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Micro-scale Cracking of Concrete due to the Alkali-Silica Reaction

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1 Introduction

Alkali-silica reaction (ASR) is a concrete disease that leads to its degradation. ASR is the reaction between the amorphous silica (SiO_2) and the alkali ions (Na^+, K^+), leading to expansion of concrete in presence of Ca^{2+}. In this study, it is assumed that the internal pressure exerted on a concrete matrix by the expanding products leads to its cracking and loss of its bearing capacity.

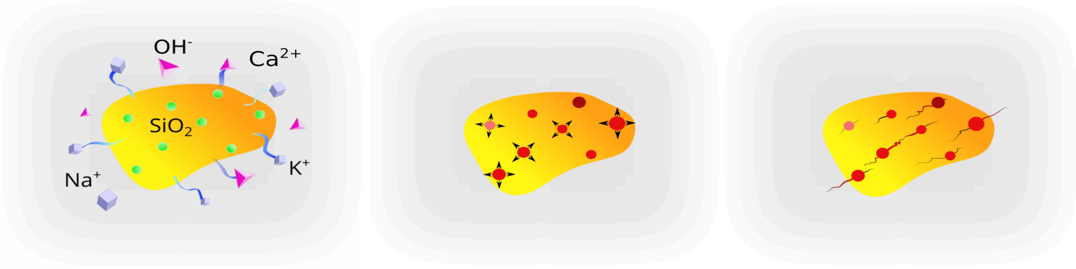

Fig. 1. Stages of ASR damaging concrete. Left) Alkalis from cement paste diffuse into aggregates and react with amorphous silica. Center) Resulting ASR products exert internal pressure on aggregates. Right) Tensile cracking of aggregates and paste.

2 Research question

Potassium-type ASR results in ASR product called **shlykovite** [1]:

$$K^+ + Ca^{2+} + 3OH^- + 4SiO_2 + 2H_2O \rightarrow KCa[Si_4O_9(OH)] \cdot 3H_2O$$

Increase in molar volume of the product in comparison with the reactants is 66%.
Could 66% expansion of a single ASR product pocket cause crack initiation and growth?

3 Method overview

The developed semi-analytical model combines Eshelby's inclusion with ring-shaped crack surrounding it. The **Eshelby's problem** [2] supposes finding the displacement field both inside and outside an expanding ellipsoidal inhomogeneity embedded in an infinite medium. Solution for stresses is:

$$\boldsymbol{\sigma}_{inc} = \boldsymbol{\sigma}^0 + \boldsymbol{C}^0[\boldsymbol{S} - \boldsymbol{I}] : \boldsymbol{\varepsilon}_{eig}$$

$$\boldsymbol{\sigma}_{med}(\boldsymbol{x}) = \boldsymbol{\sigma}^0 + \boldsymbol{C}^0\boldsymbol{G}(\boldsymbol{x}) : \boldsymbol{\varepsilon}_{eig}$$

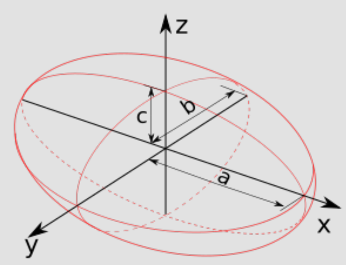

Fig. 2. Geometry for the elastic study

where σ^0 is the background stress, $\boldsymbol{C}^0$ is the medium stiffness tensor, $\boldsymbol{S}$ and $\boldsymbol{G(x)}$ are the 4th-order Eshelby tensor for interior and exterior points correspondingly [3], and ε_{eig} is the inclusion's eigen strain.

Pre-existing **ring-shaped crack** surrounding the inclusion is added in order to estimate its chances to grow. For this, **Griffith's criteria** is evaluated:

$$K_I \leq K_{IC}$$

where stress intensity factor (SIF) is obtained through the **weight function** approach [4]:

$$K_I = \int_0^l \sigma(y')h(l, y')dy'$$

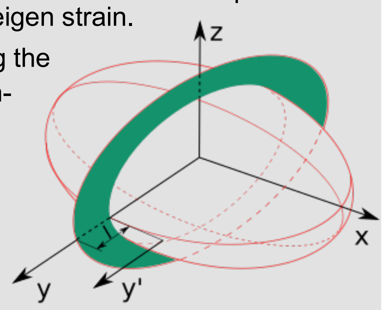

Fig. 3. Geometry for the crack growth study

This SIF is evaluated by numerically integrating a product between stresses surrounding the inclusion in a non-cracked configuration *σ(y')* and the weight function *h(l,y')* [5]:

$$h(l, y')\sqrt{l} = \frac{2}{\sqrt{\pi(1 - y'/l)(l/r + 1)}} \left(\frac{1 + y'/r}{\sqrt{2 + l/r + y'/r}} - \frac{1 - \sqrt{y'/l}}{\sqrt{l/r + 2}} \right)$$

4 Validation method

Validation of analytical results is confirmed in 2 stages by means of **finite element method** (FEM). Stress field inside and outside is computed by purely elastic simulation of 1/8 of the domain (Fig. 2 left). Crack initiation and growth is modeled by a combination of elastic finite elements and **cohesive elements** with a **strain-softening law** (Fig. 2 right).

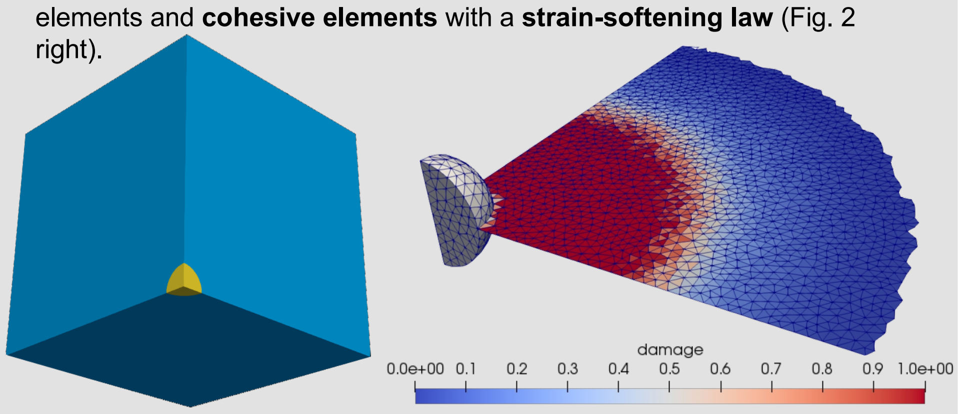

Fig. 4. Left) FE model of 1/8 of the full domain (expanding inclusion in yellow). Right) FE model of a surrounding crack represented by cohesive elements. Colors denote damage: blue is for intact elements, red for fully broken.

5 Results and discussion

The elastic solution showed that the distribution of stress inside and outside the inclusion does not depend on its size but on its shape only (Fig. 5 left). Compressive stresses inside the inclusion are overtaken by tensile values in the surrounding media. More prolonged inclusion concentrates higher stresses on its surface. Stress intensity factors on the external tip of the ring-shaped crack depends both on size and shape of the inclusion (Fig. 5 right).

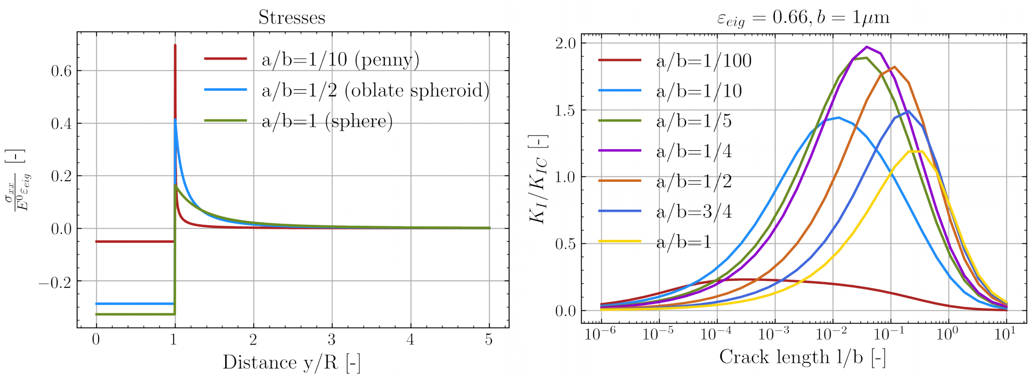

Fig. 5. Left) Distribution of normalized stress σ_{xx} along *y*-axis for 3 different inclusion shapes. Right) Normalized SIF on the external tip of the pre-existing ring-shaped crack for different semi-axes ratios of the ellipsoidal inclusion.

6 Conclusions

Semi-analytical model of crack growth resulted in following conclusions:

- Stresses inside the inclusion are homogeneous
- Stresses in the matrix intensify with reducing ratio *a/b*
- The most critical shape is a spheroid with ratio ¼
- Larger inclusion size and expansion value increase SIF
- Disk-shaped inclusions require large diameter (>10 μm) or high volume increase (> 66%) to grow a crack

7 References

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