A Simple Output Feedback PD Controller for Nonlinear Cranes*

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Abstract: A simple output feedback PD controller is proposed that stabilizes a nonlinear crane. Global asymptotic stability is achieved at any equilibrium point specified by the controller. The control scheme relies solely on the winches position and velocity and hence no cable angle measurement, or no direct measurement of the load position, is needed. The controller can be extended to many different kinds of existing cranes.

Keywords Crane control, Output feedback, PD controller, Underactuated mechanical system.

1 Introduction

Cranes constitute good examples of nonlinear oscillating systems with challenging industrial applications. Their control has been approached by various techniques, linear [4, 10], or nonlinear [5, 9, 2]. As noted by [11], the productivity of harbor cranes might be significantly improved if one could decrease the time needed to damp the oscillations of the load, without requiring the installation of fragile or complicated sensors. Indeed, measurements on all configuration variables are generally not available (especially as far as the rope angles or the load position are concerned) due to the severe operating environment. Bad weather, dust, oil, frequent shock risk restrict the panel of efficient and reliable sensors at the designer's disposal and in particular makes the use of sophisticated artificial vision systems uneasy. Consequently, state feedback techniques cannot be directly applied. In this paper, we precisely address the question of damping the load's oscillations to swiftly bring the load to its equilibrium, using only sensors (incremental encoders) mounted on the motor axes and therefore giving only an indirect information on the load's position.

We propose a simple output feedback controller of the proportional derivative type that ensures global asymptotic stability under the hypothesis that the ropes are rigid.

The proof of stability relies on the application of LaSalle invariance principle [6, 7, 1] and on the particular structure of the crane dynamics [9]. Unfortunately the Lyapunov function does not provide information on the rate of convergence and the gain tuning may be achieved using simulation owing to the reduced number of design parameters.

The paper is organized as follows. Section 2 recalls basic stability definitions and main theorems that assess this property. In Section 3, we recall from [8, 9] the model of the crane used in this study. Then Section 4 gives the controller for equilibrium stabilization with its proof of stability. Simulations confirm the good closed loop behaviour of the controlled crane, followed by some conclusions and open questions.

2 Stability definitions and theorems

Consider the system

$$\begin{aligned} \dot{x} &= f(x), \qquad x \in \mathbb{R}^n \\ f(0) &= 0 \end{aligned} \tag{1}$$

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where $f(x)$ is lipschitz continuous and let $x(t, x_0)$ denote the unique solution of the above system with initial condition $x(0) = x_0$. This material is standard and can be found in [6].

Definition 1 (Stability) *The equilibrium x=0 of (1) is stable if for all $\epsilon > 0$, there exists a $\delta > 0$, such that $\| x_0 \| < \delta \Rightarrow \| x(t, x_0) \| < \epsilon$, for all $t \geq 0$.*

Definition 2 (Asymptotic stability) *The equilibrium $x = 0$ of (1) is asymptotically stable if it is stable and if,*

$$\lim_{t \to \infty} x(t, x_0) = 0.$$

A sufficient stability condition is given by the following theorem.

Theorem 1 (Lyapunov's second method) *If there is a function $V(x)$ such that*

1. $V(x) > 0$, $\forall x \in \mathcal{U} \subset \mathbb{R}^n \backslash \{0\}$
2. $L_f V(x) < 0$, $\forall x \in \mathcal{U} \subset \mathbb{R}^n \backslash \{0\}$

where $\mathcal{U}$ is a neighborhood of 0 then 0 is locally asymptotically stable. Moreover, if $\mathcal{U} = \mathbb{R}^n$ and $V(x)$ is radially unbounded, i.e. $V(x) \to \infty$ as $\| x \| \to \infty$, then 0 is globally stable.

If $L_f V(x) = 0$ for a set of points including the origin then the stability is not guaranteed. In order to deal with this case one needs some additional definitions.

Definition 3 (Invariant set) *A set $\mathcal{I}$ is said to be invariant with respect to (1) if,*

$$\forall x_0 \in \mathcal{I} \ , \quad x(t, x_0) \in \mathcal{I}, \quad \forall t \in \mathbb{R}.$$

Definition 4 (Positively invariant set) *A set $\mathcal{I}$ is said to be positively invariant with respect to (1) if,*

$$\forall x_0 \in \mathcal{I} \ , \quad x(t, x_0) \in \mathcal{I}, \quad \forall t \geq 0.$$

Definition 5 (Approaching a set) *We say that $x(t)$ approaches a set $\mathcal{M}$ as $t \to \infty$, if for each $\epsilon > 0$, there is a $T > 0$ such that,*

$$\inf_{\bar{x} \in \mathcal{M}} \| x(t) - \bar{x} \| < \epsilon, \quad \forall t > T.$$

Theorem 2 (LaSalle invariance theorem) *Let $\mathcal{C} \subset \mathcal{U} \subset \mathbb{R}^n$ be a compact set that is positively invariant with respect to (1). Let $V : \mathcal{U} \to \mathbb{R}$ be a continuously differentiable function such that $L_f V(x) \leq 0$ for all $x \in \mathcal{U}$. Let $\mathcal{N}$ be the set of all points in $\mathcal{C}$ where $L_f V(x) = 0$. Let $\mathcal{M}$ be the largest invariant set in $\mathcal{N}$. Then every solution starting in $\mathcal{C}$ approaches $\mathcal{M}$ as $t \to \infty$.*

3 Nonlinear Crane Model

We will consider the model of an onboard disembarkment crane used by the US Navy. For simplicity of the exposition we restrict the system to evolve in a fixed vertical plane. This restriction does not impart on generality.

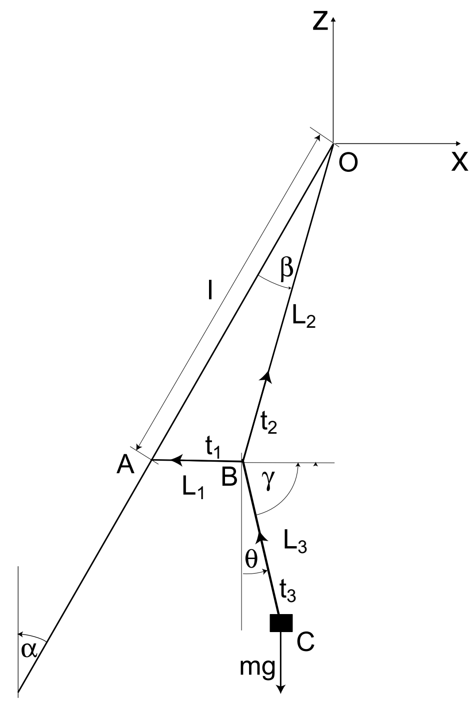

Figure 1: US Navy crane

The crane illustrated in Figure 1 consists of the following main parts:

- a pole making a fixed angle α with respect to the vertical, equipped with two winches, one located at the top, denoted by O and chosen as the origin, and the second one located at A, at a fixed distance l from O;

- a vertical rope of variable length R, starting from O, whose upper part makes an angle β with the vertical, passing through a pulley located at the point B, the lower part of the rope making an angle θ with the vertical. The length of the upper part is denoted by L_2 and the one of the lower part by L_3. Since the total length of the rope is R, we have $R = L_2 + L_3$;
- a horizontal rope of variable length L_1 relating the winch A to the pulley B;
- a load with mass m attached to the vertical rope at the point C, located at a distance L_3 from the pulley B.
- the winches at the points O and A with radii ρ_1 and ρ_2 are supposed to be torque controlled using electric motors with incremental encoders on their axes. All friction forces are supposed to be compensated.

We consider a reference orthonormal frame (O, x, z) with Oz oriented upwards. Let g denote the gravity acceleration and (x, z) the coordinates of the load C. The masses of the ropes are neglected and the ropes are assumed to be unstretchable. Also denote T_1 the modulus of the force in the rope at A and T_2 the modulus of the force in the rope at O.

The modeling of this system has been undertaken in [8] which concludes to an implicit model. The dynamics of the load are given by

$$m \begin{bmatrix} \ddot{x} \\ \ddot{z} + g \end{bmatrix} = T_3 \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}, \tag{2}$$

the force equilibrium at the pulley reads:

$$\begin{aligned} -T_1 \sin(\gamma + \theta) + T_2 \sin(\alpha - \beta) + T_3 \sin\theta = 0 \\ T_1 \cos(\gamma + \theta) + T_2 \cos(\alpha - \beta) - T_3 \cos\theta = 0 \end{aligned} \tag{3}$$

and the geometric constraints are

$$\begin{aligned} x_B &= -L_2 \sin(\alpha - \beta) \\ z_B &= -L_2 \cos(\alpha - \beta) \\ x - x_B &= L_3 \sin\theta \\ z - z_B &= L_3 \cos\theta \\ x_B + l \sin\alpha &= L_1 \sin(\theta + \gamma) \\ z_B + l \cos\alpha &= L_1 \cos(\theta + \gamma). \end{aligned} \tag{4}$$

The dynamics of the winches are given by

$$\frac{J_1}{\rho_1} \ddot{L}_1 = T_1 \rho_1 - u_1 \tag{5}$$

$$\frac{J_2}{\rho_2} \ddot{R} = T_2 \rho_2 - u_2. \tag{6}$$

Notice that the unstrechability of the ropes implies $T_2 = T_3$. Moreover, using the equations (3), it is easily verified that

$$\gamma = \frac{1}{2}(\pi + \beta - \alpha - \theta) \tag{7}$$

and that $T_1 = 2T_2 \cos\gamma$.

The crane has three degrees of freedom and a possible choice of the generalized coordinates is $q = (\gamma,\ L_1,\ R)$ which will be used in the sequel. The only external efforts are the torques u_1 and u_2 delivered by the motors.

Let $(\bar{x}, \bar{z})$ denote the coordinates of the load at equilibrium. Then one may calculate the equilibrium of the remaining variables using the following relations:

$$\begin{aligned} &\sin\bar{\beta} = \frac{\bar{x} + l \sin\alpha}{l}\ , \quad \bar{\theta} = 0 \\ &\bar{\gamma} = \frac{1}{2}\left(\pi + \arcsin\left(\frac{\bar{x} + l\sin\alpha}{l}\right) - \alpha\right) \\ &\bar{R} = l\frac{\sin\bar{\beta}}{\sin\bar{\gamma}} + \frac{\bar{z}}{\sin\bar{\gamma}} + l\frac{\sin(\bar{\gamma} - \bar{\beta})}{\sin^2\bar{\gamma}} \cos(\alpha - \bar{\beta}) \\ &\bar{L}_1 = l\frac{\sin\bar{\beta}}{\sin\bar{\gamma}} \\ &\bar{T}_1 = 2mg\cos\bar{\gamma}, \quad \bar{T}_2 = mg. \end{aligned} \tag{8}$$

Notice finally that due to the geometry of the crane, $\bar{\gamma} \in (\frac{\pi - \alpha}{2}, \frac{\pi}{2}]$.

4 PD Controller and Stability Analysis

We wish to stabilize the crane at a given equilibrium $(\bar{x}, \bar{z})$. We claim that this can be achieved using the following PD controllers:

$$u_1 = \bar{T}_1 \rho_1 + \frac{J_1}{\rho_1}\left(k_{dA} \dot{L}_1 + k_{pA}(L_1 - \bar{L}_1)\right) \tag{9}$$

$$u_2 = \bar{T}_2 \rho_2 + \frac{J_2}{\rho_2}\left(k_{dO} \dot{R} + k_{pO}(R - \bar{R})\right) \tag{10}$$

where the a priori rope tensions $\bar{T}_1$ and $\bar{T}_2$ are determined using Equation (8) and k_{pA}, k_{pO} k_{dA}, k_{dO} are constant gains, yet to be determined, so as to achieve satisfactory performance.

The crane depicted in Figure 1 has, in the absence of the controllers, kinetic and potential energy due to the load m and kinetic energy due to the inertia of the winches J_1 and J_2. Let W_{kin} denote the total kinetic energy and W_{pg} the potential gravitic energy. When the controller is present, extra energy can be stored in the controller due to the constant a priori and proportional terms. This energy will be denoted by W_{ctrl}.

Thus, the energy function consists of three terms:

$$W = W_{kin} + W_{pg} + W_{ctrl}, \tag{11}$$

with

$$\begin{aligned} W_{kin} &= \frac{1}{2}\left(m(\dot{x}^2+\dot{z}^2) + \frac{J_1}{\rho_1^2}\dot{R}^2 + \frac{J_2}{\rho_2^2}\dot{L}_1^2\right) \\ W_{pg} &= mgz \\ W_{ctrl} &= \frac{1}{2}k_{pA}(L_1-\bar{L}_1)^2 + \bar{T}_1 L_1 \\ &\qquad + \frac{1}{2}k_{pO}(R-\bar{R})^2 + \bar{T}_2 R. \end{aligned} \tag{12}$$

where, if we use generalized coordinates $\{\gamma, L_1, R\}$,

$$\begin{aligned} x &= l\sin\alpha - L_1\cos\left(\alpha+\gamma-\frac{\pi}{2}\right) \\ &\quad + \left(R - \frac{L_1^2\sin\gamma}{l\sin(\gamma-\arcsin(L_1/l\sin\gamma)}\right) \\ &\quad \times \sin\left(\pi + \arcsin(L_1/l\sin\gamma) - \alpha - 2\gamma\right) \\ z &= l\cos\alpha + L_1\sin\left(\alpha+\gamma-\frac{\pi}{2}\right) \\ &\quad + \left(R - \frac{L_1^2\sin\gamma}{l\sin(\gamma-\arcsin(L_1/l\sin\gamma))}\right) \\ &\quad \times \cos\left(\pi + \arcsin(L_1/l\sin\gamma) - \alpha - 2\gamma\right) \end{aligned} \tag{13}$$

Therefore using the Lagrangian

$$\mathcal{L} = W_{kin} - W_{pg} - W_{ctrl}, \tag{14}$$

the crane dynamics can be obtained by applying

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{q}_i} - \frac{\partial\mathcal{L}}{\partial q_i} = F_{q_i}, \tag{15}$$

where $q_1=\gamma$, $q_2=L_1$,$q_3=R$, and F_{q_i} is the associated generalized force, i.e. $F_\gamma = 0$, $F_R = -k_{dO}\dot{R}$ and $F_{L_1} = -k_{dA}\dot{L}_1$ due to the derivative terms in the controllers. Notice that the proportional term and the constant a priori forces are already in the potential function and thus absent in the generalized forces. Notice also that the actual choice of generalized coordinates does not lead to the most compact formulation of the dynamics but will make the derivation of a necessary lemma easy.

Lemma 1 *The time derivative of the energy function is*

$$\frac{d}{dt}W = -k_{dA}\dot{L}_1^2 - k_{dO}\dot{R}^2.$$

The proof is an easy adaptation of derivations appearing in most textbooks on classical mechanics that prove energy conservation in purely Lagrangian systems (no dissipation) [12, 3]. Here extra terms are present due to the derivative components in the controller.

Hence, it remains to characterize the sets of system trajectories such that $\dot{R}=0$ and $\dot{L}_1=0$. Note the usage of $x(t)\equiv\bar{x}$ to signify that the quantity x stays for all times at the value $\bar{x}$.

Lemma 2 *The only invariant trajectory compatible with $\dot{R}=0$ and $\dot{L}_1=0$ is the equilibrium trajectory, i.e. $x(t)\equiv\bar{x}$, $z(t)\equiv\bar{z}$.*

Proof: The input torques u_1 and u_2 are responsible for forces in the ropes T_1 and T_2 and motion along L_1 and R,

$$u_1 = T_1\rho_1 - \frac{J_1}{\rho_1}\ddot{L}_1 \tag{16}$$

$$u_2 = T_2\rho_2 - \frac{J_2}{\rho_2}\ddot{R} \tag{17}$$

By using the control strategy proposed, i.e. applying PD controllers on both winches, the torques u_1 and u_2 satisfy (9) and (10) where $\bar{T}_1$ and $\bar{T}_2$ are the forces corresponding to the equilibrium position $\bar{x}$ and $\bar{z}$.

Putting these equations together and under the condition that $\dot{L}_1=\ddot{L}_1=0$ and $\dot{R}=\ddot{R}=0$ (since we are interested in the trajectories compatible with $\dot{R}=\dot{L}_1=0$, i.e. L_1, R both stay at constant values say $\hat{L}_1$ and $\hat{R}$) yields,

$$\hat{T}_1 = \bar{T}_1 + k_{pA}(\hat{L}_1-\bar{L}_1) \tag{18}$$

$$\hat{T}_2 = \bar{T}_2 + k_{pO}(\hat{R}-\bar{R}). \tag{19}$$

Notice that whatever the trajectory of the load is we have that $2\cos\gamma(t) = \frac{T_1(t)}{T_2(t)}$. Since $T_1(t) \equiv \hat{T}_1$ and $T_2(t) \equiv \hat{T}_2$ are constant, so must be $\gamma(t) \equiv \hat{\gamma}$.

This shows that all configuration variables are constant if $\dot{L}_1 \equiv 0$ and $\dot{R} \equiv 0$. It follows that the only trajectory compatible with $\dot{W} = 0$ is an equilibrium of the system. Let us denote the values of the variables at this equilibria by a hat.

It remains to show that the equilibrium characterized by the hated variables coincides with the desired equilibrium given by the bared variables.

First, observe that for every equilibrium position of the load $\hat{T}_2 = \bar{T}_2 = mg$. Using (19) we conclude that $\hat{R} = \bar{R}$. The equalities $\bar{L}_1 = \hat{L}_1$ and $\bar{\gamma} = \hat{\gamma}$ will be proved by contradiction. For, suppose that $\bar{\gamma} > \hat{\gamma}$. Recall that $\hat{\theta} = \bar{\theta} = 0$, thus (7) implies $\bar{\beta} > \hat{\beta}$. Since $\hat{\gamma}, \bar{\gamma} \in (\frac{\pi-\alpha}{2}, \frac{\pi}{2}]$ it is easily verified that

$$\bar{L}_1 = l\frac{\sin\bar{\beta}}{\sin\bar{\gamma}} = l\frac{\sin(2\bar{\gamma} - \pi + \alpha)}{\sin\bar{\gamma}}$$

is a strictly increasing function of its argument, thus we conclude that $\bar{L}_1 > \hat{L}_1$. Noticing that $k_{pA} > 0$ and using (18) we have that $\bar{T}_1 > \hat{T}_1$. But then the relations $\bar{T}_1 = 2mg\cos\bar{\gamma}$ and $\hat{T}_1 = 2mg\cos\hat{\gamma}$ imply that $\bar{\gamma} < \hat{\gamma}$, a contradiction. One arrives to a similar contradiction supposing that $\bar{\gamma} < \hat{\gamma}$ thus we conclude that $\bar{\gamma} = \hat{\gamma}$ and $\bar{L}_1 = \hat{L}_1$ and the lemma is proved. ∎

We can now state our main stability theorem for the nonlinear crane together with the PD controllers given by the equations (9-10).

Theorem 3 *The crane with rigid cables equipped with PD controllers for both winches is globally asymptotically stable.*

Proof: Choose a sufficiently large w_0 such that, for both the initial condition and the equilibrium, $W < w_0$ with W being the function defined in (11). Define the set $\mathcal{C} = \{\mathbf{x} \mid W(\mathbf{x}) \leq w_0\}$. Using Lemma 1, we get $\dot{W} = -k_{dO}\dot{R}^2 - k_{dA}\dot{L}_1^2$. Since $\dot{W} \leq 0$, the system's trajectory stays in $\mathcal{C}$. Moreover W is bounded from below in the set $\mathcal{C}$ hence this latter set is positively invariant and compact. Lemma 2 characterizes the set $\mathcal{M} = \{\mathbf{x} \mid \dot{V}(\mathbf{x}) = 0\}$ as being a finite set consisting of the equilibrium point $\{\bar{x}, \bar{z}\}$. The claim follows by applying Theorem 2 with both previously defined sets $\mathcal{C}$ and $\mathcal{M}$ and $V = W$. ∎

Remark 1 *Notice that the model was obtained under the hypothesis that the cables were rigid and thus could transmit positive and negative forces to the winches which is normally not the case. As long as $\gamma < \frac{\pi}{2}$, T_1 is guaranteed to be positive and the force can be transmitted.*

When the cables are not rigid, they can get out of the pulleys due to the negative tension that cannot be delivered. Some extra mechanical device should be present to prevent such an event. Although this does not lead to an instability as such, the set of initial conditions that are handled properly by the controller is somewhat reduced as in the case of rigid cables.

5 Simulation study

Note that, though this controller has been successfully experimented on our reduced-size model of crane, we can only present simulation results since we do not have sensors to measure the position of the load or the angles of the cables and to record them. Such measurements should be made possible in the future by adding a camera.

The crane model is simulated using the following parameters: $m_1 = 0.2$ [kg], $J_1 = J_2 = 6.2510^{-3}$ [kg/m^2], $l = 0.35$ [m], $\alpha = 0.445$ [rad]. These parameters correspond to a 1/30 small-scale model of a real US-navy crane at disposal at the authors lab.

The equilibrium position is set to be $\bar{x} = -0.1$ [m] and $\bar{z} = -0.5[m]$. The gains have been set to $k_{p0} = 20$, $k_{pA} = 10$, $k_{d0} = 10$ and $k_{dA} = 20$.

The tuning of the gains has been done in simulation.

Note that the global stability of the regulator is not sensitive to the values of the design parameters as shown by Theorem 3.

6 Conclusion

Crane control is addressed using a simple output feedback PD controller, using only angular sensors

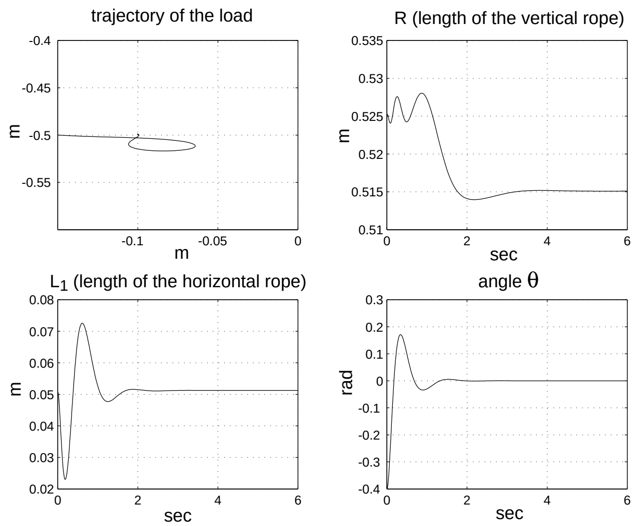

Figure 2: Closed-loop behaviour under PD control

placed at the winches. We show that it globally asymptotically stabilizes any equilibrium position under the hypothesis that the cables are rigid. Moreover, it is easy to implement and efficient if the crane model is accurate enough, or more precisely, if the frictions are satisfactorily compensated.

Note that we have not used in this work the flatness property of the crane model (see [9]) since we are only interested in equilibrium points. However, flatness might play an important role to extend this controller design in the context of tracking of trajectories that bring the load to an idle position, a question that still remains open.

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