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EDITORIAL

Numerical Mathematics and Control

Preface to a Special Issue Dedicated to Volker Mehrmann on the Occasion of his 65th Birthday

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1 Volker Mehrmann — Academic Life and Work to Date

This special issue of the *Vietnam Journal of Mathematics* honors Prof. Dr. Volker Mehrmann on the occasion of his 65th birthday in 2020. Volker Mehrmann (Fig. 1) has been a full professor at TU Berlin since 2000 and a full professor at TU Chemnitz 1993–2000, after receiving his academic education, including the *Habilitation* in Mathematics, from Universität Bielefeld. He also held several visiting positions at universities and research institutions in the USA and Germany. At the time of this writing, he is serving as the president of the European Mathematical Society (EMS) for the term 2019–2023.

In his scientific life, Volker Mehrmann has contributed to a large variety of topics in Matrix Theory, Numerical Linear Algebra, Theory and Numerical Solution of Differential-Algebraic Equations (DAEs), with applications in several areas of engineering, especially in systems and control theory as well as in vibrational analysis. Not only has he contributed genuinely to mathematical research in these disciplines, often moving forward the research frontiers in these scientific areas, but he also possesses an impressive ability to cross interdisciplinary frontiers and to link disciplines, employing ideas from various areas in his own research.

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Fig. 1 Volker Mehrmann. ©MATHEON. Source: Math+, the Berlin Mathematics Research Center, http://www.mathplus.de/

Many of Volker Mehrmann's scientific contributions have been addressed in the edited book "Numerical Algebra, Matrix Theory, Differential-Algebraic Equations and Control Theory", dedicated to him in 2015 on the occasion of his 60th birthday [3]. The preface of this book provides an account on his scientific contributions until 2015. While still contributing to the scientific fields listed above, in recent years, Volker Mehrmann has dedicated a lot of energy and effort on popularizing an energy-based modeling approach to dynamical processes, emphasizing energy dissipation as a main principle and using the port-Hamiltonian system structure as one of the main tools. In the last few years, Volker Mehrmann and his co-workers have made significant progress in the theoretical foundation of systems having port-Hamiltonian structure (thus being passive), and in particular they have shown how to gain from this in numerical discretization schemes for simulating such systems. They have also leveraged these principles in particular in the modeling and simulation of energy transportation networks, like gas and power transport or district heating networks, as well as many other areas of engineering; see, e.g., [1, 2, 4–6]. His own contribution to this special issue also falls into this category.

Volker Mehrmann has served the scientific communities in many ways: besides his role as president of the EMS, he served as president of the International Association of Applied Mathematics and Mechanics (GAMM) 2011–2013 and as chair of the excellence center MATHEON in Berlin 2008–2016. He also served the mathematical community in many editorial positions, for instance as one of the founding Editors-in-Chief of the Electronic Journal of Linear Algebra, as a long-term Editor-in-Chief of Linear Algebra and Its Applications and as member of various editorial boards, including the one of this journal. Volker Mehrmann has obtained a lot of recognition for his selfless involvement in many efforts to further the field of mathematics and its role in interdisciplinary science as well as industrial research and development. Among others, he gave the 2015 Gauss Lecture of the German Mathematical Society (DMV) and plenary lectures at many international conferences including ICIAM 2015 in Bejing. He was awarded the 2018 SIAM W.T. and Idalia Reid Prize and in 2019 the Hans Schneider Prize in Linear Algebra. He is an elected member of acatech, the German Academy of Science and Engineering, a SIAM Fellow (class of 2011), and recipient of an ERC Advanced Grant.

Volker Mehrmann's scientific leadership is underlined by the fact that he has supervised more than 30 Ph.D. students. Several of them have become professors (full, associate, and assistant levels) at top institutions in several countries, including German, Swiss, and US universities. His international recognition is further evidenced by the fact that he has hosted numerous Humboldt fellows so far.

We have no doubt that Volker Mehrmann continues to be an active researcher in applied mathematics and will make many more important contributions to the field!

2 Contents of the Special Issue

We group the contributions to this special issue into several categories, starting with the areas closest to Volker Mehrmann's recent research focus.

2.1 Energy-Based Modeling and Port-Hamiltonian Systems

In "Computation of the Analytic Center of the Solution Set of the Linear Matrix Inequality Arising in Continuous- and Discrete-Time Passivity Analysis", Daniel Bankmann, Volker Mehrmann, Yurii Nesterov, and Paul Van Dooren derive formulas for and numerical methods to compute the analytic center of the solution set of linear matrix inequalities defining passive transfer functions. It is also shown that the analytic center has beneficial robustness properties when used to represent passive systems. Passivity of a linear system is closely related to positive realness of its transfer function. In their paper, Delin Chu and Dongmei Li derive "A Structure-Preserving Method for Positive Realness Problem in Control".

A more abstract interpretation of the energy dissipation property is employed by Anke Böttcher and Herbert Egger in "Structure Preserving Discretization of Allen–Cahn Type Problems Modeling the Motion of Phase Boundaries" in order to study the systematic numerical approximation of a class of Allen–Cahn type problems modeling the motion of phase interfaces.

An extended class of port-Hamiltonian systems is introduced in "Dirac and Lagrange Algebraic Constraints in Nonlinear Port-Hamiltonian Systems" by Arjan van der Schaft and Bernhard Maschke. This is based on replacing the Hamiltonian function by a general Lagrangian submanifold of the cotangent bundle of the state space manifold leading to a new type of constraints equations called *Lagrange algebraic constraints*.

2.2 Structured Linear and Nonlinear Eigenvalue Problems

The study of dissipative and port-Hamiltonian systems is also closely related to linear and nonlinear eigenvalue problems exhibiting special symmetries. This topic has been at the heart of Volker Mehrmann's research for more than 40 years, and thus it is not surprising that numerous contributions to this special issue fall into this category.

Pole-swapping algorithms are generalizations of bulge-chasing algorithms for generalized eigenvalue problems. Structure-preserving variants of these methods for the palindromic and alternating eigenvalue problems are derived in "Pole-Swapping Algorithms for Alternating and Palindromic Eigenvalue Problems" by Thomas Mach, Thijs Steel, Raf Vandebril, and David S. Watkins. Related to this, a new family of linearizations of palindromic matrix polynomials of odd degree is suggested by Ranjan Kumar Das and Rafikul Alam in their contribution titled "Palindromic Linearizations of Palindromic Matrix Polynomials of Odd Degree Obtained from Fiedler-like Pencils". Their construction is based on Fiedler companion matrices associated with matrix polynomials and leads to low bandwidth palindromic pencils that can be useful for numerical computations.

In his paper “On the Existence of Schur-like Forms for Matrices with Symmetry Structures”, Christian Mehl develops structured Schur-like forms. The main result combines and generalizes the two well-known results that on the one hand any normal matrix can be unitarily diagonalized and on the other hand a Hamiltonian matrix can be transformed to Hamiltonian Schur form via a unitary similarity transformation if and only if its purely imaginary eigenvalues satisfy certain conditions that involve the sign characteristic of the matrix under consideration.

2.3 Linear and Multi-linear Algebra

Another set of papers in this special issue deals with certain aspects of matrices and tensors. Two of them relate to tensor calculus. In “A Note on Nonclosed Tensor Formats”, Wolfgang Hackbusch studies the difficulties arising from the fact that some tensor formats are not closed. In particular, he discusses the question whether divergence of the parameters representing tensors within the format, tending to a border tensor outside, is uniform for all border tensors. Shmuel Friedland and Stéphane Gaubert generalize some characterizations and inequalities for the eigenvalues of nonnegative matrices, such as Donsker–Varadhan, Friedland–Karlin, and Karlin–Ost inequalities, to nonnegative tensors in their paper “Spectral Inequalities for Nonnegative Tensors and Their Tropical Analogues”. In particular, they show that the logarithm of the spectral radius of a tensor is given by an entropy maximization problem over a space of occupation measures.

Richard A. Brualdi and Geir Dahl study “Permutation Matrices, Their Discrete Derivatives and Extremal Properties”. They introduce the notion of *discrete derivative* and characterize the possible derivatives of permutations. For instance, they consider permutations with distinct derivatives, and their relationship to Costas arrays.

Functions of the adjacency matrix for a network are the topic of the paper “Orthogonal Expansion of Network Functions” by Mohammed Al Mugahwi, Omar De la Cruz Cabrera, and Lothar Reichel. For instance, such functions can shed light on the relative importance of the nodes of the graph and its overall connectivity. The authors describe several approaches for generating orthonormal polynomial expansions of such network functions, and discuss their merits for network analysis.

Recent years have seen a lot of progress on numerical methods for solving matrix equations. In “Optimality Properties of Galerkin and Petrov–Galerkin Methods for Linear Matrix Equations”, Davide Palitta and Valeria Simoncini study a general framework for solving linear matrix equations, drawing on well-known optimality properties of Galerkin and Petrov–Galerkin methods in many areas of numerical analysis.

2.4 Dynamical Systems and Control Theory

Much of Volker Mehrmann’s work has been on differential equations, in particular those with algebraic constraints, and their applications in control theory. The remaining papers in this special issue are related in one way or the other to dynamical processes and systems.

The Drazin generalized inverse appears in a number of applications including the theory of linear autonomous differential-algebraic equations. In the paper “Invariant Subspaces, Derivative Arrays, and the Computation of the Drazin Inverse”, Stephen L. Campbell and Peter Kunkel discuss its robust computation.

André Eikmeier and Etienne Emmrich study the initial value problem for a particular multi-valued differential equation in their contribution "On a Multivalued Differential Equation with Nonlocality in Time". Existence of a solution to this problem is shown via a generalization of the Kakutani fixed-point theorem.

Feedback control is a recurrent topic in Volker Mehrmann's scientific vita. The "Exponential Stability for the Schlögl System by Pyragas Feedback" is analyzed by Martin Gugat, Mariano Mateos, and Fredi Tröltzsch. The Schlögl system is governed by a nonlinear reaction-diffusion partial differential equation with a cubic nonlinearity. In their paper, the authors derive feedback laws of Pyragas-type that stabilize the system in a periodic state.

Advances in optimization theory, software, and modeling environments promote widespread application of optimization-based decision making. However, the gap between the optimization model and the real-world counterpart impedes optimality of the solution and often leads to performance losses in practice. In order to overcome this barrier, the paper "A Trust-Region Framework for Real-Time Optimization with Structural Process-Model Mismatch" by Kexin Wang, Cheng Yang, Zhijiang Shao, Xiaojin Huang, and Lorenz T. Biegler proposes to reconcile the model and the real plant with iterative parameter estimation and real-time optimization based on a trust-region framework.

The iterative rational Krylov algorithm (IRKA) is a popular approach for producing locally optimal reduced-order approximations to linear time-invariant systems w.r.t. the $\mathcal{H}_2$-norm of their transfer functions. Its convergence properties are the topic of "Revisiting IRKA: Connections with Pole Placement and Backward Stability" by Christopher Beattie, Zlatko Drmač, and Serkan Gugercin. They show that the connection with pole assignment suggests refinements to the basic algorithm that can improve convergence behavior, leading also to new choices for termination criteria that assure backward stability.

Hans Georg Bock, Jürgen Gutekunst, Andreas Potschka, and María Elena Suaréz Garcés use the interpretation as a time-stepping method and the related flow to study iterative methods to solve nonlinear least-squares problems in "A Flow Perspective on Nonlinear Least-Squares Problems". They highlight the advantages of the Gauß–Newton flow and the Gauß–Newton method from a statistical and a numerical perspective in comparison with the Newton method, steepest descent, and the Levenberg–Marquardt method, which are respectively equivalent to Newton flow forward Euler, gradient flow forward Euler, and gradient flow backward Euler.

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