Correlation-based Controller Tuning

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In honor of Duncan A. Mellichamp, teacher and humanist

Outline
- Use of data for controller tuning
- Controller tuning based on the correlation approach
- Experimental illustrations
- Conclusions

Use of Data for Controller Tuning
- **Indirect** model-based approaches
  - Model identification
    - Off-line (e.g. step response)
    - On-line, fast update (indirect adaptive control)
    - Repeated, slower update (identification for control)
  - Controller design
  - Key issue: model validity
- **Direct** data-driven approaches

Direct Data-driven Controller Tuning
Framework of closed-loop output error (CLOE)

Design objective
$$\min_{\rho} J = \| \varepsilon_{cl}(\rho, t) \|$$

Model following
$$\varepsilon_{cl}(\rho, t)$$ helps compare achieved and desired closed-loop systems
$$\varepsilon_{cl}(\rho, t)$$ contains effects of
- model mismatch
- noise
**Direct Controller Tuning**

**Features**

- **Iterative update**
  \[
  \rho_{i+1} = \rho_i - \gamma_i Q^{-1}(\rho_i) J'(\rho_i) \]
  Robbins-Monro
  
  - \( \rho \): controller parameters
  - \( \gamma \): step size
  - \( Q(\rho) \): positive definite matrix
  - \( J'(\rho) \): gradient of criterion

- **Difficulties**
  - Gradient depends on unknown CL plant \( \rightarrow \) gradient estimation
  - Presence of noise \( \nu(t) \)

- **Two data-driven approaches with slow update**
  - IFT \( \rightarrow \) gradient from closed-loop data (Hjalmarsson et al., 1994)
  - CbT \( \rightarrow \) no gradient needed (Karimi et al., 2003)

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**Iterative Feedback Tuning**

**Two Experiments**

**Evaluation of criterion**

\[
J(\rho) = E[e^2(\rho,t)]
\]

Over \( N \) samples:

\[
J(\rho) = \frac{1}{N} \sum_{i=1}^{N} E_i(\rho,t)
\]

**Estimation of gradient**

\[
J'(\rho) = E[e(\rho,t)y'(\rho,t)]
\]

\[
y'(\rho,t) = G S(\rho) K'(\rho) e \quad S = \frac{1}{1 + K G}
\]


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**Correlation-based Tuning**

**Basic Idea**

**Properties**

- Unbiased model-free estimation of gradient if
  - Zero-mean disturbances
  - Disturbances in Experiments 1 & 2 are uncorrelated \( \rightarrow \) convergence to (local) minimum

- **Features**
  - Precise local information \( \rightarrow \) bias-free gradient estimation
  - Only local information \( \rightarrow \) only gradual changes possible \( \rightarrow \) slow
  - Good control \( \rightarrow \) error \( e \) small or not sufficiently rich \( \rightarrow \) poor gradient estimation (dual control problem)

**Objective:** Determine \( K(\rho) \) such that \( e_d(\rho,t) \) is uncorrelated with \( r(t) \)

\( e(\rho,t) \) affected by

- model mismatch, correlated with \( r(t) \)
- noise, uncorrelated with \( r(t) \)

A. Karimi, L. Miskovic and D. Bonvin, Iterative Correlation-based Controller Tuning with Application to a Magnetic Suspension System, Control Engineering Practice (2003)
Correlation-based Controller Tuning

- **Correlation equations**
  
  \[ f(\rho) = E[\zeta(t)\varepsilon_{cl}(\rho, t)] = 0 \]

  \( \zeta(t) \): \( n_\rho \)-dim. vector of instrumental variables
  
  - correlated with \( r(t) \), for example, \( r(t-n_z) \ldots r(t) \ldots r(t+n_z) \)
  
  - independent of noise \( v(t) \)

- **Iterative solution**
  
  \[
  \rho_{i+1} = \rho_i - \gamma_i Q^{-1}(\rho_i) \cdot f(\rho_i) \\
  \gamma_i: \text{step size} \quad Q(\rho_i): \text{positive definite matrix} \quad Q(\rho_i) = I \quad \text{substitution method}
  \]

Frequency-domain Interpretation

- For \( n_z \rightarrow \infty \) and using Parseval theorem
  \[
  \rho^* = \arg \min_\rho \int_{-\pi}^{\pi} \left[ T(e^{-j\omega}, \rho) - T_d(e^{-j\omega}) \right]^2 \Phi_\varepsilon(\omega) \, d\omega
  \]

  - Difference between achieved and desired closed-loop is minimized
  
  - Noise has no effect on the criterion (\( \Phi_\varepsilon \) is the spectrum of \( r(t) \))

- For IFT (minimization of 2-norm of \( \varepsilon_{cl} \))
  \[
  \rho^* = \arg \min_\rho \int_{-\pi}^{\pi} \left[ T(e^{-j\omega}, \rho) - T_d(e^{-j\omega}) \right]^2 \Phi_\varepsilon(\omega) + \left[ S(e^{-j\omega}, \rho) \right]^2 \Phi_v(\omega) \, d\omega
  \]

Solution of Correlation Equations

- **Newton-Raphson Algorithm**
  \[
  Q(\rho) = \frac{\partial f}{\partial \rho} = E[\zeta(t)\psi^T(\rho, t)]
  \]

  \[
  \psi^T(\rho, t) \equiv \frac{\partial \varepsilon_{cl}(\rho, t)}{\partial \rho}
  \]

  \[
  \zeta(t) = \psi^T(\rho, t)
  \]

  makes \( Q \) positive definite

- **Features**
  
  - Convergence is not affected by noise
  
  - No need for gradient with the substitution method
  
  - No need for second experiment
  
  - Existence of a solution? Perfect decorrelation might require a high-order or non-causal controller \( \rightarrow \) Minimize correlation function

Three-tank System

**Experimental Setup**

**Mathematical Model**

\[
\begin{align*}
A \frac{dh_1}{dt} &= Q_1 - Q_{13} - Q_{out} \\
A \frac{dh_2}{dt} &= Q_{13} - Q_{32} \\
A \frac{dh_3}{dt} &= Q_2 + Q_{32}
\end{align*}
\]

\[
\begin{align*}
Q_{13} &= a_{13} S \text{sgn}(h_1 - h_3) \sqrt{2g(h_1 - h_3)} \\
Q_{32} &= a_{32} S \text{sgn}(h_3 - h_2) \sqrt{2g(h_3 - h_2)} \\
Q_{out} &= a_{out}(h_1) S \sqrt{2gh_1}
\end{align*}
\]

- \( A \) - section of cylinder [m$^2$]
  
  - \( S \) - section of connecting pipe [m$^2$]
  
  - \( a \) - outflow coefficient (dimensionless)
Three-tank System
Simulated Closed-loop Response

- Identification of a linear discrete model
- Initial PID-like controller of the form
  \[ K(q^{-1}) = \frac{s_0 + s_1q^{-1} + s_2q^{-2}}{(1 - q^{-1})(1 + r_1q^{-1})} \]
designed using pole placement
- Sampling period \( T_s = 7s \)

- Reference signal
- Designed response (linear model)
- Initial CL response (model based)
- CbT after 2 iterations (data based)

Magnetic Suspension System
Experimental Setup

- Nonlinear, unstable system

Magnetic Suspension System
Approximate Model and Initial Controller

- Linearized continuous model with U/I-converter dynamics
  \[ G(s) = \frac{0.1}{0.017s + 1} \frac{15750}{s^2 - 1238} \]

- Discrete-time model
  \( T_s = 10 \text{ ms} \)
  \[ G_0(q^{-1}) = 10^{-4} (137q^{-1} + 481q^{-2} + 103q^{-3}) \]
  \[ 1 - 2.69q^{-1} + 2.19q^{-2} - 0.56q^{-3} \]

- Initial RST controller
  \[ R_0(q^{-1}) = 1 + 0.686q^{-1} + 0.163q^{-2} \]
  \[ S_0(q^{-1}) = 21.86 - 26.77q^{-1} + 8.15q^{-2} \]
  \[ T_0(q^{-1}) = 1.83 \]
Conclusions

- Role of the closed-loop output error
  - Allows easy comparison with designed closed loop
  - Expresses the effects of unmodeled dynamics, nonlinearities and noise

- Direct data-driven controller tuning
  - IFT
    - Two experiments per iteration
    - Controller depends on noise
  - CbT
    - A single experiment per iteration
    - Controller independent of noise