A Unified Approach to Model Estimation and Controller Reduction (Duality and Coherence)

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Abstract

Closed loop output error identification algorithms [7, 9]and recently developed algorithms for direct closed loop estimation of reduced order controllers [8] despite their diversity have in fact a unifying basic ground which will be enhanced.

In this paper it is shown that the plant model identification in closed loop using closed loop output error identification algorithms and the direct estimation in closed loop of a reduced order controller feature a duality character. Basic schemes, algorithms and properties of the algorithms can be directly obtained by interchanging the plant model and the controller. Additional schemes and algorithms allowing a full coverage of the various possible identification and reduction criteria are given.

The paper also will explore the coherence aspects in using closed loop plant model identification and direct estimation in closed loop of reduced order controllers. The following problem will be addressed: what closed loop plant model identification should be used when a criterion for controller reduction is given?

Keywords: System identification, closed loop identification, controller reduction.

1 Introduction

Closed loop output error (CLOE) identification algorithms [7, 9] allow an approximate design model to be identified which features a high accuracy in the critical regions for control design. Effectively the frequency distribution of the asymptotic error (bias) between the estimated and the true model is heavily weighted by the magnitude of the sensitivity function of the true system which explains why this desired property is obtained.

In fact the algorithms search for a plant model which minimizes a 2-norm error between the true closed loop transfer function and the simulated closed loop transfer function containing the final estimated model.

Asymptotic frequency bias error distribution can be obtained indicating clearly that the bias is small where some sensitivity functions are high and in addition this bias error is not asymptotically affected by the measurement noise [5, 3].

Several configurations for the closed loop output error identification can be established. While the parameter adaptation algorithm (PAA) is basically the same, the identification criterion and the asymptotic properties of the estimated model will be different.

It is well known that controller order reduction should aim to preserve the required closed loop properties as far as possible [1]. It has been shown in the paper [8] that this can be performed by estimation in closed loop of reduced order controllers either using simulated data or real data (which is a unique feature of this approach with respect to other approaches to direct controller reduction). To proceed one needs to know the nominal controller and to have an estimated plant model (either the model used for design or a model identified on site in open or closed loop). The algorithms developed for the estimation of reduced order controllers minimize a 2-norm of the difference between the nominal sensitivity function and the one obtained with the reduced order controller (i.e. they try to preserve the nominal closed loop properties). The algorithms have the property that the error between the reduced order controller and the nominal controller is small at the frequencies where some sensitivity functions are high.

Several configurations can be considered each corresponding to specific controller reduction criterion. However the PAA is in fact basically the same.

The first objective of the paper is to give a unified ground for the various schemes proposed for closed loop output error identification and direct closed loop estimation of reduced order controllers by observing that in each scheme the algorithm searches for the approximation of a specific closed loop sensitivity function. This investigation led to the observation that there were missed schemes and algorithms which are presented here for the first time.

The second objective of the paper is to show that these two problems (model estimation and controller reduction) are dual and that the effective algorithms, the stability analysis and the asymptotic properties can be obtained by a direct substitution i.e.:

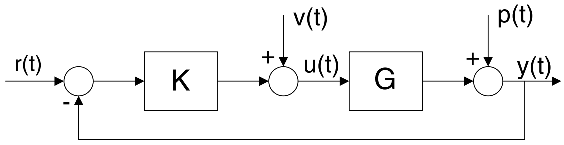

Figure 1: The closed-loop system

estimated plant model $\longrightarrow$ estimated controller;

nominal controller $\longrightarrow$ plant model.

The only differences will occur at the effective implementation of the algorithms since the controller has a direct transfer term while the plant model has at least one step delay.

The third objective of the paper is to explore the coherent use of closed loop plant model identification in direct controller order reduction. The following problem will be addressed: what closed loop plant model identification should be used when a criterion for direct controller reduction is given?

The paper is organized as follows. Section 2 will specify the notations. Section 3 will survey the various configurations for closed loop plant model identification and their properties. Section 4 will explore the various configurations for direct estimation in closed loop of reduced order controllers and their properties. Section 5 will give the basic algorithms for closed loop identification and their frequency bias distributions. The algorithms for controller order reduction will be derived from the closed loop identification algorithms via duality in Section 6. Section 7 will discuss the coherency between closed loop identification and direct controller reduction. A simulation example in Section 8 illustrates the discussed properties of the algorithms. Finally some concluding remarks will be given.

2 Notations

Consider the system shown in Fig. 1, where the plant model is given by:

$$G(z^{-1}) = \frac{z^{-d}B(z^{-1})}{A(z^{-1})} \tag{1}$$

and

$$\begin{aligned} A(z^{-1}) &= 1 + a_1 z^{-1} + \cdots + a_{n_A} z^{-n_A} \\ &= 1 + z^{-1} A^*(z^{-1}) \qquad (2) \\ B(z^{-1}) &= b_1 z^{-1} + \cdots + b_{n_B} z^{-n_B} \\ &= z^{-1} B^*(z^{-1}) \qquad (3) \end{aligned}$$

In Fig. 1, $r(t)$ is generically the external excitation (which can be applied also at other points). $v(t)$ and $p(t)$ correspond respectively to input and output disturances. It is supposed that the plant model is an exact representation of the true system. The plant model is also characterized by a parameter vector:

$$\theta^{*T} = [a_1 \cdots a_{n_A}, b_1 \cdots b_{n_B}] \tag{4}$$

The nominal controller is given by:

$$K(z^{-1}) = \frac{R(z^{-1})}{S(z^{-1})} \tag{5}$$

where

$$\begin{aligned} R(z^{-1}) &= r_0 + r_1 z^{-1} + \cdots + r_{n_R} z^{-n_R} && (6) \\ S(z^{-1}) &= 1 + s_1 z^{-1} + \cdots + s_{n_S} z^{-n_S} \\ &= 1 + z^{-1} S^*(z^{-1}) && (7) \end{aligned}$$

The nominal controller is a high-order controller computed on the basis of an available nominal plant model (and not in the basis of the true plant model (1)) that meets the control specifications for the nominal closed-loop system. The controller is also characterized by a parameter vector:

$$\theta_c^{*T} = [r_0, r_1 \cdots r_{n_R}, s_1 \cdots s_{n_S}] \tag{8}$$

The following sensitivity functions are defined:

- $S_{yp}(z^{-1}) = \dfrac{1}{1+KG} = \dfrac{A(z^{-1})S(z^{-1})}{P(z^{-1})}$;
- $S_{up}(z^{-1}) = \dfrac{-K}{1+KG} = \dfrac{-A(z^{-1})R(z^{-1})}{P(z^{-1})}$;
- $S_{yv}(z^{-1}) = \dfrac{G}{1+KG} = \dfrac{z^{-d}B(z^{-1})S(z^{-1})}{P(z^{-1})}$;
- $S_{yr}(z^{-1}) = \dfrac{KG}{1+KG} = \dfrac{z^{-d}B(z^{-1})R(z^{-1})}{P(z^{-1})}$.

where

$$P(z^{-1}) = A(z^{-1})S(z^{-1}) + z^{-d}B(z^{-1})R(z^{-1}) \tag{9}$$

is the closed loop characteristic polynomial. Note that the first subscript letter defines the output point and the second letter the input point for the evaluation of the sensitivity functions.

The system of Fig.1 will be denoted the "true closed loop system". Throughout the paper we will consider feedback systems which will use either an estimation of G (denoted $\hat{G}$) or a reduced order estimation of K (denoted $\hat{K}$). The corresponding sensitivity functions will be denoted as follows:

- S_{xy} - Sensitivity function of the true closed loop system (K, G).
- $\hat{S}_{xy}$ - Sensitivity function of the nominal simulated closed loop system (nominal controller K + estimated plant model $\hat{G}$).
- $\hat{\hat{S}}_{xy}$ - Sensitivity function of the simulated closed loop system using a reduced order controller (reduced controller $\hat{K}$ + estimated plant model $\hat{G}$).

Similar notations are used for $P(z^{-1})$: $\hat{P}(z^{-1})$ when using K and $\hat{G}$, $\hat{\hat{P}}(z^{-1})$ when using $\hat{K}$ and $\hat{G}$.

3 Closed loop identification schemes

Figures 2a, 2b and 2c summarize the various configurations belonging to the closed loop identification algorithms.

We will indicate next the identification criterion corresponding to each configuration.

a) Closed loop output error with external excitation added to the controller input (Fig. 2a): The identification criterion in this case is:

$$\hat{\theta}^* = \arg\min_{\theta} \left\| S_{yr} - \hat{S}_{yr} \right\|_2 = \arg\min_{\theta} \left\| S_{yp} - \hat{S}_{yp} \right\|_2 \quad (10)$$

$$= \arg\min_{\theta} \left\| S_{yp}(G - \hat{G})\hat{S}_{up} \right\|_2 \quad (11)$$

b) Closed loop output error with external excitation added to the plant input (Fig. 2b): The identification criterion in this case is:

$$\hat{\theta}^* = \arg\min_{\theta} \left\| S_{yv} - \hat{S}_{yv} \right\|_2 = \arg\min_{\theta} \left\| S_{yp}(G - \hat{G})\hat{S}_{yp} \right\|_2 \quad (12)$$

c) Closed loop input error with external excitation added to the controller input (Fig. 2c): The identification criterion in this case is:

$$\hat{\theta}^* = \arg\min_{\theta} \left\| S_{up} - \hat{S}_{up} \right\|_2 = \arg\min_{\theta} \left\| S_{up}(G - \hat{G})\hat{S}_{up} \right\|_2 \quad (13)$$

As one can see these three configurations cover all the possible closed loop identification criteria (one can match all the four sensitivity functions). Note that one can consider a fourth configuration corresponding to a closed loop input error with external excitation added to the plant input. However this configuration corresponds to the one of Fig. 2a where in the upper part (the true system) the place of K and G is interchanged. This, however, will not change the identification criterion for SISO systems.

In short for the configuration of Fig. 2a, 2b and 2c, if $r(t)$ is a discrete time white noise (for example a PRBS which is a good approximation) the algorithm will search for the best $\hat{G}$ which will minimize the 2-norm between

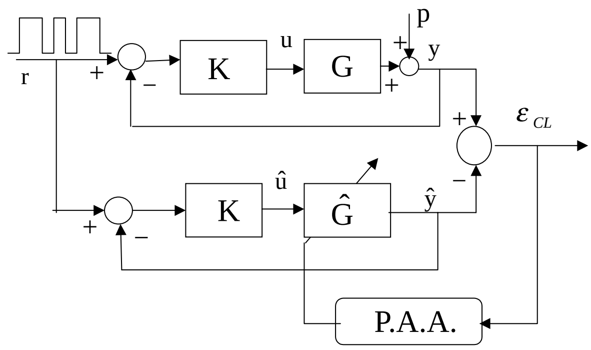

Figure 2a: Closed loop output error (CLOE) (external excitation added to the controller input)

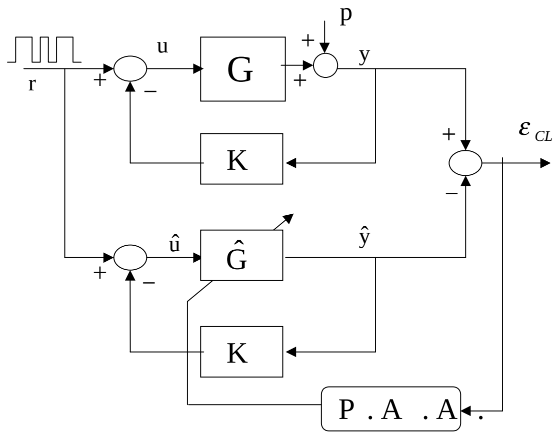

Figure 2b: Closed loop output error (CLOE) (external excitation added to the plant input)

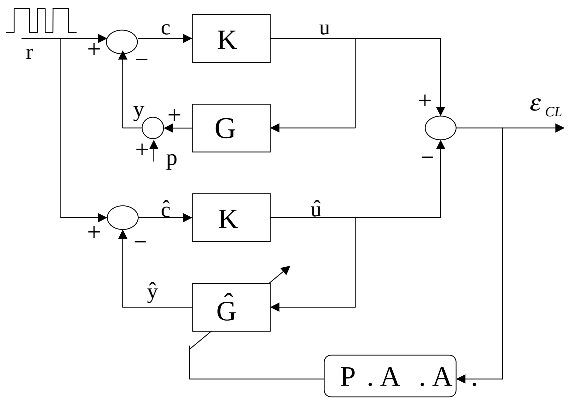

Figure 2c: Closed loop input error (CLIE) (external excitation added to the controller input)

the various sensitivity functions. In addition the differences between $\hat{G}$ and G will be minimized in the frequency regions where the sensitivity functions have large values.

The first practical consequence is that the choice of one scheme or another is related to the main performance objective of the controller design (since the model identified in closed loop in the context of this paper will be used for controller reduction of a nominal controller designed to match a desired sensitivity function). For tracking and output disturbance rejection it is preferable to use the scheme of Fig. 2a. For output rejection of a disturbance entered at the input of the plant it is preferred to use the scheme of Fig. 2b. For minimization of the effect of the output disturbance on the controller input it is more appropriate to use the scheme of Fig. 2c.

The second practical consequence of these properties is that the choice of one or other configuration should be done in relation with the type of uncertainty which is logical to be considered for controller design in a specific application. The reason is that the robust stability conditions are expressed in terms of specific sensitivity function for a given type of uncertainties and one would like to best approximate this sensitivity function.

4 Direct estimation of reduced order controllers

We will consider that a model of the plant is available (denoted by $\hat{G}$). This model can be the plant model used for controller design or an identified model (in open loop or in closed loop) which passed the validation tests. We will assume of course that the nominal controller of orders n_R and n_S is known (denoted by K).

Figures 3a, 3b and 3c summarize the various configuration for direct estimation in closed loop of reduced order controllers. We will indicate next the controller reduction criterion for each configuration.

a) Closed loop input matching with external excitation added to the plant input: The controller reduction criterion in this case is:

$$\hat{\theta}_c^* = \arg\min_{\theta_c} \left\| \hat{S}_{yr} - \hat{\hat{S}}_{yr} \right\|_2 = \arg\min_{\theta_c} \left\| \hat{S}_{yp} - \hat{\hat{S}}_{yp} \right\|_2 \tag{14}$$

b) Closed loop output matching with external excitation added to the plant input: The controller reduction criterion in this case is:

$$\hat{\theta}_c^* = \arg\min_{\theta_c} \left\| \hat{S}_{yv} - \hat{\hat{S}}_{yv} \right\|_2 \tag{15}$$

c) Closed loop input matching with external excitation added to the controller input: The controller reduction criterion in this case is:

$$\hat{\theta}_c^* = \arg\min_{\theta_c} \left\| \hat{S}_{up} - \hat{\hat{S}}_{up} \right\|_2 \tag{16}$$

These three configurations cover all possible controller reduction criteria based on preservation of the closed loop properties. Note that one can consider

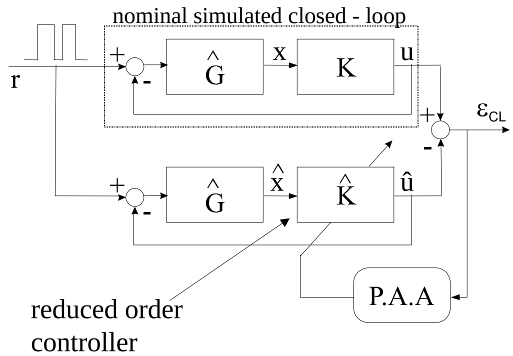

Figure 3a: Closed loop input matching (CLIM) (external excitation added to the plant input)

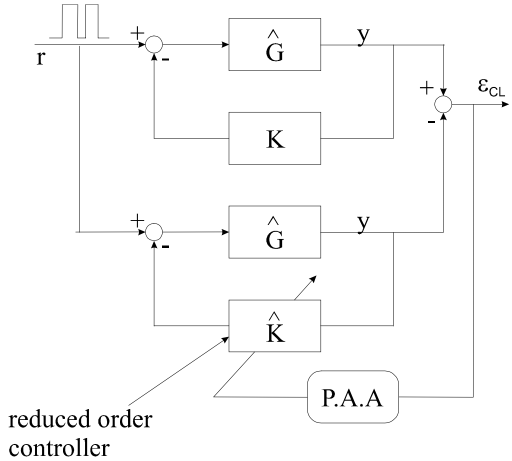

Figure 3b: Closed loop output matching (CLOM) (external excitation added to the plant input)

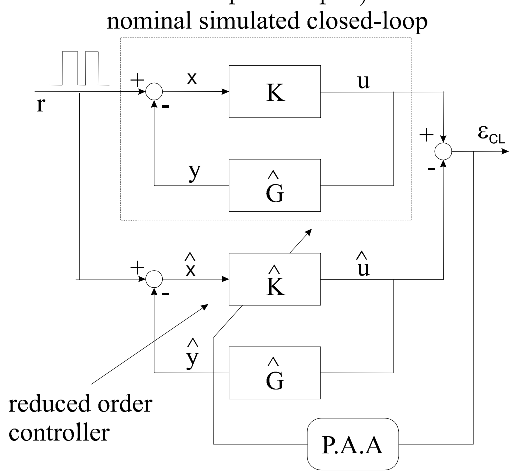

Figure 3c: Closed loop input matching (CLIM) (external excitation added to the controller input)

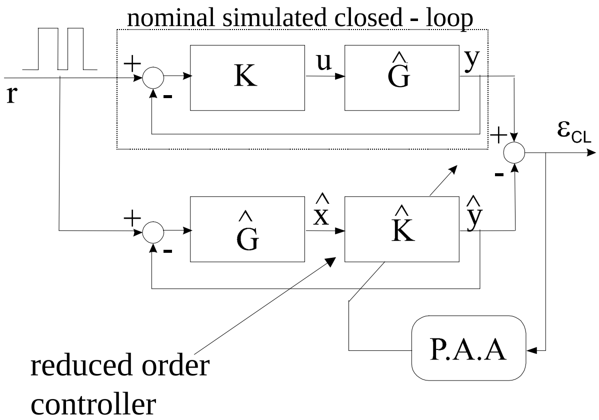

Figure 4: Closed loop output matching with external excitation added to the controller input

a fourth configuration corresponding to a closed loop output matching scheme with external excitation added to the controller input [6] (see Fig. 4). However this configuration corresponds to the one of Fig 3a where in the upper part (the nominal simulated system) the place of K and $\hat{G}$ is reverted.

When $r(t)$ is a discrete white noise (for example a PBRS) the algorithms will search for the best controller $\hat{K}$ which will minimize the 2-norm of the controller reduction criterion. Like in closed loop identification the differences between K and $\hat{K}$ will be small at the frequencies where the sensitivity functions have large values.

The above schemes can be used also with real data [8]. In this case it can be shown that the noise will not affect the minimization procedure.

5 Closed loop identification algorithms

The output of the plant is given by:

$$y(t+1) = -A^* y(t) + B^* u(t-d) + Ap(t+1) = \theta^T \psi(t) + Ap(t+1) \qquad (17)$$

where

$$\psi^T(t) = [-y(t) \ldots - y(t-n_A+1), u(t-d) \ldots u(t-d-n_B+1)] \qquad (18)$$

$$\theta^T = [a_1, \ldots, a_{n_A}, b_1, \ldots, b_{n_B}] \qquad (19)$$

$$u(t) = -\frac{R}{S}\, y(t) + r_u(t) \qquad (20)$$

and $r_u(t) = r(t)$ in the scheme of Fig. 2b and $r_u(t) = \frac{R}{S}\, r(t)$ in the scheme of Fig. 2a and 2c.

The output of the closed loop adjustable predictor is given by:

a priori:

$$\hat{y}^\circ(t+1) = -\hat{A}^*(t,q^{-1})\hat{y}(t) + \hat{B}^*(t,q^{-1})\hat{u}(t-d) = \hat{\theta}^T(t)\phi(t) \quad (21)$$

$$\hat{u}(t) = -\frac{R(q^{-1})}{S(q^{-1})}\,\hat{y}(t) + r_u(t) \quad (22)$$

a posteriori:

$$\hat{y}(t+1) = \hat{\theta}^T(t+1)\phi(t) \quad (23)$$

where

$$\hat{\theta}^T(t) = [\hat{a}_1, \ldots, \hat{a}_{n_{\hat{A}}}, \hat{b}_1, \ldots, \hat{b}_{n_{\hat{B}}}] \quad (24)$$

$$\phi^T(t) = [-\hat{y}(t) \ldots - \hat{y}(t - n_{\hat{A}} + 1), \hat{u}(t-d) \ldots \hat{u}(t - d - n_{\hat{B}} + 1)] \quad (25)$$

5.1 Closed loop output error algorithms (CLOE)

The closed loop output error is defined as:
a priori:

$$\varepsilon^\circ_{CL}(t+1) = y(t+1) - \hat{y}^\circ(t+1)$$

a posteriori:

$$\varepsilon_{CL}(t+1) = y(t+1) - \hat{y}(t+1)$$

and the parameter adaptation algorithm (PAA) is given by:

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t)\Phi(t)\varepsilon_{CL}(t+1) \quad (26)$$

$$F^{-1}(t+1) = \lambda_1(t)F^{-1}(t) + \lambda_2(t)\Phi(t)\Phi^T(t) \quad (27)$$

$$0 < \lambda_1(t) \leq 1;\ 0 \leq \lambda_2(t) < 2; \quad F(0) > 0$$

$$\varepsilon_{CL}(t+1) = \frac{\varepsilon^\circ_{CL}(t+1)}{1 + \Phi^T(t)F(t)\Phi(t)} \quad (28)$$

Specific algorithms are obtained by an appropriate choice of the observation vector $\Phi(t)$ as follows:

CLOE	$\Phi(t) = \phi(t)$
F-CLOE (Filtered CLOE)	$\Phi(t) = \frac{S(q^{-1})}{\hat{P}(q^{-1})}\,\phi(t)$
AF-CLOE (Adaptive Filtered CLOE)	$\Phi(t) = \frac{S(q^{-1})}{\hat{P}(t,q^{-1})}\,\phi(t)$

where:

$$\hat{P}(q^{-1}) = \hat{A}(q^{-1})S(q^{-1}) + q^{-d}\hat{B}(q^{-1})R(q^{-1})$$

$$\hat{P}(t,q^{-1}) = \hat{A}(t,q^{-1})S(q^{-1}) + q^{-d}\hat{B}(t,q^{-1})R(q^{-1})$$

$\hat{A}(q^{-1})$ and $\hat{B}(q^{-1})$ correspond to an *a priori* estimation of $\hat{G}$, while $\hat{A}(t,q^{-1})$ and $\hat{B}(t,q^{-1})$ correspond to the current estimates of $\hat{G}$.

When $n_{\hat{A}} = n_A$, $n_{\hat{B}} = n_B$, then the closed loop output error goes to zero (in a deterministic environment: $p(t) \equiv 0$) and unbiased estimates are obtained in a stochastic environment (when $p(t)$ is independent with respect to $r_u(t)$ and of finite power) if:

$$H'(z^{-1}) = H(z^{-1}) - \frac{\lambda}{2} \quad ; \quad \max_t \lambda_2(t) \leq \lambda < 2 \tag{29}$$

is a strictly positive real transfer function, where:

$$H(z^{-1}) = \begin{cases} S/P & \text{for} \quad \text{CLOE} \\ \hat{P}/P & \text{for} \quad \text{F-CLOE} \\ 1 & \text{for} \quad \text{AF-CLOE} \end{cases} \tag{30}$$

(in the last case this is a local result) [7, 9].

When $n_{\hat{A}} < n_A$, $n_{\hat{B}} < n_B$, it can be shown that all the signals are bounded provided that [7]:

- $r_u(t)$ is norm bounded;
- It exists a reduced order model such that:

$$y(t+1) = \quad -\hat{A}^*(q^{-1})\, y(t) + \hat{B}(q^{-1})\, u(t-d) + \eta(t+1) \tag{31}$$

  where $\eta(t+1)$ is norm bounded for a norm bounded $r(t)$;
- The passivity condition (29) is satisfied.

5.2 Closed loop input error algorithms (CLIE)

As it has already mentioned the closed loop input error scheme when the excitation signal is added to the plant input is equivalent to the CLOE algorithm with external excitation added to the controller input (they have the same identification criterion). Therefore we consider only the CLIE algorithm with external excitation added to the controller input.

The closed loop input error is in fact the closed loop output error filtered by the controller transfer function (see Eqs. 20 and 22). Thus the same algorithms can be used with the difference that the prediction error ε_{CL} is replaced with a posteriori adaptation error defined as follows:

$$\begin{aligned} \nu(t+1) &= u(t+1) - \hat{u}(t+1) = -\frac{R(q^{-1})}{S(q^{-1})}(y(t+1) - \hat{y}(t+1)) \\ &= -\frac{R(q^{-1})}{S(q^{-1})}\varepsilon_{CL}(t+1) \end{aligned} \tag{32}$$

or

$$\begin{aligned} \nu(t+1) &= -S^*(q^{-1})\nu(t) - R(q^{-1})\varepsilon_{CL}(t+1) \\ &= -S^*(q^{-1})\nu(t) - r_0\varepsilon_{CL}(t+1) - \sum_{i=1}^{n_R} r_i \varepsilon_{CL}(t-i+1) \end{aligned} \tag{33}$$

Then the parameter adaptation algorithm will be given by:

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t)\Phi(t)\nu(t+1) \tag{34}$$

$$F^{-1}(t+1) = \lambda_1(t)F^{-1}(t) + \lambda_2(t)\Phi(t)\Phi(t)^T \tag{35}$$

$$0 < \lambda_1(t) \leq 1;\ 0 \leq \lambda_2(t) < 2$$

Since $\nu(t+1)$ is not available before computing $\hat{\theta}(t+1)$, it should be computed using the a priori adaptation error defined by:

$$\nu^\circ(t+1) = -S^*(q^{-1})\nu(t) - r_0\varepsilon^\circ_{CL}(t+1) - \sum_{i=1}^{n_R} r_i\varepsilon_{CL}(t-i+1) \tag{36}$$

Subtracting Eq. 36 from Eq. 33, one obtains:

$$\begin{aligned} \nu(t+1) - \nu^\circ(t+1) &= r_0\varepsilon_{CL}(t+1) - r_0\varepsilon^\circ_{CL}(t+1) \\ &= -r_0[\hat{y}^\circ(t+1) - \hat{y}(t+1)] \\ &= -r_0[\hat{\theta}(t+1) - \hat{\theta}(t)]^T\Phi(t) \\ &= -r_0\Phi^T(t)F(t)\Phi(t)\nu(t+1) \end{aligned} \tag{37}$$

Then the relation between the a posteriori and the a priori adaptation error is given by:

$$\nu(t+1) = \frac{\nu^\circ(t+1)}{1 + r_0\Phi^T(t)F(t)\Phi(t)} \tag{38}$$

From this equation, one observes that the a posteriori adaptation error $\nu(t+1)$ may be unbounded for the negative value of r_0 when the denominator approaches to zero. In order to fix this problem the a posteriori adaptation error can be modified as follows:

$$\nu(t+1) = \text{sign}(r_0)[u(t+1) - \hat{u}(t+1)] = \text{sign}(r_0)\frac{R(q^{-1}}{S(q^{-1})}\varepsilon_{CL}(t+1) \tag{39}$$

where

$$\text{sign}(r_0) = \begin{cases} 1 & \text{if } r_0 \geq 0 \\ -1 & \text{otherwise} \end{cases}$$

The new definition leads to the following relation between the a posteriori and the a priori adaptation error:

$$\nu(t+1) = \frac{\nu^\circ(t+1)}{1 + |r_0|\Phi^T(t)F(t)\Phi(t)} \tag{40}$$

Like the CLOE algorithms, specific algorithms can be obtained by an appropriate choice of the observation vector $\Phi(t)$ as follows:

CLIE	$\Phi(t) = \phi(t)$
F-CLIE (Filtered CLIE)	$\Phi(t) = \dfrac{R(q^{-1})}{\hat{P}(q^{-1})}\phi(t)$
AF-CLIE (Adaptive Filtered CLIE)	$\Phi(t) = \dfrac{R(q^{-1})}{\hat{P}(t,q^{-1})}\phi(t)$

When $n_{\hat{A}} = n_A$, $n_{\hat{B}} = n_B$, then the closed loop input error goes to zero (in a deterministic environment: $p(t) \equiv 0$) and unbiased estimates are obtained in a stochastic environment (when $p(t)$ is independent with respect to $r_u(t)$ and of finite power) if $H'(z^{-1})$ is a strictly positive real transfer function, where:

$$H(z^{-1}) = \begin{cases} R/P & \text{for} \quad \text{CLIE} \\ \hat{P}/P & \text{for} \quad \text{F-CLIE} \\ 1 & \text{for} \quad \text{AF-CLIE} \end{cases} \tag{41}$$

Remark: The closed loop input error can also be indirectly minimized by the CLOE algorithms using the excitation signal filtered by the controller transfer function. In this case the stability and convergence condition for the algorithm is the same as the CLOE algorithms. The problems may occur when $S(q^{-1})$ is unstable: On one hand the positive real condition on S/P is no longer satisfied (this problem can be solved using F-CLOE or AF-CLOE) and on the other hand, the reference signal filtered by R/S becomes unbounded and cannot be applied to the real system.

5.3 Asymptotic frequency bias distribution

The asymptotic frequency distribution of the bias when the estimated plant model does not belong to the model set for various configurations is given by [7, 5]:

a) Closed loop output error with external excitation added to the controller input

$$\begin{aligned} \hat{\theta}^* &= \arg\min_{\theta} \int_{-\pi}^{\pi} |S_{yp}|^2 \Big[|G - \hat{G}|^2 |\hat{S}_{up}|^2 \phi_r(\omega) + \phi_p(\omega) \Big] d\omega \\ &= \arg\min_{\theta} \int_{-\pi}^{\pi} \Big[|S_{yr} - \hat{S}_{yr}|^2 \phi_r(\omega) + |S_{yp}|^2 \phi_p(\omega) \Big] d\omega \end{aligned} \tag{42}$$

b) Closed loop output error with external excitation added to the plant input

$$\begin{aligned} \hat{\theta}^* &= \arg\min_{\theta} \int_{-\pi}^{\pi} |S_{yp}|^2 \Big[|G - \hat{G}|^2 |\hat{S}_{yp}|^2 \phi_r(\omega) + \phi_p(\omega) \Big] d\omega \\ &= \arg\min_{\theta} \int_{-\pi}^{\pi} \Big[|S_{yv} - \hat{S}_{yv}|^2 \phi_r(\omega) + |S_{yp}|^2 \phi_p(\omega) \Big] d\omega \end{aligned} \tag{43}$$

c) Closed loop input error with external excitation added to the controller input[1]

$$\begin{aligned} \hat{\theta}^* &= \arg\min_{\theta} \int_{-\pi}^{\pi} |S_{up}|^2 \Big[|G - \hat{G}|^2 |\hat{S}_{up}|^2 \phi_r(\omega) + \phi_p(\omega) \Big] d\omega \\ &= \arg\min_{\theta} \int_{-\pi}^{\pi} \Big[|S_{up} - \hat{S}_{up}|^2 \phi_r(\omega) + |S_{up}|^2 \phi_p(\omega) \Big] d\omega \end{aligned} \tag{44}$$

[1] These expressions are strictly valid when using AF-CLOE (case a and b) and AF-CLIE (case c). However CLOE and F-CLOE can be viewed as approximations of AF-CLOE and CLIE and F-CLIE can be viewed as approximations of AF-CLIE

These expressions show that if the external excitation $r(t)$ is a white noise (or for example a PRBS), the algorithms will search for the best $\hat{G}$ which will minimize the 2-norm of error between the sensitivity functions of true closed loop system and of the estimated closed loop system. Furthermore the noise will not affect the asymptotic parameter estimation because output error criteria are used here.

6 Algorithms for estimation of reduced order controllers

The algorithms for controller reduction are obtained from the closed loop identification algorithms by making dual type modifications in the scheme of Figures 2a, 2b and 2c summarized in Table 1. The resulting schemes are shown in Fig. 3a, 3b and 3c (Fig. 2a → Fig. 3a ; Fig. 2b → Fig. 3c and Fig. 2c → Fig. 3b).

Table 1: Duality between plant model identification in closed loop and direct estimation of reduced order controller in closed loop

Plant model identification in closed loop Fig.2a,2b,2c		Identification of reduced order controller in closed loop Fig.3a,3c,3b
controller (K)	$\longrightarrow$	available plant model ($\hat{G}$)
true plant model (G)	$\longrightarrow$	nominal controller (K)
estimated plant model ($\hat{G}$)	$\longrightarrow$	estimated (reduced order) controller ($\hat{K}$)
y, $\hat{y}$	$\longrightarrow$	u, $\hat{u}$
u, $\hat{u}$	$\longrightarrow$	x, $\hat{x}$

6.1 Closed loop input matching algorithms (CLIM)

The signal $x(t)$ is defined as: $x(t) = r(t) - y(t)$ in Fig.3c and $x(t) = \hat{G}[r(t) - u(t)]$ in Fig.3a. using these definitions one has:
a priori:

$$\hat{u}^{\circ}(t+1) = -\hat{S}^{*}(t, q^{-1})\hat{u}(t) + \hat{R}(t, q^{-1})\hat{x}(t) = \hat{\theta}_c^T(t)\phi_c(t) \quad (45)$$

a posteriori:

$$\hat{u}(t+1) = \hat{\theta}_c^T(t+1)\phi_c(t) \quad (46)$$

where

$$\hat{x}(t+1) = -\hat{A}^{*}(q^{-1})\hat{x}(t) - \hat{B}^{*}(q^{-1})\hat{u}(t-d) + A(q^{-1})r(t) \quad (47)$$

for the scheme of Fig. 3c, and

$$\hat{x}(t+1) = -\hat{A}^*(q^{-1})\hat{x}(t) - \hat{B}^*(q^{-1})\hat{u}(t-d) + \hat{B}^* r(t-d) \tag{48}$$

for the scheme of Fig. 3a, and

$$\hat{\theta}_c^T(t) = [\hat{s}_1(t), \ldots, \hat{s}_{n_{\hat{S}}}(t), \hat{r}_0(t), \ldots, \hat{r}_{n_{\hat{R}}}(t)] \tag{49}$$

$$\phi_c^T(t) = [-\hat{u}(t), \ldots, -\hat{u}(t-n_{\hat{S}}+1), \hat{x}(t+1), \ldots, \hat{x}(t-n_{\hat{R}}+1)] \tag{50}$$

The closed loop input error will be given by:
a priori:

$$\varepsilon^\circ_{CL}(t+1) = u(t+1) - \hat{u}^\circ(t+1)$$

a posteriori:

$$\varepsilon_{CL}(t+1) = u(t+1) - \hat{u}(t+1)$$

and the same PAA algorithm described in the Eqs. (26) through (28) can be used and the corresponding specific algorithms will be:

$$\begin{array}{ll} \text{CLIM} & \Phi(t) = \phi_c(t) \\ \text{F-CLIM} & \Phi(t) = \dfrac{\hat{A}(q^{-1})}{\hat{P}(q^{-1})}\phi_c(t) \end{array}$$

where:

$$\hat{P}(q^{-1}) = \hat{A}(q^{-1})S(q^{-1}) + q^{-d-1}\hat{B}(q^{-1})R(q^{-1})$$

is a known quantity and therefore there is no need to estimate this polynomial in line. The corresponding transfer functions involved in the passivity conditions for stability become:

$$H(z^{-1}) = \begin{cases} \dfrac{\hat{A}(q^{-1})}{\hat{P}(q^{-1})} & \text{for CLIM} \\ 1 & \text{for F-CLIM} \end{cases}$$

(since the exact polynomial of the nominal simulated closed loop is known).

6.2 Closed loop output matching algorithms (CLOM)

As it has already been noted the closed loop output matching algorithm when the excitation signal is added to the controller input (See Fig. 4) is equivalent to the CLIM with external excitation added to the plant input (see Fig. 3a). Therefore the same algorithm can be used for both cases which have the same criterion.

For the CLOM algorithm when the excitation signal is added to the plant input, two choices may be considered: The first one is to filter the excitation signal through $\hat{G}$ and use the CLIM algorithm corresponding to the scheme in Fig 3a. In this case, evidently, the stability and convergence condition of the algorithm is the same as CLIM algorithm. The second choice is to derive directly an algorithm for minimizing the closed loop output error. But this algorithm encounters a technical problem so that the convergence condition becomes $\hat{B}/\hat{P}$ which is never a positive real transfer function (because of at least one step time delay in $\hat{B}$). Although this problem can be fixed using a $d+1$ step ahead prediction error, the first choice still seems to be more appropriate because in many practical systems $\hat{A}$ is stable and the necessary condition for the positive realness of $\hat{A}/\hat{P}$ is satisfied.

6.3 Asymptotic frequency bias distribution

The crucial step is to examine the properties of the estimated reduced order controller. To do this we will directly use the expressions (42) and (44) in which we will take $\phi_p(\omega) = 0$ (no noise) and we will make the substitution indicated in Table 1. One gets for the asymptotic frequency distribution of the bias the following expressions:
a) Closed loop input matching with external excitation added to the plant input

$$\begin{aligned} \hat{\theta}_c^* &= \arg\min_{\theta_c} \int_{-\pi}^{\pi} |\hat{S}_{yp}|^2 |K - \hat{K}| |\hat{\hat{S}}_{yv}| \phi_r(\omega) d\omega \\ &= \arg\min_{\theta_c} \int_{-\pi}^{\pi} |\hat{S}_{yr} - \hat{\hat{S}}_{yr}|^2 \phi_r(\omega) d\omega \end{aligned} \quad (51)$$

b) Closed loop output matching with external excitation added to the plant input

$$\begin{aligned} \hat{\theta}_c^* &= \arg\min_{\theta_c} \int_{-\pi}^{\pi} |\hat{S}_{yp}|^2 |K - \hat{K}| |\hat{\hat{S}}_{yv}| \phi_r(\omega) d\omega \\ &= \arg\min_{\theta_c} \int_{-\pi}^{\pi} |\hat{S}_{yv} - \hat{\hat{S}}_{yv}|^2 \phi_r(\omega) d\omega \end{aligned} \quad (52)$$

c) Closed loop input matching with external excitation added to the controller input

$$\begin{aligned} \hat{\theta}_c^* &= \arg\min_{\theta_c} \int_{-\pi}^{\pi} |\hat{S}_{yp}|^2 |K - \hat{K}| |\hat{\hat{S}}_{yp}| \phi_r(\omega) d\omega \\ &= \arg\min_{\theta_c} \int_{-\pi}^{\pi} |\hat{S}_{up} - \hat{\hat{S}}_{up}|^2 \phi_r(\omega) d\omega \end{aligned} \quad (53)$$

When $r(t)$ is a discrete time white noise these expressions correspond exactly to the 2-norm expression which we would like to minimize in each case. It should be noted that the resulting controller is the best reduced order controller with respect to the nominal plant model $\hat{G}$ not to the true plant model G. However, it is the case for all of the controller reduction methods. A detailed analysis when one uses real data can be found in [8]. In this case it was shown that the noise term does not affect the minimization procedure.

7 Coherency between model identification and controller reduction in closed loop

The interesting problem to address is: What closed loop plant model identification should be used when a criterion for direct controller reduction is given? To answer this question we will make reference to the iterative identification in closed loop and controller re-design methodology [4, 10, 2]. The basic rule for improving performance is to use the same criterion for identification in closed loop and controller re-design. In our case controller re-design corresponds to the reduction of the nominal controller. Therefore examining the identification criterion for the various closed loop identification schemes and the controller reduction objectives, a coherency is observed between them as indicated in Table 2.

Table 2: Coherent controller reduction and identification in closed loop

Controller reduction criterion	Controller reduction scheme	Closed loop identification scheme
$\min \|\hat{S}_{yp} - \hat{\hat{S}}_{yp}\|$	CLOM with external excitation added to the controller input	CLOE with external excitation added to the controller input
or	or	or
$\min \|\hat{S}_{yr} - \hat{\hat{S}}_{yr}\|$	CLIM with external excitation added to the plant input	CLIE with external excitation added to the plant input
$\min \|\hat{S}_{yv} - \hat{\hat{S}}_{yv}\|$	CLOM with external excitation added to the plant input	CLOE with external excitation added to the plant input
$\min \|\hat{S}_{up} - \hat{\hat{S}}_{up}\|$	CLIM with external excitation added to the controller input	CLIE with external excitation added to the controller input

8 Simulation Example

The objective is to show the improvement of performance in controller reduction when a coherent choice of closed loop identification scheme is chosen. A fifth-order discrete-time system (corresponding to the model of a flexible transmission) is considered as the true model of the plant:

$$G(z^{-1}) = \frac{0.3087z^{-3} + 0.3930z^{-4}}{1 - 1.6028z^{-1} + 1.8884z^{-2} - 1.6673z^{-3} + 1.2314z^{-4} - 0.2279z^{-5}} \quad (54)$$

and a sixth-order discrete-time system as the nominal high order controller:

$$K(z^{-1}) = \frac{0.658 - 0.82z^{-1} - 0.606z^{-2} + 1.093z^{-3} + 0.15z^{-4} - 0.168z^{-5} + 0.019z^{-6}}{1 + 0.509z^{-1} - 0.75z^{-2} - 0.699z^{-3} - 0.092z^{-4} + 0.032z^{-5}} \quad (55)$$

The controller is obtained by pole placement method and contains an integrator. The objective is to find the best reduced order model and reduced order controller (both of fourth order) for different controller reduction criteria. Two criteria are considered: the first one is to preserve the performance of the nominal controller in tracking and output disturbance rejection (the first row of Table 2) and the second one is to preserve the performance of the nominal controller in rejection of output disturbance at the plant input (the last row of Table 2).

8.1 Closed loop identification

The closed loop system formed by the true plant model and the nominal controller is excited with a PRBS of 1024 length added to the controller input. This is in fact the simulation of a noise-free real data acquisition. Two reduced order models ($n_A = 4$, $n_B = 2$ and $d = 2$) of the plant are identified using the CLOE and CLIE algorithms. In order to have the same convergence properties for two algorithms, the closed loop input error is also minimized using the CLOE algorithm but the excitation signal is filtered by the nominal controller. The CLOE model is adequate for the first objective (tracking and output disturbance rejection) whereas the CLIE model is more appropriate for the second control objective. The parameters of two identified model are as follows:

$$\hat{G}_{cloe}(z^{-1}) = \frac{0.2671z^{-3} + 0.4498z^{-4}}{1 - 1.4291z^{-1} + 1.5464z^{-2} - 1.1972z^{-3} + 0.8413z^{-4}} \quad (56)$$

$$\hat{G}_{clie}(z^{-1}) = \frac{0.2744z^{-3} + 0.4164z^{-4}}{1 - 1.515z^{-1} + 1.6779z^{-2} - 1.3594z^{-3} + 0.9423z^{-4}} \quad (57)$$

The results of identification in terms of the 2-norm error between different sensitivity functions are presented in Table 3. It can clearly be observed that the 2-norm error between the output sensitivity functions for the CLOE algorithm is less than that of the CLIE algorithm, but the 2-norm error between the input sensitivity functions is smaller for the CLIE algorithm.

Table 3: The results of closed loop identification of reduced order models

Algorithm	$\|S_{yp} - \hat{S}_{yp}\|_2$	$\|S_{up} - \hat{S}_{up}\|_2$
CLOE	0.1327	0.2016
CLIE	0.2079	0.1314

8.2 Controller order reduction

Now, the algorithms for estimation of a reduced order controller are compared via simulation. First we consider tracking and output disturbance rejection as control objective for which the error between output sensitivity functions should be minimized. The CLOM controller reduction algorithm which is conformed with the control objective is chosen with the two different closed-loop identified models ($\hat{G}_{cloe}$ and $\hat{G}_{clie}$). The simulation results in Table 4 (column 2, simulated data) for two identified models ($\hat{G}_{cloe}$ and $\hat{G}_{clie}$) show that for the coherent algorithms (CLOM and CLOE), the two norm of error between the output sensitivity function is smaller than that of non-coherent algorithms (CLOM and CLIE). It can be observed that when the real data are used for controller reduction (obtained using the true plant model, column 3) the results are further improved. It should be noted that the true plant model is used only for data generation and it is not used in the closed loop predictor. The similar results are obtained when input disturbance rejection is considered as control objective. Table 5 shows that the two norm of error between the input sensitivity function is smaller for the coherent algorithms (CLIM and CLIE). Again, the use of real data can improve the results. It is worthy to mention that the use of real data for controller reduction is a unique feature of the proposed approach.

Table 4: The results of controller reduction by output matching method

	$\|S_{yp} - \hat{\hat{S}}_{yp}\|_2$	
	Simulated data	real data
CLOM($\hat{G}_{cloe}$)	0.1324	0.1102
CLOM($\hat{G}_{clie}$)	0.2078	0.2555

Table 5: The results of controller reduction by input matching method

	$\|S_{up} - \hat{\hat{S}}_{up}\|_2$	
	Simulated data	real data
CLIM($\hat{G}_{cloe}$)	0.2062	0.1750
CLIM($\hat{G}_{clie}$)	0.1326	0.1201

9 Conclusions

A unified approach to closed-loop plant identification and direct controller reduction by closed-loop identification has been proposed. The main idea is to consider the control objective in all of the model based control design steps (i.e. in model identification step, in high order controller design and in the controller reduction step). The control criterion can be matched with the identification criterion by plant model identification in closed-loop with the appropriate choice of the criterion (output error or input error) as well as the place where the excitation signal is added (to the plant input or to the controller input). The control objective can also be considered in the controller reduction step by direct identification of a reduced order controller in closed-loop. In this step the choice of the criterion and the excitation point play again an important role.

The paper has emphasized the duality character of the algorithms, allowing basically with only one algorithm to cover several closed loop plant model identification and controller reduction. It also enhanced the importance of the use of appropriate closed loop identification schemes for obtaining the plant model to be used for controller reduction

Experimental results reported in [8] have clearly shown the potential of the algorithms to solve practical closed loop identification and controller reduction problems.

References

[1] B. D. O. Anderson. Controller design: Moving from theory to practice. *IEEE Control Magazine*, August 1993.

[2] P. Bendotti, B. Codrons, C.M. Falinower, and M. Gevers. Control oriented low order modeling of a complex PWR plant: a comparison between open loop and closed loop methods. In *IEEE-CDC 1998 (FAO 7-2)*, Tampa, Florida USA, December 1998.

[3] U. Forssell and L. Ljung. Closed loop identification revisited. *Automatica*, 35(7):1215–1242, 1999.

[4] M. Gevers. Towards a joint design of identification and control? In H. L. Trentelman and J.C. Willems, editors, *Essays on Control, Perspectives in the Theory and its Applications*. Birkhäuser, Boston, USA, 1993.

[5] A. Karimi and I. D. Landau. Comparison of the closed loop identification methods in terms of the bias distribution. *Systems and Control Letters*, (34):159–167, 1998.

[6] A. Karimi and I. D. Landau. Controller order reduction by direct closed loop identification (output matching). In *3rd Rocond IFAC*, Prague, June 2000.

[7] I. D. Landau and A. Karimi. Recursive algorithms for identification in closed loop - a unified approach and evaluation. *Automatica*, 33(8):1499–1523, August 1997.

[8] I. D. Landau, A. Karimi, and A. Constantinescu. Direct controller order reduction by identification in closed-loop. *Automatica*, 37(11):1689–1702, 2001.

[9] I. D. Landau, R. Lozano, and M. M'Saad. *Adaptive Control*. Springer-Verlag, London, 1997.

[10] P. M. J. Van den Hof and R. R. Schrama. Identification and control - Closed-loop issues. *Automatica*, 31(12):1751–1770, December 1995.