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SECTION DE GÉNIE CIVIL

Interactive, Real-Time Structural Simulation

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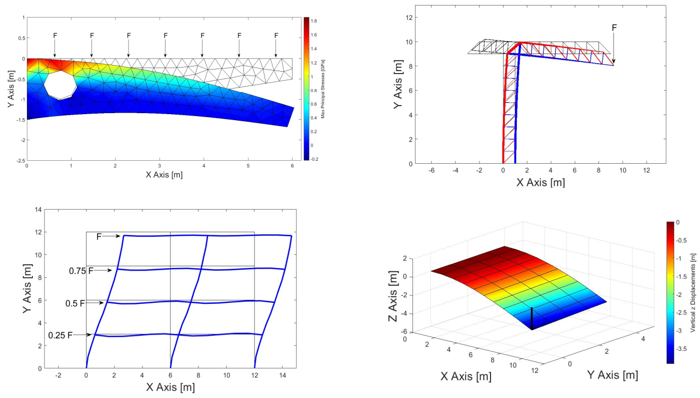
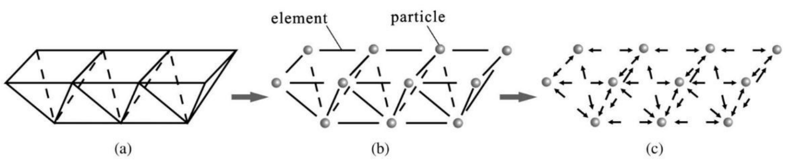
I. Abstract

The aim of this thesis is to develop a model that provides a fast analysis of structures subjected to large displacements. This is done by transforming the structure in a set of finite particles and by applying a method called dynamic relaxation to it. The latter simulates a dynamic behaviour of the structure by applying Newton's second law to the set of particles until the equilibrium is reached. This can be accomplished using only explicit methods and it does not require to build a global stiffness matrix nor to inverse it. The position and rotations of the elements are followed during the process and their forces are always computed locally by using their linear stiffness matrix. This methodology is applied to multiple type of elements and the accuracy and computation time of the model are finally tested through various examples.


Multiple Model Applications



Dynamic Relaxation Principle

II. Dynamic Relaxation

The dynamic relaxation is a method in which the structure is modelled by a set of finite particles. The mass of the system is lumped on those particles and a fictitious dynamic behavior is simulated. In order to achieve this, the residual or out-of-balance forces are applied to the particles. Those are defined as the difference between the reaction forces of the deformed structure and the external loading.

$$Residulas = [K]\{d\} - F_{external}$$

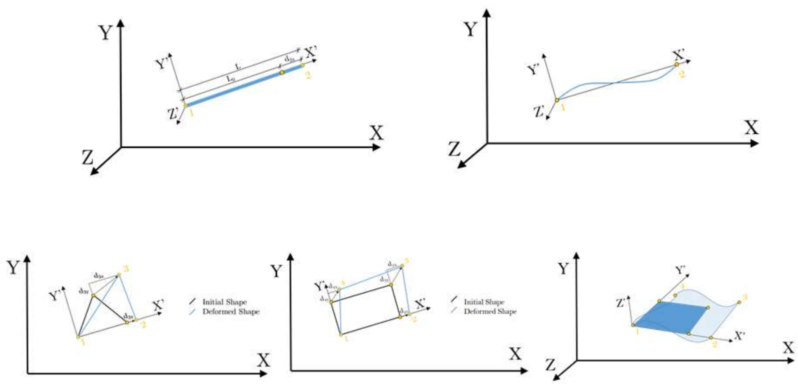
Once the residual forces are computed, we use a central differencing scheme to update the position and orientation of each particle. The translations and rotations are treated separately, and the integration continues until a criteria is reached.

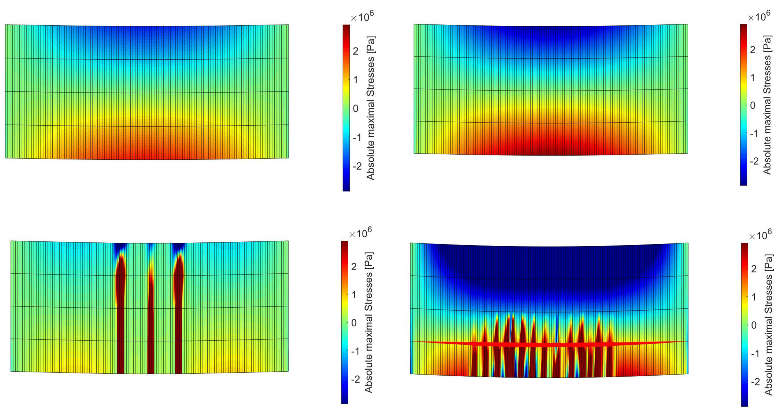
	Translational movement	Rotational movement
Compute the velocity	$v_{ij}^t = v_{ij}^{t-2} + 2(\Delta t)\frac{Res_{ij}^{t-1}}{M_{ij}}$	$\omega_{ij}^t = \omega_{ij}^{t-2} + 2(\Delta t)\frac{Res_{ij}^{t-1}}{I_{ij}}$
Compute the displacements	$\Delta d_{ij} = 2(\Delta t)v_{ij}^t$	$\Delta\theta_{ij} = 2(\Delta t)\omega_{ij}^t$
Update the position/orientation	$d_{ij}^{t+1} = d_{ij}^{t-1} + \Delta d_{ij}$	$R^{t+\Delta t} = Rod(\overrightarrow{\Delta\theta})R^t$

Central Differencing Scheme for Translations and Rotations

III. Co-rotational Formulation

Co-rotational formulation is the name given to the methodology used to compute the forces. Each element of the structure is attached to a local reference frame and the forces are computed with respect to it. The reference frame is chosen in order to set some of the degrees of freedom of the element to zero. That enable to avoid some operations and thus, save computation time. The linear stiffness matrix [K] of the element is used to calculate the local forces applied by the element on its nodes due to the local displacements {d}. This formulation has been applied to multiple elements such as the linear bar and beam, two plane stress/strain 2D elements and the 3D shell element.


Co-rotational Formulation of 5 Element Types



Model Application to a Reinforced Concrete Beam

IV. Conclusion and further work

We managed to apply this methodology to multiple types of elements. Multiple example have been analysed and the results were compared with those of a commercial software. For bars and beams, this formulation has been shown to be very effective. The accuracy of the model was less than 2% for those elements and the gain of computation time was up to 88%. For other elements, the accuracy was not as good but still less than 5%. We managed to obtain a smaller computation time than the commercial software, except for the shell element.

In further works, we could add a plastic behaviour to the model. As the method is iterative and explicit, the stress within the elements may be computed at every step and a plastic model could be applied to change the stiffness of the elements.