LABORATOIRE D'AUTOMATIQUE

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

# Iterative Tuning of Restricted-Complexity Controllers

# A Perspective


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*International Workshop on:*

**Design and Optimization of Restricted Complexity Controllers**

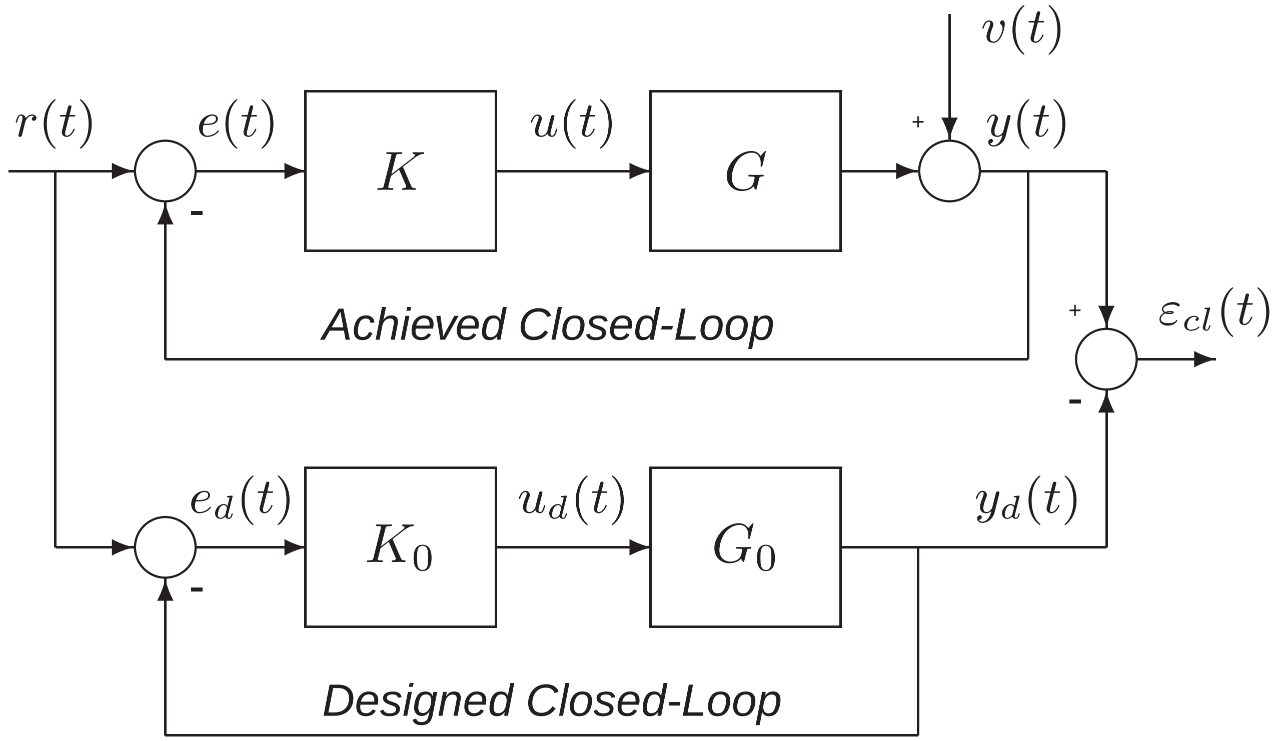
**Grenoble, January 2003**

# Outline

1. Introduction
   - Identification for control
   - Iterative Feedback Tuning
2. Iterative Correlation-Based Controller Tuning
   - Correlation approach using Instrumental Variables
   - Convergence and Frequency Analysis
   - Simulation and Experimental Results
3. Iterative Controller Tuning by Minimizing a Frequency Criterion
   - Simple relay tests for gain and phase margins measurement
   - Model-free gradient estimation using Bode's integrals
4. Concluding remarks

# Introduction

**Iterative identification and control**




Because of unmodeled dynamics the designed performance cannot be achieved on the real system.

**Solution:** Closed-loop identification and Controller redesign

# Introduction

## Direct Controller Tuning (controller order is independent of model order)

**Objective:** Minimizing

$$J(\rho) = ||\varepsilon_{cl}(\rho, t)||_2^2$$

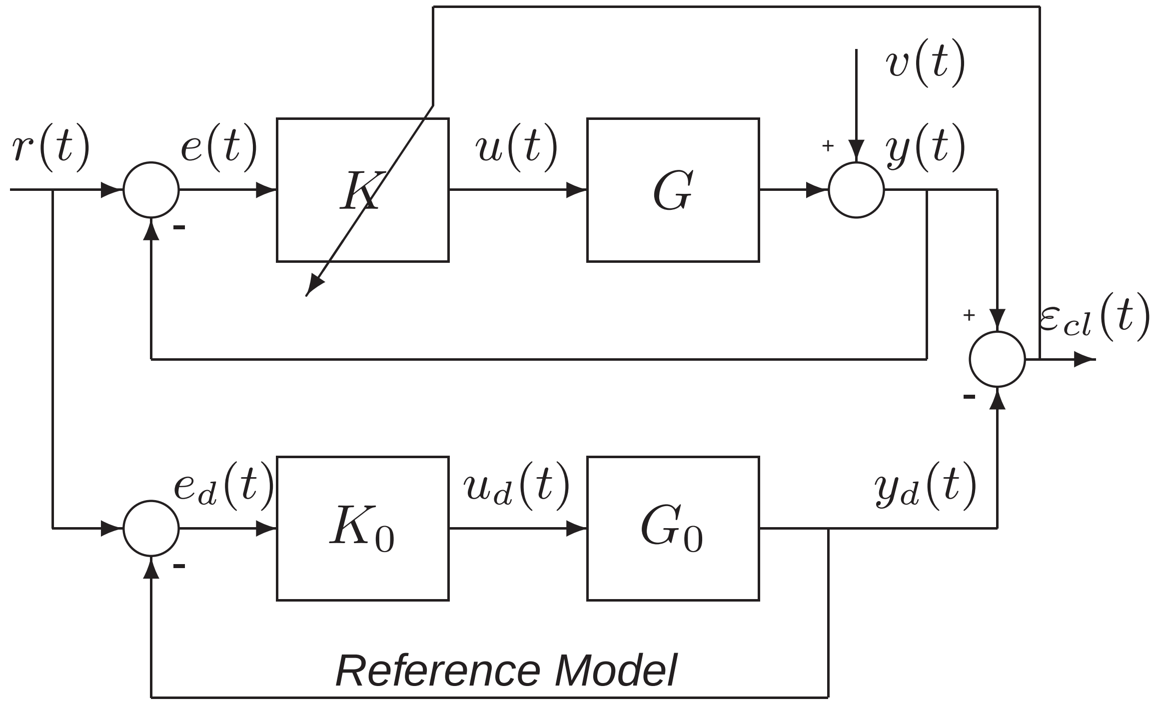
using an iterative tuning algorithm:

$$\rho_{i+1} = \rho_i - \gamma_i [Q(\rho_i)]^{-1} J'(\rho_i)$$

$\gamma_i$: Step size

$Q(\rho_i)$: A positive definite matrix

$J'(\rho_i)$: Gradient of the criterion




**Problems:**

1. Gradient estimation (Gradient depends on unknown $G$)

2. Noise effects

# Introduction

## Gradient estimation

Gradient depends on the *true* closed-loop transfer function

**Different approaches:**

- Model Reference Adaptive Control (MRAC)[ MIT rule, 1958]:

  *reference model* $\longrightarrow$ *gradient*

- Self Tuning Regulation (STR)[ Astrom, Wittenmark 1973]:

  *identified model* $\longrightarrow$ *gradient*

- Iterative Controller Tuning (Adaptation period $\gg$ Sampling period) [ Trulsson, Ljung 1985]:

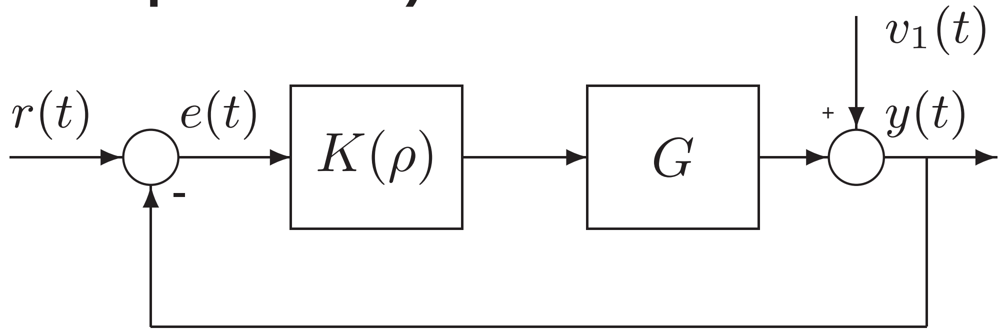
  *better identified model* $\longrightarrow$ *better gradient estimate*

- Iterative Feedback Tuning (IFT) [Hjalmarsson, Gunnarsson, Gevers 1994]:

  *closed-loop data* $\longrightarrow$ *gradient (model-free, unbiased)*

# Introduction

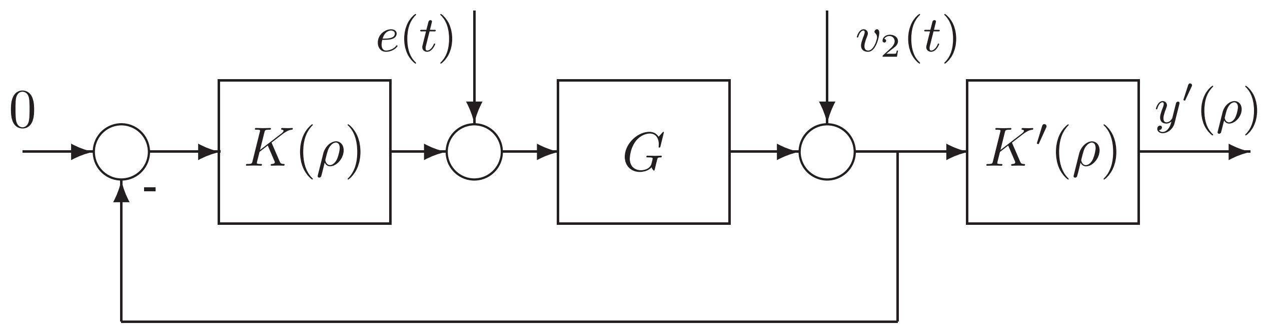
## Iterative Feedback Tuning (two experiments)




1. Normal experiment

   (criterion evaluation)

$$J(\rho) = \frac{1}{2N} \sum_{t=1}^{N} E\{[y(\rho,t) - y_d(t)]^2\}$$




2. Gradient experiment

   (gradient estimation)

$$J'(\rho) = \frac{1}{N} \sum_{t=1}^{N} E\{[y(\rho,t) - y_d(t)]y'(\rho,t)\}$$

Unbiased model-free estimation of gradient $\longrightarrow$ convergence to a local minimum of the criterion

# Introduction

## Iterative Feedback Tuning: Properties

- Possibility of minimizing a generalized criterion:

$$J(\rho) = \frac{1}{2N} \sum_{t=1}^{N} F(q^{-1}) E\{[y(\rho,t) - y_d(t)]^2 + \lambda u^2(\rho,t)\}$$

- Application to non-linear and MIMO systems

- Tuned controller depends on the reference signal and noise characteristics

$$\begin{aligned} \rho^* &= \arg\min_{\rho} \frac{1}{2N} \sum_{t=1}^{N} E\{[y(\rho,t) - y_d(t)]^2\} \\ &= \arg\min_{\rho} \int_{-\pi}^{\pi} [|T(e^{-j\omega},\rho) - T_0(e^{-j\omega})|^2 \Phi_r(\omega) + |S(e^{-j\omega},\rho)|^2 \Phi_v(\omega)] d\omega \end{aligned}$$

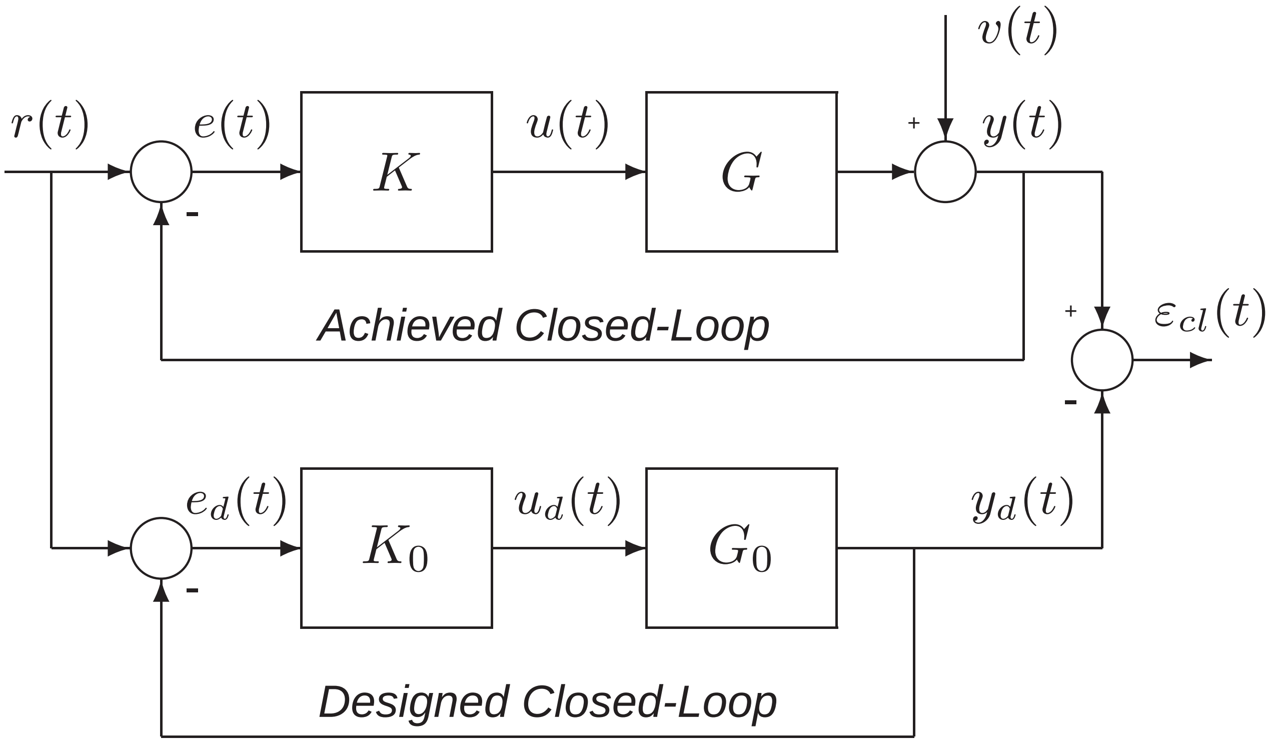
$$S(\rho) = \frac{1}{1+KG} \quad , \quad T(\rho) = \frac{KG}{1+KG} \quad , \quad T_0 = \frac{K_0 G_0}{1+K_0 G_0}$$

# Iterative Correlation-Based Controller Tuning

- Correlation Approach
- Convergence and Consistency
- Simulation Results
- Reduced-Order Controller Tuning
- Frequency Analysis
- Application to a Magnetic Suspension System

# Correlation Approach

## Main idea




If $K = K_0$ then $\varepsilon_{cl}(t)$ contains:

1. filtered noise $v(t)$
2. filtered unmodeled dynamics $(G - G_0)$

**Control objective:** Find $K$ that makes $\varepsilon_{cl}(t)$ *uncorrelated* with $r(t)$

*New controller should compensate for the unmodeled dynamics*

# Correlation Approach

Correlation Equation:

$$f(\rho) = E\{\zeta(t)\varepsilon_{cl}(\rho, t)\} = 0$$

$\rho$: Vector of controller parameters $\qquad$ $\varepsilon_{cl}(\rho, t)$: Closed-loop output error

$\zeta(t)$: Vector of instrumental variables, correlated with $r(t)$ and independent of noise $v(t)$

Iterative Solution to the Correlation Equation

$$\rho_{i+1} = \rho_i - \gamma_i [Q_N(\rho_i)]^{-1} \hat{f}(\rho_i)$$

where:

$$\hat{f}(\rho) = \frac{1}{N} \sum_{t=1}^{N} \zeta(\rho, t)\varepsilon_{cl}(\rho, t)$$

$N$: number of data $\qquad$ $\gamma_i$: scalar positive step size $\qquad$ $Q_N(\rho_i)$: a positive definite matrix

*No gradient, no additional experiment and no model is required !!!*

# Correlation Approach

**Newton-Raphson method:** can be used in order to improve the convergence speed

$$Q_N(\rho_i) = \left.\frac{\partial \hat{f}}{\partial \rho}\right|_{\rho_i} = \frac{1}{N}\sum_{t=1}^{N}\left\{ \left.\frac{\partial \zeta(\rho,t)}{\partial \rho}\right|_{\rho_i} \varepsilon_{cl}(\rho_i,t) + \zeta(\rho_i,t)\left.\frac{\partial \varepsilon_{cl}(\rho,t)}{\partial \rho}\right|_{\rho_i} \right\}$$

$$\approx \frac{1}{N}\sum_{t=1}^{N}\zeta(\rho_i,t)\left.\frac{\partial \varepsilon_{cl}(\rho,t)}{\partial \rho}\right|_{\rho_i}$$

**Choice of Instrumental Variables:**

Let define:

$$\psi^T(\rho,t) = \frac{\partial \widehat{\varepsilon_{cl}}(\rho,t)}{\partial \rho} \qquad\qquad \zeta(\rho,t) = \hat{\psi}(\rho,t)$$

$\psi^T(\rho,t)$: gradient estimate based on an identified model of the plant and real data

$\hat{\psi}(\rho,t)$: noise-free part of the gradient estimate, based on the identified model and simulated data

# Convergence and Consistency

## Assumptions

**A1)** The system to be controlled is SISO, LTI, finite order and strictly causal

**A2)** The reference signal $r(t)$ is persistently exciting of sufficiently high order and uncorrelated with zero-mean finite power disturbance signal $v(t)$

**A3)** The controller Computed at each iteration stabilizes the closed-loop system

**A4)** The solution $\rho^*$ exists and is unique:

$$K^* = K_0 \frac{G_0}{G}$$

- $G_0$ contains the unstable zeros of $G$
- the order of the estimated controller is large enough to compensate the unmodeled dynamics

# Convergence and Consistency

*The Controller parameters $\rho_i$, when $N \to \infty$ and $i \to \infty$, converge to the solution of the correlation equation with probability one if:*

$$Q(\rho_i) = E\{\zeta(\rho_i, t)\hat{\psi}^T(\rho_i, t)\} \text{ exists and is nonsingular (w.p.1)}$$

**Nonsingularity of $Q(\rho_i)$**

**Theorem:** Consider the instrumental variables based on the identified model $\zeta(\rho_i, t)$ and $H(z^{-1})$ as follows:

$$H(z^{-1}) = \frac{\hat{A}(z^{-1})}{\hat{P}(z^{-1})} \frac{P(z^{-1})}{A(z^{-1})}$$

where $A(z^{-1})$ and $P(z^{-1})$ are denominator of the plant and the closed loop system respectively, and $\hat{A}(z^{-1})$ and $\hat{P}(z^{-1})$ are identified ones used in constructions of the IV.

**(a)** If $r(t)$ is persistently exciting of order $\rho$ (or more) and $H(z^{-1})$ is SPR then $Q$ is nonsingular.

**(b)** If $r(t)$ is a deterministic periodic signal with period $\rho$ and persistently exciting of order $\rho$ and $H(z^{-1})$ has no pole on the unit circle, then $Q$ is nonsingular.

# Convergence and Consistency

**Simulation Example (effect of modeling error)**

True system:

$$y(t) = \frac{q^{-1} + 0.5q^{-2}}{1 - 1.5q^{-1} + 0.7q^{-2}} u(t)$$

Reference model:

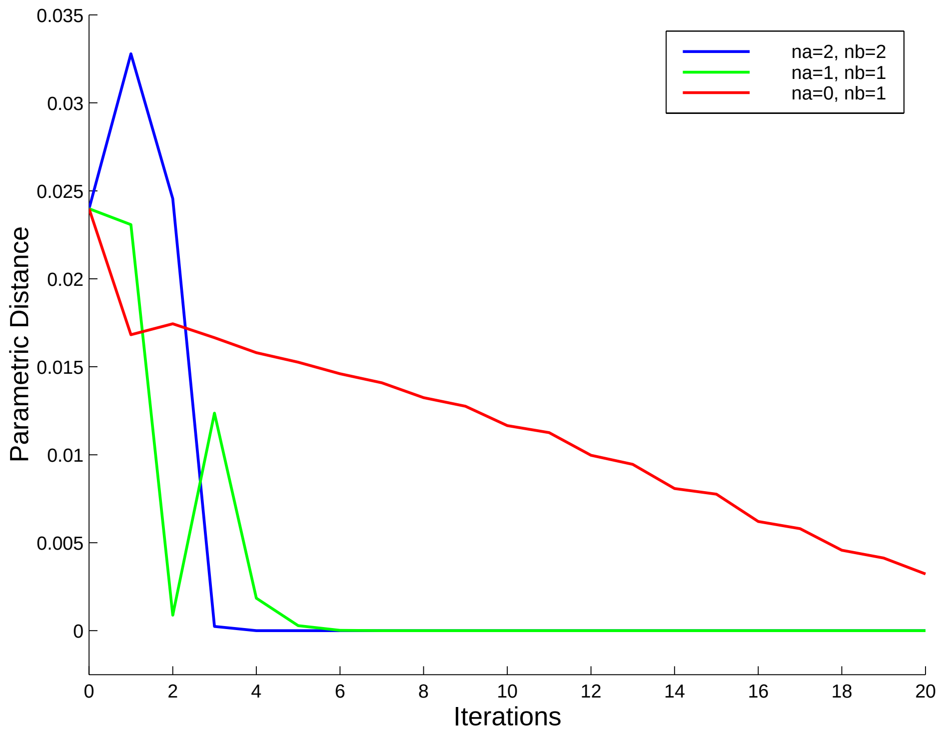
$$\frac{B_m}{A_m} = \frac{-10^{-3}[78q^{-1} + 63q^{-2} + 12q^{-3}]}{1 - 1.578q^{-1} + 0.638q^{-2} - 0.012q^{-3}}$$

The optimal controller:

$R^* = 1$ and $S^* = -0.0781 - 0.0234q^{-1}$

The initial controller:

$R_0 = 1$ and $S_0 = 0.075 + 0.0q^{-1}$




*Modeling error does not affect the convergence of algorithm*

# Convergence and Consistency

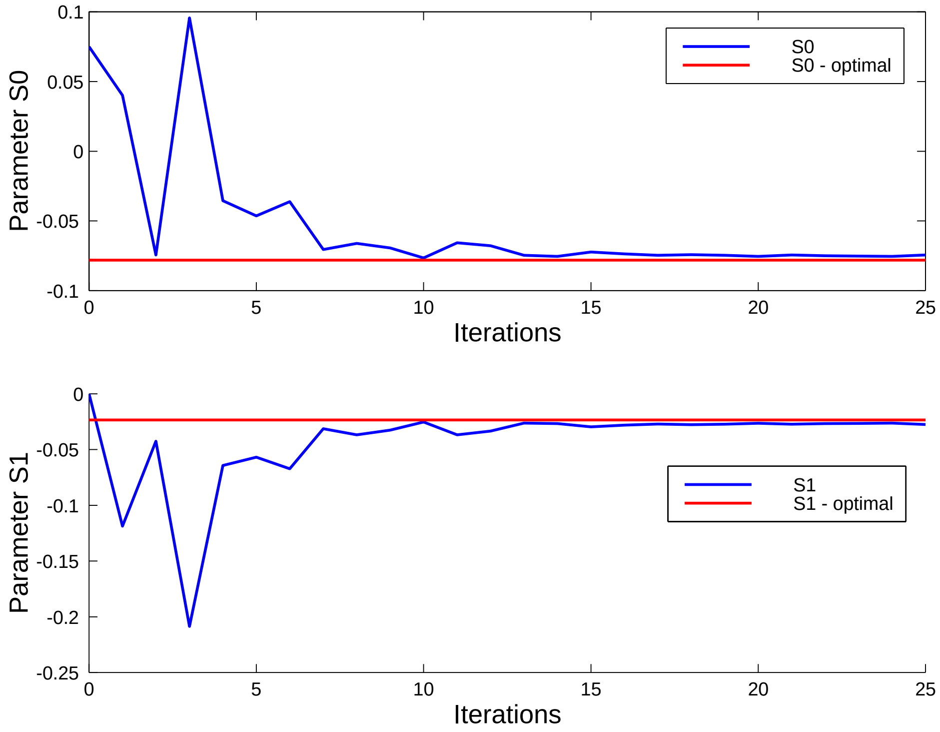
## Simulation Example (effect of noise)

True system:

$$y(t) = \frac{q^{-1} + 0.5q^{-2}}{1 - 1.5q^{-1} + 0.7q^{-2}} u(t) + v(t)$$

$$v(t) = \frac{1 + 0.5q^{-1} + 0.5q^{-2}}{1 - 1.5q^{-1} + 0.7q^{-2}} e(t)$$

- Monte-Carlo simulation with 100 runs
- Noise/Signal ratio of $7.5\%$ in terms of variance
- 11-bit PRBS of length 2047 as reference signal
- 25 iterations per simulation
- Plant model is identified with $n_{\hat{A}} = n_{\hat{B}} = 1$




*Noise does not affect the convergence of algorithm*

# Reduced-Order Controller Tuning

*In this case there is no solution to the correlation equation*

**A new criterion is defined as follows:**

$$\rho^* = \arg\min_{\rho} J(\rho) = ||f(\rho)||_2^2 = f^T(\rho)f(\rho)$$

Iterative Solution:

$$\rho_{i+1} = \rho_i - \gamma_i [Q(\rho_i)]^{-1} J'(\rho_i)$$

$\gamma_i$: Step size $\quad Q(\rho_i)$: A positive definite matrix $\quad J'(\rho_i)$: Gradient of the criterion

This algorithm converges to a local minimum of the criterion provided that (Robbins-Monro):

- $\sum_{i=0}^{\infty} \gamma_i = \infty \qquad \sum_{i=0}^{\infty} \gamma_i^2 < \infty$
- $r(t)$ and $v(t)$ are independent stationary stochastic processes.
- $y(t)$ is bounded at each iteration (closed-loop system is stable).

# Frequency Domain Analysis

Consider the following instrumental variables:

$$\zeta^T(t) = [r(t+n_z), r(t+n_z-1), \ldots, r(t), r(t-1), \ldots, r(t-n_z)]$$

So the criterion becomes:

$$J(\rho) = f^T(\rho) f(\rho) = \sum_{\tau=-n_z}^{n_z} R_{\varepsilon r}^2(\tau)$$

where: $R_{\varepsilon r}(\tau) = E\{\varepsilon_{cl}(\rho, t) r(t-\tau)\}$

Using the Parseval's relation for the criterion ($n_z \to \infty$), we have:

$$\rho^* = \arg\min_{\rho} \int_{-\pi}^{\pi} |\Phi_{\varepsilon r}(\omega)|^2 \, d\omega = \int_{-\pi}^{\pi} |T(e^{-j\omega}, \rho) - T_0(e^{-j\omega})|^2 \Phi_r^2(\omega) d\omega$$

For the methods which minimize the two norm of closed-loop output error (like IFT, STR), we have:

$$\rho^* = \arg\min_{\rho} \int_{-\pi}^{\pi} [|T(e^{-j\omega}, \rho) - T_0(e^{-j\omega})|^2 \Phi_r(\omega) + |S(e^{-j\omega}, \rho)|^2 \Phi_v(\omega)] d\omega$$

# Application to a Magnetic Suspension System

Linearized model:

$$G(s) = \frac{0.1}{0.017s + 1}\,\frac{15750}{s^2 - 1238}$$

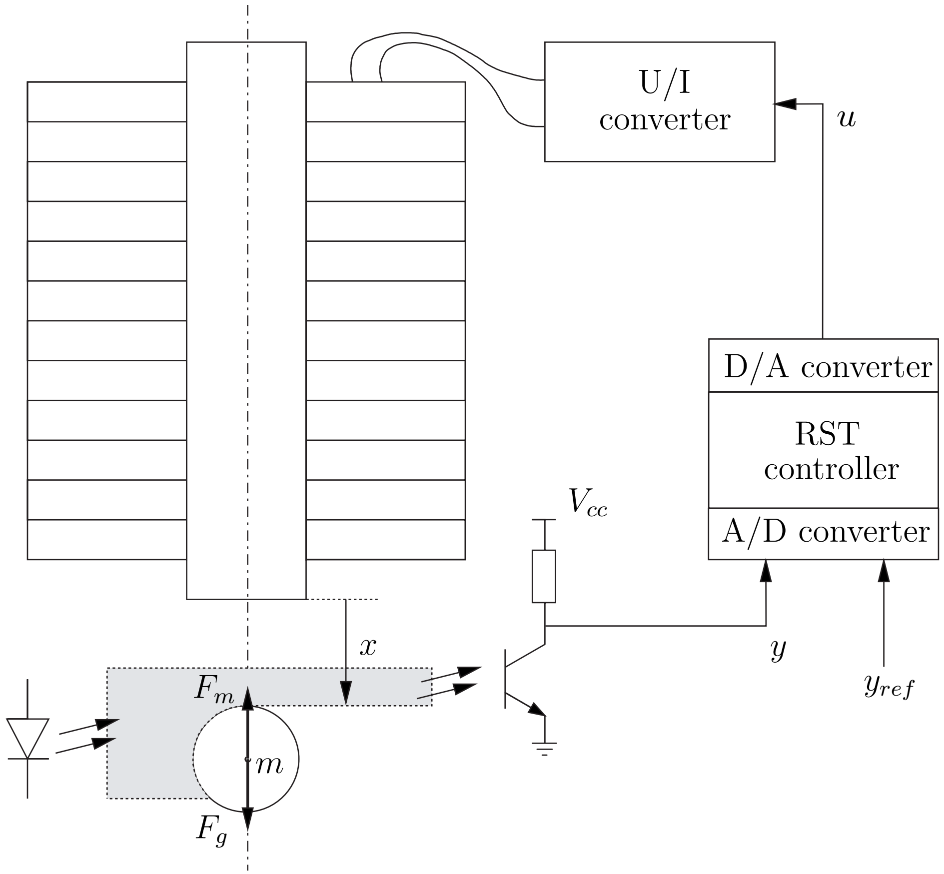
Discrete-time model ($f_s = 100Hz$):

$$G_0(q^{-1}) = \frac{10^{-4}(137q^{-1} + 481q^{-2} + 103q^{-3})}{1 - 2.69q^{-1} + 2.19q^{-2} - 0.56q^{-3}}$$

Initial RST controller:

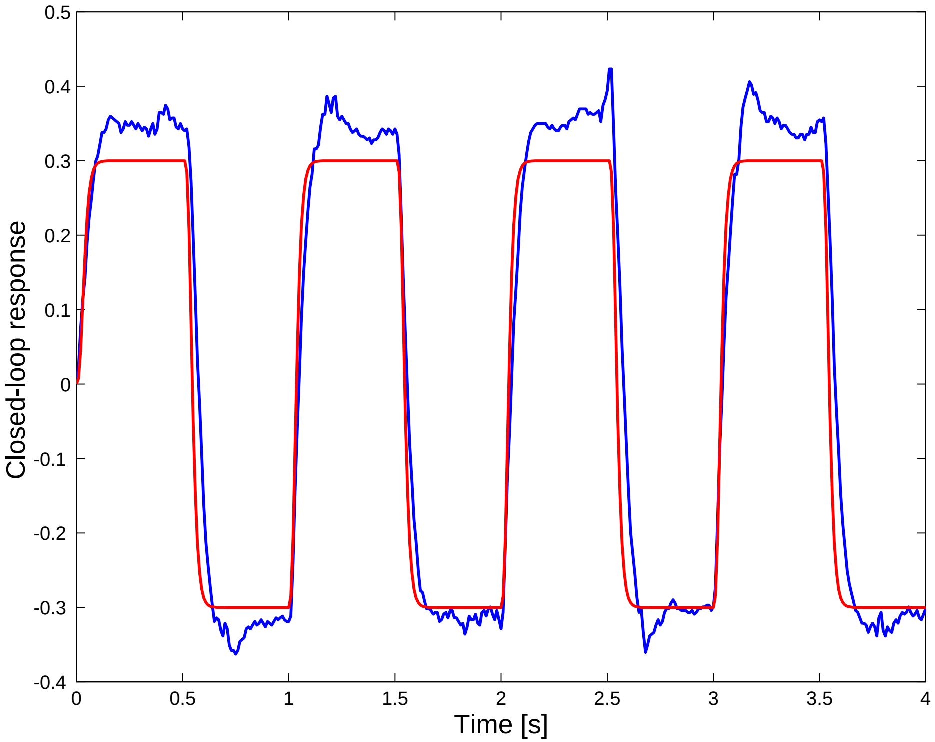
$$\begin{aligned} R_0(q^{-1}) &= 1 + 0.686q^{-1} + 0.163q^{-2} \\ S_0(q^{-1}) &= 21.86 - 26.77q^{-1} + 8.15q^{-2} \\ T_0(q^{-1}) &= 1.83 \end{aligned}$$

**Magnetic Suspension System**

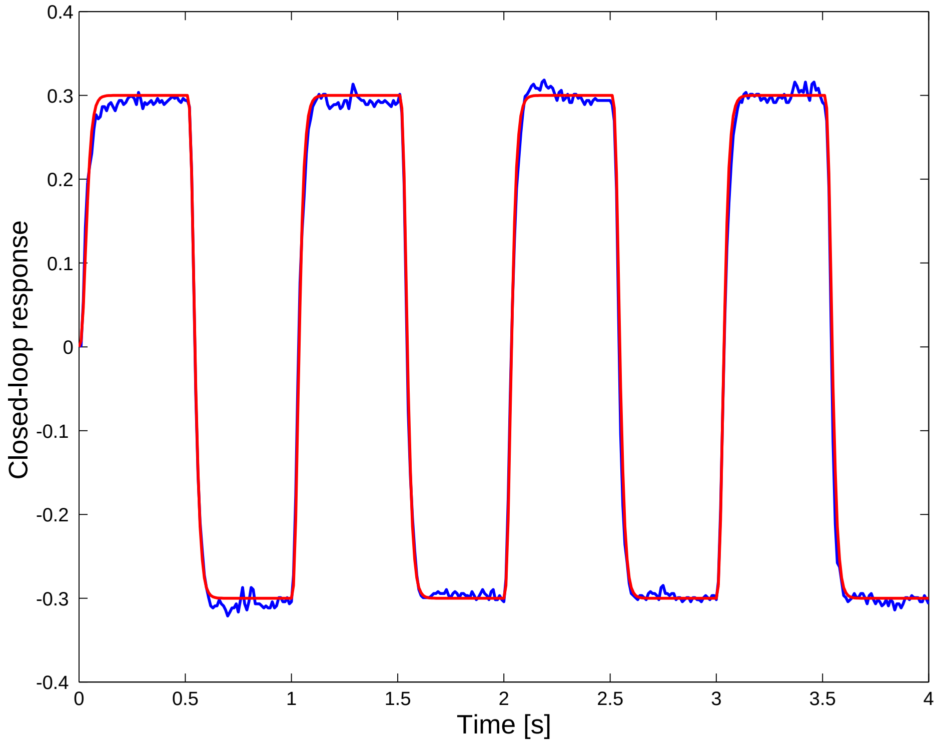


# Application to a Magnetic Suspension System

**Closed-Loop Response**




Initial RST controller and designed response




After 6 iterations using the proposed approach

# Iterative Controller Tuning Using Bode's Integrals

- Minimizing a Frequency Criterion in Terms of Phase Margin, Gain Margin and Cross-Over Frequency
- Relay Feedback Tests for Measuring the Robustness Margins
- Using Bode's Integrals for Gradient and Hessian Estimation
- Simulation and Experimental Results

# Frequency criteria

*The objective is to tune a controller by minimizing iteratively a frequency criterion with the Gauss-Newton method*

- **Criteria:**

$$J_1(\rho) = \frac{1}{2}\left[\frac{1}{\omega_d^2}(\omega_c - \omega_d)^2 + \frac{1}{\Phi_d^2}(\Phi_m - \Phi_d)^2\right]$$

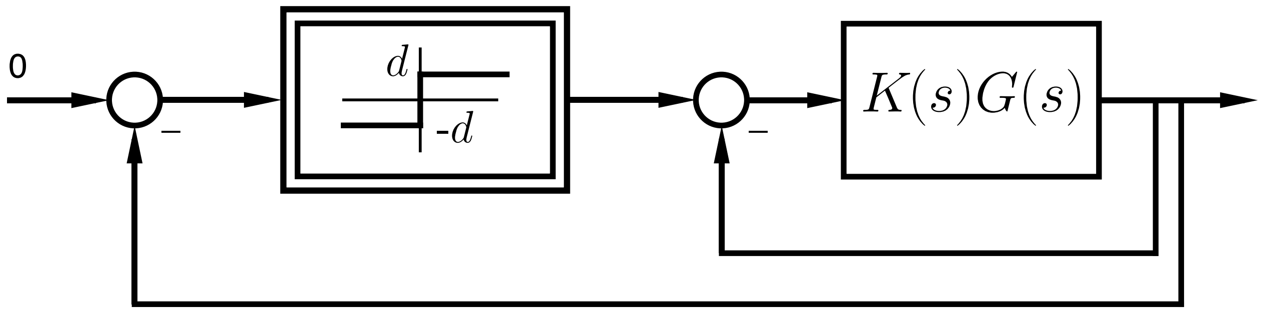
$$J_2(\rho) = \frac{1}{2}\left[\frac{1}{\omega_d^2}(\omega_c - \omega_d)^2 + \frac{1}{\Phi_d^2}(\Phi_m - \Phi_d)^2 + \frac{1}{K_d^2}(K_u - K_d)^2\right]$$

- **Iterative solution:** $\rho_{i+1} = \rho_i - H_{1,2}^{-1} J'_{1,2}(\rho_i)$

| | | | |
|---|---|---|---|
| $\omega_{c(d)}$: | Measured (desired) crossover frequencies | $\rho$: | Vector of the controller parameters |
| $\Phi_{m(d)}$: | Measured (desired) phase margin | $i$: | iteration number |
| $K_{u(d)}$: | Inverse of measured (desired) gain margin | $J'_{1,2}$: | Gradient of the criterion $J_1$ or $J_2$ |
| $H_{1,2}$: | Hessian of the criterion $J_1$ or $J_2$ | | |

# Closed-loop relay test (1)

**Gain margin and ultimate frequency $\omega_u$ measurement** [Aström, Hägglund]




- Relay output is the reference signal of the closed-loop system
- Condition for a limit cycle: $\frac{K(j\omega)G(j\omega)}{1+K(j\omega)G(j\omega)} = -\frac{\pi a}{4d}$
- Identified point: $K(j\omega_u)G(j\omega_u) = -\frac{\pi a}{4d+\pi a} \in (-1, 0)$

# Closed-loop relay test (2)

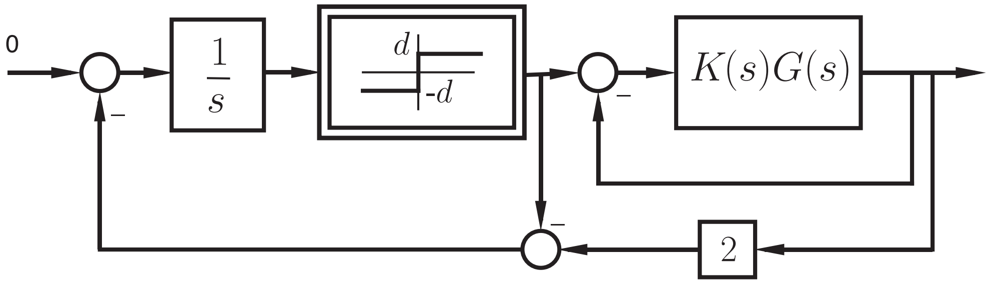
**Phase margin and crossover frequency $\omega_c$ measurement** [Schei, Longchamp, Piguet]

- Condition for a limit cycle:

$$\frac{1}{j\omega}\frac{K(j\omega)G(j\omega)-1}{K(j\omega)G(j\omega)+1} = -\frac{\pi a}{4d}$$

- Identified point:

$$\begin{aligned} K(j\omega_c)G(j\omega_c) &= \frac{1-j\frac{\omega_c\pi a}{4d}}{1+j\frac{\omega_c\pi a}{4d}} \\ |K(j\omega_c)G(j\omega_c)| &= 1 \\ \angle K(j\omega_c)G(j\omega_c) &= -2\arctan(\frac{\pi a\omega_c}{4d}) \end{aligned}$$

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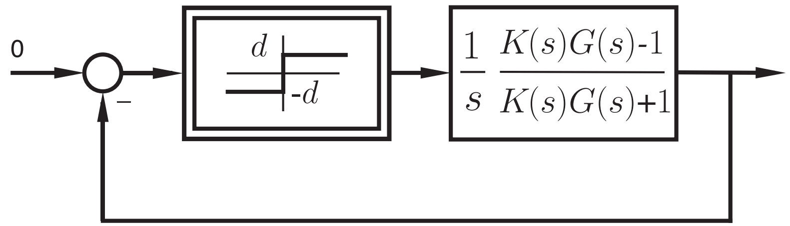


# Procedure for phase margin adjustment (1)

- **Criterion:** $$J(\rho) = \frac{1}{2}\left[\frac{1}{\omega_d^2}(\omega_c - \omega_d)^2 + \frac{1}{\Phi_d^2}(\Phi_m - \Phi_d)^2\right]$$
- **Iterative solution:** $$\rho_{i+1} = \rho_i - H^{-1} J'(\rho_i)$$

$\rho$: Vector of the controller parameters

$i$: iteration number

$\omega_{c(d)}$: Measured (desired) crossover frequencies

$\Phi_{m(d)}$: Measured (desired) phase margin

- Gradient: $$J'(\rho) = \frac{1}{\omega_d^2}(\omega_c - \omega_d)\frac{\partial \omega_c}{\partial \rho} + \frac{1}{\Phi_d^2}(\Phi_m - \Phi_d)\Phi'_m$$

- Hessian: $$\begin{aligned} H(\rho) &= J''(\rho) = \frac{1}{\omega_d^2}\frac{\partial \omega_c}{\partial \rho}\left(\frac{\partial \omega_c}{\partial \rho}\right)^T + \frac{1}{\Phi_d^2}\Phi'_m(\Phi'_m)^T \\ &\quad + \frac{1}{\omega_d^2}(\omega_c - \omega_d)\frac{\partial^2 \omega_c}{\partial \rho^2} + \frac{1}{\Phi_d^2}(\Phi_m - \Phi_d)\Phi''_m \\ &\approx \frac{1}{\omega_d^2}\frac{\partial \omega_c}{\partial \rho}\left(\frac{\partial \omega_c}{\partial \rho}\right)^T + \frac{1}{\Phi_d^2}\Phi'_m(\Phi'_m)^T \end{aligned}$$

# Procedure for phase margin adjustment (2)

**Approximation of $\frac{\partial \omega_c}{\partial \rho}$:**

$$\frac{d \ln |L(j\omega_c)|}{d\rho} = 0 \qquad \Rightarrow \frac{\partial \ln |L(j\omega_c)|}{\partial \rho} + \left.\frac{\partial \ln |L(j\omega)|}{\partial \omega}\right|_{\omega_c} \frac{\partial \omega_c}{\partial \rho} = 0$$

$$\Rightarrow \frac{\partial \omega_c}{\partial \rho} = -\frac{\partial \ln |L(j\omega_c)|}{\partial \rho} \boxed{\left[ \left.\frac{\partial \ln |L(j\omega)|}{\partial \omega}\right|_{\omega_c} \right]^{-1}}$$

**Approximation of $\Phi'_m$:**

$$\Phi'_m = \frac{\partial \Phi_m}{\partial \rho} + \left.\frac{\partial \Phi_m}{\partial \omega}\right|_{\omega_c} \frac{\partial \omega_c}{\partial \rho} = \frac{\partial \angle L(j\omega_c)}{\partial \rho} + \left.\frac{\partial \angle L(j\omega)}{\partial \omega}\right|_{\omega_c} \frac{\partial \omega_c}{\partial \rho}$$

$$\left.\frac{\partial \angle L(j\omega)}{\partial \omega}\right|_{\omega_c} = \left.\frac{\partial \angle K(j\omega)}{\partial \omega}\right|_{\omega_c} + \boxed{\left.\frac{\partial \angle G(j\omega)}{\partial \omega}\right|_{\omega_c}}$$

# Bode's integrals: Derivative of amplitude

- For a stable minimum-phase transfer function $G(s)$, the phase of the system at any $\omega_0$ is given by:

$$\angle G(j\omega_0) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{d\ln|G(j\omega)|}{d\nu} \ln\coth\frac{|\nu|}{2} d\nu$$

where $\nu = \ln\frac{\omega}{\omega_0}$

- Since: $\ln\coth\frac{|\nu|}{2}$ decreases rapildy as $\omega$ deviates from $\omega_0$

The slope of the Bode plot is almost constant in the neighborhood of $\omega_0$

$$\angle G(j\omega_0) \approx \frac{\pi}{2} \left.\frac{d\ln|G(j\omega)|}{d\nu}\right|_{\omega_0}$$

$$\Rightarrow \quad \boxed{s_a(\omega_0) = \left.\frac{d\ln|G(j\omega)|}{d\nu}\right|_{\omega_0} \approx \frac{2}{\pi}\angle G(j\omega_0)}$$

# Bode's integrals: Derivative of phase

- For a stable minimum-phase transfer function $G(s)$, the logarithm of the system amplitude at any $\omega_0$ is given by:

$$\ln|G(j\omega_0)| = \ln|K_g| - \frac{\omega_0}{\pi}\int_{-\infty}^{+\infty} \frac{d(\angle G(j\omega)/\omega)}{d\nu} \ln\coth\frac{|\nu|}{2} d\nu$$

where $\nu = \ln\frac{\omega}{\omega_0}$, and $K_g$ is the static gain of the plant

- In the same way

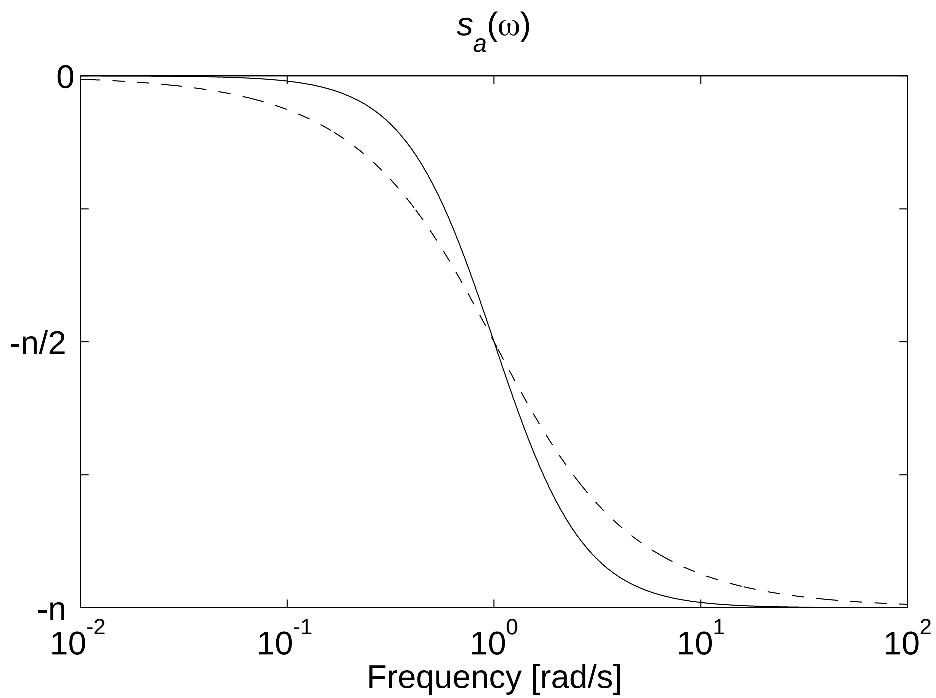
$$\ln|G(j\omega_0)| \approx \ln|K_g| - \frac{\omega_0}{\pi}\left.\frac{d(\angle G(j\omega)/\omega)}{d\nu}\right|_{\omega_0}\frac{\pi^2}{2}$$

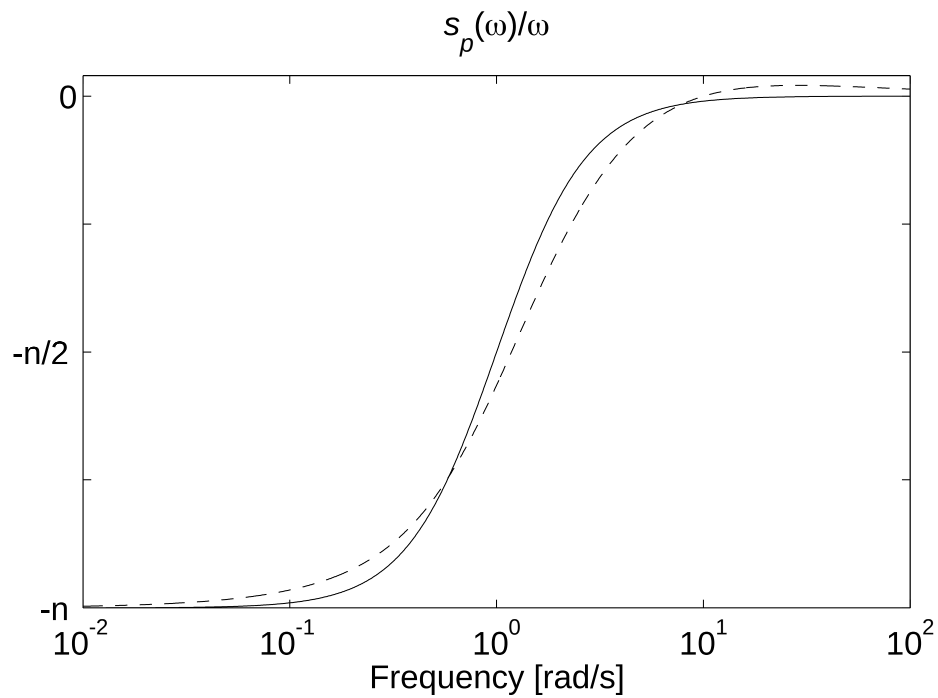
$$\Rightarrow \quad \boxed{s_p(\omega_0) = \omega_0 \left.\frac{d\angle G(j\omega)}{d\omega}\right|_{\omega_0} \approx \angle G(j\omega_0) + \frac{2}{\pi}[\ln|K_g| - \ln|G(j\omega_0)|]}$$

# Bode's Integrals (Precision of estimates)

Comparison of true $S_a(\omega)$ and $S_p(\omega)/\omega$ and estimates based on Bode's integrals for:

$$G(s) = \frac{1}{(s+1)^n}$$




True values (solid line) and estimates (dashed line)

# Procedure for phase and gain margin adjustment (1)

- **Criterion:** $J(\rho) = \frac{1}{2}[\frac{1}{\omega_d^2}(\omega_c - \omega_d)^2 + \frac{1}{\Phi_d^2}(\Phi_m - \Phi_d)^2 + \frac{1}{K_d^2}(K_u - K_d)^2]$
- **Iterative solution:** $\rho_{i+1} = \rho_i - H^{-1}J'(\rho_i)$

$\omega_{c(d)}$: Measured (desired) crossover frequencies

$\Phi_{m(d)}$: Measured (desired) phase margin

$K_{u(d)}$: Inverse of measured (desired) gain margin

$\rho$: Vector of the controller parameters

$i$: iteration number

- Gradient: $J'(\rho) = \frac{1}{\omega_d^2}(\omega_c - \omega_d)\frac{\partial \omega_c}{\partial \rho} + \frac{1}{\Phi_d^2}(\Phi_m - \Phi_d)\Phi'_m + \frac{1}{K_d^2}(K_u - K_d)K'_u$

- Hessian: $H(\rho) = J''(\rho)$

$$\approx \frac{1}{\omega_d^2}\frac{\partial \omega_c}{\partial \rho}(\frac{\partial \omega_c}{\partial \rho})^T + \frac{1}{\Phi_d^2}\Phi'_m(\Phi'_m)^T + \frac{1}{K_d^2}K'_u(K'_u)^T$$

# Procedure for phase and gain margin adjustment (2)

**Approximation of** $K_u'$:

$$K_u' = \frac{\partial |L(j\omega_u)|}{\partial \rho} + \left.\frac{\partial |L(j\omega)|}{\partial \omega}\right|_{\omega_u} \frac{\partial \omega_u}{\partial \rho}$$

$$\left.\frac{\partial |L(j\omega)|}{\partial \omega}\right|_{\omega_u} = |L(j\omega_u)| \left.\frac{\partial \ln |L(j\omega)|}{\partial \omega}\right|_{\omega_u} \approx K_u \frac{2\angle L(j\omega_u)}{\pi \omega_u} = -\frac{2K_u}{\omega_u}$$

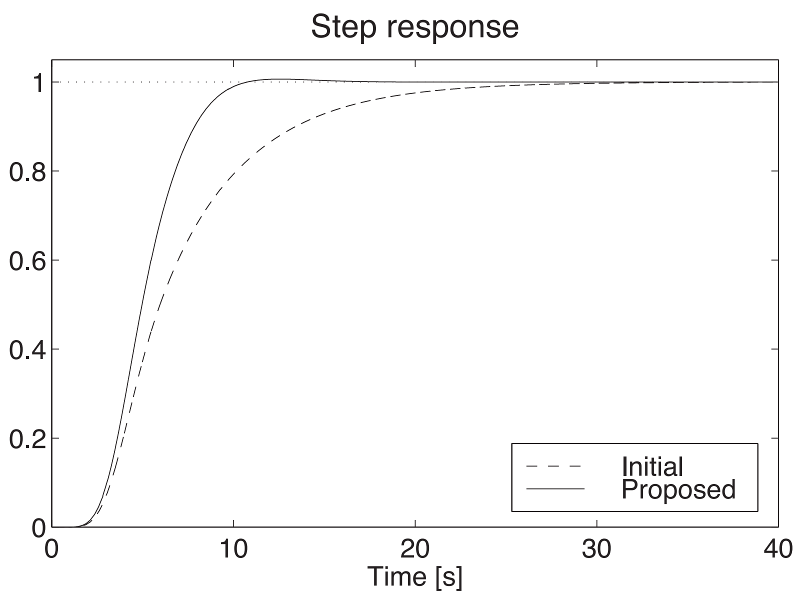
**Approximation of** $\frac{\partial \omega_u}{\partial \rho}$**:**

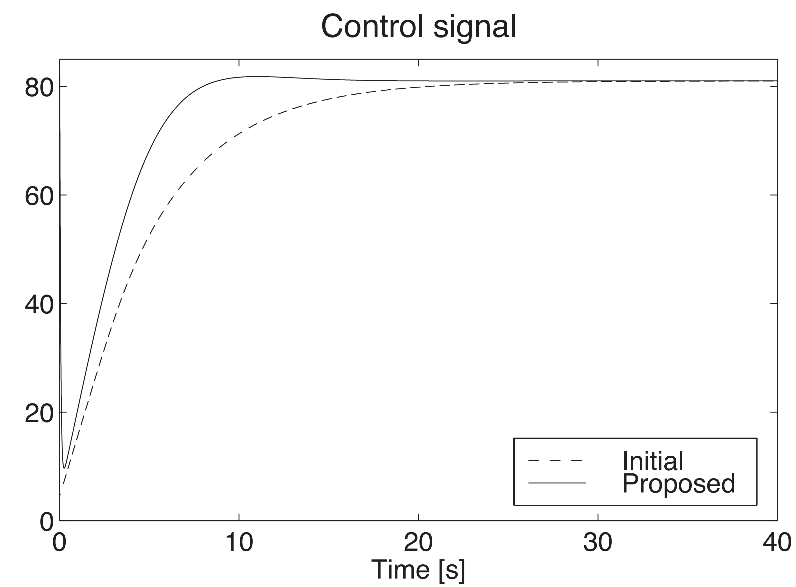
$$\frac{\partial \angle L(j\omega_u)}{\partial \rho} + \left.\frac{\partial \angle L(j\omega)}{\partial \omega}\right|_{\omega_u} \frac{\partial \omega_u}{\partial \rho} = 0 \quad \Rightarrow \frac{\partial \omega_u}{\partial \rho} = -\frac{\partial \angle L(j\omega_u)}{\partial \rho} \left( \left.\frac{\partial \angle L(j\omega)}{\partial \omega}\right|_{\omega_u} \right)^{-1}$$

$$\left.\frac{\partial \angle L(j\omega)}{\partial \omega}\right|_{\omega_u} = \left.\frac{\partial \angle K(j\omega)}{\partial \omega}\right|_{\omega_u} + \left.\frac{\partial \angle G(j\omega)}{\partial \omega}\right|_{\omega_u} \approx \left.\frac{\partial \angle K(j\omega)}{\partial \omega}\right|_{\omega_u} + \frac{s_p(\omega_u)}{\omega_u}$$

# Simulation results

- Model: $G(s) = \frac{e^{-0.3s}}{(s^2+2s+3)^3(s+3)}$
- Initial Controller: $K(s) = 4.5\,(1 + \frac{1}{0.41s} + 0.033s)$ ($\kappa$-$\tau$ tuning rule)
- Measured performances: $\Phi_m = 78.5^\circ$, $\omega_c = 0.139$ rad/s, $K_u = \frac{1}{4.39}$
- Specifications: $\Phi_d = 70^\circ$, $\omega_d = 0.2$ rad/s, $K_d = \frac{1}{3}$
- New controller (1 iteration): $K(s) = 4.93\,(1 + \frac{1}{0.316s} + 0.125s)$
- Obtained performance: $\Phi_m = 66^\circ$, $\omega_c = 0.199$ rad/s, $K_m = \frac{1}{2.97}$

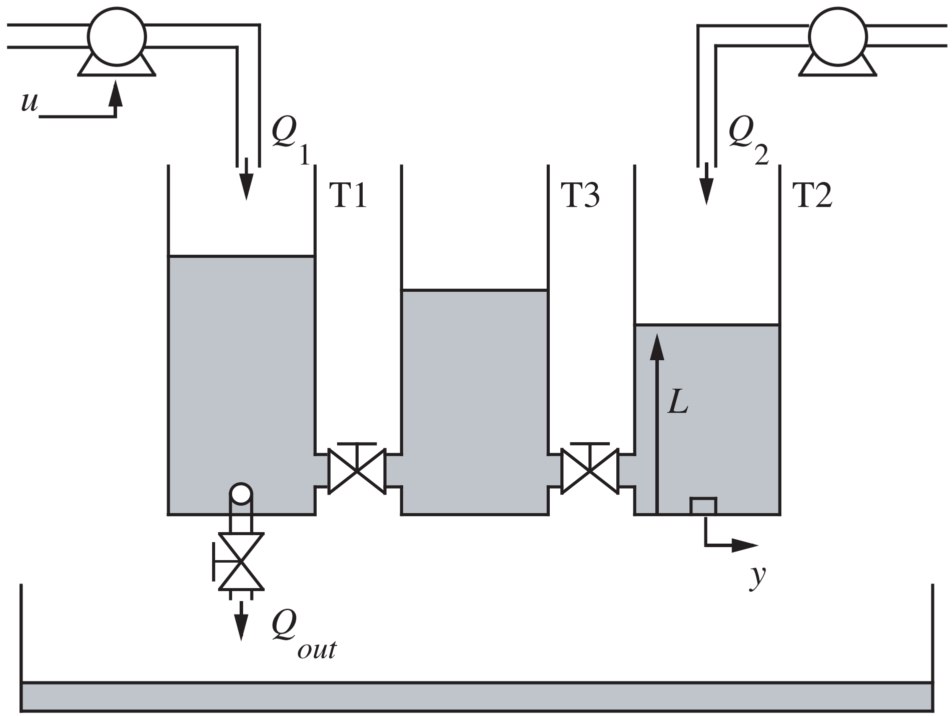


# Experimental Results (1)

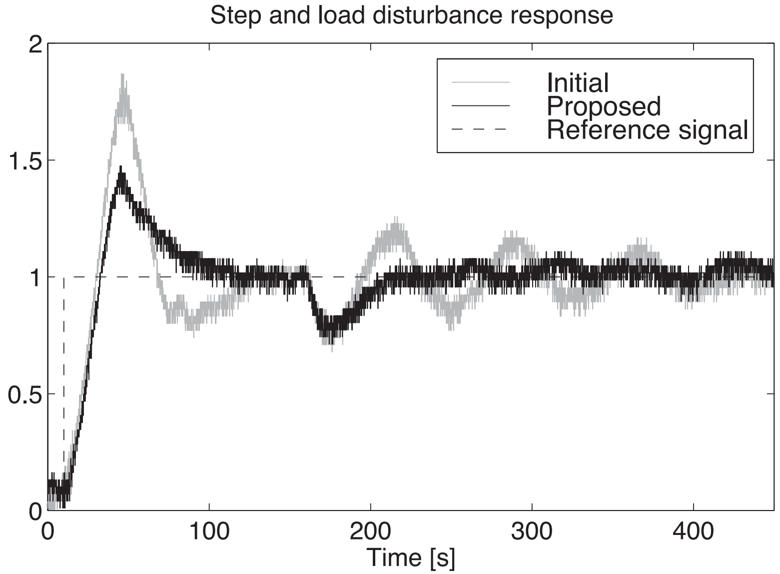
## Three-tank system

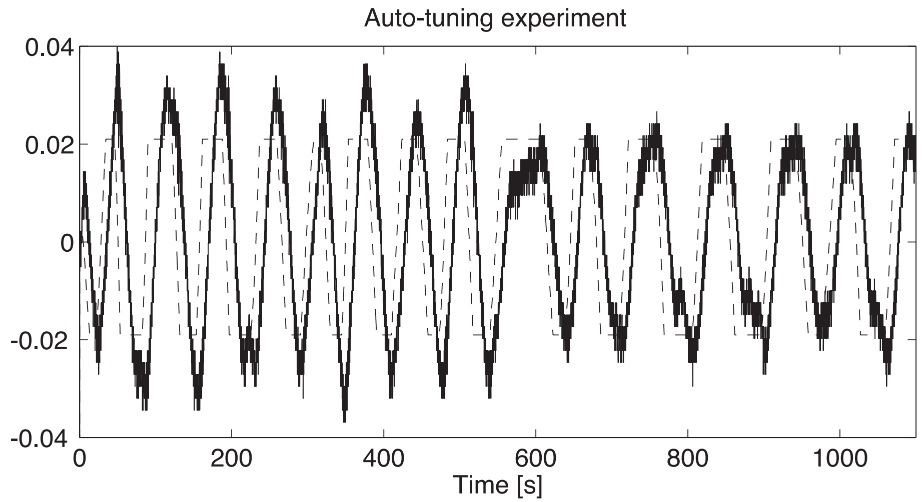
- T1, T2 and T3: interconnected cylinders
- $L$: water level of T2
- $Q_1$, $Q_2$ and $Q_{out}$: flow rates
- Input $u$: controls the flow rate $Q_1$
- Output $y$: proportional to the level $L$
- Disturbance: flow rate $Q_2$



# Experimental Results (2)

- Initial Controller: $K(s) = 29.3\,(1 + \frac{1}{20.84s} + 4.72s)$ ($\kappa$-$\tau$ tuning rule)
- Measured performances: $\Phi_m = 64^\circ$ $\omega_c = 0.097$ rad/s
- Specifications: $\Phi_d = 80^\circ$, $\omega_d = 0.08$ rad/s, $T_i = 4T_d$
- New controller (1 iteration): $K(s) = 20.4\,(1 + \frac{1}{31.5s} + 7.88s)$
- Obtained performance: $\Phi_m = 86.2^\circ$, $\omega_c = 0.085$ rad/s



# Concluding Remarks

- Experiment-based tuning methods are appropriate for restricted complexity controller design. No model is required or the model is not directly involved in controller tuning.

- Two new approaches for iterative controller tuning are proposed:

**Correlation approach**

- Making the closed-loop output error *uncorrelated* with the reference signal, can be used as an *objective* for controller tuning.

- Parametric convergence of the controller is not affected by noise and modeling errors.

**Frequency approach**

- Takes advantage of the Bode's integrals to estimate the gradient of a frequency criterion

- Converges in a few iterations to the minimum of the frequency criterion

- No parametric model is required

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