Ecole polytechnique fédérale de Lausanne
Master Cours fall 2020
CIVIL-492 Laboratoy GC

NUMERICAL SIMULATION OF A LAYERED ELASTIC SYSTEM USING THE BOUNDARY ELEMENT METHOD

STUDENT

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I. Topic
  i. Context
  ii. Last time

II. Model implementation
  i. Phase 3

III. Validation with two surfaces
  i. Line of concentrated force
  ii. Error

IV. Results

V. Conclusion

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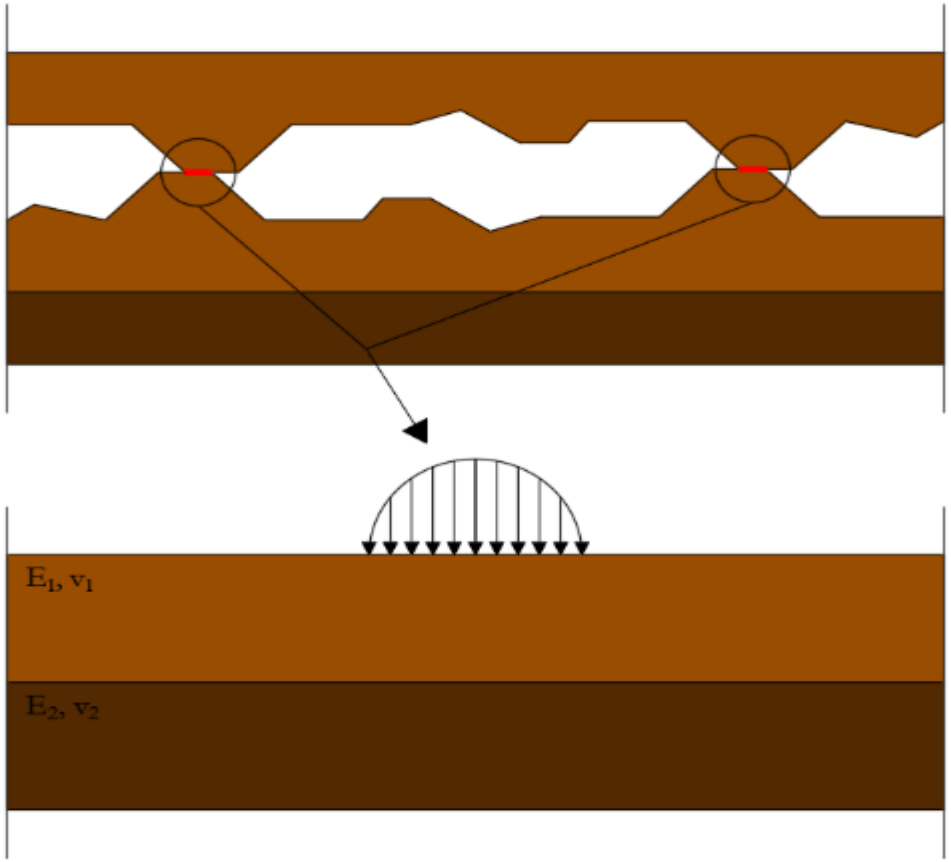

Surface contact pressure

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Phase 1

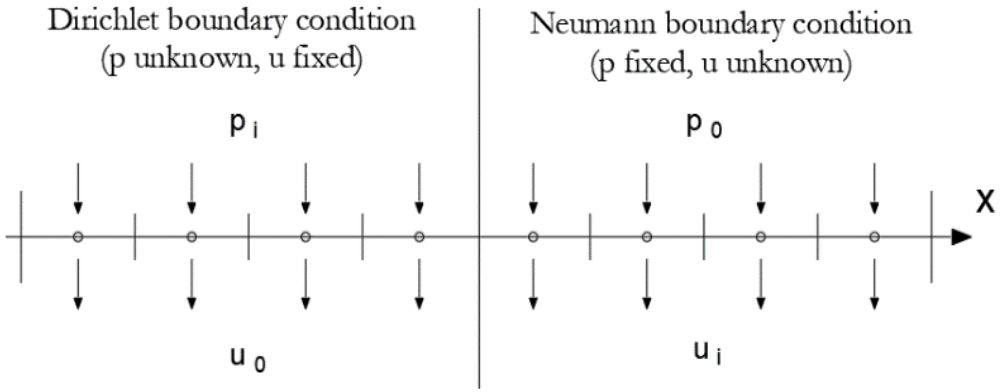
$$p = Pq \quad \rightarrow \quad q = P^{-1}p$$

$$u = Uq \quad \rightarrow \quad u = UP^{-1}p$$

$$K = PU^{-1}$$

$$\mathbf{u = K^{-1}p}$$

Phase 2



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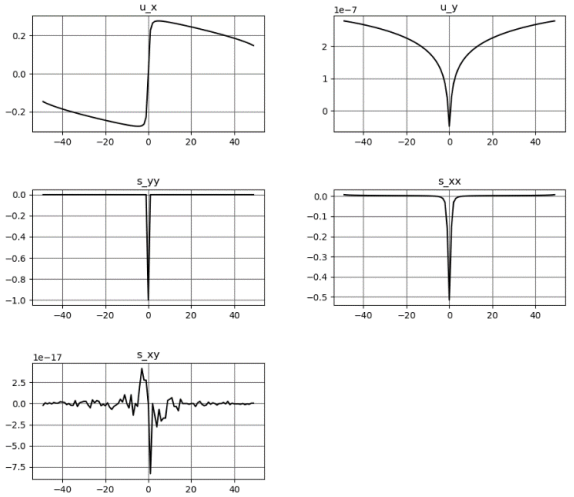
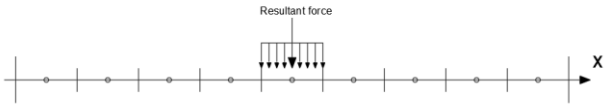
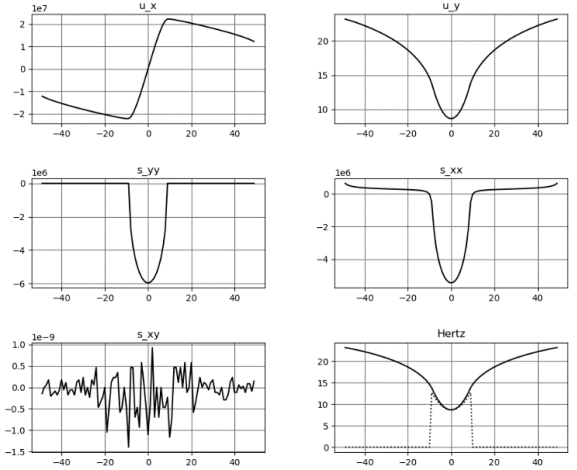
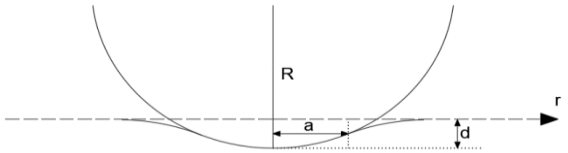
Line of concentrated force (constant over line segment)



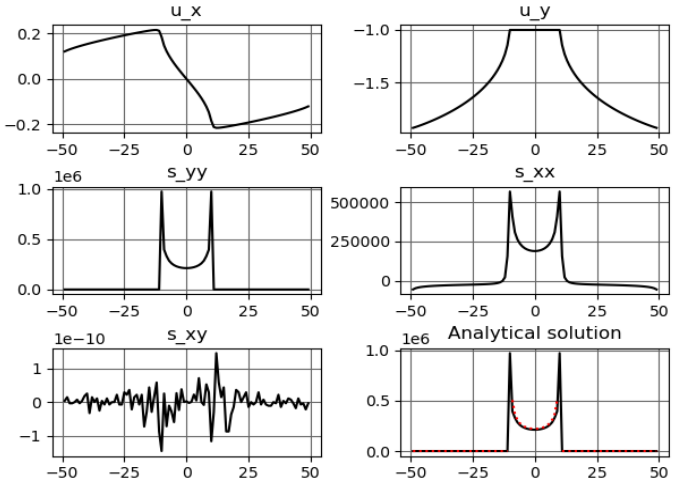


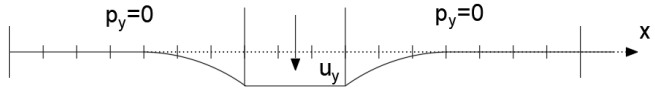
Hertzian stress





Imposed displacement





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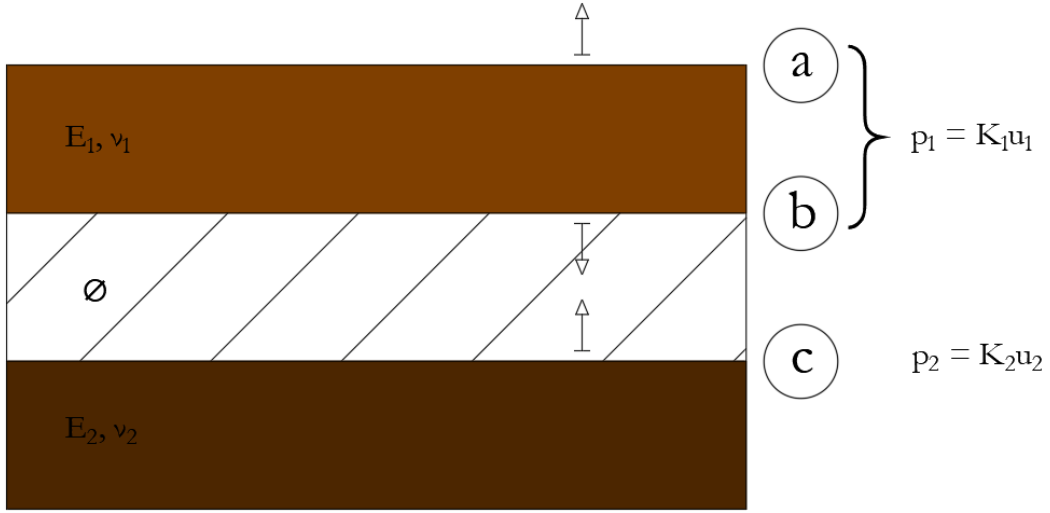
Phase 3



$$\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix} \times \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} p_a \\ p_b \\ p_c \end{bmatrix} = \begin{bmatrix} K_{aa} & K_{ab} & 0 \\ K_{ba} & K_{bb} & 0 \\ 0 & 0 & K_{cc} \end{bmatrix} \times \begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix}$$

$$p_a = (K_{aa} + K_{ab}(-K_{cc} - K_{bb})^{-1} K_{ba}) u_a = K_a' u_a$$

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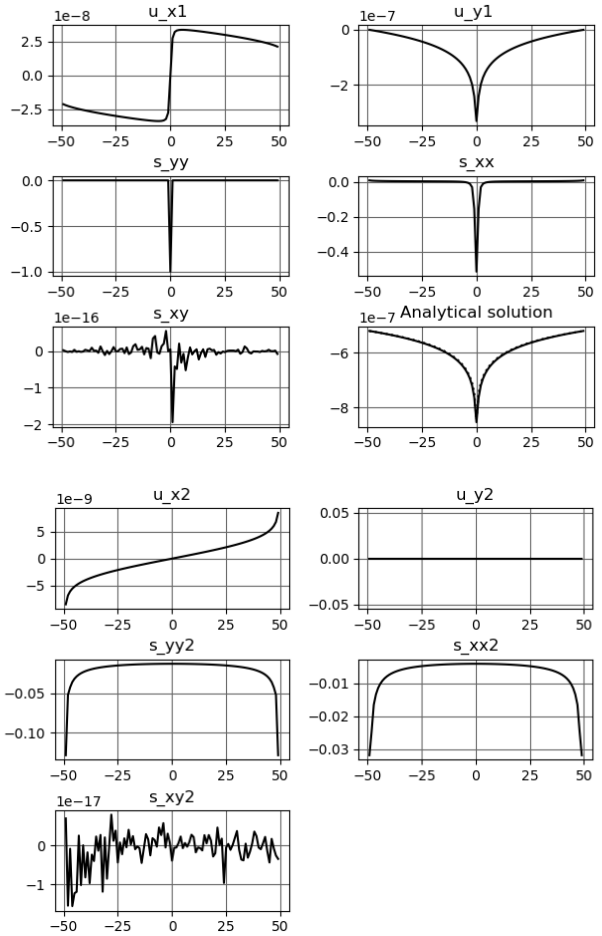
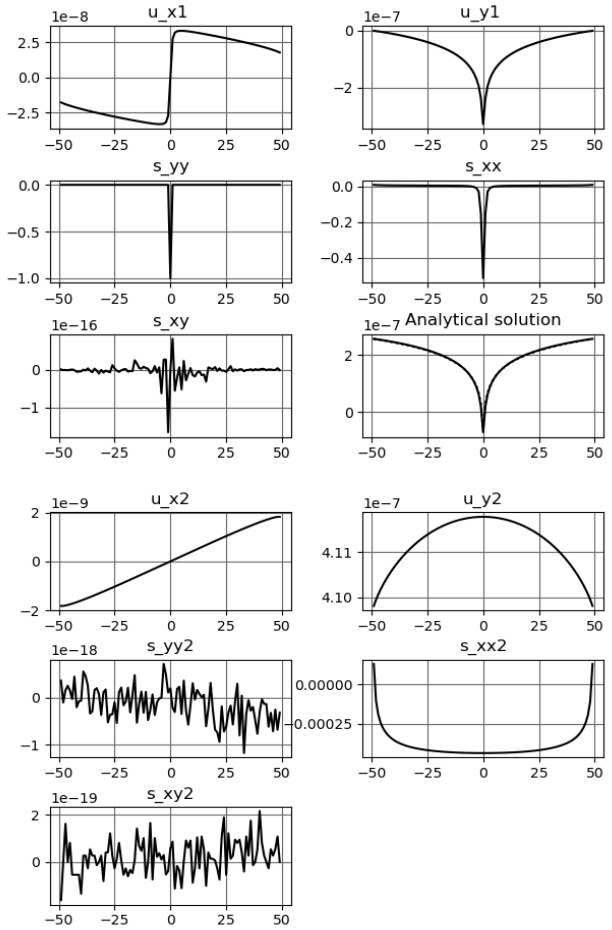
Line of concentrated force (constant over line segment)

Imposed vertical displacement on second surface



Imposed vertical pressure on second surface



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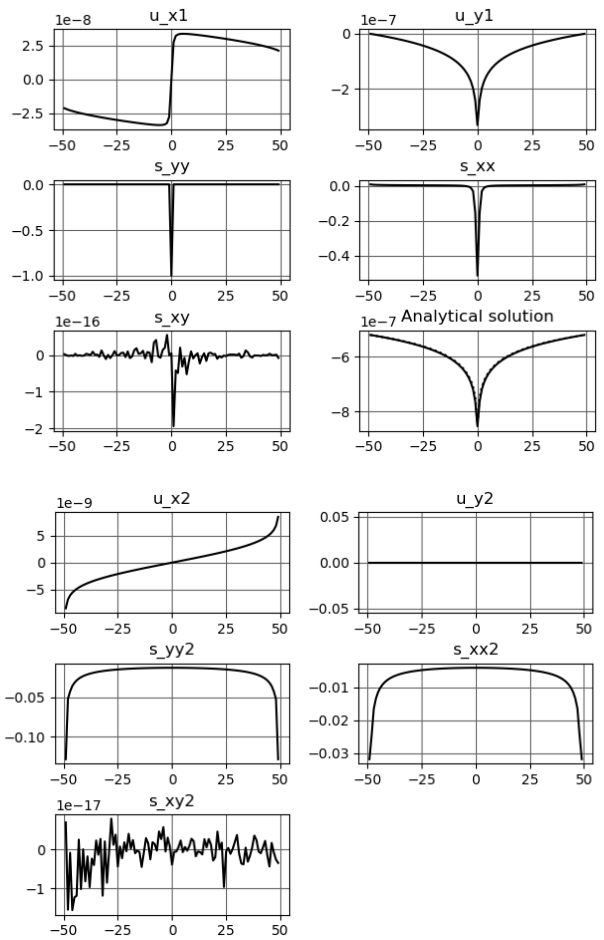
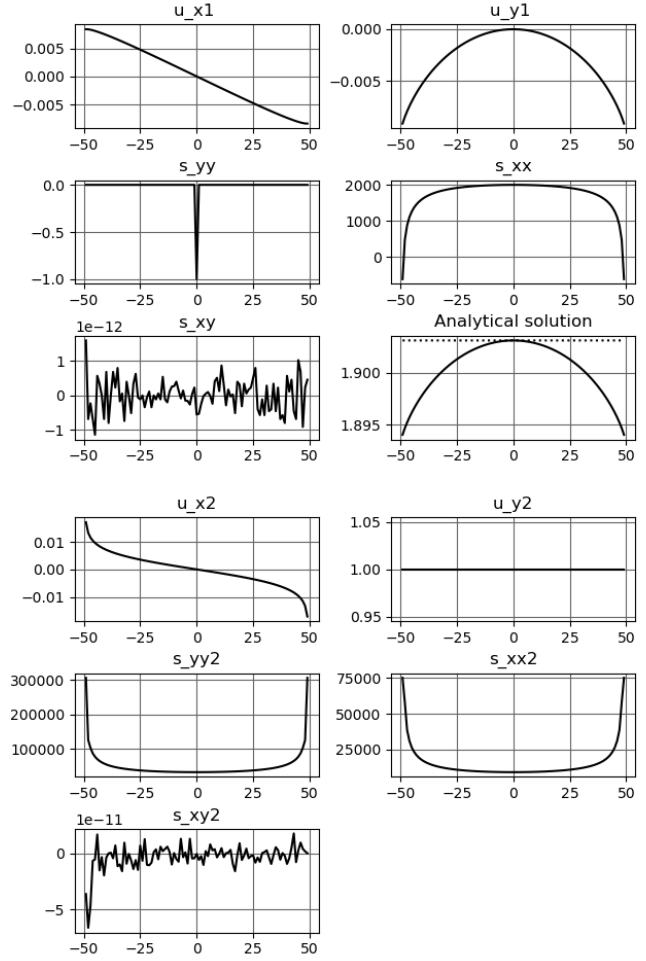
V. Conclusion

Imposed vertical displacement on second surface

$u_{y2} = 0$



$u_{y2} = 1$



- Relative displacements
- Sum of pressures of the system is not imposed to be zero

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Master Cours fall 2020
CIVIL-492 Laboratoy GC
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  i. Context
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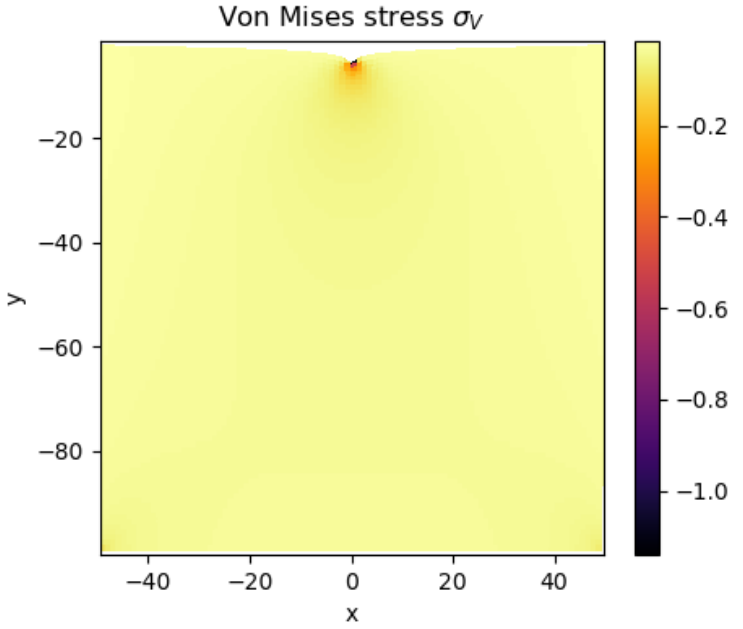
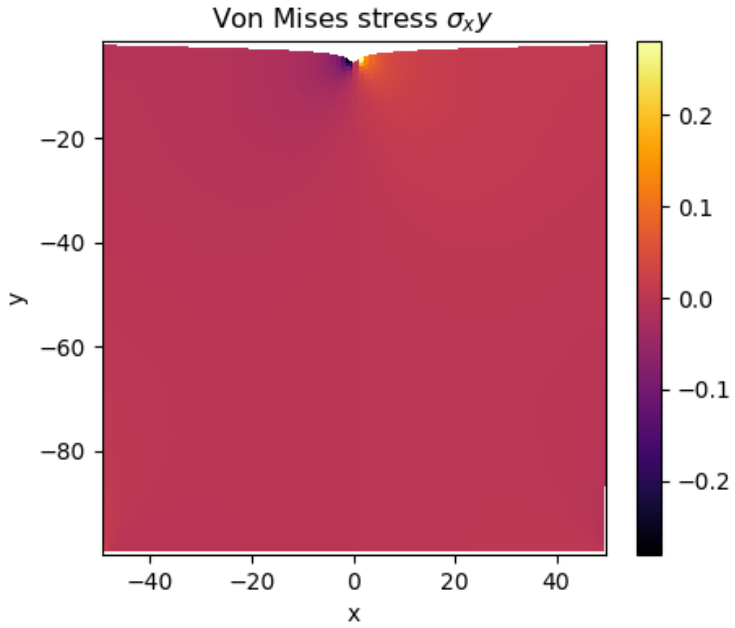
  ii. Error

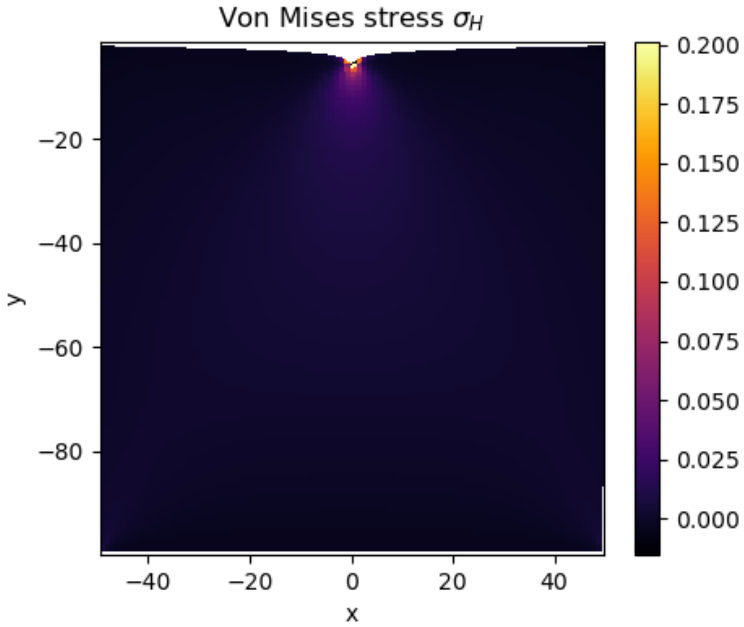
IV. Results

V. Conclusion

Line of concentrated force







Undetermined error in assembling the stress matrices.. Results a little different than with 2 surfaces.

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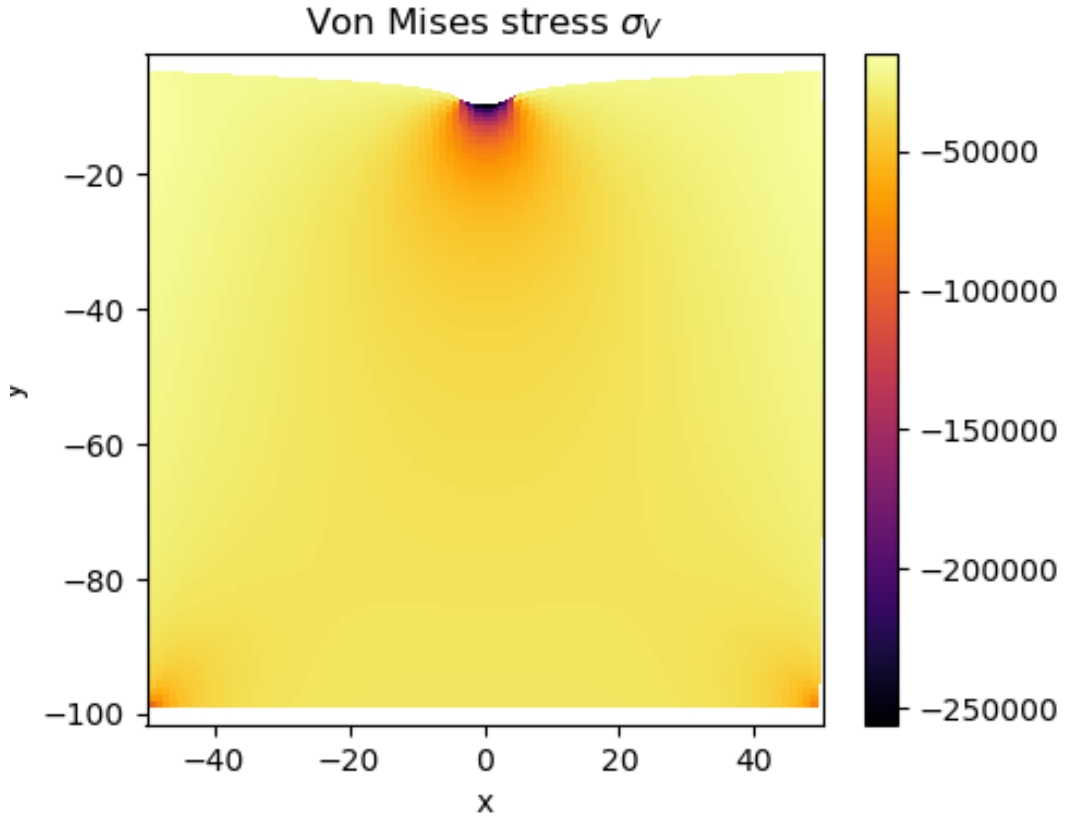
Hertzian stress



- Sum of pressures of the top and bottom surfaces different from zero

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Master Cours fall 2020
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- For an exterior problem of this type, the boundary element method is relatively simple to develop and have a great efficiency
- Relative displacements may be a problem when we simulate mixed boundary value systems
- It is necessary to impose the sum of the pressures of two surfaces equal to zero
- Important to define a point of reference

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References

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Thank you

for your attention