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How to Market Smart Products: Design and Pricing for Sharing Markets

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ABSTRACT

This paper introduces joint product design and non-linear pricing in the context of sharing markets. Product ecosystems enable user sensing, setting the stage for the control of post-purchase consumption patterns. By varying the degree to which products can be reused and transferred among peers, a company can engineer their shareability, which together with a capacity of aftermarket control, allows for flexible non-linear pricing that involves charging for the initial purchase and for subsequent collaborative transfers separately. Using a dynamic model with heterogeneous consumers and asymmetric information, we analyze a firm's economic strategy, including ecosystem design and flexible pricing, for long-term profitability. We show that an optimal product design balances durability-driven demand and price effects. Furthermore, for any given product design a profit-maximizing non-linear pricing schedule features retail price and sharing tariff in a robustly quadratic relationship, independent of the specifics of the consumer distribution. Various extensions, relating to the interaction of the firm's policy with a sharing market and the possibility of time-varying sales distributions, are also considered.

KEYWORDS

Aftermarket control; collaborative consumption; durability design; non-linear pricing; product ecosystems; sharing economy; smart products; product design; product pricing

Introduction

By creating product ecosystems rather than mere standalone solutions, companies have gained unprecedented information transparency and post-purchase influence over the operation of their appliances. The resulting "aftermarket control," enabled by an engineered product durability and user-sensing capability, can be used—among other applications—to meter usage-transitions in sharing markets. Thus, a company is able to monetize collaborative consumption by charging product owners whenever they would like to share their products with their peers. Its revenues may be further increased by offering a flexible combination of retail price and sharing tariff to consumers with different (and for the firm unknown) expected needs. For example, a consumer with an anticipated high need for the product may prefer to pay a lower retail price in conjunction with a more expensive sharing tariff, whereas a consumer with a lower expected need would opt for a purchase contract with a more expensive retail price and a lower sharing tariff. The latter combination of prices reduces the commitment in ownership the lower-need consumer experiences (in terms of the irreversible investment into the purchase price) compared with the higher-need consumer who does not mind an elevated sharing tariff. In this paper, we analyze this flexible (non-linear) pricing scheme and ask the additional question of how the

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company would want to design the product's level of durability and user-sensing ability so as to earn an optimal profit that is sustainable in steady state.

Shareability and aftermarket control

A *shareable* product is such that it can be transferred from its owner to another agent for temporary use without significant degradation. Thus, a completely disposable product is not shareable because it would be completely degraded after a single use. All else equal, a product becomes all the more shareable the more durable it is.[1] Thus, a commodity such as toilet paper is not shareable while a computer program usually is. The company has *aftermarket control* over a shareable product if it has the ability of *user sensing* (i.e., being able to detect and recognize users) and *blocking* (i.e., being able to exclude a user). The latter is usually implied by the former. Indeed, given an agreement with the original owner about sharing the product with others,[2] the ability of user sensing is sufficient for implementing aftermarket control. Figure 1 shows the "shareability-control matrix," which categorizes products according to their respective shareability and susceptibility to aftermarket control. The matrix entries are based on intrinsic properties: for instance, a timed transportation pass is usually non-transferable due to arbitrary restrictions imposed by the issuer; yet, because it is *intrinsically* transferable (without degradation—except for loss), it does appear above the shareability midline.

From an economic point of view the dimensions of the shareability-control matrix naturally align with a variety of pricing models. For example, low shareability and low aftermarket control yield "standard retail"—at least in the absence of additional product attributes such as quality (here viewed orthogonal to product durability). When shareability increases but aftermarket control remains low, the retail-pricing model stays largely in place, but the retailer is able to charge a "sharing premium" [43], which monetizes users' additional expected net benefit from being able to rent out the product on the sharing market whenever it is not needed privately. On the other hand, when shareability remains low and aftermarket control increases, one usually obtains a "subscription model" with printers and capsule-fed

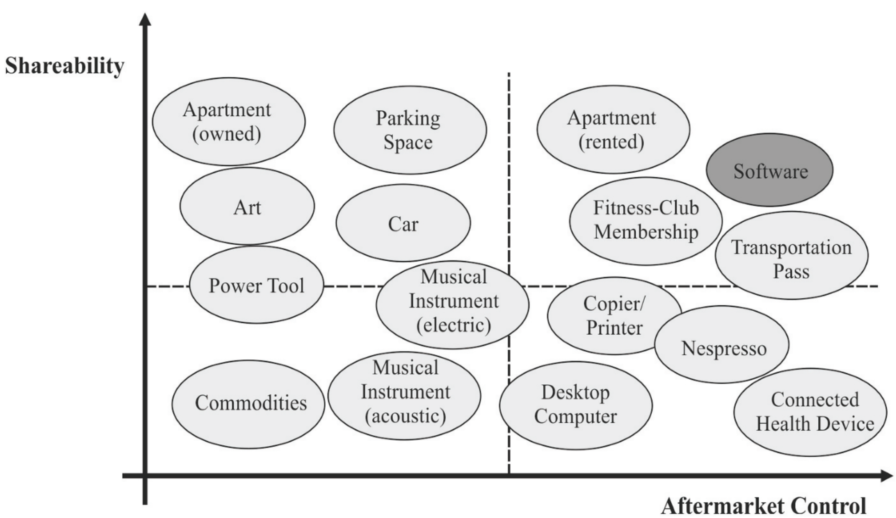

Figure 1. Shareability-control matrix.

espresso machines as two prime examples. Another very timely instance of extremely high user sensing and aftermarket control is a connected health device which enables personalized medicine but cannot usually be shared across the boundaries of the owner's (or primary user's) home. Finally, at sufficiently high levels of both, shareability and aftermarket control, the firm is able to use flexible pricing. Not only are the products sufficiently smart to support the application of a sharing tariff [39], but the firm is also able to use its retail price and sharing tariff as two separate instruments to create a menu of flexible options. By customers' selecting their individually preferred price combinations, the firm is able to distinguish the different types and commensurately extract different rents. Figure 2 summarizes the pricing models in the shareability-control matrix.

Product durability and user sensing

The very nature of a *durable* product (e.g., a car, a musical instrument, a personal computer, or a software package) is that it can be consumed over an extended period. A finite product lifetime is synonymous with a limited (or imperfect) product durability: products break or become otherwise obsolete. A consumer usually thinks of the *product lifetime* in probabilistic (*stochastic*) terms due to random product failure or imprecise foresight. From a firm's point of view durability is very much a design variable [11],[3] which in the case of its artificial reduction was referred to by Gregory [21] as "purposeful obsolescence." But a finite product lifetime can also be non-probabilistic (*deterministic*), for example in the case of a transportation pass or a magazine subscription; more subtly, the product lifetime can be finite unbeknownst to the consumer, for instance when a device (e.g., a printer) stops working as soon as an internal counter reaches a predefined number of impressions. Overall, product obsolescence (i.e., the end of a product's lifetime) may be due to physical failure, preset term expiry, or the psychological need for a substitute [21].

At the origin of the contextual awareness an appliance may have of a user's state lies the notion of remote sensing, which originally referred to observational inference without physical contact in military applications [15]. Over the past two decades, wearable

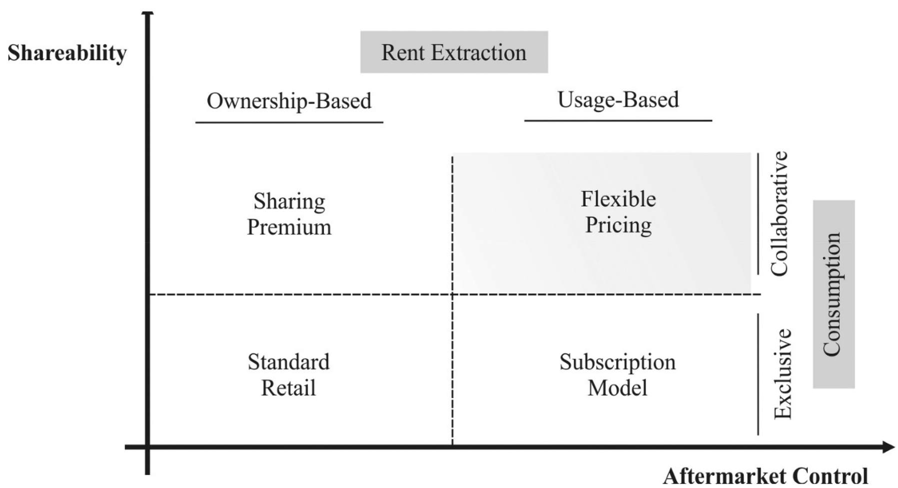

Figure 2. Pricing models in the shareability-control matrix.

computing has become increasingly able to naturally nest gadgets (belonging to the class of "cyberphysical systems" [12]) that are capable of following human movement, gestures, and physiology [33]. For the mere purpose of detecting usage transitions such tracking is quite subtle compared to the widespread biometric access-control technologies (e.g., fingerprint readers or face recognition) already deployed in today's portable phones. Even more subtle is algorithmic tracking which has been in use on portable devices for at least a decade [24]. More recently, mobile sensing technologies have also been employed for consequential classification tasks such as the measurement of relative job performance [27]. Taken together, these developments can leave us with a sense of confidence that detecting usage transitions is a fairly easy task that can be accomplished on any reasonably enabled device. For some goods, this "user sensing" may be more easily achieved than for others.

The choice of product durability and engineered user sensing determines the pricing model available for the firm's product in the shareability-control matrix. By increasing either durability or sensing capability the company can extend the scope of flexible pricing, as shown in Figure 3. Here we decompose the firm's profit-maximization problem into a product design problem (the solution to which yields an optimal level of durability) and the flexible pricing problem (the solution to which yields an optimal non-linear pricing schedule).

Literature

The need for sharing durable goods arises because of intertemporal consumption uncertainty. Consumers do not know *ex ante* whether they will need a product in any given period. Sharing markets allow for mutual insurance among the consumers whose payoffs

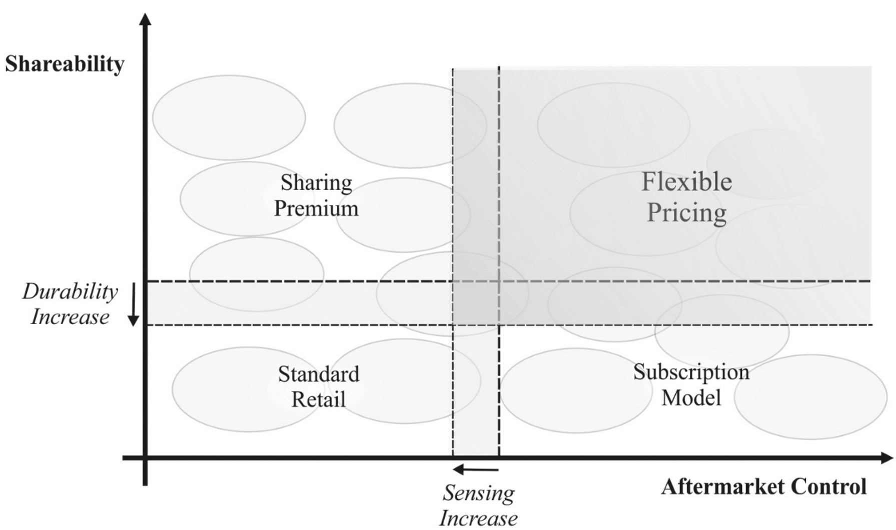

Figure 3. Durability and sensing increase the scope of flexible pricing.

depend on the realizations of their uncertain needs, much in the spirit of Arrow [3]. The sharing economy, with its many facets as laid out in Sundararajan [34] and Trenz *et al.* [36], has become the subject of active investigation [4–6, 8, 43, 44]. Generally speaking, the sharing aftermarket helps consumers adapt their investedness in the product, when faced with the binary upfront decision of becoming an owner or not. While manufacturers have largely come to accept the existence of sharing markets, a wide-scale adjustment of product design and pricing, which can be viewed as a second-order effect of sharing markets, has not yet taken place. New models for effective surplus extraction need to be found, where the firm can participate over a longer time horizon in the extraction of the unfolding consumer utility. For this a control of shareability is necessary [41]. Such aftermarket control had been previously contemplated by firms mainly through the active design of compatibility as well as complementarity, resulting in proprietary product ecosystems that lock in consumers; Adams and Brock [2] mark a beginning of that literature. Controlling the shareability of an item means to offer a one-time "license to share" at the time when the current owner has a low need for the item. Using a standard model, Weber [39] determines the optimal fixed sharing tariff to charge consumers for the right to share perfectly durable products.

Given any level of durability, we examine the construction of a profit-maximizing non-linear pricing scheme. The idea of non-linear pricing goes back to the seminal contribution by Mirrlees [28] on optimal taxation. Mussa and Rosen [29] adapted these findings for second-degree price discrimination on a continuum of consumer types. Our approach follows the general model in Weber [42], but includes a more detailed discussion on how to determine the boundary values for the firm's optimal schedule, as well as the robustness of the results. To the best of our knowledge, the application of non-linear pricing to the specifics of sharing markets (in conjunction with product design) is entirely new.

In this paper, we think of "durability" in general terms, as a scalar variable $q \in [0,1]$ that indexes the likelihood of a product's becoming obsolete through *either* failure (including predetermined expiry) *or* the fact that the consumer has been given an incentive by the firm to replace it by an updated version (as noted for cars and TVs already in the 1950s [25]). At one extreme, zero durability ($q = 0$) means a disposable product that lasts exactly one consumption period. At the other extreme, perfect durability ($q = 1$) describes a product that never fails from period to period. For intermediate $q \in (0,1)$, increases in durability mean less failure (in the sense of first-order stochastic dominance) without us worrying too much about the precise structure (e.g., stochastic or deterministic) and cause of the failure.[4] This type of approach has been used in the literature to describe firms' motivations to provide durability.

Swan [35] notes that the cheapest way for a firm to provide a consumer with a stream of service from a durable good is to refrain from artificially shortening the product's lifetime, since any action precipitates the need for the firm to provide another carrier (a new version of the product) to deliver the service. Using a simple two-period model, Bulow [9] finds that a firm may prefer to plan obsolescence, since it is unable to commit to a long product lifetime.[5] Yet, this logic seems flawed in a regime with going concern (and infinite horizon) where overlapping generations of consumers may well have rational expectations for durability, leading back to Swan's logic, at least when the per-unit production cost is significant; when marginal production cost is low, the firm may be best off to provide completely disposable products so as to artificially disable trading in an aftermarket, as shown recently by Razeghian and Weber [32]. We opt for a similar approach to model durability, namely as the probability of the product's being available in the next period. The optimization of product durability in the presence of flexible pricing and a sharing market has not been investigated before.

Model

We consider a dynamic economy with a continuum of infinitely lived consumers (also referred to as "agents"); see Appendix 2 for a summary of notation. Without loss of generality, the total number of agents is normalized to 1. This number can be scaled as needed without affecting any of the results. At the beginning of each time period $t \in \{0, 1, \ldots\}$, a consumer of type $\theta \in \Theta = [0,1]$ observes the realization of his random binary need state $\tilde{s}_t(\theta) \in \{0, 1\}$ for a given durable good (e.g., a power tool, a computer, a piece of software, or a car), which is distributed so that

$$\mathbb{P}(\tilde{s}_t(\theta) = 1) = \theta, \tag{1}$$

for all $\theta \in \Theta$, without any correlation across periods. When the need state is high (i.e., $s_t(\theta) = 1$), the agent has a consumption utility of $v > 0$ for the item; on the other hand, when the need state is low (i.e., $s_t(\theta) = 0$), his consumption utility in the current period is zero. To focus our attention on the sharing-related type-attribute θ, we assume that v is identical across all consumers and common knowledge, and that the consumer types θ are distributed on the type space Θ according to the smooth cumulative distribution function (cdf) F: $\Theta \to [0,1]$, such that its derivative $f = F'$ is increasing on Θ. This means that almost all types are available with positive probability density. In general, the durability $q \in [0,1]$ of the firm's product is not necessarily perfect. As in Razeghian and Weber [32], the product fails in any given period with probability $1 - q$. Hence, for $q = 0$ the good becomes completely disposable, while for $q = 1$ it never breaks. We assume that product durability can be chosen by the firm and that the marginal production cost $c(q)$ increases in q, at least weakly.

Payoffs

Consumers

When his need state is high, an agent of type θ can get access to the product by *either* purchasing it from a monopolistic firm at the retail price r *or* borrowing it on a peer-to-peer market at the sharing price p. Once the agent becomes an owner by purchasing the item, he remains an owner until the item breaks, which—as long as $q < 1$—happens with probability 1 at some finite time in the future. As an owner, the agent can use the item when in need or rent it out on the sharing market for a net revenue of $p - \tau$, where the "sharing tariff" τ is imposed by the firm for authorizing the transfer of usership for one period, as in Weber [43]. Hence, any agent can find himself in four possible states, being either an owner or a non-owner, and at the same time, being in either a high-need or a low-need state. The per-period discount factor for all agents is $\delta \in (0,1)$.[6] With a functioning sharing market, it is clear that agents will consider acquiring ownership of the product only if they are non-owners in a high-need state. In that state, some agents may still prefer renting the item from the sharing market rather than committing to ownership. Let

$$\hat{p}(\theta) \overset{\Delta}{=} p - \tau(\theta), \tag{2}$$

denote the effective transaction price for lenders in the sharing market. With this, we consider first non-owners and then owners. The payoffs are derived recursively, considering the consumption value in the current period plus the discounted future consumption value starting next period.

Non-owners

The (discounted) value for non-owners in the low-need state is

$$U_0 = 0 + \delta\,(\theta U_1 + (1-\theta)U_0), \tag{3}$$

where "0" marks the absence of any consumption benefit in the current period. For non-owners in the high-need state the value is

$$U_1 = \max\{A, B\}, \tag{4}$$

where the agent chooses between the expected non-ownership value,

$$A = v - p + \delta\,(\theta U_1 + (1-\theta)U_0), \tag{5}$$

obtained via access on the sharing market, and the expected ownership value,

$$B = v - r + \delta\,(1-q)(\theta U_1 + (1-\theta)U_0) + \delta\,q\,(\theta V_1 + (1-\theta)V_0), \tag{6}$$

obtained via purchase of the product from the retailer. In Equation (5), the expected non-ownership value A is the agent's consumption value v minus the sharing price p (for access to the product) plus the expected value of being a non-owner ($\theta\ U_1 + (1-\theta)\ U_0$), discounted to the present by multiplication with δ. By contrast, in Equation (6) the agent's expected ownership value B is equal to current value $v - r$, where r is the purchase price, plus the expected present value of future ownership. The latter is $\delta\ (\theta\ U_1 + (1-\theta)U_0)$ in the case of failure (which happens with probability $1-q$), and it is $\delta\ (\theta\ V_1 + (1-\theta)V_0)$ in the case of non-failure (which happens with probability q). When the product does not fail, the agent remains an owner (with expected value of V_0 or V_1 in the next period, depending on the need-state realization then; see the following section), whereas upon product failure the agent returns to being a non-owner (with expected value of U_0 or U_1 in the next period, depending on the need-state realization). Naturally, all expected payoffs discussed thus far (i.e., U_0, U_1, A, B) depend on the consumer type θ.

Owners

The (discounted) value for owners in the low-need state is

$$V_0 = \hat{p} + \delta\,(1-q)(\theta U_1 + (1-\theta)U_0) + \delta\,q\,(\theta V_1 + (1-\theta)V_0), \tag{7}$$

which is equal to the expected non-ownership payoff B, except that in the current period the owner obtains—by Equation (2)—the effective transaction price $\hat{p} = p - \tau$ for lending the item out on the sharing market, instead of the current value $v - r$ in Equation (6) that a non-owner would obtain. In the high-need state, an owner's expected value becomes

$$V_1 = v + \delta\,(1-q)(\theta U_1 + (1-\theta)U_0) + \delta\,q\,(\theta V_1 + (1-\theta)V_0), \tag{8}$$

which differs from the expected non-ownership value B only by the current value, as owners enjoy the consumption value v whenever their need-state is high (so they actually want to use their product) and provided the product has not yet failed. Analogous to the non-ownership payoffs above, the ownership payoffs (i.e., V_0, V_1) also depend on the consumer type θ.

Firm

The firm's (non-negative) marginal cost $c(q)$ for delivering a new product to a purchaser depends on the durability q. It is such that $c(\cdot)$ is smooth, increasing, and convex, with $c(0) < v$. Clearly, if for some quality the firm's marginal cost reaches the consumption value (v), it is best not to sell the product at all. By introducing the "maximum viable durability,"

$$\bar{q} \equiv \sup\{q \in [0,1] : c(q) < v\}, \quad (>0) \tag{9}$$

we can—without any loss of generality—restrict attention to the (non-empty) durability interval

$$\mathcal{Q} = [0, \bar{q}]. \tag{10}$$

Thus, by construction, given any product design q in the durability interval $\mathcal{Q}$, the marginal production cost $c(q)$ does not exceed the high-need agents' willingness-to-pay v for one-period access to the product. Let $r(\theta)$ be the retail price offered to a consumer of type θ, and let $\tau(\theta)$ be the sharing tariff offered to that same type. Then the firm's expected steady-state per-period profit for serving a given participating type θ is

$$\pi(r(\theta), \tau(\theta), \theta|q) = \bar{y}_1(\theta|q)(r - c(q)) + y_0(\theta|q)\tau, \quad q \in \mathcal{Q}, \tag{11}$$

where $\bar{y}_s$ (resp., y_s) denotes the steady-state probabilities of a non-owner (resp., an owner) being in the need state $s \in \{0,1\}$. The firm needs to balance two streams of benefit, namely its net income $\bar{y}_1(\theta|q)\ (r - c(q))$ from retail and its receipts $y_0(\theta|q)\ \tau$ from validating sharing transactions. A tradeoff arises because, all else equal, an increase in product durability would decrease steady-state retail sales as fewer products break, while at the same time the rise in reliability would increase the number of product owners in steady state and thus also the number of products available on the sharing market because they are temporarily not needed.

Consumer choice

What consumers would like to do at any point in time (e.g., purchase the product or participate in the sharing market) depends on the respective realizations of their random needs. An agent's type θ defines the base rate of high-need realizations according to Equation (1). The agent's equilibrium payoff at time t is characterized by one of four scenarios he can find himself in, being either an owner or a non-owner, and in either a high-need or a low-need state. Being a non-owner now, could have resulted *either* from not ever wanting to buy the product (for low-θ types) *or* from having been an owner in the last period whose item failed after use (for high-θ types). Important for the firm's problem are the non-owners' payoffs in the high-need state (U_1), as the non-owners are the only agents who would be candidates for purchasing the product.

An agent in this scenario obtains *either* $U_1 = A$ in Equation (5) if he prefers not to purchase the item, *or* $U_1 = B$ in Equation (6) if he prefers to purchase the item. Thus, solving Equations (3), (4), (7), and (8), together with *either* (5) (for $U_1 = A$) *or* (6) (for $U_1 = B$), it is straightforward to obtain the consumers' state-contingent payoffs in a stationary regime.

Lemma 1

Consider a stationary regime in the sharing economy. Non-owners in the high-need state obtain the expected payoff $U_1 = \max\{A, B\}$, *with*

$$A(\theta) = \left(1 + \frac{\delta\theta}{1-\delta}\right)(v - p), \tag{12}$$

when choosing product access via the sharing market, and

$$B(r, \tau, \theta) = v - r + \frac{\delta}{1-\delta}\left(\theta v + (1-\theta)\hat{p} - \frac{(1-q)(\theta r + (1-\theta)\hat{p})}{1-(1-\theta)\delta q}\right)\Bigg|_{\hat{p}=p-\tau}, \tag{13}$$

when choosing to purchase the product from the firm.

All proofs are provided in Appendix 1. The preceding result specifies non-owners' payoffs A in the high-need state when they do not purchase the product and payoffs B when they do purchase it. Thus, the firm's retail demand consists of exactly those consumer types θ for whom B exceeds A at least weakly. The corresponding inequality appears in the firm's pricing problem below, as individual-rationality constraint (IR).

Note that Lemma 1 is also important for the firm's product design problem. Indeed, a completely disposable product reduces the ownership payoffs to

$$B|_{q=0} = \left(1 + \frac{\delta\theta}{1-\delta}\right)(v - r) = \left(\frac{v-r}{v-p}\right)A.$$

That is, provided $r = p$ (which is the only equilibrium when $q = 0$) any consumer's payoff from a completely disposable product equals the payoff of not purchasing the product at all.

On the other hand, a perfectly durable product provides the ownership payoff of

$$B|_{q=1} = v - r + \frac{\delta}{1-\delta}\ (\theta v + (1-\theta)\hat{p}).$$

Thus, the value of perfect durability is obtained as the difference between these two values:

$$B|_{q=1} - B|_{q=0} = \frac{\delta}{1-\delta}\ (\theta r + (1-\theta)\hat{p}).$$

High likelihood types prefer durable products because they avoid having to purchase the item again, whereas low likelihood types expect extra payoff from being able to rent out a perfectly durable item on the sharing market in any low-need state in the future.

Flexible pricing

By offering different combinations of retail price $r(\theta)$ and sharing tariff $\tau(\theta)$ to different consumer types θ, the firm is able to increase its expected revenues. A side-benefit is that it is able to separate the different consumer types. While we do provide the precise mathematical formulation of the firm's pricing problem below, the details of how to set up this problem have been relegated to Appendix 1.1, so that we can focus here on a discussion of the underlying intuition. The general idea is for the firm to provide sufficient incentives to each purchaser of a new product to reveal his true type θ. Given this detailed knowledge about each consumer, it is not possible for the firm to extract the entire surplus, e.g., by charging a retail price exactly equal to the consumers' expected benefits $B - A$, as this would annihilate any incentives to reveal the private type information truthfully. We first note that a candidate price schedule (r, τ) implies a set of participating types $\Theta_0 \subset \Theta$, whereby $\theta \in \Theta_0$ if and only if

$$B(r(\theta), \tau(\theta), \theta) \geq A(\theta), \tag{IR}$$

with A and B as in Lemma 1. The following result characterizes that participation set as an interval of the form $\Theta_0 = [\theta_0,1]$, which means that all consumer types above a certain "marginal type" θ_0 participate.

Lemma 2

Let $r' \leq 0 \leq \tau'$. Without loss of generality,[7] *the participation set is of the form $\Theta_0 = [\theta_0,1]$, with $(r_0, \tau_0) = (r(\theta_0), \tau(\theta_0))$ and*[8]

$$\theta_0 = \left[1 - \frac{p - (1-\delta q) r_0}{\delta q \tau_0}\right]_{[0,1]}, \tag{14}$$

as long as $q\tau_0 > 0$. For $q = 0$, it is $\theta_0 = \mathbf{1}_{\{r_0 > p\}}$; for $\tau_0 = 0$, it is $\theta_0 = \mathbf{1}_{\{r_0 > p/(1-\delta q)\}}$.

The marginal type θ_0 is fully determined by the starting point (r_0, τ_0) of the firm's schedule (r, τ). Note also that $\theta_0 = 1$ effectively corresponds to an empty participation set, as its (Lebesque-) measure vanishes in that case. This means that whenever the firm is choosing a price schedule that is too expensive even for the most enthusiastic consumer type, without loss of generality one can set

$\theta_0 = 1$ to obtain zero profits, the result of no participation whatsoever. Note also that Lemma 2 is consistent with the fact that whenever a type decides to purchase the firm's product, all higher types also purchase. Thus, the type space is partitioned into two disjoint convex sets of buyers (Θ_0) and non-buyers ($\bar{\Theta}_0 = \Theta \setminus \Theta_0$).

The participating agents' truth-telling constraint requires the first-order necessary optimality condition for maximizing the purchase utility $B(r(\hat{\theta}), \tau(\hat{\theta}), \theta)$ with respect to the announcement $\hat{\theta}$ (so as to obtain the contract $(r(\hat{\theta}), \tau(\hat{\theta}))$ to be satisfied at $\hat{\theta} = \theta$, resulting in the incentive-compatibility constraint

$$r'(\theta) = -\left(\tfrac{(1-\theta)\delta q}{1-\delta q}\right)\tau'(\theta), \quad \theta \in \Theta_0. \tag{IC}$$

Using the second-order necessary optimality condition for truth-telling implies the implementability constraint,

$$\tau'(\theta) \geq 0, \quad \theta \in \Theta_0. \tag{M}$$

That is, for the schedule (r, τ) to be implementable, the firm's sharing tariff τ needs to be non-decreasing. By virtue of (IC) the sharing-tariff schedule implies the retail-price schedule r on the participation set Θ_0, up to a constant. Introducing the control variable

$$u = \tau'$$

in the class of bounded measurable functions, we can now formulate the firm's *pricing problem* as an optimal control problem of the form

$$V(r, \tau, \theta_0|q) = \int_{\theta_0}^{1} \pi(r, \tau, \theta|q) dF(\theta) \;\; \rightarrow \;\; \max_{r,\tau,\theta_0}, \tag{*}$$

subject to $r(\theta_0) = r_0$, $\tau(\theta_0) = \tau_0$, Equation (14), and

$$r'(\theta) = -\left(\frac{(1-\theta)\delta q}{1-\delta q}\right)u(\theta), \tag{15}$$

$$\tau'(\theta) = u(\theta), \tag{16}$$

$$u(\theta) \in [0, \rho], \tag{17}$$

$$\theta \in [\theta_0, 1]. \tag{18}$$

Equations (15) and (16) encapsulate the incentive-compatibility constraint (IC) and the definition of the control u in Equation (16) as the slope of the sharing-tariff schedule. Together these two equations define the evolution of the firm's pricing schedule $(r, \tau) : \Theta \rightarrow \mathbb{R}^2$ as a function of the consumer type θ. Equation (17) incorporates the implementability constraint (M) by requiring a sign-semidefinite and bounded control, which by

Equation (16) means that the slope of the sharing tariff τ is non-negative and bounded. The positive constant ρ denotes the upper bound for the control, and we assume that[9]

$$\rho < \infty. \tag{19}$$

The boundedness of the control variable is usually required to apply standard optimality conditions. Finally, Equation (18) restricts the firm's attention to types in the participation set Θ_0. The pricing problem is solved for any given product design $q \in \mathcal{Q}$; the following result notes that the resulting family of pricing problems can indeed be solved.

Lemma 3

There exists a solution $(r^*, \tau^*, \theta_0^*)$ (together with an optimal control u^*), defined on $\Theta_0^* = [\theta_0^*, 1]$, to the optimal control problem (*) subject to Equations (14)-(18).

The solution of the (steady-state) pricing problem will be obtained in the next section, after a discussion of the expected stationary sales as a function of consumer type and product design. The (by Lemma 3 guaranteed) existence of a solution, together with a unique pricing schedule that satisfies a complete set of necessary optimality conditions, as developed in the following section, instills the necessary confidence that indeed a solution to the firm's pricing problem has been found.

Steady-state sales distribution

In steady state, the number of potential buyers, for any given consumer type $\theta \in \Theta_0$, is constant. It is composed of those non-owners who are in a high-need state (i.e., those who observed a realization $s_t(\theta) = 1$ at the beginning of the current time period t). Current non-owners may have been owners (in the previous period) whose items failed (which happens with probability $1 - q$). On the other hand, they may also have been non-owners. In order to find the steady-state distribution of buyers (corresponding to the firm's sales) we need to determine the stationary distribution of the underlying Markov process with state transitions from non-owners (in low- or high-need state) to owners (in low- or high-need state). By construction, a non-owner of type $\theta \in \Theta_0$ in a high-need state will purchase the product and therefore transition to being an owner. The corresponding Markov transition matrix for the shifts among the four possible states is given by

$$\mathbf{P} = \begin{bmatrix} 1-\theta & \theta & 0 & 0 \\ (1-\theta)(1-q) & \theta(1-q) & (1-\theta)q & \theta q \\ (1-\theta)(1-q) & \theta(1-q) & (1-\theta)q & \theta q \\ (1-\theta)(1-q) & \theta(1-q) & (1-\theta)q & \theta q \end{bmatrix}. \tag{20}$$

The stationary distribution $\mathbf{y} = (\bar{y}_0, \bar{y}_1, y_0, y_1)^\top$ is such that $\mathbf{y}^\top \mathbf{P} = \mathbf{y}^\top$, which yields

$$\mathbf{y}(\theta|q) = \left(\frac{(1-\theta)(1-q)}{1-(1-\theta)q}, \frac{\theta(1-q)}{1-(1-\theta)q}, \frac{(1-\theta)\theta q}{1-(1-\theta)q}, \frac{\theta^2 q}{1-(1-\theta)q} \right)^\top, \tag{21}$$

as the steady-state probabilities p_s of being a non-owner or owner in the states s ∈ {0,1}. As a result, the probability of a random agent in the current period of the given type $\theta \in \Theta_0$ to be a buyer is $\bar{y}_1$, and to be a lender it is y_0. For perfectly durable goods (when q = 1) it is $\mathbf{y} = (0, 0, 1-\theta, \theta)^\top$, while for completely disposable products (when q = 0) it is $\mathbf{y} = (1-\theta, \theta, 0, 0)^\top$.

Optimal pricing schedule

We now develop optimality conditions for the firm's pricing problem and use them to derive a full characterization of the solution, resulting in a simple quadratic schedule. Eliminating the consumer types as independent variable yields a so-called "type-free schedule" that can be directly used by the firm to advertise its flexible pricing to consumers. We then turn to identification and robustness issues.

Optimality conditions

In order to solve the firm's variational problem (*) subject to the constraints (14)-(18) we use the Pontryagin maximum principle (PMP) as necessary optimality condition [31, 45].[10] For this, we first introduce the Hamiltonian

$$H(r, \tau, u, \theta, \psi|q) = \pi(r, \tau, \theta|q)f(\theta) - \psi_r\left(\frac{(1-\theta)\delta q}{1-\delta q}\right)u + \psi_\tau u,$$

where $\psi = (\psi_r, \psi_\tau) : [\theta_0, 1] \to \mathbb{R}^2$ denotes an adjoint variable (or "co-state"), which quantifies the "shadow value" of changes in the schedule (r, τ) as a function of the independent variable θ (corresponding to the consumer's type). The components of the co-state (ψ_r and ψ_τ) capture the unit volumes of retail sales and sharing authorizations, respectively; see Equations (25) and (26). Intuitively, the Hamiltonian represents the firm's profit flow (π f) at the current type θ plus a shadow profit $(\psi \cdot (r', \tau') = \psi_r\, r' + \psi_\tau\, \tau')$ related to the marginal evolution of the schedule in the direction of increasing types as specified in Equations (15) and (16). Applying the PMP, the firm's optimal pricing schedule (r^*, τ^*), given an optimal marginal type θ_0^*, must satisfy the following necessary optimality conditions.

Theorem 1 (PMP)

For a given durability $q \in \mathcal{Q}$ let $(r^*, \tau^*, \theta_0^*)(\cdot|q)$, together with the optimal control $u^*(\cdot|q)$ be a solution to the optimal control problem (*) subject to (14)-(18). Then there exists an absolutely continuous adjoint variable $\psi(\cdot|q) = (\psi_r, \psi_\tau)(\cdot|q)$, defined on $\Theta_0^* = [\theta_0^*, 1]$, such that the following optimality conditions are satisfied:

(i) Maximality:

$$u^*(\theta|q) \in \arg\max_{u\in[0,\rho]} H(r^*(\theta|q), \tau^*(\theta|q), u^*(\theta|q), \theta, \psi(\theta|q)|q), \quad \theta \in \Theta_0^*; \tag{22}$$

(ii) Adjoint Equation:

$$(\psi'_r, \psi'_\tau)(\theta|q) = -\left(\frac{\partial H(r^*(\theta|q), \tau^*(\theta|q), u^*(\theta|q), \theta, \psi(\theta|q)|q)}{\partial(r,\tau)}\right), \quad \theta \in \Theta_0^*; \tag{23}$$

(iii) Transversality:

$$(\psi_r(1|q), \psi_\tau(1|q)) = 0, \quad q \in \mathcal{Q}. \tag{24}$$

Along an optimal schedule (r^*, τ^*) the Hamiltonian must be maximal, as a function of the control variable u, which implies a "maximality" condition. The change of the adjoint variable (ψ_r, ψ_τ) in the type direction corresponds to the (negative) gradient of the Hamiltonian in the direction of the "state" (r, τ),[11] which implies the "adjoint equation." Consistent with the fact that at the upper end of the participation set $\Theta_0 = [\theta_0, 1]$ (i.e., at $\theta = 1$) there cannot be any shadow profit related to the marginal evolution of the schedule (which has reached its limit), the adjoint variables must vanish for the highest type. This provides an intuitive justification of the "transversality condition."

The three optimality conditions (22)-(24) provided by Theorem 1 need to be supplemented by appropriate boundary conditions so as to fix the initial values $r_0^*(q) \triangleq r^*(\theta_0^*|q)$ and $\tau_0^*(q) \triangleq \tau^*(\theta_0^*|q)$, as well as an optimality condition for the optimal type threshold $\theta_0^*(q)$. While the boundary conditions can be found based on economic viability of the sharing market in conjunction with the individual rationality in Equation (14), the marginal type $\theta_0^*(q)$ will be derived by subsequent global optimization with respect to the type threshold.

Solution

We first use the necessary optimality conditions in Theorem 1 to derive the parametrized "shape" of the firm's optimal schedule $(r^*, \tau^*)(\cdot|q)$, and then pin down the unique solution by an initial condition for the schedule, followed by a solution to the problem of finding the optimal marginal type $\theta_0^*(q)$.

Shape ($(r^*, \tau^*)(\cdot|q)$)

Considering the transversality condition (24) at the boundary $\theta = 1$, the adjoint Equation (23) yields closed-form expressions for the adjoint variables:

$$\psi_r(\theta|q) = \int_\theta^1 \bar{y}_1(\vartheta|q)dF(\vartheta), \tag{25}$$

and

$$\psi_\tau(\theta|q) = \int_\theta^1 y_0(\vartheta|q)dF(\vartheta), \tag{26}$$

for all $\theta \in \Theta_0^*$. This provides the economic intuition of the adjoint-variable components, ψ_r as unit volume of retail sales (represented by the aggregate number of non-owners in the high-need state) and ψ_τ as unit volume of sharing authorizations (represented by the aggregate number of owners in the low-need state), both defined on the (hypothetical) participation set [θ, 1]. Because of the linearity of the Hamiltonian in u, the maximality condition (22) implies that *either* the control is singular (when $\partial H/\partial u = 0$) *or* the control is extremal (when $\partial H/\partial u \neq 0$).[12] The possibility of a singular control can be excluded because neither the schedule (r, τ) nor the control u feature in the gradient $\partial H/\partial u$, which we denote by

$$\eta(\theta|q) \triangleq \frac{\partial H(r^*(\theta|q), \tau^*(\theta|q), u, \theta, \psi(\theta|q)|q)}{\partial u} = \psi_\tau(\theta|q) - \psi_r(\theta|q)\left(\frac{(1-\theta)\delta q}{1-\delta q}\right). \tag{27}$$

Consistent with the sign of η, we obtain an extremal (or "bang-bang") optimal control,[13]

$$u^*(\theta|q) = \rho\, \mathbf{1}_{\{\eta(\theta|q)\geq 0\}}, \quad \theta \in \Theta_0^*(q), \tag{28}$$

which is *either* equal to the maximum control value ρ (whenever $\eta(\theta|q) \geq 0$) *or* equal to zero (whenever η(θ) < 0). For any durability $q \in \mathcal{Q}$ the optimal control $u^*(\cdot|q)$ lies on the boundary of the control-constraint set [0, ρ]. Combining the value of the control bound in Equation (19) with the optimal slope $u^*(\theta|q)$ of the sharing tariff in Equation (28), we can use Equation (16) to obtain

$$\tau^*(\theta|q) = \tau_0^*(q) + \rho\left(\sigma(\theta|q) - \sigma(\theta_0^*|q)\right), \tag{29}$$

where

$$\sigma(\theta|q) \triangleq \int_0^\theta \mathbf{1}_{\{\eta(\vartheta|q)\geq 0\}} d\vartheta \in [0, 1], \quad \theta \in \Theta, \tag{30}$$

denotes the number of types (below θ) for whom the slope of the sharing tariff is best positive (i.e., where the gradient η is non-negative). For example, if $\eta(\theta|q) \geq 0$ for all θ ∈

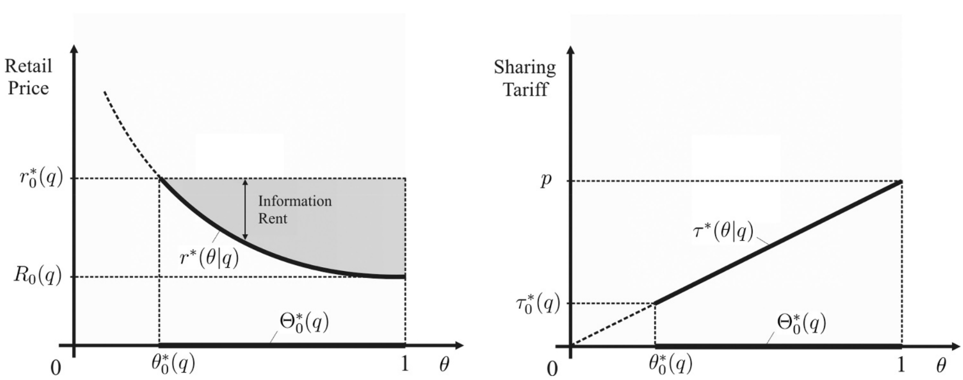

Figure 4. Optimal schedules of retail price $r^*(\theta|q)$ and sharing tariff $\tau^*(\theta|q)$, for $\theta \in \Theta_0^*(q)$.

Θ, then $\sigma(\theta|q)\equiv\theta$, so that the optimal sharing tariff becomes affine in the customer type, i.e., $\tau^*(\theta|q)\equiv\tau_0^*(q)+p$; see Figure 4.

By virtue of Equation (15), the firm's optimal retail-price schedule becomes

$$r^*(\theta|q)=r_0^*-\frac{\delta pq}{1-\delta q}\left(\left(\sigma(\theta|q)-\sigma(\theta_0^*|q)\right)-\left(\hat{\sigma}(\theta|q)-\hat{\sigma}(\theta_0^*|q)\right)\right), \tag{31}$$

where

$$\hat{\sigma}^*(\theta|q)\triangleq\int_{\theta_0^*}^{\theta}\mathbf{1}_{\{\eta(\vartheta|q)\geq 1\}}\vartheta d\vartheta, \quad \theta\in\Theta, \tag{32}$$

is an average consumer type (below θ) for whom the gradient η is non-negative.

Initial value $((r_0^*,\tau_0^*))$

By Equation (16) the optimal sharing tariff $\tau^*(\cdot|q)$ increases (piecewise) linearly at the rate ρ. Its largest value must therefore be attained at the highest type $\theta = 1$. From Lemma 2 we know that this type always participates (without loss of generality). Hence, the sharing tariff $\tau^*(\theta|q)$ needs to be equal to the highest extractable transaction value p for the highest type and, therefore

$$\tau^*(1|q)=p. \tag{33}$$

The highest type owner does not mind paying a sharing tariff that leaves nothing on the table (corresponding to a zero effective transaction price $\hat{p}$), because he needs the item in each period with probability $\theta = 100\%$, such that the contingency of wanting to rent it out on the sharing market never arises. From Equations (29) and (33) we obtain the initial value of the sharing tariff:

$$\tau_0^*(q)=\left(1-\sigma(1|q)+\sigma(\theta_0^*|q)\right)p. \tag{34}$$

However, we can then combine the initial value in Equation (34) with the expression for the marginal type in Equation (14) to obtain the initial value of the retail-price schedule:

$$r_0^*(q)=\left(\frac{1-(1-\theta_0^*)(1-\sigma(1|q)+\sigma(\theta_0^*|q))\delta q}{1-\delta q}\right)p. \tag{35}$$

Marginal type (θ_0^*)

The firm's profit as a function of the marginal type θ_0^* is

$$V^*(q)=\int_{\theta_0^*(q)}^{1}\left(\bar{y}_1(\theta|q)(r^*(\theta|q)-c(q))+y_0(\theta|q)\tau^*(\theta|q)\right)dF(\theta), \quad q\in\mathcal{Q}. \tag{36}$$

Thus, using Equations (15) and (16) together with the Leibniz rule, it is

$$\frac{dV^*(q)}{d\theta_0^*} = -\pi(r_0^*, \tau_0^*, (\theta_0^*|q)f(\theta_0^*) + \frac{\delta pq}{1-\delta q}\left(1-\sigma(1|q)+\sigma(\theta_0^*|q)\right)\psi_r(\theta_0^*|q) \stackrel{\Delta}{=} g(\theta_0^*|q). \tag{37}$$

The preceding equation illustrates a tradeoff when increasing the marginal type θ_0^* (which decreases participation): on the one hand, it decreases the profit flow (represented by the term – π *f*); but, on the other hand, it increases the firm's retail-price schedule (represented by the second term) while the firm's sharing-tariff schedule remains unaffected. Thus, the effect of making the participation set smaller (e.g., by charging larger retail prices) must be balanced against the consequence that the firm needs to capture its two revenue streams from less customer types.

When searching for the profit-maximizing marginal type $\theta_0^* \in \Theta = [0, 1]$, clearly $\theta_0^* = 1$ can be excluded, as it leads to zero profit for the firm. For an interior marginal type $\theta_0^* \in (0, 1)$, optimality requires that the standard first-order necessary optimality condition (also known as Fermat's Lemma [45, p. 247]) holds, that is,

$$\theta_0^* \in (0, 1) \;\Rightarrow\; g(\theta_0^*|q) = 0.$$

Alternatively, $\theta_0^* = 0$ may be optimal, resulting therefore in the "complementary slackness" condition,

$$\theta_0^* \, g(\theta_0^*|q) = 0. \tag{38}$$

Weber [40] provides necessary and sufficient optimality conditions for the underlying global optimization problem on an interval, which can easily be solved numerically by simply tracking V^* for $\theta_0^* \in [0, 1)$, so as to choose

$$\theta_0^*(q) \in \arg\max_{\theta_0 \in \Theta} \pi(r^*(\theta_0|q), \tau^*(\theta_0|q), \theta_0|q). \tag{39}$$

Summary

The following result condenses the preceding developments regarding a full characterization of the solution to the firm's pricing problem.

Theorem 2 (Optimal Pricing Schedule)

Let ρ = p. A solution $(r^*, \tau^*, \theta_0^*)$ to the firm's pricing problem (*), subject to Equations (14)-(18), is given by

$$r^*(\theta|q) = \frac{p - \left((1-\theta_0^*)\left(1-\sigma(1|q)+\sigma(\theta_0^*|q)\right) + \left(\sigma(\theta|q)-\sigma(\theta_0^*|q)\right) - \left(\hat{\sigma}(\theta|q)-\hat{\sigma}(\theta_0^*|q)\right)\right)\delta pq}{1-\delta q}, \tag{40}$$

and

$$\tau^*(\theta|q) = (1 - \sigma(1|q) + \sigma(\theta|q))p, \tag{41}$$

for all participating consumer types $\theta \in \Theta_0^*(q)$. The optimal marginal type θ_0^* is characterized by Equation (39)*, and it must satisfy the necessary optimality condition (38).*

To compute the solution to the firm's pricing problem in practice, one would usually start by determining the adjoint variable ψ_r in Equation (25) [using the expression for $\bar{y}_1$ in Equation (21)], followed by η in Equation (27), σ in Equation (30), and $\hat{\sigma}$ in Equation (32), to be able to obtain the gradient g in Equation (37). Then, one would determine all candidates for the optimal marginal type θ_0^* based on the necessary optimality condition (38), to satisfy Equation (39).[14] Finally, Equations (40) and (41) yield the firm's optimal pricing schedule.

Remark

While the expressions in Theorem 2 may seem complicated, they tend to belie the fact that the shape of the optimal schedule is quite simple, which can be seen by considering the special case where $g(\theta|q) \geq 0$ for all $\theta \in \Theta$, when $\sigma(\theta|q) \equiv \theta$ and $\hat{\sigma}(\theta|q) \equiv \theta^2/2$. Thus, in that case

$$r^*(\theta|q) = \frac{p}{1-\delta q}\left(1 + \frac{\delta q}{2}\left(\theta^2 - 2\theta + (\theta_0^*)^2\right)\right), \quad \theta \in \Theta_0^*, \tag{42}$$

and

$$\tau^*(\theta|q) = p\ \theta, \tag{43}$$

for all consumer types $\theta \in \Theta_0^*(q)$; see Figure 4. The quadratic shape of the optimal price schedule is driven by the fact that the slope of the sharing-tariff schedule $\tau^*(\theta|q)$ is constant and maximal ($\rho = p$, so the highest type $\theta = 1$ gets no surplus from a sharing transaction). This implies, by the incentive compatibility condition (15) [equivalent to (IC)] for $u = \tau'(\theta|q) = \rho = p$, that $r'(\theta)$ is linear in θ and, therefore, $r(\theta|q)$ must be quadratic in θ. This result is important from a practical point of view, as it reduces the search for the best pricing curve to a very simple class of functions. An additional implication of Theorem 2 is that the search for the best quadratic schedule amounts to determining the best marginal consumer type, which is found by solving Equation (39).

Type-free pricing schedule

In practice, pricing schedules do not come as a function of a model parameter such as a "consumer type," but rather as a relation of one instrument to the other; in our case, retail price as a function of the sharing tariff. With this, a customer can easily pick the most fitting combination of prices, according to his preferences. To determine the corresponding type-free schedule, we need to eliminate the type parameter from the firm's optimal schedule $(r^*(\theta|q), \tau^*(\theta|q))$ in Equations (40) and (41). Indeed, $\tau^*(\theta|q) = \tau$ (together with the assumption that $\rho = p$) yields the mapping $\tau \mapsto \theta(\tau|q)$ that is implicitly defined by

$$\int_{\theta(\tau|q)}^{1} \mathbf{1}_{\{\eta(\vartheta|q) \geq 0\}} d\vartheta = 1 - \frac{\tau}{p}, \quad \tau \in [\tau_0^*(q), p].$$

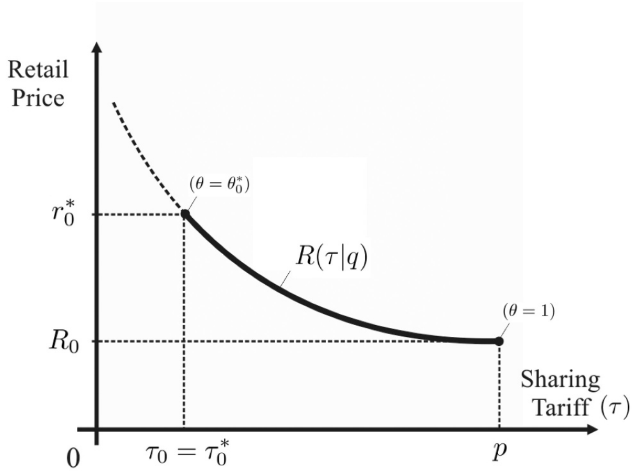

Figure 5. Type-free schedule.

In the special case where $g \geq 0$ on Θ, we get an explicit expression (where $\tau_0^*(q) = p\theta_0^*(q)$),

$$\theta(\tau|q) = \frac{\tau}{p} \in \Theta_0^*(q), \quad \tau \in [\tau_0^*(q), p],$$

which in turn implies the "type-free" schedule $R(\tau|q) = r^*(\theta(\tau|q)|q)$ of the purchase price as a function of the desired sharing premium. In the aforementioned special case (see Figure 5), this is

$$R(\tau|q) = \frac{p}{1-\delta q}\left(1 + \frac{\delta q}{2}\left(\left(\frac{\tau}{p}\right)^2 - 2\left(\frac{\tau}{p}\right) + (\theta_0^*(q))^2\right)\right), \quad \tau \in [\tau_0^*(q), p].$$

The type-free schedule is decreasing, as

$$R'(\tau|q) = -\frac{\delta q}{1-\delta q}\left(1 - \frac{\tau}{p}\right) < 0, \quad \tau \in [\tau_0^*(q), p],$$

and it is convex, as

$$R''(\tau|q) = \frac{1}{p}\frac{\delta q}{1-\delta q} > 0, \quad \tau \in [\tau_0^*(q), p].$$

Note that the sharing tariff τ indexes the firm's menu of contracts: for any given $\tau \in [\tau_0^*(q), p]$, the contract $(R(\tau|q), \tau)$ is chosen by consumers of type $\theta(\tau|q)$. Since τ—as a prospective drain on future rents—is *not* a desirable product attribute, its relative absence does constitute an attractive contract feature. More specifically, it is useful to interpret

$$P = p - \tau \in [0, \bar{P}]$$

as a (desirable) “premium” that comes with the shareable good, which is bounded from above by $\bar{P} = p - \tau_0^*(q)$. The corresponding direct schedule

$$\hat{R}(P) = R(p - P), \quad P \in [0, \bar{P}]$$

is increasing and concave in P, as one would expect in a standard non-linear pricing scheme with quantity discounts. That means that from the firm’s perspective the valuable “retail types,” that is, those who select the contract with the highest product premium (reflected by the lowest sharing tariff), are in fact those consumers in the participation set $\Theta_0^*(q)$ with the lowest types. The consumers with the lowest need for the item in the future must be afforded the largest product premium. On the other hand, the firm also values its “sharing types,” that is, the large-θ purchasers of products who will occasionally be sharing the products with other consumers on the sharing market at high sharing tariffs.

Robustness and identification

Robustness

It is remarkable that usually the shape of the optimal schedule (r^*, τ^*), or—equivalently—the shape of the corresponding type-free schedule $R(\tau|q)$, does not depend on very detailed information about the consumers’ type distribution. Indeed, only the marginal type θ_0^* (and thus the initial value (r_0^*, τ_0^*)) depends on F. The cdf is used for computing the adjoint variable $\psi = (\psi_r, \psi_\tau)$ in Equations (25) and (26), which can be interpreted as the volume of purchase and sharing transactions, respectively. Estimating the type distribution F, which *a priori* is an infinite-dimensional object, would in principle require non-parametric identification. This in turn would require a significant amount of data to produce an acceptable statistical fit. Thus, the company would need to gather (or procure) user-centric longitudinal information by observing customers’ usage patterns for durable goods. This type of estimation may be feasible when the firm already has a significant installed base, much in contrast to a situation without significant service experience. However, the parametric form of the optimal non-linear pricing scheme suggests that a detailed estimation of the type distribution is not really necessary, as the firm can instead focus on the much simpler task of tuning the schedule’s main distribution-sensitive parameter (θ_0^*).

Identification

Without loss of generality, consider the firm’s type-free schedule which can be written in the parametric form

$$R(\tau|q) = R_0 + a(p - \tau) + b\frac{(p - \tau)^2}{2}, \quad \tau \in [\tau_0, p],$$

where a and b are non-negative parameters, and R_0 is the product’s “stand-alone” retail price when sharing is disallowed (i.e., for $\tau = p$). Since $R'(p|q) = 0$, we find that $a = 0$.

In addition, comparing the second derivative $R''(\tau|q) = b$ with the result, the type-free schedule yields

$$b = \frac{1}{p}\frac{\delta q}{1-\delta q}.$$

Hence, we can rewrite the firm's type-free schedule as

$$R(\tau|q) = R_0 + \frac{1}{2}\left(\frac{\delta pq}{1-\delta q}\right)\left(1-\frac{\tau}{p}\right)^2, \quad \tau \in [\tau_0, p], \tag{44}$$

where the parameters R_0 and τ_0 need to be identified (e.g., by data-driven trial and error, or learning algorithms), whenever a reliable estimate of the type distribution F is not available, or more generally, when the user prefers not to engage in the modeling details.

Remark

The value for R_0 may be fairly easy to bootstrap by observing the prices for extant non-shareable substitutes. In our model, this is the price paid by the highest likelihood type ($\theta = 1$) who has a constant need for the item. This consumer type is provided with an efficient sharing tariff $\tau = p$ extracting the full surplus from the zero-probability after-market transactions, and he obtains in return the largest "information rent" in the retail price as surplus over his outside option; see Figure 4.

Optimal product design

Given an optimal non-linear pricing schedule $(r^*, \tau^*)(q)$ as the solution to the firm's pricing problem $(^*)$ for a fixed durability $q \in \mathcal{Q}$, the firm's *product design problem* is to determine[15]

$$q^* \in \arg\max{}_{q\in\mathcal{Q}} V^*(q). \tag{**}$$

To better understand the properties of an optimal durability design as solution to $(^{**})$, let us first recall that the firm's profit for a given participating type $\theta \in \Theta_0^*(q)$ in Equation (36) is composed of the retail margin $r^*(\theta|q) - c(q)$ times the steady-state probability $\bar{y}_1$ of the agent's current state to be a non-owner in the high-need state ($s_t(\theta) = 1$), plus the sharing-authorization revenue $\tau^*(\theta|q)$ times the steady-state probability $y_0(\theta|q)$ of the agent's current state to be an owner in the low-need state ($s_t(\theta) = 0$). Based on Equation (36), the change of the firm's expected profit $V^*(q)$ as a function of the durability q features a demand effect and a price effect, which are now examined in turn before characterizing the solution to the product design problem $(^{**})$.

Demand effect

The demand effect with respect to changes in durability describes how the volume of transactions, comprised of the participating owners in the low-need state and the participating non-owners in the high-need state, evolve—all else equal. Specifically, the dependence of $y_0(\theta|q)$ and $\bar{y}_1(\theta|q)$ on changes in the product's durability is such that

$$\frac{\partial \bar{y}_1(\theta|q)}{\partial q} = -\frac{\theta^2}{(1-(1-\theta)q)^2} < 0 < \frac{(1-\theta)\theta}{(1-(1-\theta)q)^2} = \frac{\partial y_0(\theta|q)}{\partial q}.$$

Hence, the steady-state probability of non-owners in a high-need state decreases in q, whereas the steady-state probability of owners in a low-need state increases in q. That is, while an increase in the product's durability tends to reduce the unit volume of sales, it also tends to increase the unit volume of aftermarket sharing transactions. This describes a natural tradeoff. Since the optimal sharing tariff, $\tau^*(\theta|q) = p\theta,$ does not actually depend on the product's durability, increasing q also augments the total expected sharing-related revenue from the consumer type θ, provided the type's participation status remains unchanged.

Price effect

The price effect with respect to durability variations refers to the sensitivity of the firm's optimal pricing schedule $(r^*, \tau^*)(\cdot|q)$ to changes in q. In general, the change of $r^*(\cdot|q)$ can easily be found numerically. In the simple case where $g \geq 0$ on Θ, based on Equation (42) we find

$$\frac{\partial r^*(\theta|q)}{\partial q} = \frac{2(1-\theta) + \theta^2 + (\theta_0^*)^2}{(1-\delta q)^2} \frac{\delta p}{2} > 0,$$

for all $q \in \mathcal{Q}$. That is, the retail-price schedule increases in the durability, at least partially offsetting the aforementioned volume decrease as well as an increase in the marginal production cost.

Durability optimization

The preceding discussion leads to an optimality condition for a solution to the firm's durability design problem (**). For this it is useful to consider "interior" durability levels $q \in (0,1)$ and "extreme" durability levels $q \in \{0,1\}$ separately.

Interior durability

Combining the demand effect and the price effect discussed earlier, we obtain an optimality condition for the firm's product design problem (**).

Theorem 3 (Optimal Durability)

An optimal (interior) durability $q^* \in (0,1)$ is such that

$$\left[\int_{\theta_0^*(q)}^{1}\left((1-\theta)\tau^*(\theta|q)-\theta(r^*(\theta|q)-c(q))+\frac{\partial r^*(\theta|q)}{\partial q}-c'(q)\right)\frac{\theta dF(\theta)}{(1-(1-\theta)q)^2}\right]\Bigg|_{q=q^*}=0.$$

An optimal level of durability balances the tradeoff between the firm's two revenue streams and the direct effect of the per-transaction profit increase. It is interesting to note that the effect of choosing the durability q on the participation set $\Theta_0^*(q)$ does *not* enter the optimality condition in Theorem 3. The reason for this is that the marginal type $\theta_0^*(q)$ has been optimized as solution to (38) for any admissible $q \in \mathcal{Q}$, such that by the envelope theorem [26] the dependence of the optimized objective function $V^*(q)$ on $\theta_0^*(q)$ vanishes.

Extreme durability

A completely disposable product is the cheapest to produce and may therefore be the most attractive to the firm. It also eliminates the revenue stream from the sharing market.[16] Indeed, for $q = 0$, it is $r^*(\theta|0) = p$, and the corresponding expected per-period profit is

$$V^*(0) = (p - c(0))\int_{\theta_0^*}^{1}\theta dF(\theta).$$

Conversely, perfect durability ($q = 1$, provided that $\bar{q} = 1$) eliminates the possibility of retail revenues in the long run, so that the firm needs to rely entirely on its revenues from authorizing sharing transactions. The resulting expected per-period profit is

$$V^*(1) = p\int_{\theta_0^*}^{1}(1-\theta)\theta dF(\theta).$$

The two preceding expressions immediately imply that full participation is optimal, so

$$\theta_0^*(0) = \theta_0^*(1) = 0.$$

Comparing the expressions leads to a description of solutions involving "extreme" levels of durability.

Lemma 4

Assume that $\bar{q} = 1$. The firm prefers disposable products to perfectly durable products in a stationary regime if and only if

$$p \geq \frac{\mu_\theta\, c(0)}{\mu_\theta^2 + \sigma_\theta^2},$$

where $\mu_\theta = E[\tilde{\theta}]$ is the mean and $\sigma_\theta^2 = E[(\tilde{\theta} - \mu_\theta)^2]$ the variance of the type distribution.

When the sharing price is sufficiently high and/or production cost sufficiently low, the firm finds it beneficial to concentrate on retail sales of completely disposable products. By contrast, in situations with high production cost it is better to increase the level of durability. This is qualitatively consistent with the findings in Razeghian and Weber [32], although there the model is different and the insight is driven by the possibility of strategically disabling the sharing market—without the firm's capacity of controlling usage transitions.

Ecosystem implications

At the outset, we noted that firms are increasingly designing product ecosystems rather than standalone solutions. The sensing of usage transitions for hardware may be accomplished via software updates or authorization of critical plug-ins. For example, Universal Audio, one of the premier suppliers of recording gear for professional music studios, couples its audio interfaces with software plugins (e.g., related to the emulation of high-budget analogue processors). The latter represent a much larger source of revenue than the carrier hardware, such that the plug-ins need to be restricted to individuals. To meter sharing transitions between users, the company may want to link access authorization to personal information that users would typically not want to share with others (e.g., credit-card information, social security numbers, or biometric information such as fingerprints).

As pointed out in our introduction, the capacity of user sensing—when paired with sufficient durability—enables a flexible pricing policy. By simultaneously offering different combinations of retail price and sharing tariff the firm is able to extract type-specific rents from its users. Enabling the products technologically to detect and control usage transitions (e.g., by adding a fingerprint reader), as required for the meaningful implementation of a positive sharing tariff, may be costly; see Weber [39].[17] The main difference in cost between durability and sensing features is that the former usually increase the marginal production cost while the latter may require a fixed upfront investment I and a per-period management or monitoring expense γ, largely decoupled from the sales volume.

Lemma 2 implies that the optimal monopoly price without sharing rent (i.e., for $\tau = \tau_0 = 0$) is $r^m = p/(1-\delta q)$, consistent with the (stationary) per-period profit

$$V^m(q) \triangleq V(r^m, 0, 0|q) = \left(\frac{p}{1-\delta q} - c(q)\right) \int_0^1 \frac{(1-q)\ \theta dF(\theta)}{1-(1-\theta)q}.$$

Provided the required investment I for the detection technology and a per-period cost γ related to the monitoring and administration of usage transitions, for the sharing control to be economically viable, the following "ecosystem viability condition" needs to be satisfied:[18]

$$I + \frac{\gamma}{1-\delta} \leq \frac{V^*(q) - V^m(q)}{1-\delta}, \tag{45}$$

where q corresponds to the desired level of durability. In principle, q should be equal to the optimal level of durability q^* determined earlier, but to ensure ecosystem viability an *augmented product design problem*, consisting of (**) subject to the ecosystem viability condition (45), needs to be solved. The constrained solution (which we denote by q^{**})

differs from the optimal design q^* only in the case where the inequality constraint (45) is in fact binding, which means that the firm obtains no benefit from implementing a complicated flexible pricing solution over standard retail pricing.[19] A decision to still go for a costly ecosystem-based solution with flexible pricing would need to be justified by other ecosystem benefits (not contained in our single-product analysis) and/or strategic considerations.[20] Finally, we note that the firm's standalone profit V^m vanishes for perfectly durable goods (i.e., when $q = 1$), rendering an ecosystem solution with active sharing authorization a strictly improving strategy in that case.

Illustration

Consider the case of a uniform type distribution where $F(\theta) = \theta$ for all $\theta \in \Theta = [0,1]$.[21] Based on Equation (43), the sharing tariff is proportional to the consumer type and independent of $\theta_0^*: \ \tau^*(\theta) \equiv p\theta$. Given the uniformity of the type distribution (and in the absence of market-clearing considerations), it is best for the firm to be all-inclusive (i.e., $\theta_0^* = 0$) which we illustrate for the special cases of perfectly durable products ($q = 1$) and completely disposable products ($q = 0$).[22]

For $q = 1$, the payoff-relevant steady-state probabilities become $\bar{y}_1 = 0$ (as nobody will need to purchase a product ever again) and $y_0 = 1 - \theta$. Hence,

$$V^* = p\int_{\theta_0^*}^{1} (1-\theta)\theta d\theta = \left(\frac{1-(3-2\theta_0^*)(\theta_0^*)^2}{6}\right) p,$$

which is strictly decreasing in $\theta_0^* \in (0,1)$, such that full inclusivity ($\theta_0^* = 0$) is optimal, resulting in a profit of $V^* = p/6$—entirely from sharing authorizations.

For $q = 0$, the payoff-relevant steady-state probabilities are $\bar{y}_1 = \theta$ and $y_0 = 0$. Moreover, $r^*(\theta) \equiv p$, and the firm's profit,

$$V^*(0) = (p - c(0))\int_{\theta_0^*}^{1} \theta \, d\theta = (p - c(0))\left(\frac{1-(\theta_0^*)^2}{2}\right),$$

is strictly decreasing in the marginal type, so that again $\theta_0^* = 0$ is optimal. This generates the profit $V^* = (p - c(0))/2$—exclusively from retail.

Comparing the firm's optimal profits for $q \in \{0,1\}$ suggests that a disposable product yields a payoff up to three times as high, depending on the cost $c(0)$.[23] However, using the market-clearing condition (46) to determine the marginal type (yielding $\theta_0^* = 1/2$ for $q = 1$, and $\theta_0^* = 0$ for $q = 0$) increases the profit ratio to a maximum of six [for $c(0) = 0$].

Extensions

Market clearing

How does the model interact with the sharing market? With uniform consumption value v across all agents, the clearing price $p \in [0, v]$ on the sharing market is exogenous. When the firm is a monopolist both in the primary (retail) and the secondary (sharing) market, the sharing price may be induced by the firm's optimal schedule, and naturally the firm would have an incentive to set $p = v$, i.e., at the highest possible value, as this guarantees maximum rent extraction from the aftermarket, without compromising retail sales. On the other hand, in steady state the sharing demand (from non-owners in the high-need state),

$$D(\theta_0^*) \triangleq \int_0^{\theta_0^*} \theta \, dF(\theta),$$

would need to be equal to the sharing supply (from owners in the low-need state), using Equation (25),

$$S(\theta_0^*) \triangleq \int_{\theta_0^*}^1 y_0(\theta) dF(\theta) = \psi_\tau(\theta_0^*),$$

for any $\theta_0^* \in \Theta$. The resulting market-clearing condition,

$$D(\theta_0^*) = S(\theta_0^*), \tag{46}$$

determines the marginal type θ_0^* as a function of the product durability q. We note that because of the uniform contingent use value v across consumer types, the sharing price $p \in [0, v]$ cannot be determined by market clearing alone.[24]

Non-stationary regime

What if owners and non-owners appear in proportions different from the steady-state? The preceding developments were limited to the regime where the different states were distributed according to the stationary distribution $\mathbf{y}$. However, the firm may care substantially about the non-stationary behavior of the consumers, before reaching steady state. Given the initial distribution $\mathbf{y}(0) = \mathbf{y}_0$ at time $t = 0$, the distribution at time $t > 0$ is given by

$$\mathbf{y}^\top(t) = \mathbf{y}_0^\top \mathbf{P}^t. \tag{47}$$

Hence, the firm's net present value of the schedule (r^*, τ^*),

$$\Pi^*(\mathbf{y}_0) = \int_{\theta_0^*}^1 \sum_{t=0}^{\infty} \delta^t \left(\mathbf{y}_0^\top \mathbf{P}^t\right)(0, r^*, \tau^*, 0)^\top dF(\theta),$$

depends on the initial distribution $\mathbf{y}_0$.

Examples

(i) Starting at the stationary distribution, $\mathbf{y}(0) = \mathbf{y}$, the firm's discounted payoff is

$$\Pi^*(\mathbf{y}) = \sum_{t=0}^{\infty} \delta^t V^* = \frac{V^*}{1 - \delta},$$

where the per-period profit V^* is given by $\bar{y}_1 r^* + y_0 \tau^*$.

(ii) For perfectly durable products (q = 1), there are no repeat purchases. Given the initial distribution $\mathbf{y}_0 = (1-\theta, \theta, 0, 0)^\top$, the firm's discounted payoff is

$$\Pi^*(\mathbf{y}_0|q) = \sum_{t=0}^{\infty} \delta^t \int_{\theta_0^*(q)}^{1} \Big(\bar{y}_{1,t}\ r^*(\theta|q) + y_{0,t}\ \tau^*(\theta|q)\Big) dF(\theta),$$

where $\bar{y}_{1,t} \triangleq \theta(1-\theta)^t$ and $y_{0,t} \triangleq (1-\theta)(1-(1-\theta)^t)$.[25] Compared with our illustrative example earlier, a substantial portion of the firm's profit now stems from retail, which in the stationary regime is not a source of benefit.

Generalized solution

Theorem 2 generalizes to non-stationary state distributions, by replacing condition (39) for the optimal marginal type as follows:

$$\theta_0^* \in \arg\max_{\theta_0 \in \Theta} \hat{\pi}(r^*, \tau^*, \theta_0), \tag{48}$$

where, for any $\theta_0 \in \Theta$,

$$\hat{\pi}(r^*, \tau^*, \theta_0) \triangleq {}_{\theta_0}^{1}(\bar{Y}_1\ r^*(\theta) + Y_0\ \tau^*(\theta))dF(\theta),$$

with $\bar{Y}_1 \triangleq \sum_{t=0}^{\infty} \delta^t \bar{y}_{1,t}$ and $Y_0 \triangleq \sum_{t=0}^{\infty} \delta^t y_{0,t}$, and $\mathbf{y}(t) = (\bar{y}_{0,t}, \bar{y}_{1,t}, y_{0,t}, y_{1,t})$, for $t \geq 0$, determined according to Equation (47) for $\mathbf{y}(0) = \mathbf{y}_0$.

Theorem 4 (Generalized Pricing)

Under the conditions of Theorem 2, a solution $(r^*, \tau^*, \theta_0^*)$ to the problem of maximizing the firm's discounted profit $\Pi^*(\mathbf{y}_0|q)$ for any given initial distribution $\mathbf{y}_0$, subject to Equations (14)-(18), is characterized by Equations (40) and (41) when replacing $\bar{y}_1$ with $\bar{Y}_1$ and y_0 with Y_0 in Equations (25), (26) (for ψ_r and ψ_τ), and in Equation (37) (for g), in conjunction with Equation (48) (with a necessary optimality condition analogous to Equation (38)).

The preceding result describes the firm's best stationary schedule, when faced with a non-stationary distribution of states starting with initial distribution $\mathbf{y}_0$. This result looks at the full intertemporal evolution of the consumer base when started away from the long-run equilibrium distribution. That is important, since at the time of product launch, for example, there would not be any users such that the initial distribution is $\mathbf{y}_0 = (1-\theta, \theta, 0, 0)^\top$, as noted earlier.

Conclusion

Managerial insights

Consumers have uncertain future needs for products. The possibility of peer-to-peer collaborative consumption decreases the perceived risk of investing in ownership through mutual insurance, provided the product is sufficiently shareable. In periods of low need, a sharing market presents a revenue opportunity offsetting an owner's "utility vacuum" in those periods. By retaining some form of aftermarket control over who uses a given item, as is now commonplace for numerous products (e.g., long-term transportation tickets, software subscriptions, studio equipment, phones, or computers), the original seller (or manufacturer) can charge the original buyer *ex post* for usage transitions.[26]

The shareability of a durable product (i.e., its reusability and transferability) is subject to design; the level of durability is an important instrument to increase shareability. Aftermarket control (i.e., the ability to meter and ration post-purchase consumption) is also subject to design; finally, user sensing is a key lever to increase aftermarket control. The shareability-control matrix in Figure 2 provides a simple tool to create a portfolio view of products, consistent with economically dominant mechanisms of rent extraction (ownership-based vs. usage-based) and modes of consumption (private vs. collaborative). It indicates that with sufficiently high shareability and aftermarket control, a flexible pricing strategy may be used, which amounts to offering different combinations of retail price and sharing price as the two instruments of an economic screening mechanism. Thus, different consumers end up paying different amounts over time, each type of consumer having selected a different pricing option. We demonstrated that the pricing schedule can be viewed as a graph that shows for each consumer type the rent extraction on the sharing market. For example, a consumer who needs the item with 20% probability in any given period pays 20 percent of the sharing price (corresponding to the effective transaction price on the sharing market, net of any intermediary's commission) and a retail price, which is decreasing and quadratic in the consumer's need likelihood (his "type"), independent of how the consumer types are distributed. The shape of this non-linear pricing curve is a simple and robust takeaway from our model.[27]

One can interpret the flexible rent-extraction mechanism more broadly as an example of "servitization" [37], often viewed as a service add-on to hardware.[28] Indeed, given its user-sensing capability the company is adding on the service of authorizing sharing transactions by allowing transfers of its products between different users on a case-by-case basis, or in general. A blanket sharing permission would amount to charging a sharing premium (see Figure 2) which is the company's default option if aftermarket control is weak or very expensive to implement. Flexible pricing allows for different payment arrangements for different users, with low-likelihood types exerting a positive externality on high-likelihood types, in the sense that a relative increase in the frequency of lower types will decrease the retail prices paid by the higher types.

Regarding insights for product design, it is important to realize that augmenting the user-sensing capability in the product ecosystem requires an upfront investment, in addition to a monitoring and technology cost for each period of operation. Moreover, any increase in the product durability may result in a higher marginal production cost. Since product-design investments can benefit an entire product ecosystem,[29] to employ the language by Woodard et al. [47], "design moves" can generate "design capital," in the sense that a product-ecosystem infrastructure with user sensing includes the real option of applying aftermarket control to other products and extending flexible pricing to other metering dimensions. For example, instead of a usage transition being measured as a binary event (that either happened or not), one could measure the intensity of sharing in terms of time and service level. Thus, a tariff for sharing a video-on-demand subscription could depend on both the number of movies watched and the selected service level (e.g., in terms of screen resolution or bandwidth).

Directions for future research

In the main text, we have abstracted from the possibility of user-tracking between purchases, where a consumer type might already be known because of an earlier purchase with the company. This user-centric traceability would generically lead to incentives to report types untruthfully so as to retain anonymity and thus a potential for information rent in the future. To exclude this substantial complication, we assumed that consumers may anonymize themselves, for example, by regenerating user accounts, such that the firm does not know more about repeat buyers than about first-time buyers. In future research, it would be interesting to allow for consumer tracking and learning, leading to semi-strategic interactions between consumers and the firm. The term *semi*-strategic applies because despite having a private incentive to limit the personal information to a firm that tracks behavior over time, each customer type remains atomistic, without any power to unilaterally influence the firm's actions.

Other interesting extensions of the present model include serial need correlation as well as heterogeneity with respect to need-contingent use value. Introducing additional metering dimensions gives rise to a multiattribute non-linear pricing model, which would tend to relax the implementability constraint (M); see Weber [42]. The model as it stands illustrates the possibilities a firm gains when equipping their products with technology that allows for the detection of usage transitions. The present findings are robust in the sense that the basic shape of the optimal schedule is independent of the precise distribution of types.

Notes

1. In addition to reusability, shareability requires the transferability of items from one user to another. For example, a toothbrush can be reused without significant degradation but is difficult to transfer among different users, such that it is not very shareable in its standard mechanical form. By contrast, an electric toothbrush with different heads might well be shared among different family members, as it separates the transferable part (the holder) from the non-transferable part (the head).
2. Here we assume the possibility of legal enforcement by a benevolent court of law. Economic rationality may still prevail in the absence of standard legal enforcement mechanisms [14].
3. In this context, Fox [17, p. 111] noted firms' tendency to "cause more changes in the form of the product than is justified by the content of such change," such that it remains difficult to disentangle true innovation (representing "content") and planned obsolescence [1] (representing "form") [23, p. 86].
4. The view taken here is that intermediate values of $q \in (0,1)$ continuously connect the two extremes of complete disposability ($q = 0$) and perfect durability ($q = 1$). Such homotopic connections have been employed in Optimization [18] as well as in Information Systems [46].
5. In line with the Coase conjecture [13, 22], a monopolist loses all market power when offering a perfectly durable good (in continuous time). With strategic consumers the firm's price-

commitment ability increases when durability decreases. But there are many real-life reasons (e.g., reputational consequences and adjustment costs) for why in practice firms do enjoy significant commitment power.

6. The per-period discount factor $\delta = 1/(1+i) \in (0,1)$ describes the present value of one dollar when obtained one period from now, where $i \in (0,\infty)$ is the corresponding per-period interest rate.
7. For $\theta_0 = 1$, the measure of the participation set $\Theta_0 = \{1\}$ vanishes, so that the actual participation of the highest type $\theta = 1$ is of no consequence for the firm and for the solution to its pricing problem.
8. We use the shorthand $[x]_{[0,1]} \triangleq \max\{0, \min\{1,x\}\}$ for the "restriction" of $x \in \mathbb{R}$ to the interval [0,1].
9. More specifically, we posit a "full price flexibility" ($\rho = p$) assumption meaning that even at the largest slope it is in principle possible to obtain a full separation of types (without "bunching"). That is, every type may be offered a different combination of prices. By increasing the allowable slope, there is generic bunching with intervals of types being non-separable for the firm. In the extreme, the firm could charge $\tau \in \{0, p\}$ for all participating types, which reverts back to a situation without flexible pricing, close to the model discussed by Weber [39].
10. The boundedness of the control, as required in Equation (17), is usually needed in the proofs of standard versions of the PMP (which do not allow for impulse controls and jumps in the state variables—here (r, τ)) [31, 45]; in our setup, boundedness of the control implies a rate constraint on the sharing tariff.
11. For a more precise interpretation of the adjoint equation, starting with the Hamilton-Jacobi-Bellman equation as a sufficient condition for optimality, see Weber [45, p. 95].
12. A "singular" control arises when the gradient η of the Hamiltonian (with respect to the control variable u) vanishes over a non-trivial type interval along an optimal trajectory, in which case the control variable would usually be recovered from the values the states need to take when the gradient of the Hamiltonian vanishes. This occurs frequently in screening problems; see, for example, Weber [42, 45]. Here, however, the Hamiltonian does not even depend on the state variable (r, τ), so that generically it does not vanish, leading to the "non-singular" extremal bang-bang control we observe for the pricing of smart products.
13. $\mathbf{1}$ denotes the standard indicator function.
14. The optimal marginal type can be found by comparing the value of the objective function V^* in Eq. (39) for all candidates that satisfy the necessary optimality condition (38).
15. Since the objective function $V^*(\cdot)$ is continuous and the durability domain $\mathcal{Q}$ is compact, by the Weierstrass theorem a solution to the optimization problem $(^{**})$ does exist; see, for example, Bertsekas [7, p. 540].
16. Since the price p for product access in the sharing market is assumed to be exogenous, zero durability cannot be used here to strategically disable the sharing market as in Razeghian and Weber [32].
17. The costs could be very low; for example, sharing a train ticket requires merely an authorization update.
18. For notational simplicity (and by sheer coincidence) the firm's discount factor δ here is the same as the consumers' discount factor δ. No simplification results from this, and there is no loss of generality.
19. In this situation, retail pricing is likely to perform strictly better than flexible pricing because the "standalone product design problem" can be solved by optimizing $V^m(q)$, which yields the "optimal standalone durability" q^m, where generically $V^m(q^m) > V^*(q^{**})$.
20. For example, the firm may decide that a more sophisticated solution may be worth a financial investment in view of the strategic side-benefits by being a first-mover in this domain, obtaining additional consumer information (which may be leveraged for other products) and being viewed as a technological leader.
21. The per-period unit volume of retail is $\psi_r(\theta_0^*) = \frac{1-q}{q}\left[1 - \theta_0^* - \frac{1-q}{q}\ln(1-(1-\theta_0^*)q)\right]$ for $q \in (0,1]$, and $\psi_r(\theta_0^*) = (1-(\theta_0^*)^2)/2$ for $q = 0$. On the other hand, the per-period unit volume of

sharing licenses is $\psi_\tau(\theta_0^*) = \frac{(1-\theta_0^*)(2-(1+\theta_0^*)q)}{2q} + \frac{1-q}{q^2}\ln(1-(1-\theta_0^*)q)$ for $q \in (0,1]$, and $\psi_\tau(\theta_0^*) = 0$ for $q = 0$.

22. For both of these cases, it is $\eta \geq 0$ on Θ, leading to substantial simplifications (including Equations (42) and (43) for the optimal schedule, instead of the general expressions in Theorem 2), as $\sigma(\theta|q) \equiv \theta$ and $\hat{\sigma}(\theta|q) \equiv \theta^2/2$.
23. For high marginal cost (in the vicinity of the sharing price p), steady-state profits for disposable products may become much lower than steady-state profits for perfectly durable products.
24. In practice, a given firm would need to take the sharing price p as an exogenous factor, especially in situations where the sharing supply is composed of a variety of substitute products, most of which the firm has no control over.
25. At time $t > 0$, the probability of not yet having bought the item over the previous t periods (from 0 to $t-1$) is $(1-\theta)^t$, such that $\bar{y}_{1,t} = \theta(1-\theta)^t$. Summing up the purchasers of the previous periods yields (by the geometric-series formula) $(1-(1-\theta)^t)$, such that in the current period the fraction of owners in the low-need state becomes $y_{0,t} = (1-\theta)(1-(1-\theta)^t)$. Note also that, as $t \to \infty$, one obtains $\bar{y}_{1,t} \to 0$ and $y_{0,t} \to 1-\theta$, consistent with our remarks at the end of our discussion of the steady-state sales distribution.
26. The technological limitations of effective sharing control are receding rapidly for almost all durable goods, thus persistently expanding the scope of this paper to a wide variety of products. Most limits of aftermarket sharing control thus far may be due to cost (e.g., a fingerprint reader or retinal scan to unlock a drone) but are usually not fundamentally related to infeasibility.
27. There exists a blunt extraction mechanism as well, namely to charge each consumer the full benefit p of each sharing transaction, which does not discriminate between consumers, and in real life would have a minimal chance of success (being prone to failure even in the presence of only very small transaction costs).
28. In the literature on servitization there are two different interpretations of the term, notably as a goods-dominant notion (with the service as an add-on) or as a service-dominant concept (with the service as the main product) [20].
29. In this context, the fixed costs (in our discussion on ecosystem design) may be shared among a portfolio of products, in addition to being part of a strategically necessary investment—for which traditional investment-return criteria may not apply due to the potential downsides of not investing.

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References

1. Abbott, L.A. Old Hats—and New. *Challenge 5*, 9/10 (June-July 1957), 32–36.
2. Adams, W.; and Brock, J.W. Integrated monopoly and market power: System selling, compatibility standards, and market control. *Quarterly Review of Economics and Business 22*, 4 (Summer 1982), 29–42.
3. Arrow, K.J. (1953) Le rôle des valeurs boursières pour la répartition la meilleure des risques. *Économetrie, Centre National de la Recherche Scientifique, 11* (1953), 41–47. [Reprinted: The role of securities in the optimal allocation of risk bearing. *Review of Economic Studies 31*, 2 (April 1964), 91–96.]

4. Bardhi, F.; and Eckhardt, G.M. Access-based consumption: The case of car sharing. *Journal of Consumer Research 39*, 4 (December 2012), 881–898.
5. Belk, R. You are what you can access: Sharing and collaborative consumption online. *Journal of Business Research 67*, 8 (August 2014), 1595–1600.
6. Benkler, Y. Sharing nicely: On shareable goods and the emergence of sharing as a modality of economic production. *Yale Law Journal 114*, 2 (November 2004), 273–358.
7. Bertsekas, D.P. *Nonlinear Programming*, Belmont, MA: Athena Scientific, 1995.
8. Botsman, R.; and Rogers, R. *What's Mine Is Yours: How Collaborative Consumption Is Changing the Way We Live*, London, UK: HarperCollins, 2010.
9. Bulow, J. An economic theory of planned obsolescence. *Quarterly Journal of Economics 101*, 4 (November 1986), 729–749.
10. Cesari, L. *Optimization—Theory and Applications: Problems with Ordinary Differential Equations*. New York, NY: Springer, 1983.
11. Chamberlin, E.H. The product as an economic variable. *Quarterly Journal of Economics 67*, 1 (February 1953), 1–29.
12. Clemons, E.K.; Dewan, R.M.; Kauffman, R.J.; and Weber, T.A. Understanding the information-based transformation of strategy and society. *Journal of Management Information Systems 34*, 2 (Fall 2017), 425–456.
13. Coase, R.H. Durability and monopoly. *Journal of Law and Economics 15*, 1 (April 1972), 143–149.
14. Dixit, A.K. *Lawlessness and Economics: Alternative Modes of Governance*. Princeton, NJ: Princeton University Press, 2004.
15. Feder, A.M. Remote sensing for engineering: A review. *Military Engineer 60*, 397 (September-October 1968), 364–370.
16. Filippov, A.F. On certain questions in the theory of optimal control. *Journal of the Society for Industrial and Applied Mathematics Series A: Control* (*SIAM Journal on Control*) *1*, 1 (January 1962), 76–84.
17. Fox, A.H. A theory of second-hand markets. *Economica (New Series) 24*, 94 (May 1957), 99–115.
18. Garcia, C.B.; and Zangwill, W.I. *Pathways to Solutions, Fixed Points, and Equilibria*. Englewood Cliffs, NJ: Prentice-Hall, 1981.
19. Gibbard, A. Manipulation of voting schemes: A general result. *Econometrica 41*, 4 (July 1973), 587–601.
20. Green, M.H.; Davies, P.; and Ng, I.C.L. Two strands of servitization: A thematic analysis of traditional and customer co-created servitization and future research directions. *International Journal of Production Economics 192* (October 2017), 40–53.
21. Gregory, P.M. A theory of purposeful obsolescence. *Southern Economic Journal 14*, 1 (July 1947), 24–45.
22. Gul, F.; Sonnenschein, H.; and Wilson, R.B. Foundations of dynamic monopoly and the Coase conjecture. *Journal of Economic Theory 39*, 1 (June 1986), 155–190.
23. Hattwick, R.E.; and Sailors, J.W. Some aspects of the theory of planned obsolescence: An inquiry into the supposed benefits of oligopolistic product competition. *Social Science Quarterly 49*, 1 (June 1968), 78–86.
24. Lane, N.D.; Miluzzo, E.; Lu, H., Peebles, D.; Choudhury, T.; and Campbell, A.T. A survey of mobile phone sensing. *IEEE Communications Magazine 48*, 9 (September 2010), 140–150.
25. Maloney, H.B. Planned obsolescence and TV. *Clearing House 33*, 1 (September 1958), 58–59.
26. Mas-Colell, A.; Whinston, M.D.; and Green, J.R. *Microeconomic Theory*. Oxford, UK: Oxford University Press, 1995.
27. Mirjafari, S.; Masaba, K.; Grover, T.; Wang, W.; Audia, P.; Campbell, A.T.; …; and Striegel, A. Differentiating higher and lower job performers in the workplace using mobile sensing. *Proceedings of the ACM on Interactive, Mobile, Wearable and Ubiquitous Technologies 3*, 2 (June 2019), Article 37.

28. Mirrlees, J.A. An exploration in the theory of optimum income taxation. *Review of Economic Studies 38*, 2 (April 1971), 175–208.
29. Mussa, M.; and Rosen, S. Monopoly and product quality. *Journal of Economic Theory 18*, 2 (August 1978), 301–317.
30. Myerson, R.B. Incentive compatibility and the bargaining problem. *Econometrica 47*, 1 (January 1979), 61–74.
31. Pontryagin, L.S.; Boltyanskii, V.G.; Gamkrelidze, R.V.; and Mishchenko, E.F. *The Mathematical Theory of Optimal Processes*. New York, NY: Wiley Interscience, 1962.
32. Razeghian, M.; and Weber, T.A. Strategic durability with sharing markets. *Sustainable Production and Consumption 19* (July 2019), 79–96.
33. Starner, T.E. Wearable computing and contextual awareness, Ph.D. Thesis, Media Arts and Sciences, Massachusetts Institute of Technology, Cambridge, MA, 1999.
34. Sundararajan, A. *The Sharing Economy*, Cambridge, MA: MIT Press, 2016.
35. Swan, P.L. Optimum durability, second-hand markets, and planned obsolescence. *Journal of Political Economy 80*, 3 (May-June 1972), 575–585.
36. Trenz, M.; Frey, A.; and Veit, D. Disentangling the facets of sharing: A categorization of what we know and don't know about the sharing economy. *Internet Research 28*, 4 (2018), 888—925.
37. Vandermerwe, S.; and Rada, J. Servitization of business: Adding value by adding services. *European Management Journal 6*, 4 (Winter 1988), 314–324.
38. Weber, T.A. Nonlinear pricing of shareable products. *Proceedings of the 53rd Annual Hawaii International Conference on System Sciences* (HICSS), January 2020, 6023—6032.
39. Weber, T.A. Smart products for sharing. *Journal of Management Information Systems 34*, 2 (Fall 2017), 341–368.
40. Weber, T.A. Global optimization on an interval. *Journal of Optimization Theory and Applications 182*, 2 (February 2017), 684–705.
41. Weber, T.A. Controlling and pricing shareability. *Proceedings of the 50th Annual Hawaii International Conference on System Sciences* (HICSS), January 2017, 5572–5581.
42. Weber, T.A. Optimal multiattribute screening. *Ural Mathematical Journal 2*, 2 (December 2016), 87–107.
43. Weber, T.A. Product pricing in a peer-to-peer economy. *Journal of Management Information Systems 33*, 2 (Fall 2016), 573–596.
44. Weber, T.A. Intermediation in a sharing economy: Insurance, moral hazard, and rent extraction. *Journal of Management Information Systems 31*, 3 (Winter 2014), 35–71.
45. Weber, T.A. *Optimal Control Theory with Applications in Economics*. Cambridge, MA: MIT Press, 2011.
46. Weber, T.A.; and Zheng, E. A model of search intermediaries and paid referrals. *Information Systems Research 18*, 4 (December 2007), 414–436.
47. Woodard, C.J.; Ramasubbu, N.; Tschang, F.T.; and Sambamurthy, V. Design capital and design moves: The logic of digital business strategy. *MIS Quarterly 37*, 2 (June 2013), 537–564.

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Appendix 1. Mathematical Details

1.1 Mechanism Design

To extract the type information from the otherwise anonymous consumers, the firm uses a screening mechanism $\widehat{\mathbf{M}} = (\hat{\mathcal{M}}, \hat{\alpha})$, where $\hat{\mathcal{M}}$ is a message space and $\hat{\alpha} = (\hat{r}, \hat{\tau}) : \hat{\mathcal{M}} \to \mathbb{R}^2_+$ is an allocation function which maps any message $m \in \hat{\mathcal{M}}$ a given agent sends to a tuple $\hat{\alpha}(m) = (\hat{r}(m), \hat{\tau}(m))$, which means that the agent pays the retail price $\hat{r}(m)$ and accepts the sharing tariff $\hat{\tau}(m)$ for future use transfers. We assume that the firm, which is also referred to as the "principal" in the context of the problem of designing its mechanism, can commit to $\widehat{\mathbf{M}}$. That is, consumers do not have to worry about the firm's not fulfilling its pre-agreed obligations *ex post*, for example by renegotiating the sales contract based on the revealed information. The revelation principle [19, 30] allows the principal, without any loss of generality, to restrict attention to so-called *direct* revelation mechanisms $\mathbf{M} = (\mathcal{M}, \alpha)$, where the message space $\mathcal{M}$ is equal to the type space Θ and all agents announce their types truthfully. This principle holds because any agent of type θ, when maximizing the payoff $B(\hat{r}(m), \hat{\tau}(m), \theta)$ (given in Lemma 1) from buying the product under the (indirect) mechanism $\widehat{\mathbf{M}}$, would solve

$$\max_{m \in \mathcal{M}} \left\{ -\frac{\delta}{1-\delta} \left((1-\theta)\hat{\tau}(m) + \frac{(1-q)(\theta \hat{r}(m) - (1-\theta)\hat{\tau}(m))}{1-(1-\theta)\delta q} \right) - \hat{r}(m) \right\},$$

with solution $\hat{m}^*(\theta)$. In the agent's optimization problem, all terms in $B(\hat{r}, \hat{\tau}, \theta)$ that are constant with respect to $(\hat{r}, \hat{\tau})$ have been omitted, without any loss of generality. However, the same allocation can be achieved with the direct mechanism $\mathbf{M} = (\Theta, \alpha)$ where $\alpha = (r, \tau) : \Theta \to \mathbb{R}^2_+$, with $r(\theta) = \hat{r}(\hat{m}^*(\theta))$ and $\tau(\theta) = \hat{\tau}(\hat{m}^*(\theta))$, for all $\theta \in \Theta$. Any type-θ agent participating in $\mathbf{M}$ would therefore happily report

$$\hat{\theta}^*(\theta) = \theta,$$

where $\mathbf{M}$ is indeed a direct revelation mechanism as claimed. Agents participating in the principal's mechanism must expect a payoff B from buying that exceeds the payoff A expected when getting access to the product on the sharing market, thus leading to the type-dependent individual-rationality (IR) constraint given in the main text. This participation constraint defines the participation set $\Theta_0 \subset \Theta$ for a given schedule (r, τ). To convert the principal's mechanism design problem into a tractable optimal control problem, we note that the participation set is convex, provided natural monotonicity of the principal's schedule (implied by (IC) and (M) as shown).

The participating agents' truth-telling constraint requires the first-order necessary optimality condition for maximizing the purchase utility $B(r(\hat{\theta}), \tau(\hat{\theta}), \theta)$ with respect to the announcement $\hat{\theta}$ (so as to obtain the contract $(r(\hat{\theta}), \tau(\hat{\theta}))$ to be satisfied at $\hat{\theta} = \theta$), and, therefore:

$$r'(\theta) + \frac{\delta}{1-\delta} \left(\frac{q(1-\theta)(1-(1-\theta)\delta)\tau'(\theta) + (1-q)\theta r'(\theta)}{1-(1-\theta)\delta q} \right) = 0,$$

resulting in the incentive-compatibility constraint (IC) given in the main text. Using the second-order necessary optimality condition for truth-telling implies that

$$r''(\theta) + \left(\frac{(1-\theta)\delta q}{1-\delta q} \right) \tau''(\theta) \geq 0, \quad \theta \in \Theta_0.$$

Conversely, differentiating the incentive-compatibility condition (IC) yields

$$r''(\theta) + \left(\frac{(1-\theta)\delta q}{1-\delta q}\right)\tau''(\theta) - \left(\frac{\delta q}{1-\delta q}\right)\tau'(\theta) = 0,$$

for all $\theta \in \Theta_0$, which (together with the preceding inequality) shows the implementability constraint (M) given in the main text. For the schedule (r, τ) to be implementable, the firm's sharing tariff τ needs to be non-decreasing. By virtue of (IC) the sharing-tariff schedule implies the retail-price schedule r on the participation set Θ_0, up to a constant. In the main text, we then formulate the firm's pricing problem as the optimal control problem (*), subject to Equations (14)–(18).

1.2 Proofs

Proof of Lemma 1

As pointed out in the main text, the (discounted) value for non-owners in the low-need state is

$$U_0 = 0 + \delta\,(\theta U_1 + (1-\theta)U_0),$$

while for non-owners in the high-need state it is

$$U_1 = \max\{A, B\},$$

where the agent chooses between the expected non-ownership value,

$$A = v - p + \delta\,(\theta U_1 + (1-\theta)U_0),$$

obtained via access on the sharing market, and the expected ownership value,

$$B = v - r + \delta(1-q)(\theta U_1 + (1-\theta)U_0) + \delta q(\theta V_1 + (1-\theta)V_0),$$

obtained via buying the product from the retailer. Naturally, all of the payoffs (i.e., U_0, U_1, A, B) depend on the consumer type θ. The (discounted) value for owners in the low-need state is

$$V_0 = \hat{p} + \delta(1-q)(\theta U_1 + (1-\theta)U_0) + \delta q(\theta V_1 + (1-\theta)V_0),$$

whereas in the high-need state it becomes

$$V_1 = v + \delta(1-q)(\theta U_1 + (1-\theta)U_0) + \delta q(\theta V_1 + (1-\theta)V_0).$$

For consumer types θ below the marginal type θ_0, it is $U_1 = A$ and for those types θ above θ_0, it is $U_1 = B$. Thus, we can now solve the four equations (for U_0, U_1, V_0, V_1): first for $U_1 = A$ (i.e., $\theta \in [0, \theta_0]$) and then for $U_1 = B$ (i.e., $\theta \in [\theta_0, 1]$), which yields the given expressions for $A(\theta)$ and $B(r(\theta), \tau(\theta), \theta)$.

Proof of Lemma 2

The result follows from examining the sign of the difference $B - A$. For $q\tau_0 > 0$, it is positive, as long as the term $(p - r + (r - (1-\theta)\,\tau)\,\delta\,q)$ is positive. However, the latter has a positive slope due to the assumed monotonicity of the firm's pricing schedule.

Proof of Lemma 3

See Cesari [10] and Filippov [16].

Proof of Theorem 1

See Pontryagin et al. [31] and Weber [45].

Proof of Theorem 2

The proof is provided in the main text.

Proof of Theorem 3

Differentiation $V^*(q)$ with respect to q, using the expressions for the demand effect and the price effect with respect to changes in durability, together with the steady-state demand distribution $\mathbf{y}(\theta|q)$, yields the given first-order necessary optimality condition for an interior optimum $q^* \in (0, 1)$.

Proof of Lemma 4

The claim follows directly from a comparison of $V^*(0)$ and $V^*(1)$.

Proof of Theorem 4

As pointed out in the main text, the non-stationary problem is isomorphic to the stationary problem when instead of the steady-state demands $\bar{y}_1$ and y_0 on uses the discounted sums $\bar{Y}_1$ and Y_0 over the non-stationary demands $\bar{y}_{1,t}$ and $y_{0,t}$ (over $t \in \{0, 1, \ ...\}$). Thus, the characterization result in Theorem 2 remains essentially intact, only that one needs to replace the condition (39) determining the optimal marginal type θ_0, based on maximizing the firm's per-period profit $V(r^*, \tau^*, \theta_0)$, by condition (48), based on maximizing the firm's (discounted) lifetime profit $\hat{\pi}(r^*, \tau^*, \theta_0)$.

Appendix 2. Notation

Table 2.1 Summary of notation.

Symbol	*Definition*	*Domain*
$c(q)$	Marginal production cost (at durability q)	$\mathbb{R}_+$
D	Sharing demand	$\mathbb{R}_+$
f / F	Probability density / distribution function ($f = F'$)	$\mathcal{L}_1[0,1]$
i	Per-period interest rate	$\mathbb{R}_{++}$
I	Technology investment	$\mathbb{R}_+$
p	Sharing price	$\mathbb{R}_+$
$\hat{p}$	Effective transaction price ($\hat{p} = p - \tau$)	$[0, p]$
$\mathbf{P}$	Markov transition matrix for sales distribution $\mathbf{y}$	(4×4)-matrix
q	Durability (i.e., probability that product survives until next period)	$[0,1]$
$\bar{q}$	Maximum viable durability; see Eq. (9)	$[0,1]$
$\mathcal{Q}$	Durability interval (contains q such that $c(q) \in [0, \nu]$)	$[0, \bar{q}]$
$r(\theta)$	Retail price for consumer type θ	$\mathbb{R}_+$
$R(\tau)$	Retail price at sharing tariff τ	$\mathbb{R}_+$
$s_t(\theta)$	Need-state realization for consumer type θ at time t	$\{0,1\}$
S	Sharing supply	$\mathbb{R}_+$
t	Time period	$\{0, 1, \ldots\}$
u	Control variable ($u = \tau'$)	$\mathcal{L}_\infty[0,1]$
U_s	Discounted value for nonowners in need state $s \in \{0,1\}$	$\mathbb{R}$
V_s	Discounted value for nonowners in need state $s \in \{0,1\}$	$\mathbb{R}$
V	Firm's expected per-period profit	$\mathbb{R}$
y_s / $\bar{y}_s$	Probability for owner / nonowner to be in need state $s \in \{0,1\}$	$[0,1]$
$\mathbf{y}$	Sales distribution ($\mathbf{y} = (\bar{y}_0, \bar{y}_1, y_0, y_1)$)	$\{x \in [0,1]^4 : \|x\|_1 = 1\}$
δ	Per-period discount factor ($\delta = 1/(1+i)$)	$(0,1)$
θ	Consumer type (likelihood of need)	$\Theta = [0,1]$
θ_0	Marginal type (indifferent between purchasing and not)	$\Theta = [0,1]$
$\tau(\theta)$	Sharing tariff for consumer type θ	$[0, p]$
μ_θ	Mean of the type distribution ($\mu_\theta = \mathbb{E}[\tilde{\theta}]$)	$[0,1]$
ν	Consumption value (contingent on high-need state)	$\mathbb{R}_{++}$
π	Per-period profit	$\mathbb{R}$
Π	Firm's expected lifetime profit (discounted)	$\mathbb{R}$
σ_θ	Standard deviation of the type distribution ($\sigma_\theta^2 = \mathbb{E}\big[(\tilde{\theta} - \mu_\theta)^2\big]$)	$[0,1]$