Correlation-based Controller Tuning

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In honor of Duncan A. Mellichamp, teacher and humanist

Outline

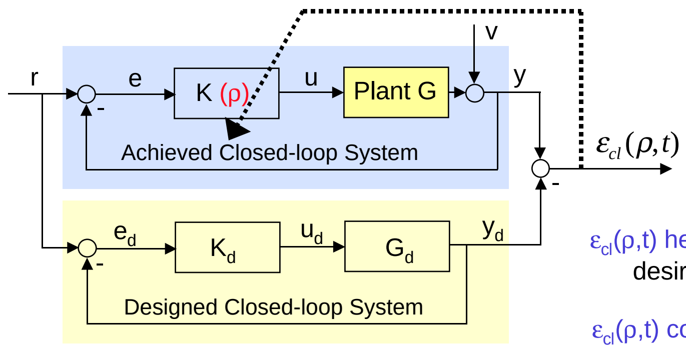
- Use of data for controller tuning
- Controller tuning based on the correlation approach
- Experimental illustrations
- Conclusions

Use of Data for Controller Tuning

- Indirect model-based approaches
  - Model identification
    - Off-line (e.g. step response)
    - On-line, *fast* update (indirect adaptive control)
    - Repeated, *slower* update (identification for control)
  - Controller design
  - Key issue: model validity
- Direct data-driven approaches

Direct Data-driven Controller Tuning

Framework of closed-loop output error (CLOE)



Model following

$\varepsilon_{cl}(\rho,t)$ helps compare achieved and desired closed-loop systems

$\varepsilon_{cl}(\rho,t)$ contains effects of
- model mismatch
- noise

Design objective $\min_{\rho} \; J = \left\| \varepsilon_{cl}(\rho,t) \right\|$

Direct Controller Tuning
Features

- Iterative update $\rho_{i+1} = \rho_i - \gamma_i \; Q^{-1}(\rho_i) \; J'(\rho_i)$ Robbins-Monro

  ρ : controller parameters
  γ_i : step size
  $Q(\rho_i)$: positive definite matrix
  $J'(\rho_i)$: gradient of criterion

- Difficulties
  - Gradient depends on unknown CL plant $\rightarrow$ gradient estimation
  - Presence of noise *v(t)*

- Two data-driven approaches with slow update
  - IFT $\rightarrow$ gradient from closed-loop data (Hjalmarsson et al., 1994)
  - CbT $\rightarrow$ no gradient needed (Karimi et al., 2003)

Iterative Feedback Tuning
Two Experiments

Evaluation of criterion

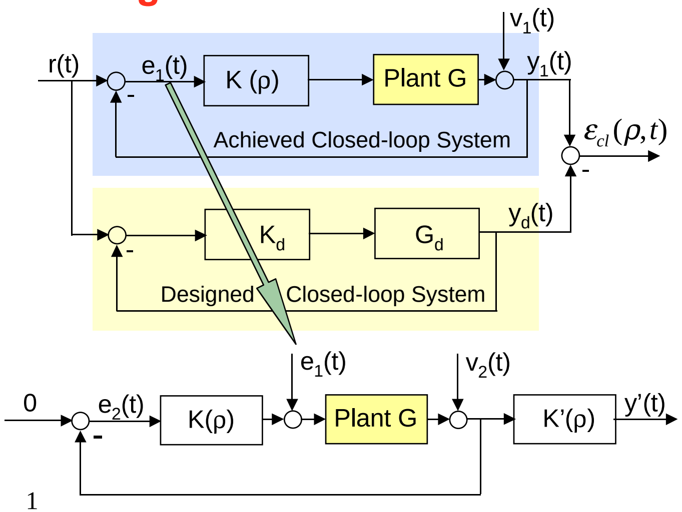
$$J(\rho) = E\left[\varepsilon_{cl}^2(\rho,t)\right]$$

Over N samples:

$$J(\rho) = \frac{1}{N}\sum_{t=1}^{N}\varepsilon_{cl}^2(\rho,t)$$

Estimation of gradient

$$J'(\rho) = E\left[\varepsilon_{cl}(\rho,t)y'(\rho,t)\right]$$



$$y'(\rho,t) = G\,S(\rho)K'(\rho)e \qquad S = \frac{1}{1+KG}$$

H. Hjalmarsson, Iterative Feedback Tuning: An Overview, Int. J. Adapt. Control Signal Process., pp 373-95 (2002)

Iterative Feedback Tuning
Properties

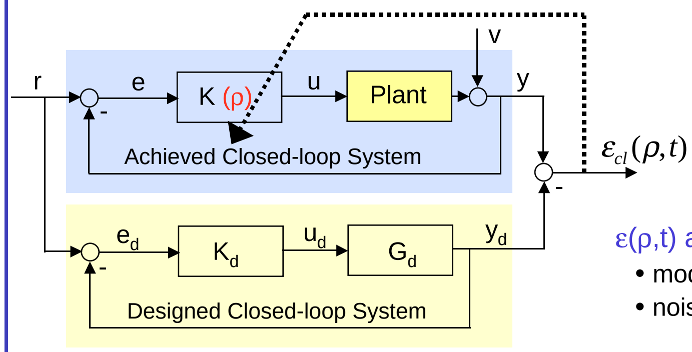
- Unbiased model-free estimation of gradient if
  - Zero-mean disturbances
  - Disturbances in Experiments 1 & 2 are uncorrelated

    $\rightarrow$ convergence to (local) minimum

- Features
  - Precise local information $\rightarrow$ bias-free gradient estimation
  - Only local information $\rightarrow$ only gradual changes possible $\rightarrow$ slow
  - Good control $\rightarrow$ error e small or not sufficiently rich

    $\rightarrow$ poor gradient estimation (dual control problem)

Correlation-based Tuning
Basic Idea



ε(ρ,t) affected by
- model mismatch, correlated with r(t)
- noise, uncorrelated with r(t)

Objective: Determine K(ρ) such that $\varepsilon_{cl}(\rho,t)$ is uncorrelated with r(t)

$\rightarrow$ controller compensates the effect of model mismatch

A. Karimi, L. Miskovic and D .Bonvin, Iterative Correlation-based Controller Tuning with Application to a Magnetic Suspension System, Control Engineering Practice (2003)

Correlation-based Tuning

- Correlation equations

$$f(\rho) = E[\zeta(t)\varepsilon_{cl}(\rho,t)] = 0$$

$\zeta(t)$: n_ρ-dim. vector of instrumental variables
- correlated with $r(t)$, for example, $r(t-n_z)$ … $r(t)$ … $r(t+n_z)$
- independent of noise $v(t)$

- Iterative solution

$$\rho_{i+1} = \rho_i - \gamma_i \; Q^{-1}(\rho_i) \; f(\rho_i)$$ Gauss-Newton

γ_i : step size $Q(\rho_i)$: positive definite matrix

$Q(\rho_i)$ = I substitution method

Solution of Correlation Equations

- Newton-Raphson Algorithm: $Q(\rho_i) = \left.\dfrac{\partial f}{\partial \rho}\right|_{\rho_i} = E[\zeta(t)\psi^T(\rho_i,t)]$

$$\psi^T(\rho,t) \equiv \frac{\partial \varepsilon_{cl}(\rho,t)}{\partial \rho} \qquad \zeta(t) = \hat{\psi}^T(\rho,t)$$ makes Q positive definite

- Features
  - Convergence is not affected by noise
  - No need for gradient with the substitution method
  - No need for second experiment
  - Existence of a solution ? Perfect decorrelation might require a high-order or non-causal controller → Minimize correlation function

Frequency-domain Interpretation

- For $n_z \to \infty$ and using Parseval theorem

$$\rho^* = \arg\min_\rho \int_{-\pi}^{\pi} \left|T(e^{-j\omega},\rho) - T_d(e^{-j\omega})\right|^2 \Phi_r^2(\omega)\, d\omega$$

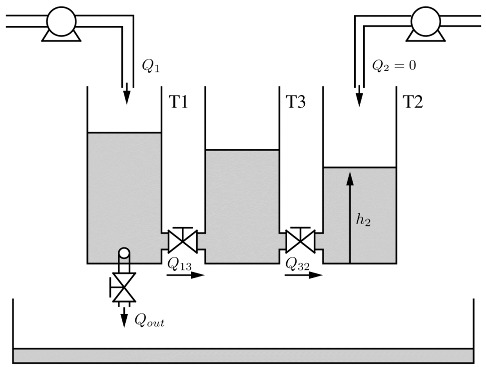
- Difference between achieved and desired closed-loop is minimized
- Noise has no effect on the criterion (Φ_r is the spectrum of $r(t)$)

- For IFT (minimization of 2-norm of ε_{cl})

$$\rho^* = \arg\min_\rho \int_{-\pi}^{\pi} \left[\left|T(e^{-j\omega},\rho) - T_d(e^{-j\omega})\right|^2 \Phi_r(\omega) + \left|S(e^{-j\omega},\rho)\right|^2 \Phi_v(\omega)\right] d\omega$$

Three-tank System

Experimental Setup



Manipulated input: Q_1
Measured output: h_2

Mathematical Model

$$A\frac{dh_1}{dt} = Q_1 - Q_{13} - Q_{out}$$

$$A\frac{dh_2}{dt} = Q_{13} - Q_{32}$$

$$A\frac{dh_3}{dt} = Q_2 + Q_{32}$$

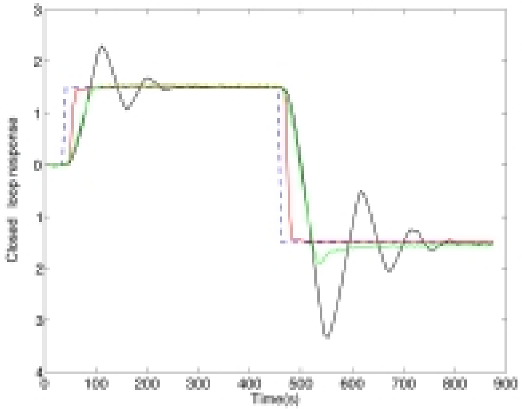
$$Q_{13} = a_{13}\, S\, \mathrm{sgn}(h_1 - h_3)\sqrt{2g|h_1 - h_3|}$$

$$Q_{32} = a_{32}\, S\, \mathrm{sgn}(h_3 - h_2)\sqrt{2g|h_3 - h_2|}$$

$$Q_{out} = a_{out}(h_1)\, S\, \sqrt{2gh_1}$$

A - section of cylinder $[m^2]$
S - section of connecting pipe $[m^2]$
a - outflow coefficient (dimensionless)

Three-tank System
Simulated Closed-loop Response



◇ Identification of a linear discrete model

◇ Initial PID-like controller of the form

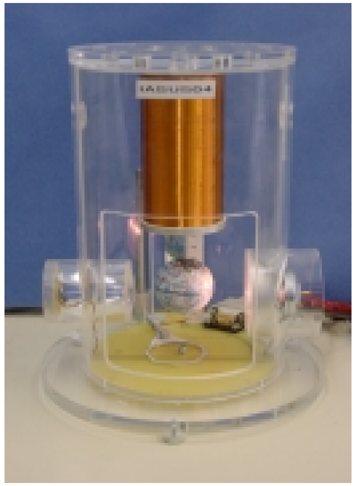
$$K(q^{-1}) = \frac{s_0 + s_1 q^{-1} + s_2 q^{-2}}{(1-q^{-1})(1+r_1 q^{-1})}$$

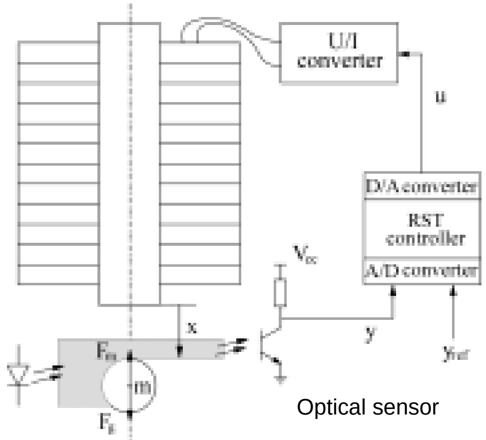
designed using pole placement

◇ Sampling period $T_s = 7s$

- - - Reference signal
— Designed response (linear model)
— Initial CL response (model based)
— CbT after 2 iterations (data based)

Magnetic Suspension System
Experimental Setup





Nonlinear, unstable system

Magnetic Suspension System
Approximate Model and Initial Controller

- Linearized continuous model
  with U/I-converter dynamics

$$G(s) = \frac{0.1}{0.017s+1}\,\frac{15750}{s^2-1238}$$

- Discrete-time model
  T_s = 10 ms

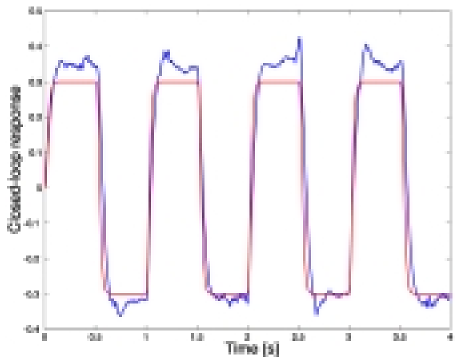
$$G_0(q^{-1}) = \frac{10^{-4}(137q^{-1}+481q^{-2}+103q^{-3})}{1-2.69q^{-1}+2.19q^{-2}-0.56q^{-3}}$$

- Initial RST controller

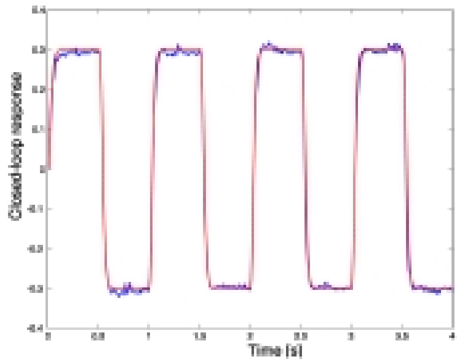
$$R_0(q^{-1}) = 1+0.686q^{-1}+.163q^{-2}$$
$$S_0(q^{-1}) = 21.86-26.77q^{-1}+8.15q^{-2}$$
$$T_0(q^{-1}) = 1.83$$

Magnetic Suspension System
Correlation-based Tuning

Closed-loop Response



Initial RST controller
(model based)

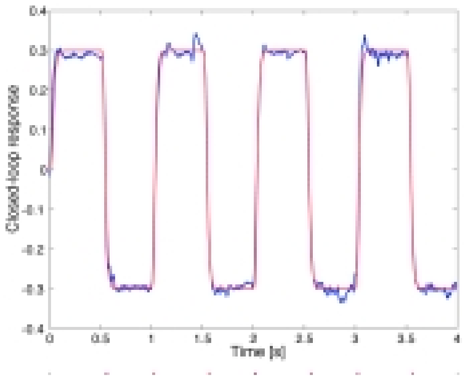


CbT after 6 iterations
(data based)

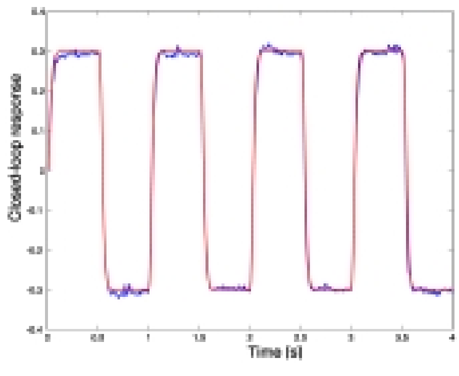
Magnetic Suspension System

IFT vs. CbT

Closed-loop Response



IFT after 24 experiments



CbT after 6 experiments

Conclusions

- Role of the closed-loop output error
  - Allows easy comparison with designed closed loop
  - Expresses the effects of unmodeled dynamics, nonlinearities and noise
- Direct data-driven controller tuning
  - IFT
    - Two experiments per iteration
    - Controller depends on noise
  - CbT
    - A single experiment per iteration
    - Controller independent of noise