

SiML: Sieved Maximum Likelihood for Array Signal Processing

Matthieu Simeoni^{*}, Paul Hurley[†]

^{*} École Polytechnique Fédérale de Lausanne (EPFL), Lausanne

[†] Western Sydney University (WSU), Parramatta



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Abstract— We propose the **Sieved Maximum Likelihood (SiML)** method for imaging purposes in array processing. It uses a functional data model, allowing for an **unrestricted number** of arbitrarily-shaped sources. As the likelihood problem elicits many solutions, we restrict the search space. We show that SiML results in much better accuracy than the traditional spectral-based methods.

1. A Functional Data Model

Array signal processing is concerned with the **estimation of random wavefields** in the *far-field*. The Fraunhofer equation is used to model the phenomenon:

$$Y(\mathbf{p}_i) = \int_{\mathbb{S}^2} S(\mathbf{r}) \exp \left[-j \frac{2\pi}{\lambda} \langle \mathbf{r}, \mathbf{p}_i \rangle \right] d\mathbf{r} + n_i.$$

The **amplitude function** S of the field is modelled as a complex *Gaussian random field*. We can formulate the data model more compactly using a **sampling operator**:

$$\mathbf{Y} = \Phi^* S + \mathbf{n}.$$

The data and noise are *independent* and follow a *centered, multivariate complex Gaussian distribution*. They are fully determined by their second order moments:

$$\mathbb{E}[\mathbf{Y}\mathbf{Y}^H] := \Sigma = \Phi^* \mathcal{T}_\kappa \Phi + \sigma I, \quad \sigma > 0.$$

\mathcal{T}_κ is the **covariance operator** of the random field.

2. Maximum Likelihood Function

In practice Σ is estimated as $\hat{\Sigma} = \frac{1}{N} \sum_{n=1}^N \mathbf{y}_n \mathbf{y}_n^H$. The *negative log-likelihood* of the data is given by:

$$\ell(\kappa, \sigma | \hat{\Sigma}) = \text{tr} \left[(\Phi^* \mathcal{T}_\kappa \Phi + \sigma I)^{-1} \hat{\Sigma} \right] + \ln |\Phi^* \mathcal{T}_\kappa \Phi + \sigma I|$$

Since the sampling operator is **rank deficient**, the likelihood elicits *infinitely many solutions*!

3. Sieved Maximum Likelihood

To make the likelihood problem well-posed, we resort to **sieved maximum likelihood** and assume:

$$\mathcal{T}_\kappa = \Psi R \Psi^* = \sum_{i,j=1}^M R_{ij} \psi_i \bar{\psi}_j,$$

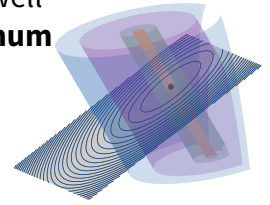
for some basis functions ψ_i . For **identifiability**, we require $M < L$.

A suitable dimension M is chosen according to a *BIC criterion*.

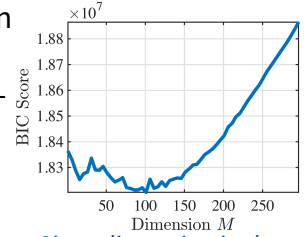
With the sieve approach, closed-form formulae are available:

$$\hat{\sigma} = \frac{\text{tr} \left[\hat{\Sigma} - G G^\dagger \hat{\Sigma} \right]}{L - M},$$

$$\hat{R} = G^\dagger \left[\hat{\Sigma} - \hat{\sigma} I \right] G^{\dagger H}.$$

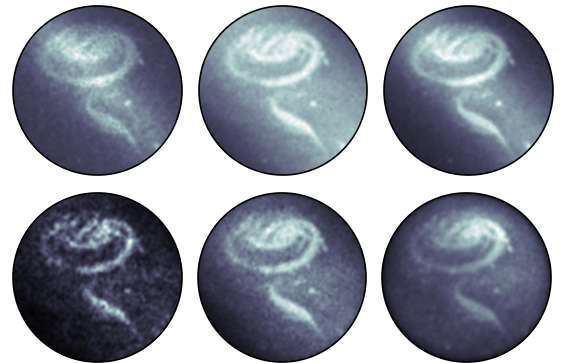


Sieving restricts the search space dimension

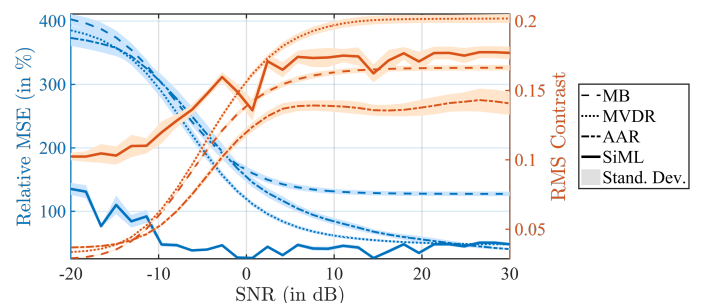


Sieve dimension is chosen with the BIC criterion

4. Results



Top: Spectral methods (DAS, MVDR, AAR); Bottom: SiML with $M=15, 101, 296$.



Performances of the different algorithms for various SNR