Control of a Reduced Size Model of US Navy Crane Using Only Motor Position Sensors*

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Abstract. Two control problems related to a particular underactuated mechanical system, the reduced size US Navy crane, are addressed. The *open-loop* motion planning problem is solved by showing that the model of the crane is differentially flat with a flat output comprising the coordinates of the load as its first components. The *closed-loop* global asymptotic stabilization of equilibria is achieved using an *output* feedback regulator of proportional-derivative type. The extension of this approach to tracking is analyzed based on simulation results.

1 Introduction

Cranes constitute good examples of nonlinear oscillating pendulum- like systems with challenging industrial applications. Their control has been approached by various techniques, linear [1,7,8,15] and nonlinear [6,9,14].

Cranes present two interesting properties from the control engineering point of view. They are underactuated, i.e. the number of actuators is less than the number of degrees of freedom (see [2]). Moreover, only partial information can be used for closed-loop control, i.e. measurement of the whole state is unavailable (especially as far as the rope angles or the load position are concerned) (see [13]).

In this paper the particular example of the reduced size US Navy crane is studied. (All presented methods can be extended to a large class of similar equipment [10].)

Two control problems are addressed: *open-loop* real-time motion planning and *closed-loop* stabilization. The solution presented to the *open-loop* motion planning problem allows to calculate the necessary control input as to move the load along any (sufficiently smooth) trajectory in the working space using the flatness property [3,4,5] of the system. The second control problem is the *closed-loop* stabilization of both an equilibrium and a trajectory. Since the only measurements available are the motor positions (recall that the load position or the rope angles are not measured) this problem can not be solved using full state feedback. Instead, a classical PD output regulation

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http://www.supelec.fr/lss/NCN/crane.html#PDP

is proposed. Global closed-loop stability of equilibria can be proved using LaSalle's invariance principle [12] and the particular structure of the crane dynamics. Note that this result is of particular practical interest to reduce the time to damp the oscillations of the load during harbor operations [16].

Simulation results show that the same regulator may also be used for tracking. Based on experimental considerations, it appears that our PD regulator together with flatness based trajectory planning outperforms the globally stabilizing regulator, though no proof is presented herein.

The remaining part of the paper is organized as follows. The next section is devoted to the general description of the experimental setup. Modeling equations are given in Section 3. The solution of the *open-loop* motion planning problem is presented in Section 4 based on the flatness property of the model. Asymptotic global stability of equilibria in *closed-loop* using output feedback regulators of proportional-derivative type is studied in Section 5. Simulation results of an extension of the same controller with open-loop trajectory planning for tracking are presented in Section 6.

2 General description of the experimental setup

The reduced scale (1:80 size) model[1] of the US-Navy crane is depicted in Figure 1. Four DC motors (three of them winching ropes) are mounted on the structure allowing to manipulate the load in a three dimensional workspace.

The control objective is to move the load swiftly from an initial position to a desired final position without sway and avoiding obstacles. Since the accelerations of the motors tend to create oscillations of the load, simultaneously fast and swayless displacements are hard to realize.

The reduced size model comprises:

- A load (maximal nominal mass: 800g)
- A mobile pulley guiding the rope which hoists the load.
- A rotate platform actuated by the DC motor no.4 using a synchronous belt transmission.
- A hoisting system mounted on the rotate platform comprising three ropes winched by three DC motors (motors no.$1, 2, 3$):
  - A *horizontal* rope attached to the mobile pulley and ending up on the winch of motor 1.
  - a *vertical* rope attached to the mobile pulley and ending up on the winch of motor 2. This rope prevents the mobile pulley from falling.
  - A rope attached to the load passing *through* the mobile pulley and ending up on the winch of motor 3.
- A power electronics unit[2]. It receives sensor signals from the incremental encoders mounted on the motor axes and transmits them to a computer. It also provides the necessary power amplification to the DC motors.

[1] the reduced scale model was made by Walter Rumsey, Paris, France

[2] the power electronics unit was made by the Institut d'Automatique of the École Polytechnique Fédérale de Lausanne, Switzerland

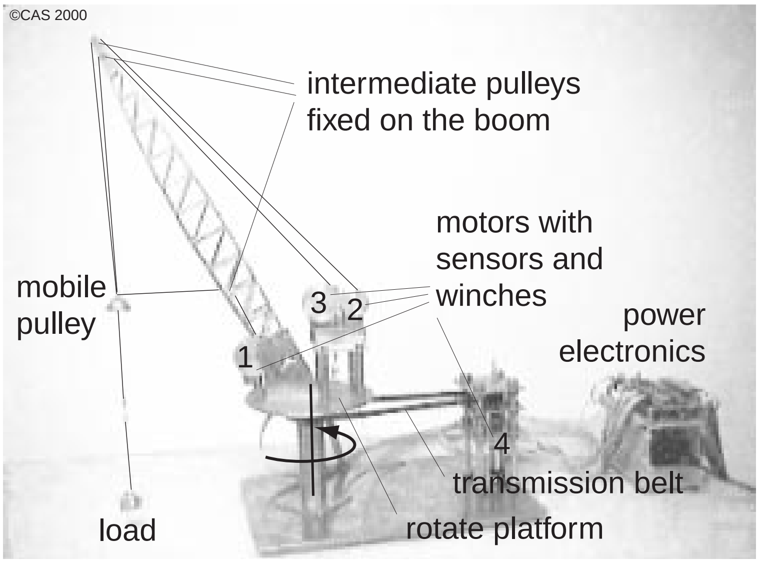

Fig. 1. The reduced (1:80) size US Navy crane in the authors' lab.

The control algorithm is implemented on a personal computer equipped with a standard data acquisition card. The measured signals are the rotation angles of the motor axes which allow to calculate directly the rope lengths and the rotation angle of the platform. The corresponding velocities are calculated using numerical derivation.

3 Model Equations

Figure 2 gives the schematic representation of the crane. The electronic time constants are negligible w.r.t. the mechanical time constants. Consequently, the input variables of the model are the torques T_1, T_2, T_3, T_4 delivered by the motors no.$1-4$ respectively.

Observe that along each rope there is an intermediate pulley fixed to the boom. Since the length of the rope sections between these pulleys and the winches of the motors are constant, we consider that the motors drive directly the axis of the intermediate pulleys and we reduce all rotating inertias along each rope on these axes. The same simplification is made concerning the belt transmission, i.e. we consider that motor 4 drives directly the axis of the rotate platform and we reduce all rotating inertias to this axis. All ropes are considered to be rigid.

The following variables and inertia parameters are introduced:

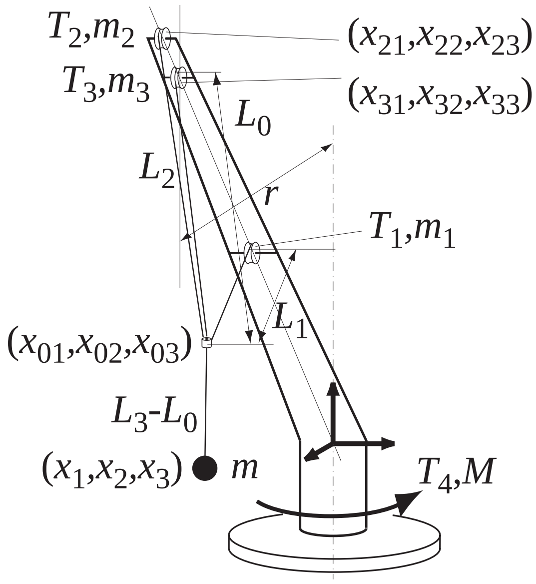

Fig. 2. Simplified representation of the 3D US Navy crane

- x_1, x_2, x_3: position of the load,
- x_{01}, x_{02}, x_{03}: position of the mobile pulley,
- x_{11}, x_{12}, x_{13}: position of the pulley winding the *horizontal* rope attached to the mobile pulley,
- x_{21}, x_{22}, x_{23}: position of the pulley winding the *vertical* rope attached to the mobile pulley,
- x_{31}, x_{32}, x_{33}: position of the pulley hoisting the load,
- L_0: length of the rope section between the pulley hoisting the load and the mobile pulley,
- L_1: length of the *horizontal* rope attached to the mobile pulley,
- L_2: length of the *vertical* rope attached to the mobile pulley,
- L_3: length of the rope attached to the load,
- m: mass of the load,
- m_0: mass of the mobile pulley,
- m_1, m_2, m_3: rotating inertias reduced to the intermediate pulleys' axis,
- M: rotating inertia reduced to the axis of the rotate platform

The construction of the crane is such that the three intermediate pulleys are aligned. Thus we introduce two geometric parameters α_2 and α_3 such that $x_{ij} = \alpha_i x_{1j}$ for $= 2,3$ and $j = 1,2,3$. Observe also that $x_{i3} = constant$, $i = 1,2,3$.

Denote by $q = (q_1 \dots q_{12})^T = (x_1,\ x_2,\ x_3,\ x_{01},\ x_{02},\ x_{03},\ x_{11},\ x_{12},\ L_0,\ L_1,\ L_2,\ L_3)^T$ the vector of system variables.

Geometric constraints are present due to the various cable distances that need to be compatible with each other.

Theorem 1. *The dynamics associated to the US Navy crane are*

$$
\begin{aligned}
m\ddot{x}_1 &= \lambda_1(x_1 - x_{01}) \\
m\ddot{x}_2 &= \lambda_1(x_2 - x_{02}) \\
m\ddot{x}_3 &= \lambda_1(x_3 - x_{03}) - mg \\
m_0\ddot{x}_{01} &= -\lambda_1(x_1 - x_{01}) - \lambda_2(x_{01} - x_{11}) - \lambda_3(x_{01} - \alpha_2 x_{11}) - \lambda_4(x_{01} - \alpha_3 x_{11}) \\
m_0\ddot{x}_{02} &= -\lambda_1(x_2 - x_{02}) - \lambda_2(x_{02} - x_{12}) - \lambda_3(x_{02} - \alpha_2 x_{12}) - \lambda_4(x_{02} - \alpha_3 x_{12}) \\
m_0\ddot{x}_{03} &= -\lambda_1(x_3 - x_{03}) - \lambda_2(x_{03} - x_{13}) - \lambda_3(x_{03} - \alpha_2 x_{13}) - \lambda_4(x_{03} - \alpha_3 x_{13}) - m_0 g \\
0 &= \lambda_1(L_3 - L_0) - \lambda_4 L_0 \qquad (1) \\
m_1\ddot{L}_1 &= -\lambda_2 L_1 + T_1 \\
m_2\ddot{L}_2 &= -\lambda_3 L_2 + T_2 \\
m_3\ddot{L}_3 &= -\lambda_1(L_3 - L_0) + T_3 \\
M\ddot{x}_{11} &= -\lambda_2(x_{01} - x_{11}) - \alpha_2\lambda_3(x_{01} - \alpha_2 x_{11}) - \alpha_3\lambda_4(x_{01} - \alpha_3 x_{11}) - \lambda_5 x_{11} - T_4 x_{12} \\
M\ddot{x}_{12} &= -\lambda_2(x_{02} - x_{12}) - \alpha_2\lambda_3(x_{02} - \alpha_2 x_{12}) - \alpha_3\lambda_4(x_{02} - \alpha_3 x_{12}) - \lambda_5 x_{12} + T_4 x_{11}
\end{aligned}
$$

subject to the constraints

$$
\begin{aligned}
C_1 &= \frac{1}{2}\left(\sum_{i=1}^{3}(x_i - x_{0i})^2 - (L_3 - L_0)^2\right) = 0 \\
C_2 &= \frac{1}{2}\left(\sum_{i=1}^{3}(x_{0i} - x_{1i})^2 - L_1^2\right) = 0 \\
C_3 &= \frac{1}{2}\left(\sum_{i=1}^{3}(x_{0i} - \alpha_2 x_{1i})^2 - L_2^2\right) = 0 \qquad (2) \\
C_4 &= \frac{1}{2}\left(\sum_{i=1}^{3}(x_{0i} - \alpha_3 x_{1i})^2 - L_0^2\right) = 0 \\
C_5 &= \frac{1}{2}\left(x_{31}^2 + x_{32}^2 - r^2\right) = 0
\end{aligned}
$$

The multipliers $\lambda_1 \dots \lambda_5$ are associated to the constraints $C_1 \dots C_5$. Recall that the kinetic and the potential energy of the system read

$$
\begin{aligned}
W_k &= \frac{1}{2}\left(\sum_{i=1}^{3}\left(m\dot{x}_i^2 + m_0\dot{x}_{0i}^2\right) + \sum_{i=1}^{2} M\dot{x}_{1i}^2 + m_1\dot{L}_1^2 + m_2\dot{L}_2^2 + m_3\dot{L}_3^2\right) \\
W_p &= mgx_3 + m_0 g x_{03},
\end{aligned}
$$

hence the Lagrangian is defined by $\mathcal{L} = W_k - W_p$. A proof of this result can be found in [10].

4 Flatness and Motion Planning

4.1 Flatness

A flat output is given by $Y = (Y_1, \ldots, Y_4)^T = (x_1, x_2, x_3, x_{03})^T$, the coordinates of the load and the height of the mobile pulley. As long as we omit free-fall reference trajectories, i.e. $\ddot{x}_3 \neq g$, the first three equations of (1) give λ_1, x_{01} and x_{02} as functions of Y and $\ddot{Y}$. Equations 5 to 8 of (1) and Constraints C_1, C_4 and C_5 of (2) can then be used to express $\{\lambda_2, \lambda_3, \lambda_4, x_{11}, \lambda_{12}, L_3, L_0\}$ as expressions of $\lambda_1, x_1, x_2, x_3, x_{01}, x_{02}, x_{03}$ and derivatives up to order 2 (thus giving expressions involving Y, $\dot{Y}$, $\ddot{Y}$, $Y^{(3)}$ and $Y^{(4)}$). Next, the constraints C_2 and C_3 give L_1 and L_2. The other equations of (1) give the inputs T_1, T_2, T_3 and T_4 (T_4 is obtained after solving the last two equations for T_4 and λ_5). The inputs are expressions of Y and its derivatives up to order 6.

4.2 Motion planning

Assume that the position, velocity, acceleration, jerk and all derivatives up to $6th$ order of the flat output (including the position of the load) at the start time t_I are given by $(Y_I, \dot{Y}_I, \ddot{Y}_I, \ldots, Y_I^{(5)}, Y_I^{(6)})$ and the desired final configuration of the flat output at time t_F is $(Y_F, \dot{Y}_F, \ddot{Y}_F, \ldots, Y_F^{(5)}, Y_F^{(6)})$. We can construct 13 th degree polynomials,

$$Y_{ci}(t) {=} Y_{Ii} + (Y_{Fi} - Y_{Ii}) \sum_{j=1}^{13} a_{ji} \left(\frac{t - t_I}{t_F - t_I} \right)^j , \tag{3}$$

where the coefficients a_{ji}, $j = 1 \ldots 13$ and $i = 1 \ldots 3$, are computed by solving linear equations, whose entries are combinations of the initial and final conditions. In particular, motion planning between two different equilibria $\overline{Y}_I$ and $\overline{Y}_F$ can be obtained simply by setting $Y_I = \overline{Y}_I$, $\dot{Y}_I = \ddot{Y}_I = \ldots = Y_I^{(5)} = Y_I^{(6)} = 0$ and $Y_F = \overline{Y}_F$, $\dot{Y}_F = \ddot{Y}_F = \ldots = Y_F^{(5)} = Y_F^{(6)} = 0$. The input to be applied that results in the above trajectories is then computed using the flatness property as described in the previous subsection.

5 Output Feedback Regulation

We wish to stabilize the crane at a given equilibrium point of the load $(\bar{x}_1, \bar{x}_2, \bar{x}_3)$ and at a given height of the mobile pulley $\bar{x}_{03}$.

Using the constraints and the dynamic equations at equilibrium, we find the equilibrium values of the remaining variables: $\bar{x}_{11}$, $\bar{x}_{12}$, $\overline{L}_0$, $\overline{L}_1$, $\overline{L}_2$, $\overline{L}_3$ and the corresponding input torques to be applied: $\overline{T}_1$, $\overline{T}_2$, $\overline{T}_3$ and $\overline{T}_4$. (Observe that $\overline{T}_4 = 0$ for all equilibria). Define the error variables as $e_{q_i} = \bar{q}_i - q_i$ where q_i stands for ith component of q. Additionally define $\xi = \arctan(\frac{x_{12}}{x_{11}})$, the rotation angle of the rotate platform. Then the corresponding error variable is $e_\xi = \bar{\xi} - \xi$.

Recall that the measured variables are: L_1, L_2, L_3 and ξ.

Theorem 2. *The four PD controllers,*

$$\begin{aligned} T_1 &= \overline{T}_1 + k_{d1}\dot{e}_{L_1} + k_{p1}e_{L_1} \\ T_2 &= \overline{T}_2 + k_{d2}\dot{e}_{L_2} + k_{p2}e_{L_2} \\ T_3 &= \overline{T}_3 + k_{d3}\dot{e}_{L_3} + k_{p3}e_{L_3} \\ T_4 &= k_{d4}\dot{e}_\xi + k_{p4}e_\xi, \end{aligned} \tag{4}$$

applied to the crane dynamics (1) with Constraints (2) assure closed-loop global stability of the equilibrium $(\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_{03})$.

The proof relies on two lemmas as in [11]. Let us define the following energy-like function:

$$W = W_k + W_p + W_{ctrl}, \tag{5}$$

with

$$W_{ctrl} = \frac{1}{2}\left(\sum_{i=1}^{3} k_{pi}e_{L_i}^2 + k_{p4}e_\xi^2\right) + \sum_{i=1}^{3} \bar{T}_i e_{L_i} + \bar{T}_4 e_\xi \tag{6}$$

the "potential" energy stored in the controllers.

Lemma 1. *The derivative of* W *along closed-loop trajectories is given by:*

$$\dot{W} = -k_{d1}\dot{e}_{L_1}^2 - k_{d2}\dot{e}_{L_2}^2 - k_{d3}\dot{e}_{L_3}^2 - k_{d4}\dot{e}_\xi^2. \tag{7}$$

Lemma 2. *The only invariant trajectory compatible with* $\dot{W} = 0$ *is the equilibrium trajectory, i.e.* $x_1(t) \equiv \bar{x}_1$, $x_2(t) \equiv \bar{x}_2$, $x_3(t) \equiv \bar{x}_3$ *and* $x_{03}(t) = \bar{x}_{03}$.

6 Extension to Tracking

Assume that a reference trajectory is constructed so as to steer the load from an idle point to another idle point with obstacle avoidance. This can be done using polynomials as described in Section 4. Denote the polynomial reference trajectory of the flat output by Y_c.

Based on flatness, one can calculate the reference trajectory of all other variables in the system as functions of $Y_c, \dot{Y}_c, \ddot{Y}_c, \ldots, Y_c^{(6)}$. Denote by $q_c = (q_{1c} \ldots q_{12c})^T = (x_{1c}, x_{2c}, x_{3c}, x_{01c}, x_{02c}, x_{03c}, x_{11c}, x_{12c}, L_{0c}, L_{1c}, L_{2c}, L_{3c})^T$ the reference trajectory of all system variables and by T_{1c}, T_{2c}, T_{3c}, T_{4c} the reference inputs.

We investigate in this section the *closed-loop* behavior of the system using the same PD regulator as before but fed by the above references. This modified controller is referred to as the tracking controller.

Define $e_{q_i} = q_{ic} - q_i$ where q_i is the ith component of the vector q and q_{ic} is the ith component of q_c. The tracking PD controller is given by:

$$\begin{aligned} T_1 &= T_{1c} + k_{d1}\dot{e}_{L_1} + k_{p1}e_{L_1} \\ T_2 &= T_{2c} + k_{d2}\dot{e}_{L_2} + k_{p2}e_{L_2} \\ T_3 &= T_{3c} + k_{d3}\dot{e}_{L_3} + k_{p3}e_{L_3} \\ T_4 &= T_{4c} + k_{d4}\dot{e}_{\xi} + k_{p4}e_{\xi}. \end{aligned} \tag{8}$$

Note that for equilibrium trajectories we get the same PD regulator as before.

Theorem 3. *Let the final point $q_c(t_F)$ of the reference trajectory be an equilibrium of the system and suppose that all derivatives along the reference trajectory are bounded. Then $q_c(t_F)$ is asymptotically stable in closed-loop using the tracking PD controller.*

The stabilization property of the tracking controller given by (8) has been validated using simulation. Comparison of the closed-loop behavior of the two controllers during point to point steering is undertaken. The global stabilizing controller is fed by the equilibrium reference of the desired final point and the tracking controller is fed by the reference trajectory obtained by flatness-based motion planning.

Two reference trajectories connecting the same initial and final points with transit time of 2.5 seconds are envisaged. The first trajectory (Figures 3-6) is a horizontal displacement. The second one (Figures 7-9) is a parabola in the vertical plane determined by the two points. The globally stabilizing controller produces the same behavior in both cases with damped oscillations, while the tracking controller stabilizes the desired reference and arrives at the equilibrium faster and with less oscillations. The same gains k_{di}, k_{pi} $(i = 1 \dots 4)$ are used for both controllers.

Notice that the tracking controller outperforms the global one, hence decreasing both the residual sway and the reaching time.

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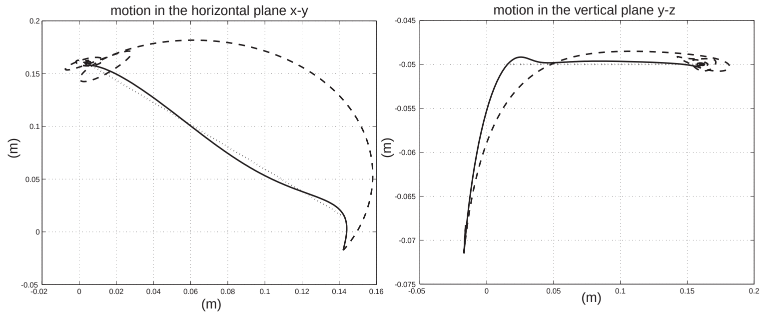

Fig. 3. Closed-loop tracking behavior under PD control. Trajectories of the load in the horizontal and vertical planes: *i*) global stabilizing equilibrium controller (hashed line); *ii*) tracking controller with motion planning; *iii*) reference to steer to equilibrium along a straight line (dotted).

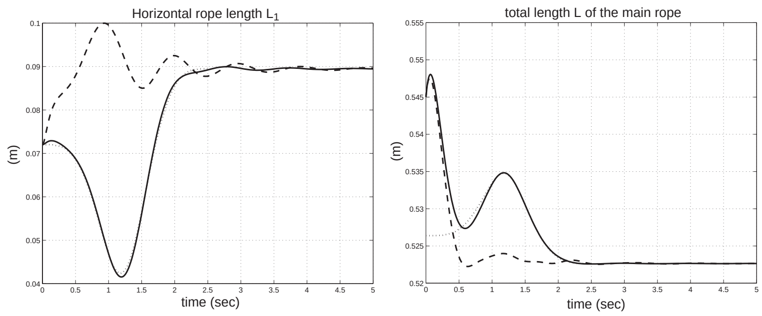

Fig. 4. Closed-loop tracking behavior, rope lengths: *i*) global stabilizing equilibrium controller (hashed line); *ii*) tracking controller with motion planning; *iii*) reference to steer to equilibrium along a straight line (dotted).

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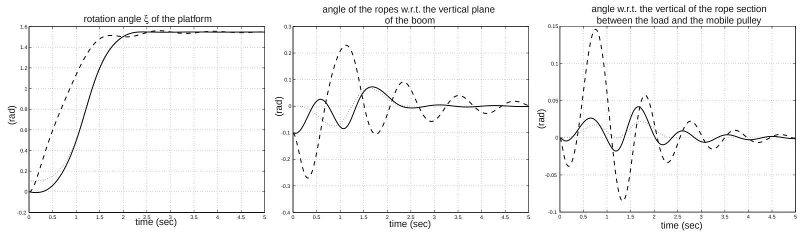

Fig. 5. Closed-loop tracking behavior, angles: *i*) global stabilizing equilibrium controller (hashed line); *ii*) tracking controller with motion planning; *iii*) reference to steer to equilibrium along a straight line (dotted).

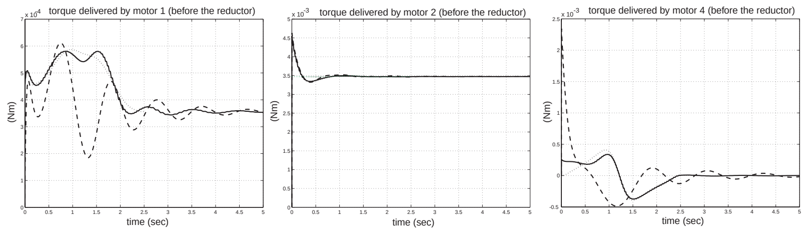

Fig. 6. Closed-loop tracking behavior, motor tensions: *i*) global stabilizing equilibrium controller (hashed line); *ii*) tracking controller with motion planning; *iii*) reference to steer to equilibrium along a straight line (dotted).

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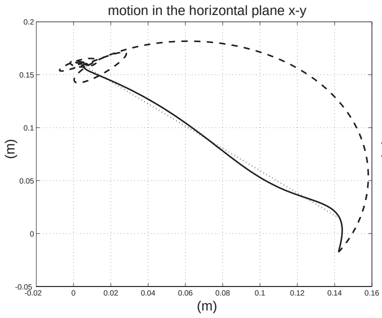

Fig. 7. Closed-loop tracking behavior under PD control. Trajectories of the load in the horizontal and vertical planes: *i*) global stabilizing equilibrium controller (hashed line); *ii*) tracking controller with motion planning; *iii*) reference to steer to equilibrium along a parabola (dotted).

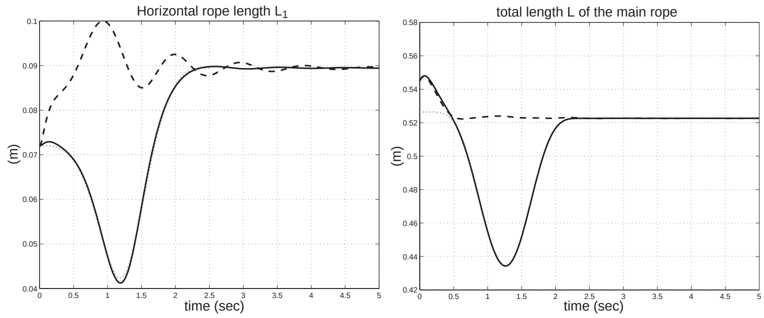

Fig. 8. Closed-loop tracking behavior, rope lengths: *i*) global stabilizing equilibrium controller (hashed line); *ii*) tracking controller with motion planning; *iii*) reference to steer to equilibrium along a parabola (dotted).

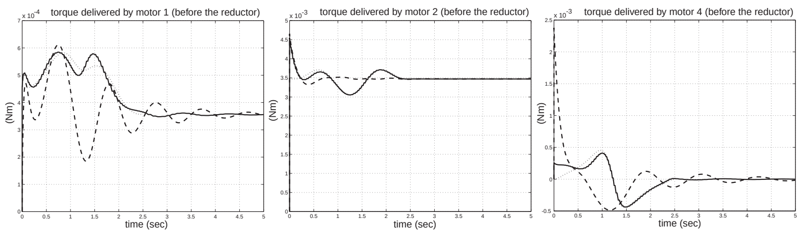

Fig. 9. Closed-loop tracking behavior, motor tensions: *i*) global stabilizing equilibrium controller (hashed line); *ii*) tracking controller with motion planning; *iii*) reference to steer to equilibrium along a parabola (dotted).