} \tag{9}$$

$$ODL'(i) = 1 + ODL_{o^*} + ODL_{d^*} \tag{10}$$

Similar to Repoux et al. (2019), the omitted demand loss is based on a 2-h time window. As we consider short term omitted demand losses, some incentives may have a negative effect in the long run. This can be reduced by choosing a longer estimation window. However, as the estimations do not incorporate future incentives nor relocations and contain a lot of uncertainty, the estimation quality decreases as the length of the estimation window increases. Most importantly, we aim to improve the system in the short term. The reason for this is that demand is highly asymmetric and stations that require additional vehicles in the short term may no longer require these in the long term. Our user-based relocations focus to solve short-term imbalances in the system, for which a 2-h time window has shown to be suitable.

4.1.1. Staff-based relocations

Besides the incentivizing method, we consider predictive staff-based relocations. We consider the Markovian relocation policy as described by Repoux et al. (2019) as a benchmark for the performance of our policy. As soon as a staff member is not occupied, his next job is determined by considering all origin and destination stations, which we denote by s_1 and s_2 respectively. The origin and destination stations are selected such that the weighted expected omitted demand loss is maximized. We weight the *ODL* by the time it takes to get to the origin location, $move(s_1)$, and the time needed to execute the relocation, $drive(s_1, s_2)$. This leads to the following maximization problem:

$$(o^*, d^*) = \arg \max_{s_1, s_2} \frac{ODL_{s_1} + ODL_{s_2}}{move(s_1) + drive(s_1, s_2)} \tag{11}$$

To reduce the number of staff-based relocations, we extend this policy by introducing a threshold value. We assume that the relocation is only executed if the expected omitted demand loss $ODL_{s_1} + ODL_{s_2}$ is higher than some threshold value τ . By introducing this threshold value, staff members no longer perform relocations that bring forth little additional demand.

User-based and staff-based relocations have some fundamental differences. Staff-based relocations can only be performed whenever a staff member is available, which limits the total number of relocations. On the other hand, user-based relocations can in theory be performed by every user and therefore does not have this limitation. In terms of feasibility, user-based relocations can only be performed if the user can reach the station without walking too far and they can always decline a request for a change. A staff member does not have this restriction and can therefore do any relocation at the time he is available.

4.2. Welfare maximization

Although car-sharing operators mainly focus on profit maximization, the problem can be alternatively formulated to aim for welfare maximization. Rather than only considering the cost of the operator, we now also regard the cost of current and future users. This changes the objective of the subproblem to the following:

$$\begin{aligned} & f_i(\Delta_{cost}(i)) \\ = & \max_{\Delta_{cost}(i) \geq 0} \hat{P}_{acc}(\Delta_{time}(i), \Delta_{cost}(i)) \cdot [wODL(i) - \Delta_{cost}(i) + \Delta_{cost}(i) - vot\Delta_{time}(i) + w_2ODL(i)] \\ = & \max_{\Delta_{cost}(i) \geq 0} \hat{P}_{acc}(\Delta_{time}(i), \Delta_{cost}(i)) \cdot [wODL(i) - vot\Delta_{time}(i) + w_2ODL(i)] \end{aligned} \tag{12}$$

This objective now represents the expected gain in total welfare of offering the incentive, compared to not offering one. The first two terms represent the benefit and cost of the operator, and are therefore similar to that in Eq. (1). The third and fourth component model the benefit and cost of the current user respectively, where vot represents the value of time of the current user. The last component considers the benefit future users experience for not being lost, through parameter w_2 . The value of w_2 depends on the alternative transportation modes users have access to and is therefore highly system-specific. Note that, similar to the profit maximization objective, we only consider incentives offered to the current customer and disregard the possibility to offer incentives in the future.

As a consequence of maximizing welfare in stead of profit, the nested objective function now only depends on the offered discount $\Delta_{cost}(i)$ through the acceptance probability. Intuitively, welfare does not change if the price of the trip is split differently between user and operator. It can be easily verified that if $(w + w_2)ODL(i) > vot\Delta_{time}(i)$ a maximal discount is offered and no incentive is offered otherwise. The maximal discount can be chosen arbitrarily as the price of the trip or such that the acceptance probability is equal to some threshold and is referred to as $\Delta_{cost}^{max}(i)$. Theorem 3 follows directly from this property and is therefore presented without further proof:

Theorem 3. Under the objective of welfare maximization as in Eq. (12), the optimal discount value for incentive i is given as follows:

$$\Delta_{cost}(i) = \begin{cases} \Delta_{cost}^{max}(i) & \text{if } (w + w_2)ODL(i) > vot\Delta_{time}(i) \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

The optimal discount and corresponding welfare for a given incentive i can be determined efficiently using Theorem 3. The optimal incentive can then be computed using the obtained values in Eq. (2).

4.3. Customer decision

To model the customer decisions, we define the probability function P_{acc} . We assume that the probability with which a customer accepts an incentive depends on the discount value of the incentive and the additional access time that is experienced because of this incentive. This type of customer decisions is commonly modelled using a binomial logistic (logit) model. A similar model has been used by Di Febraro et al. (2018). The acceptance probability is defined as follows:

$$P_{acc}(X) = \frac{1}{1 + e^{-(\beta X)}}, \tag{14}$$

with $X = [\Delta_{time}, \Delta_{cost}]^T$. As βX may be negative, $P_{acc}(X)$ varies between 0 and 1. Two important notes have to be made considering this probability function. First, customers are heterogeneous in the sense that they value time differently. This suggests that the parameters $\beta = [\beta_{time}, \beta_{cost}]$ are customer-specific. Second, the actual shape of the probability function P_{acc} is unknown to the operator. The operator can, however, use previously observed data to create an estimation of the parameters $\hat{\beta}$ through learning over time and thereby estimate the probability function \hat{P}_{acc} . The estimation of this function is discussed in detail in Section 4.4.

A customer's value of time can be determined as the relative importance of the coefficients β_{cost} and β_{time} , which can be estimated by taking the ratio of the two. The higher the ratio of these coefficients, the higher a customer's value of time. A customer's value of time can then be interpreted as the additional discount a customer wishes to receive for one minute of

extra access time. If the offered discount is exactly equal to the value of time, the customer is indifferent between accepting and not accepting such that the acceptance probability is equal to 0.5. We note that it is also possible to directly model the value of time of a customer by considering the fraction $\frac{\Delta_{cost}}{\Delta_{time}}$.

In addition to this probability model, some hard constraints may apply to the choice to accept an incentive. It is commonly assumed that customers are not willing to walk too far to pick up their vehicle or reach their destination after delivering their vehicle. For example, [Schulte and Voß \(2015\)](#) assume that customers choose an alternative mode of transportation if they have to walk for more than 500 m to reach their vehicle. We use a similar assumption, that says that customers never accept an incentive that requires them to increase their one-way access time by 7 min (≈ 450 m). This constraint can be easily incorporated in the definition of the set I . A major advantage of this is that it reduces the computation time of problem (2). Low computation time is of importance to the operator, as an incentive has to be offered immediately after customers reveal their preference. We emphasize that any other relevant constraints on the feasibility of representatives can also be implicitly incorporated in the set I . This constraint requires that the density of stations should be quite high, so that a number of alternatives within this walking distance exists. While this might be the case for the city centers of major cities with car-sharing services, lower density of stations might exist in the suburbs deteriorating the rebalancing power of this policy. For this reason we will test policies that consider a mixture of incentives and staff relocations.

4.3.1. Truthfulness

An important property of an incentivization policy is that it forces the customers to be truthful. An untruthful customer purposefully reports wrong information for her own benefit. In our case, this means that she specifies a wrong pickup or delivery station, as she knows she will receive a discount for her actual preferred station. As untruthful behaviour can have negative effects on the revenue collected by the operator, the policy should avoid untruthful behaviour. In this section, we elaborate on the truthfulness of users under the designed policy.

A customer can gain from being untruthful if she purposefully report a wrong pickup or delivery station and receive an incentive for her actual preferred station, thereby reducing her cost. On the contrary, she loses from being untruthful if this incentive is not offered and therefore her access time is increased. Based on this intuition, the expected gain of being untruthful is defined as the expected incentive value multiplied by the probability of that incentive being offered. The expected loss of being untruthful is the additional time the customer needs to walk if the incentive is not offered, multiplied by her value of time (vot) and the probability of no incentive being offered. That is,

$$\mathbb{E}(gain) = \mathbb{E}(\Delta_{cost}) \cdot P(\text{desired incentive offered}), \quad (15)$$

$$\mathbb{E}(loss) = \Delta_{time} \cdot vot \cdot P(\text{desired incentive not offered}). \quad (16)$$

Of course, customers can cancel their reservation if the incentive is not offered and make a new reservation for their actual preferred station. However, this behaviour can be recognized by the reservation tool as untruthful. If a customer is recognized to behave untruthfully, no incentive will be offered to this customer in the future. Under the assumption that customers are risk-neutral, a customer will be untruthful if:

$$\mathbb{E}(gain) \geq \mathbb{E}(loss). \quad (17)$$

By rewriting this equation, we observe that a customer will be untruthful if:

$$\mathbb{E}(\Delta_{cost}) \geq \Delta_{time} \cdot vot \cdot \frac{P(\text{desired incentive not offered})}{P(\text{desired incentive offered})}. \quad (18)$$

Eq. (18) can be seen as a condition for truthfulness. We emphasize that the desired incentive is unknown to the operator but the corresponding probability can be bounded as follows:

$$P(\text{desired incentive offered}) \leq P(\text{any incentive offered}), \quad (19)$$

$$P(\text{desired incentive not offered}) \geq P(\text{no incentive offered}). \quad (20)$$

Thereby, our results show that in general, Δ_{cost} is not much higher than $\Delta_{time} \cdot vot$. In addition to this, customers are in general risk-averse, which means they value losses higher than gains. Following these arguments, we conclude that these conditions are in general satisfied and are therefore not included in the simulation model.

This restriction can be enforced either on a customer-based, station-based or a system-based level. In case a violation of the truthfulness restriction is observed, it can either be enforced by reducing the number of offered incentives or by limiting the maximum discount value offered. Both conditions can be incorporated in the optimization problem in [Section 4.1](#). Intuitively, some incentives are easier to anticipate than others. For example, experienced users can identify incentives offered when the original trip is not available more easily compared to other incentives. How and within what time-span strategic users are able to predict incentives is an interesting topic of further research.

Incentives can create a new way for individuals to earn money. As individuals are paid to relocate vehicles, this may attract new users that are solely looking to create some income without interest for a specific travel. As these users are

new to the system and are only offered those incentives from which the system benefits, they cannot have a negative effect on the performance of the system and may only improve performance. Nevertheless, a demand model to integrate these actions is beyond the scope of the paper and it might require data that are not readily available. Thus, our focus remains only on travelers that are willing to change their origin or destination for some discount in their trip.

4.4. Learning from customer behaviour

The efficiency of the proposed service depends on the willingness of travelers to accept the offered incentive. Nevertheless, a high value of discount might create losses for the operator. Thus, learning the customer behavior and having a model that adequately predicts the acceptance probability as a function of the value of incentive is an important aspect of the framework. In this section, we describe the estimation method of the acceptance probability function. The acceptance probability function has the shape of a binomial logistic (logit) model. The operator does not have any information on the values of the coefficients in β , but it does have full knowledge of the offered incentive and therefore the values of Δ_{time} and Δ_{cost} . In addition to this, the operator can observe the outcomes of the offered incentives. That is, whether the incentive is accepted or not. Using this, we can estimate the probability function using a maximum likelihood estimate of the coefficients in β . As both the dependent (acceptance choice, hereafter also referred to as y) and independent variables (value of the incentive and additional access time, hereafter also referred to as X) are known, they can be used to estimate the corresponding values of the coefficients. The likelihood function corresponding to the binary logit model with n observations is written as follows:

$$L(\beta) = \prod_{i=1}^n P(X_i)^{y_i} (1 - P(X_i))^{1-y_i}. \quad (21)$$

The optimal value of β is the one that maximizes the likelihood function. Instead of maximizing the likelihood function, it is easier to maximize the log-likelihood function which is given as follows.

$$l(\beta) = \sum_{i=1}^n y_i \ln(P(X_i)) (1 - y_i) \ln(1 - P(X_i)). \quad (22)$$

As no analytical solution exists, we use a numerical optimization approach to find the optimal value for β . We use a steepest-descent algorithm with decaying step-size. This estimation method suggests that we can train our model using previously observed data and use this to forecast the probability that a customer accepts the offered incentive. An advantage of the described methods is that, besides the origin and destination location of a customer request, no other information is required. This limits the possibilities for customers to be untruthful about personal information to maximize their own profit and therefore contributes to the truthfulness of our method. This method can be used to obtain customer-specific estimates or one estimate for the entire population. If a customer-specific estimate is obtained, only those observations corresponding to that customer are used to train the logit model. If a single estimate is obtained for the entire population, all observations are used. In this case, our method is used to estimate a sample average value of β .

The use of this learning method in combination with the optimization with the optimized incentives as described in Section 4.1 will create a measurement bias. The reason for this is that the input variable Δ_{cost} is optimized based on the same acceptance probability function we try to estimate. Experiments show that this generally leads to an overestimation of the value of time. Therefore, we first use a training period to estimate the value of the coefficients using the described maximum likelihood methods. During this training period, the optimal incentive is determined using the methods described in Section 4.1, but the discount value Δ_{cost} is randomly drawn from a uniform distribution on the interval $[0, w \cdot ODL]$. After the training period, the performance of the incentivization method is evaluated using the optimized discount using the estimated coefficients $\hat{\beta}$. In case customer-specific parameter estimates have to be obtained, newly arriving customers are treated in a similar way. The first discounts are determined randomly during a training period until an adequate estimation can be made. Alternatively, discounts for newly arriving customers can be determined using estimates of a set of existing customers.

In reality, estimates can be further improved by grouping users with similar features. Travellers generally have to create an account to utilize the car-sharing system. They can then be grouped according to relevant features as age or occupancy, such that group-specific estimates can be obtained. For example, it is likely that students have a lower value of time than elderly people. By using group-specific estimates, acceptance probability estimations can be improved.

5. Experimental results

The relative performance of the incentivization method described in the previous sections is evaluated using a case study of the Grenoble car-sharing system. The details of this case study and the cost structures we use in our evaluations are described in Section 5.1. In Section 5.2 we describe the simulation model. In the following sections, experimental results are provided that give insights into the relative performance of the described methods.

5.1. Case study: grenoble car-sharing system

In our case study, we consider the Grenoble car-sharing system, which has been previously studied by Repoux et al. (2019). The system was operational between September 2014 and November 2017 and was based on a complete journey reservation policy, as described in Section 3. The system consisted of 27 stations with a total of 121 parking spots (each station had between 3 and 8 parking spots). In our simulation framework, 40 electric vehicles are available in the system every day. The maximum speed of the vehicles is equal to 50 km/h, corresponding to the speed limit in urban areas in France. In this case study, we disregard the battery restrictions of the vehicles, as previous studies have shown that in station-based systems these influence the results only marginally. As we compare our methods to the Markovian staff-based relocation policy designed by Repoux et al. (2019), we use similar settings for this policy.

Our simulation is based on demand data of the actual car-sharing system. Every simulation run consists of 10 consecutive days. We generate 100 random synthetic demand realizations per day, based on the observed distribution of demand in the actual system. As the exact itinerary of a trip is unknown, trip distance and trip duration are assumed to be independent of incentives. This distribution is based on trip transaction data from the operational period. The system is operational 24 h per day, but the majority of the trips occur between 7 a.m. and 8 p.m. As no customer information was collected by the car-sharing system, every demand realization is randomly assigned to one of 50 customers, which allows us to evaluate the effect of our learning procedure within the set time-horizon. We note that customer information is only required for our learning procedure. Each customer has a specific value of time. The values of time are drawn from a normal distribution with mean € 0.30 per min and standard deviation of 0.10. We assume β_{time} is fixed at -0.75 (in min) such that β_{cost} follows directly from the value of time. We emphasize that the number of customers does not influence any of the obtained results other than the learning procedure. The choice of parameter w depends on the importance of the service level relative to the profit. We choose the value for w equal to the average revenue earned for a single demand unit, which is equal to approximately € 15.

Walking and public transport times between stations for the city of Grenoble have been extracted from Google (2019). The public transport time comprises walking time to reach public transport and time spent in public transport. In case walking from origin to destination is the least time-consuming option, walking time is used as the full travel time. Staff members also either walk or use public transport, depending on which is faster, to move between stations when they are not relocating. For consistency, we use the same moving times as considered by Repoux et al. (2019).

Finally, the profit is based on various costs similar to those defined by Boyacı et al. (2015). The profit is calculated as the user revenue based on a cost of € 0.20 per min minus the cost of relocators (€ 18 per h), fixed vehicle cost (€ 20 per day) and a cost of € 0.01 per kilometer travelled by both users and relocators. In practice, users pay € 3 for every 15 min, so their trip duration is rounded up to 15 min.

5.2. Simulation model

Our experimental results are obtained using an event-based simulation framework. The simulation framework is an extended version of the developed framework by Repoux et al. (2015) and later updated by Repoux et al. (2019). For a detailed description of the framework, the reader is referred to these papers. The framework simulates the actual situation of the Grenoble car-sharing system as described in Section 3. The event-based simulator models vehicle reservations, pickups and drop-offs. Thereby, it keeps track of the status of vehicles at stations and on the road and staff members.

The network of stations is taken directly from the Grenoble car-sharing system. Travel times for users between stations are extracted from Google (2019). We emphasize that we incorporate asymmetries in both walking and transit times. Synthetic data is used to model the demand for vehicles and parking spaces. Arrival rates for origin-destination pairs are estimated based on observed demand during the period when the system was active.

We extend the simulation framework by the described incentivization procedure. For every customer entering the system, we solve the problem described in Section 4.1 to determine the optimal pickup and delivery location and discount value corresponding to the incentive, if any beneficial incentive exists. After the incentive is offered, the response of the customer to this incentive is randomly drawn corresponding to the logistic distribution described in Section 4.3. In addition to this, we implement a learning procedure which allows the operator to learn from previously observed customer behaviour to determine unobserved customer preferences. This procedure is described in detail in Section 4.4.

5.3. Model evaluation

While user-based relocations only change the origin or the destination station within the proximity of the original trip, they can help to locally rebalance the system. Staff-based relocation can perform any movement of an empty vehicle between two stations, but they might increase the operational cost. Thus, we are interested in the performance of the system for different combinations of user-based and staff-based relocations.

We first consider the general user-based relocation policy as described in Section 4.1. We evaluate the relative performance of this policy under the assumption that the operator has perfect information on the value of time of customers, that is $\hat{P}_{acc} = P_{acc}$. The maximum one-way access time is equal to 7 min (≈ 450 m). The total additional access time a user experiences may therefore be at most 14 min, but this is not commonly observed. The average results of 100 simulations

Table 1
Simulation results.

Staff	Incentives	% served	# relocations	% accepted	# incentives	KM	Profit
0	No	59.7	0.0	0.0	0.0	5.82	62.79
	Yes	71.5	0.0	56.7	21.5	5.78	183.66
1	No	84.6	33.9	0.0	0.0	6.36	206.93
	Yes	91.8	30.8	45.9	21.4	6.31	263.74
2	No	89.6	62.1	0.0	0.0	6.97	61.17
	Yes	94.7	58.4	40.8	18.6	6.90	95.77
3	No	91.0	86.0	0.0	0.0	7.49	-135.92
	Yes	95.4	81.4	38.5	17.2	7.40	-106.52

The first two columns describe the relocation policy in place (i.e. number of staff members and whether incentives are used). The third column denotes the percentage of served customers. The fourth column contains a daily average of the number of relocations performed. The fifth and sixth column contain the percentage of offered incentives that are accepted and the actual number of incentives accepted respectively. The KM travelled is measured as an average per served demand unit and includes both user and staff KM travelled. The profit is denoted in euros per day.

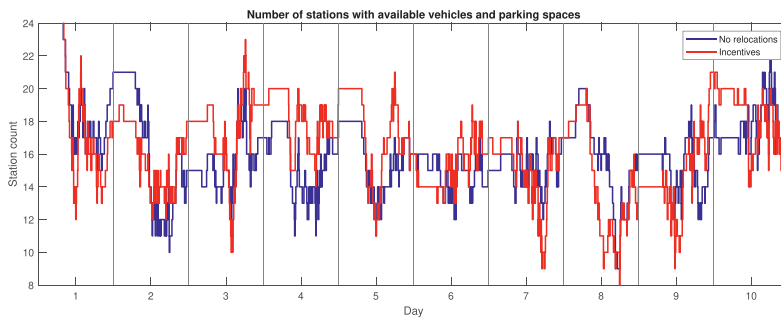


Fig. 2. Graphical representation of number of stations with available vehicles and parking spaces over time for one simulation.

are reported in Table 1. We present different performance measures that can ease our understanding of the system from the perspective of the users and the operators. The number of active personnel varies between 0 and 3 with or without incentives.

The results indicate that by offering incentives, the service level can be increased significantly. Thereby, by only offering incentives if they are expected to be profitable, the profit also significantly increases. We also observe that incentives are much more sustainable compared to staff-based relocations. Whereas staff-based relocations significantly increase the average kilometers travelled, this is not the case for incentives.

Due to staff-based relocations, the service level can be increased to a percentage between 84.6% and 91%, depending on the number of staff members, but the profit decreases if the number of personnel is higher than 1 (and it becomes negative for 3 or more). By using incentives without any personell, this is only 71.5%. The main reason for this is that user-based relocations are limited to short-distance relocations, while staff members can also do long-distance relocations. By combining the two policies (one staff member and incentives), the profit is optimized and the service level is higher than the one with three staff members and no incentives. Interestingly, offering incentives with one personnel is capable of serving more customers compared to two personnel with no incentives, which also has a significantly higher operational cost.

Daily, approximately 20 incentives are offered and accepted, depending on the policy that is used. Note that the total number of offered incentives can be obtained directly from the number of accepted incentives and the percentage of accepted incentives. As daily demand is equal to 100, this means an incentive is accepted by approximately 20% of the arriving customers. This supports the truthfulness of our policy, as discussed in Section 4.3.1. Approximately 55% of the offered incentives are accepted if only incentives are used, which decreases if it is combined with staff members. As proven in Theorem 2 in the Appendix, the discount value is non-decreasing in the expected omitted demand loss. As staff-members reduce imbalances in the systems, the expected omitted demand loss of incentives tends to decrease. In turn, this decreases the offered discounts. As a consequence, the acceptance probability of those incentives decreases and thereby the percentage of accepted incentives decreases. This also means that if the original trip is not available, the offered incentive is much more likely to be accepted as the lost customer is incorporated in the objective function.

By offering incentives or performing relocations, less critical situations (i.e. no available vehicles or no available parking places) at stations arise. Fig. 2 displays the number of stations with at least one available vehicle and at least one available parking space. We compare the scenario where no relocations are performed to the scenario where incentives are used, obtained using a single simulation of 10 days. No staff members are used in both scenarios. The results indicate that, by

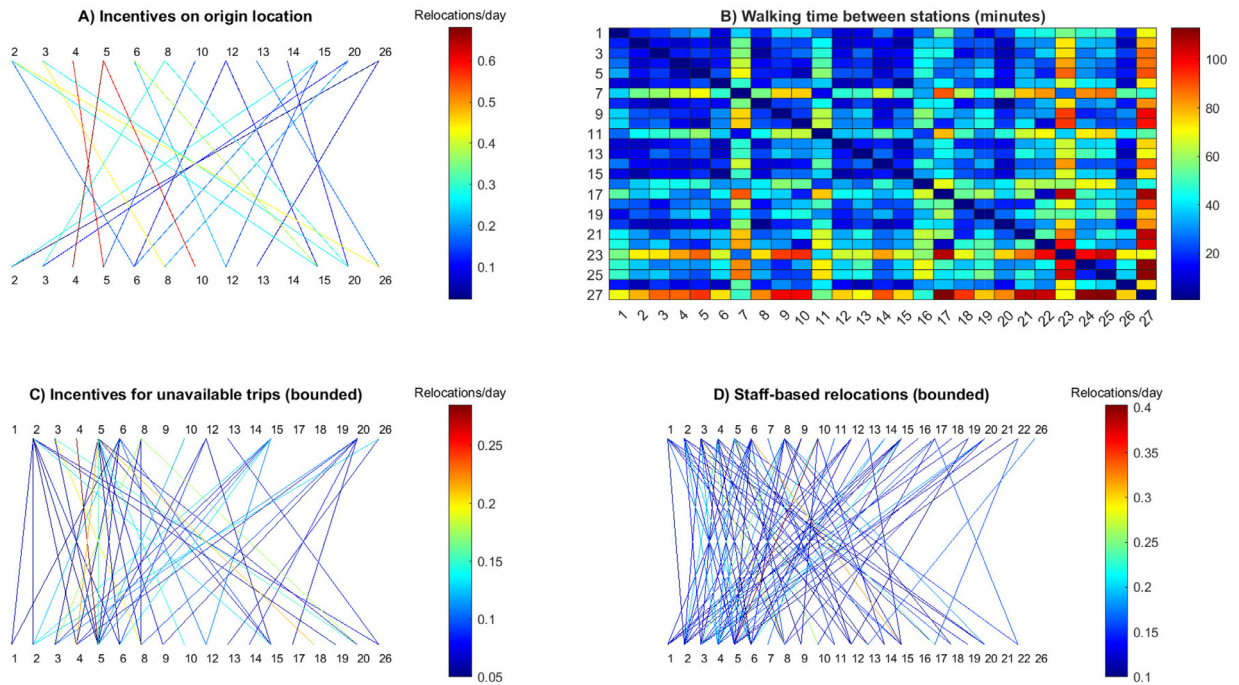


Fig. 3. Graphical representation of relocations.

using incentives, slightly more stations have both available vehicles and parking spaces. Because less critical situations arise, more demand can be served which is in line with the results in Table 1.

Fig. 3 graphically represents the relocations performed by staff members and due to incentives. Fig. 3A displays the incentives on origin locations (origin locations changed because of incentives), Fig. 3B displays the average walking time between two stations in minutes, Fig. 3C displays the incentives for which the original trip was unavailable and 3 D displays the staff-based relocations. Fig. 3C and D display those relocations that occur at least once every 20 or 10 days respectively. The relocations correspond to the simulations for which either only incentives or only staff is used and are an average of 100 simulation runs.

The results confirm the intuition that incentives are used for short-distance relocations. The relocations are solely between stations that are within 7 min walking distance from each other. Staff-based relocations, on the other hand, can relocate vehicles between any two stations. A similar graph can be obtained for incentives on the destination location. If the original trip is unavailable, the relocations look more like the staff-based relocations as either the origin or destination can be outside the maximum access time range. Incentives on unavailable trips are mostly used to change the origin location of the trip, which can be seen from the stations that are selected as origins in Fig. 3C. The reason for this is that, due to the high number of parking spaces (121) compared to the number of vehicles (40), unavailability of vehicles at the origin station is more problematic than unavailability of parking spaces at the destination station. If we reduce the number of parking spaces, we observe that the number of incentives regarding an unavailable vehicle and those regarding an unavailable parking space become roughly similar. By combining staff and user-based relocations, we are using a hybrid operator-user based relocation policy. In this policy, user-based relocations are used for short-distance relocations and are extremely effective if the original trip is unavailable. Thereby, staff-based relocations can be used to cover imbalances over longer distances such as between suburbs and the city center, as is illustrated in Fig. 3. Many incentives apply to stations 4, 5 and 10. These stations are located close to the train station of Grenoble, with 5 located approximately between 4 and 10. Not coincidentally, station 5 is also the station with the highest demand.

One of the advantages of our method is that it is applicable in real-time operation. For an incentivization method to be applicable in real-time operation, it should be able to determine the optimal incentive (if any) within seconds. Our simulation results illustrate that this condition is satisfied and our method is computationally very efficient. By limiting the number of feasible incentives and using the property of the subproblem that has at most one stationary point, the optimal discount can be found very fast. This suggests that our model can also be applied to larger cities, where the number of feasible incentives is typically much higher, because stations are located closer together. As the number of feasible incentives is higher in larger cities, our user-based relocation approach is expected to perform even better in these cities.

Demand rates may change due to the used relocation policies. In general, if a customer is not served she is less likely to return in the future. In addition to this, low availability of vehicles may decrease the demand rates at those stations whereas high availability at other stations may increase the demand rates there. The pricing policy may therefore change the demand

Table 2
Simulation results with learning method.

Staff	Estimation	% served	% accepted	# incentives	discount/minute	Profit
0	Exact	70.4	56.8	21.3	41.08	173.74
	Learning	69.9	54.1	20.5	42.86	168.76
	Underestimate	68.8	43.6	17.6	33.05	167.48
1	Exact	91.7	45.5	21.3	37.05	267.66
	Learning	91.2	43.8	20.5	38.66	260.80
	Underestimate	90.2	35.4	17.4	30.26	260.15
2	Exact	94.6	40.7	18.5	35.61	99.80
	Learning	94.3	39.0	17.7	36.77	94.71
	Underestimate	93.5	31.6	15.2	29.46	94.48
3	Exact	95.3	38.2	16.9	34.52	-102.83
	Learning	95.1	37.6	16.7	36.07	-107.63
	Underestimate	94.4	29.8	14.2	28.69	-106.43

The first column describes the relocation policy in place (i.e. number of staff members). The second column denotes the method used to estimate the value of time: exact, learning or underestimated. The third column denotes the percentage of served customers. The fourth and fifth column contain the percentage of offered incentives that are accepted and the actual number of accepted incentives respectively. The average discount per minute of accepted incentives is given in cents. The profit is given in euros per day.

rates (as may any staff- or user-based policy). To anticipate this change, demand rates can be re-estimated and the omitted demand loss estimations can be updated accordingly. Using such an iterative process, dynamically changing demand can be anticipated indirectly. To directly anticipate dynamically changing demand a proper demand model is required (depending on service level, pricing and other features), which is outside the scope of this paper.

5.4. Learning evaluation

In this section, we evaluate the performance of our learning algorithm. We use our learning algorithm described in Section 4.4 to estimate a single acceptance probability function for the entire population. We assume that the value for β_{time} is fixed and known for all customers, whereas the value for β_{costs} is drawn randomly and unknown. We emphasize that, if enough customer-specific data is gathered, the exact same procedure can be used to obtain a customer-specific acceptance probability function. If enough data is gathered, the performance using customer-specific estimates will attain the performance under perfect information. We compare the performance of the learning algorithm to the performance when the value of time is known exactly and when the value of time is underestimated by 30%. For the latter case, we approximate the value of time by a single estimate which is 30% lower than the population average.

Table 2 presents the simulation results for this experiment. A training period of 3 days is used. The simulation results are therefore an average of the last 7 days. We observe that the performance of the learning algorithm increases with the length of the training period. After 3 days, the performance does not increase significantly.

The results indicate that in case the operator does not have perfect information about the customer's value of time, incentives are still effective. The observed differences mainly occur because customer heterogeneity is ignored and all customers are treated as if their value of time is equal. We observe that, even though the average discount per minute is higher, the percentage of accepted incentives is lower. Consequently, fewer incentives are offered and the profit and service level decrease.

If the value of time is underestimated, the percentage of accepted incentives decreases significantly and so does the average discount. As a consequence, the service level decreases. Similar results can be obtained when the value of time is overestimated. In this case, the average discount value will be higher, causing the profit to decrease. This emphasizes the importance of a correct estimate of the value of time, as a wrong estimate can decrease both the profit and the service level.

Our experiments indicate that customers with a higher value of time are offered higher discounts. A regression of the discount value on the actual value of time of a customer indicates that the value of time has a significant positive effect on the discount value. From the experiments in this section, we conclude that our learning methods enable the operator to obtain a good estimate of the acceptance probability function of customers. Naturally, as dispersion among the customers in terms of their value of time increases, the performance of the learning method decreases. However, when more data is gathered, customer-specific estimates can be obtained which are not influenced by dispersion. A more detailed analysis of learning the distribution is beyond the scope of this work, as no real data was available for specific users. This can be a research priority for a demand-oriented analysis.

5.5. Increasing customer flexibility

In the previous experiments, we assumed the maximum one-way time customers were willing to walk towards their pickup location and from their destination location was 7 min. In this section, we perform a sensitivity analysis to investigate

Table 3
Simulation results for multiple maximum walking times.

Access time	Staff	Incentives	% served	# relocations	# incentives	KM travelled	Profit
5	0	No	59.7	0.0	0.0	5.82	62.79
		Yes	65.1	0.0	11.0	5.79	118.68
	1	No	84.6	33.9	0.0	6.36	206.93
		Yes	88.3	32.2	11.9	6.33	239.59
	2	No	89.6	62.1	0.0	6.97	61.17
		Yes	92.2	59.9	10.7	6.94	80.88
3	No	91.0	86.0	0.0	7.49	-135.92	
	Yes	93.4	83.5	10.1	7.44	-115.82	
7	0	No	59.7	0.0	0.0	5.82	62.79
		Yes	71.5	0.0	21.5	5.78	183.66
	1	No	84.6	33.9	0.0	6.36	206.93
		Yes	91.8	30.8	21.4	6.31	263.74
	2	No	89.6	62.1	0.0	6.97	61.17
		Yes	94.7	58.4	18.6	6.90	95.77
3	No	91.0	86.0	0.0	7.49	-135.92	
	Yes	95.4	81.4	17.2	7.40	-106.52	
10	0	No	59.7	0.0	0.0	5.82	62.79
		Yes	79.4	0.0	32.4	5.76	252.32
	1	No	84.6	33.9	0.0	6.36	206.93
		Yes	94.1	29.7	26.6	6.31	276.15
	2	No	89.6	62.1	0.0	6.97	61.17
		Yes	95.9	57.1	22.7	6.89	98.10
3	No	91.0	86.0	0.0	7.49	-135.92	
	Yes	96.4	80.0	20.5	7.37	-104.39	

The first column denotes the maximum extra access time in minutes. The following two columns describe the relocation policy in place (i.e. number of staff members and whether incentives are used). The fourth column denotes the percentage of served customers. The fifth and sixth column are a daily average of the number of relocations and the number of accepted incentives respectively. The KM travelled is measured as an average per served demand unit and includes both user and staff KM travelled. The profit is given in euros per day.

the effect of increasing customer flexibility. We assume the operator has perfect information on the customers' value of time. First, we consider three scenarios where the maximum walking time is either 5, 7 or 10 min (again, this applies separately to origin and destination). The higher the maximum walking time, the more flexible customers are. The results of this experiment are displayed in Table 3.

We observe that as the maximum walking time increases, more incentives are offered, as more profitable incentives are found. As a consequence, the percentage of demand served and profit increase significantly. As flexibility increases, the percentage of served demand and effectiveness of incentives increases. This result is in line with those provided by Correia et al. (2014). Due to the increasing number of incentives, the number of staff-based relocations also decreases slightly.

We note that walking for 10 min to pick up or drop off a vehicle can be rather undesirable for customers. Therefore, we explore the use of public transportation modes to transport customers before pick up or after delivery. The public transport data for the city of Grenoble has been extracted from Google (2019). The travel time comprises walking time to reach public transport and time spent in public transport. In case walking from origin to destination is the least time-consuming option, walking time is used as the full travel time. On top of that, the customer may wish to be compensated for the inconvenience of public transport (which comprises among others waiting time and scheduling delay). We assume users expect to be compensated for inconvenience comparable to five minutes of walking. This is incorporated in the acceptance probability function and therefore indirectly leads to higher compensations in case public transport is used.

The results of this experiment are presented in Table 4. We consider two scenarios, where the maximum time to get to or from the vehicle is either 7 or 10 min. For the sake of comparison, we assume the value of time is similar for both experiments. We note that the value of time in public transport is likely to be higher than that for walking, as the price of the public transport ticket needs to be paid, but this is omitted in the current work.

The results indicate that if the maximum one-way access time in public transport is 7 min, the results are better than walking for 10 min. The reason for this is that if customers are willing to use public transport, they can reach a higher number of stations within their maximum access time. When the maximum access time is equal to 10 min in public transport, the use of incentives outperforms the use of 3 staff members. The main reason for this is that if a customer requests an unavailable trip, an alternative origin or destination can almost always be reached within the maximum access time. Therefore, by offering a discount this customer can often be saved without the need to relocate vehicles. Although the use of incentives outperforms the use of staff members in this case, 41 incentives per day are needed to achieve this. This means that almost half of the arriving customers change their preferred origin or destination station. This has a negative effect on truthfulness and, despite the fact that users are compensated, it is likely to decrease customer satisfaction. Thereby, if users

Table 4
Simulation results using public transport.

Access time	Staff	Incentives	% served	# relocations	# incentives	KM travelled	Profit
7	0	No	59.7	0.0	0.0	5.82	62.79
		Yes	80.6	0.0	32.0	5.77	263.96
	1	No	84.6	33.9	0.0	6.36	206.93
		Yes	95.0	30.4	26.9	6.31	282.00
	2	No	89.6	62.1	0.0	6.97	61.17
		Yes	96.8	58.0	22.9	6.89	105.75
3	No	91.0	86.0	0.0	7.49	-135.92	
	Yes	97.2	81.0	20.6	7.37	-97.69	
10	0	No	59.7	0.0	0.0	5.82	62.79
		Yes	91.9	0.0	41.6	5.76	370.87
	1	No	84.6	33.9	0.0	6.36	206.93
		Yes	97.5	29.2	30.9	6.30	289.08
	2	No	89.6	62.1	0.0	6.97	61.17
		Yes	98.2	56.7	25.6	6.87	106.42
	3	No	91.0	86.0	0.0	7.49	-135.92
		Yes	98.5	79.4	23.5	7.34	-96.23

The first column denotes the maximum extra access time in minutes. The following two columns describe the relocation policy in place (i.e. number of staff members and whether incentives are used). The fourth column denotes the percentage of served customers. The fifth and sixth column are a daily average of the number of relocations and the number of accepted incentives respectively. The KM travelled is measured as an average per served demand unit and includes both user and staff KM travelled. The profit is given in euros per day.

Table 5
Simulation results for profit versus welfare maximization.

Staff	Optimize	% served	# relocations	% accepted	# incentives	Avg discount	Profit
0	Profit	71.3	0.0	56.1	21.3	40.23	181.91
	Welfare	71.5	0.0	95.2	29.4	48.05	136.03
1	Profit	91.8	30.9	45.2	21.1	37.26	265.60
	Welfare	92.1	29.9	95.1	34.1	47.29	204.59

The first column describes the relocation policy in place (i.e. number of staff members). The second column denotes the optimization function (i.e. profit or welfare maximization) The third column denotes the number of staff-based relocations. The fourth and fifth column contain the percentage of offered incentives that are accepted and the actual number of accepted incentives per day respectively. The average discount per minute of accepted incentives is given in cents. The profit is given in euros per day.

demand higher compensations for the inconvenience of using public transportation, the price of incentives will increase. As a consequence, profit goes down and service level goes down as less profitable incentives exist.

The results of this experiment are promising in the sense that if customers are willing to combine multiple transportation modes, i.e. public transport and car-sharing, the balancing problem can be solved efficiently without using any staff members. This implies that incentives are a sustainable alternative to staff-based relocations. A downside of this is that in practice customers may choose to waive their car-sharing request when they are already in public transport. A thorough customer survey is required to evaluate whether the increased service level outweighs the potential demand lost to public transport.

5.6. Welfare versus profit maximization

In this section we compare the results obtained under the profit maximization objective to the results under the welfare maximization objective. As w_2 generally depends on the availability of alternative transportation modes (both public and private) which we are unaware of, we choose w_2 equal to w which is the average price paid for a trip and therefore the minimum value a user attributes to the trip. We note that, due to the redistribution of costs between operator and user, a difference in discount value or profit does not change the welfare. What does change the welfare is the service level and the number of incentives (the latter through inconvenience perceived by the users). Therefore, these are the metrics we use to evaluate the welfare. We again assume the operator has perfect information about the value of time of the current user. The maximum price $\Delta_{cost}^{max}(i)$ is chosen relative to Δ_{time} such that the acceptance probability is equal to roughly 95%. For the sake of comparison, we impose the same upper bound to the discounts offered under the profit maximization objective (unlike the other experiments where no upper bound was used). The results of this experiment are displayed in Table 5.

We observe that using a welfare optimization objective increases the number of accepted incentives and, as a direct consequence of the changed objective, increases the average discount offered. As an effect, we observe a decrease in profit. In line with the chosen maximum price, the percentage of accepted incentives is equal to approximately 95% for the case of welfare maximization, whereas this is much lower for profit maximization. We also see that the service level only increases marginally despite the fact that significantly more incentives were offered. For the profit maximization objective, higher dis-

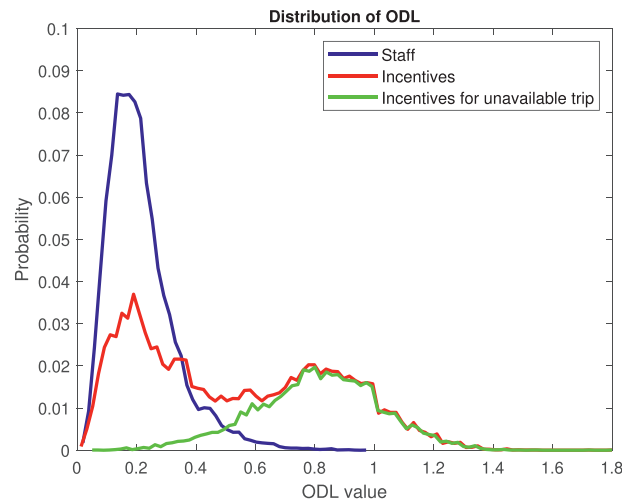


Fig. 4. Distribution of ODL by incentive and staff relocation.

counts are offered to incentives that lead to a higher ODL (in line with Theorem 2). For welfare maximization, the discount value is independent of ODL. Therefore, under the welfare maximization objective, also incentives with lower ODL values are accepted.

While the service level is similar, welfare maximization does not actually improve the welfare compared to profit maximization. This is mainly due to the increased number of incentives and thereby access time inconvenience perceived by the users. The reason for this is the myopic nature of the incentives. Under welfare maximization, more and smaller incentives are offered that require customers to walk. Although this has a direct positive effect on the welfare, offering this or a similar incentive to another user arriving in the near future may be even better. For profit maximization, although not incorporated explicitly, this is indirectly avoided through the shape of the objective function. Small incentives that only marginally increase the ODL and thereby the service level, are either not profitable or only receive small discounts and are therefore less likely to be accepted. As soon as the ODL value increases and the incentive becomes more “urgent” (over time imbalances tend to increase) the discount increases and the incentive is more likely to be accepted. As an indirect effect, unnecessary walking time through these myopic incentives is partially avoided and thereby welfare is higher. If we only offer incentives to users whose original trip is available (a policy that is more resilient against untruthful behaviour, as is discussed in the next section), welfare-maximization actually leads to a higher service level (66.6 versus 68.0%) at the cost of some additional incentives and therefore increases the welfare at the cost of a decrease in profit.

From the results we can conclude that, as both policies base their choice for incentives largely on omitted demand loss, their service levels are rather similar. However, using targeted discounts as in the profit maximization policy leads to significantly better performance for the company in terms of profit. Due to the relatively small increase in service level, the welfare maximization objective does not seem to be more desirable from a government point of view compared to profit maximization.

5.7. Adapting staff-based relocation policy

As we consider incentives and staff-based relocations that both aim to maximize the omitted demand loss, it is interesting to consider the distribution of the ODL obtained by a relocation due to an incentive and that obtained by a staff-based relocation. Fig. 4 depicts the distribution of the ODL values for these two types of relocations. The distribution of the ODL values for incentives has two peaks. The reason for this is that incentives can be classified as one of two types: incentives if the original trip is available and incentives if the original trip is unavailable. For the second type, one lost customer is omitted with certainty if the incentive is accepted, which can be seen from the plus 1 term in Eq. (10). Therefore, the ODL corresponding to this type of relocation is generally high, causing the second peak.

The two distributions indicate that staff-based relocations on average bring forth lower omitted demand losses than relocations due to incentives. This is partially caused by the second type of incentives for unavailable trips. Another reason for this is that due to the trade-off in the optimization problem, incentives with small ODL values have lower discounts and therefore lower acceptance probabilities or they may not even be offered. This causes the ODL values to be higher in general. We emphasize that the omitted demand loss is based on a 2-h time interval. In the long term, relocations generally bring forth higher ODL values except for those for which the original trip is unavailable. In that case, if the ODL value is smaller than 1, this value is likely to decrease in the long term.

By only offering incentives when the original trip is unavailable, the service level only decreases slightly whereas the profit may even increase. Of course, these incentives do not actually contribute to the rebalancing of the system as only

Table 6
Simulation results for different staff thresholds.

τ	% served	# relocations	% occupancy	# incentives	KM travelled	Profit
0.00	91.8	31.0	98.3	21.2	6.32	267.58
0.05	91.9	30.6	98.2	21.2	6.32	268.22
0.10	91.7	28.7	96.4	21.7	6.31	270.96
0.15	91.4	25.0	89.0	21.8	6.25	280.54
0.20	90.5	20.8	77.5	22.2	6.18	290.78
0.25	89.1	16.4	63.5	23.3	6.10	297.64

We consider the case where one staff member is used. The first column denotes the value of τ . The second column denotes the percentage of served customers. The third and fifth column are a daily average of the number of relocations performed and the number of incentives accepted respectively. The fourth column denotes the average occupancy of the staff member as a percentage of the total workday. The KM travelled is measured as an average per served demand unit and includes both user and staff KM travelled. The profit is given in euros per day.

Table 7
Simulation results for different customer value of time.

$\nu\alpha t$	% served	# relocations	% accepted	# incentives	discount/minute	Profit
20	92.0	30.4	55.1	26.2	25.11	273.52
30	91.8	31.0	45.4	21.2	37.42	267.58
40	91.6	31.4	39.4	17.7	49.29	257.83
50	91.3	31.9	35.0	15.2	60.58	250.83

We consider the case where one staff member and incentives are used. The first column describes the scenario, where the value of time is given in cents per minute. The second column denotes the percentage of served customers. The third column denotes the percentage of offered incentives that is accepted and the fourth column denotes the actual number of accepted incentives. The fifth column denotes the average discount value per minute for all accepted incentives. The profit is given in euros per day.

customers that would have otherwise been lost are redirected to a different station. These incentives are also more likely to induce untruthful behaviour, as they are easier to identify and compensations are higher. On the other hand, by offering incentives only when the original trip is available, the service level and profit are decreased but are still significantly higher compared to the case when no incentives are offered. Contrary to the other type of incentives, a policy where incentives are only offered in case the original trip is available is more resilient to untruthful behaviour and still leads to a system that is properly rebalanced. Overall, offering incentives independent of the availability of the original trip is highly effective and easier to implement in reality. An additional subtlety for originally unavailable trips is that users are in generally more likely to accept a small detour, but this highly depends on their alternative transportation modes. This means that in reality profit can be even higher by reducing the discounts for those trips. We can adapt the staff-based relocation policy to the use of incentives by changing the threshold value τ , the minimum expected omitted demand loss for a staff-based relocation to be performed, as defined in Section 4.1.1. By changing this value we can reduce the number of relocations performed by staff members and thereby reduce their overall activity. This can be convenient as in practice staff members often perform maintenance jobs and other tasks if they are not relocating vehicles. We consider various threshold values varying between 0.00 and 0.25. The results of this experiment are provided in Table 6. For this experiment, we assume relocators are only paid for the number of hours they effectively worked, corresponding to their occupancy rate.

The results indicate that by increasing the threshold value, we reduce the number of relocations and the percentage of time the staff member is relocating vehicles. A positive finding is that for a value of $\tau = 0.25$, the number of staff-based relocations is almost half compared to $\tau = 0$, but the served demand decreases by only 2.5%. We also observe that the number of incentives increases slightly as the threshold increases. The reason for this is that some of the relocations that are not executed by staff members are now (implicitly) executed by users. By increasing the threshold, the profit increases at the cost of a small decrease in the percentage of served customers. The results of this experiment strengthen the idea of a hybrid operator-user-based relocation approach.

5.8. Sensitivity analysis: customer value of time

In all previous experiments, the customer value of time is assumed to be equal to 30 cents per minute of additional walking time. In this section, we perform a sensitivity analysis on the customer value of time to illustrate the robustness of our results. The results of this experiment are displayed in Table 7. We consider average values of time which range from 20 to 50 cents per minute. Other than that, the simulation settings are similar to those used in the previous experiments.

The results indicate that as the value of time increases, fewer incentives are profitable and therefore fewer incentives are offered. In addition to this, because of the higher value of time, the average discount value relative to the value of time is

lower, which means that the percentage of accepted incentives decreases. For the accepted incentives, we observe that the average discount value increases proportionally with the average value of time.

As fewer incentives are accepted, the service level and profit both decrease. Interestingly, the service level and profit do not decrease significantly even though the number of accepted incentives is almost halved. This is partially caused by the use of staff-based relocations which can replace the incentives as well as later users that are offered a similar incentive. In addition to this, those incentives that are no longer profitable to offer are typically those for which the influence on profit and service level was rather small.

We emphasize that this representation of profit is not realistic, as we use a constant rental price. In cities where the average value of time is higher, a higher rental price can be imposed. Thereby, the value for w (monetary value per unit of lost demand) increases and the effect of value of time will be negligible.

6. Conclusion

In this paper, we proposed a predictive user-based relocation policy for one-way car-sharing systems. Our method relies on user-based relocations that are stimulated by offering discounts to customers. By performing an alternative and less convenient trip, users implicitly contribute to the redistribution of vehicles throughout the system. Our policy uses information on the current state of the system as well as expected future demand to determine appropriate relocations, to reduce expected future demand losses. Our policy is adaptive to the value of time of customers. As customer preferences such as their value of time are generally unknown, they have to be estimated. We developed a learning algorithm that allows the operator to learn from previously offered incentives and adjust the future offers accordingly.

Our simulation results indicate that, by using our incentivization approach, we can partially solve the balancing problem of vehicles throughout the network and thereby increase the service level. In addition to this, our methods allow the operator to use fewer staff members while attaining a higher service level and thereby increase the profit. Specifically, by using a hybrid operator-user-based relocation policy, service level and profit can be maximized. In this case, user-based relocations perform short-distance relocations, while long-distance relocations are executed by staff members. We also observe that by using user-based relocations, the average KM travelled by staff and users per unit of served demand decreases, suggesting our method is environmentally more sustainable than staff-based policies.

Using a learning algorithm, we can accurately approximate the customers' acceptance probability functions. Therefore, we can obtain results that are close to those under the assumption of perfect information. A sensitivity analysis indicates that we can further increase the service level in case customers are more flexible. That is, if customers are willing to walk further to pick up or deliver their vehicle or even use public transport, the effectiveness of our incentivization method increases.

In future work, our model can be extended to include competition between users for incentives. If one user declines the offered incentive, a similar incentive may be offered to the next arriving user who may then choose to accept it. Including future decisions on incentives would result in a computationally expensive recourse problem and is therefore omitted in the current work, but is an interesting direction of future work for mobility systems with higher demand. By incorporating this type of competition for incentives in the optimization problem, offered discounts may be lower without decreasing the performance of the system.

Other topics of further research include the extension of our methods to free-floating vehicle-sharing systems. In these systems, vehicles are not required to be parked at stations but can be parked at any legal parking place within the perimeter. Furthermore, we aim to implement our methods in a real field experiment, to evaluate participation levels of customers. Finally, we note that our method to determine incentives and learn from customer behaviour can be applied in various other fields. We aim to apply and adapt our developed methods to other modes of transportation, such as ride-sharing and crowd-shipping.

Declaration of Competing Interest

No.

Appendix A. Proofs

For notational convenience we denote the optimization problem as follows:

$$f_i(\Delta_{cost}(i)) = \max_{\Delta_{cost}(i) \geq 0} g_i(\Delta_{cost}(i)) \tag{23}$$

In addition to this, we substitute $\Delta_{cost}(i)$ by x and $w \cdot ODL(i)$ by K . This reduces the function to be optimized to the following:

$$g(x) = P(x)(K - x) \tag{24}$$

Theorem 4. *If for a given incentive i a profitable discount value $\Delta_{cost}(i)$ exists, there exists a unique most profitable (optimal) discount value $\Delta_{cost}^*(i)$ which satisfies $\frac{dg(\Delta_{cost}^*(i))}{d\Delta_{cost}^*(i)} = 0$ $g'(\Delta_{cost}^*(i)) = 0$.*

Proof. We first note that the optimal discount value should lie somewhere on the interval $[0, +\infty)$, as any value below 0 violates the definition of an incentive. Also, we note that as $g(0) \geq 0$ and $g(x) < 0$ for each $x > K$, such that we can reduce the interval to the closed and bounded interval $[0, K]$. Additionally, we know that $g(K) = 0$.

A global optimum for function g on a closed and bounded interval can occur either on the boundary points, a non-differentiable point or a stationary point. As this function is differentiable on the defined interval, only the boundary points and stationary points need to be identified.

Using Fermat's theorem, a first order stationary point requires for the derivative $\frac{dg(x)}{dx} = 0$

$$\frac{dg(x)}{dx} = \frac{dP(x)}{dx}(K - x) - P(x) = \beta P(x)P(-x)(K - x) - P(x) \text{ where we use the fact that } \frac{dP(x)}{dx} = \beta P(x)P(-x)$$

By rearranging the terms, we obtain the following requirement: $\frac{dg(x)}{dx} = P(x)[\beta P(-x)(K - x) - 1] = 0$. As $0 < P(x) < 1$ by definition, we can reduce this to: $\beta P(-x)(K - x) = 1$.

By further rearranging the terms and substituting $y = -x$, it should hold that $P(y)(K + y) = \frac{1}{\beta}$.

As $P(y)(K + y)$ is strictly increasing in y there exists at most one stationary point to which we refer as x^* .

This suggests that, using Weierstrass extreme value theorem, if a profitable incentive value exists, i.e. there exists some $x \geq 0$ for which $g(x) > 0$, there exists an $x^* \geq 0$ which is the unique optimum. \square

Theorem 5. The optimal discount $\Delta_{cost}^*(i)$ is non-decreasing in the value of ODL(i)

Proof. Using the changed notation, the theorem follows directly from the proof that x^* is increasing in K . We consider two incentives i and j for which $K_i < K_j$ and all other variables are equal. The corresponding optimal discounts are x_i^* and x_j^* respectively. We distinguish between the optimal discount being at a boundary point 0 or at a stationary point. Note that we ignore the boundary point at K as this incentive will not be offered. Therefore, we consider the following three cases:

(i) x_i^* and x_j^* are both at a stationary point

Given the first order necessary condition derived in Theorem 1, it holds that $P(-x_i^*)K_i - P(-x_i^*)x_i^* = \frac{1}{\beta}$.

Rewriting this equation in terms of K_i yields: $K_i = \frac{\frac{1}{\beta} + P(-x_i^*)x_i^*}{P(-x_i^*)}$ (and similar for K_j).

Given $K_i < K_j$, it follows that $K_j - K_i > 0$ which after some rewriting implies that

$$\frac{P(x_j^*) - P(x_i^*)}{\beta} + P(-x_i^*)P(-x_j^*)(x_j^* - x_i^*) > 0 \text{ which can only hold if } x_j^* \geq x_i^*.$$

(ii) $x_i^* = 0$ (i.e. i at a boundary point)

By definition, $x_j^* \geq 0$, so $x_j^* \geq x_i^*$

(iii) $x_j^* = 0$ (i.e. j at a boundary point)

If the discount is optimal at the boundary point, the following relationship must hold: $\max_x P(x)(K_j - x) \leq P(0)K_j$.

Consider specifically $K_i = K_j - \tau$ with $\tau > 0$.

$$\max_x P(x)(K_i - x) = \max_x P(x)(K_j - x - \tau) = \max_x P(x)(K_j - x) - \tau P(x) \leq \max_x P(x)(K_j - x) - \tau P(0) \leq P(0)K_j - \tau P(0) = P(0)(K_j - \tau) = P(0)K_i.$$

As $\max_x P(x)(K_i - x) \leq P(0)K_i \rightarrow x_i^* = 0 = x_j^*$

We note that in each of these three cases it holds that $x_j^* \geq x_i^*$. As the only variable change is $K_j > K_i$, it follows that x is non-decreasing in K and therefore the optimal discount is non-decreasing in the ODL value. \square

CRedit authorship contribution statement

Patrick Stokkink: Conceptualization, Methodology, Investigation, Validation, Writing - review & editing, Software, Writing - original draft. **Nikolas Geroliminis:** Conceptualization, Methodology, Investigation, Validation, Writing - review & editing, Supervision, Funding acquisition.

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