# On the Influence of an Elevated Terrain on the Grounding Resistance of a Vertical Rod 

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#### Abstract

In this paper, we analyze the grounding resistance of a vertical grounding rod located in an elevated terrain. Tall structures such as wind turbines and mobile phone base stations are often installed in remote and hilly locations to gain more power or to facilitate line of sight to end users. Those locations are at an increased risk of being struck by lightning due to the greater elevation compared to their surroundings. These hilly areas are usually very rocky with low soil conductivity. A proper grounding system is therefore of paramount importance. In this study, we analyze the influence of the elevated terrain considering a simplified cone and a truncated cone geometries as a function of the apex angle and top radius. It is shown that the grounding resistance can be significantly increased both, in the case of small mounds of only a few meters height, and in the case of a high hill or a mountain.


Index Terms-Lightning, grounding resistance, hilly, elevated, irregular terrain, non-flat, vertical rod, FEM

## I. Introduction

Tall structures are often installed in remote and hilly locations. For instance, wind turbines can be erected on mountains to increase to gain unobstructed access to wind and, thus, to more power, and mobile phone base stations can be installed on hills to improve line of sight to the end users. Tall structures on hills or mountains are more likely to be struck by lightning due to the geographical elevation $[1,2]$ and by the tall structures themselves [3,4] that can initiate upward lightning discharges [5]. The probability of incidence can be further increased for the case of wind turbines due to the rotation of the blades [6].
These hilly areas are usually very rocky with low soil conductivity ( $0.001 \mathrm{~S} / \mathrm{m}$ and lower), making the requirement of a proper grounding system all the more important. This study is an extension of [7], in which an analysis of the response of a grounding electrode located on top of a mountain was presented, and analytical formulas were derived for the case of a hemispheric electrode buried on the top of a cone- shaped mountain. It was shown that the grounding resistance can increase significantly for steep mountains.
In the present study, we analyze the influence of a non-flat terrain for the case of a vertical grounding rod for different configurations.

## II. Computation Method

In the analysis, the numerical simulations will be carried out using the AC/DC module of the commercial tool COMSOL [8]. In all of the simulations, a current source of 1 A is applied on the top surface of a vertical rod. The Dirichlet boundary condition $\varphi=0 \mathrm{~V}$ is set at the bottom of the domain and the Neuman boundary condition $\vec{J}_{\text {normal }}=0$ at the surface of the cone-shape mountain.

## III. Vertical Grounding Rod

## A. Vertical Grounding Rod in a Cone-Shaped Ground

We will first examine the worst-case scenario with the vertical grounding rod located in the apex of a cone-shaped mountain. The geometry of the problem is shown in Fig. 1. Note that we consider a relatively thick wire of radius 10 cm so that the FEM results converge faster. This can be seen as a first approximation of a grounding rod with ground enhancement material (GEM). In the case of a flat ground, the resistance can be calculated simply as [9,10,11]:

$$
\begin{equation*}
R_{\text {flat }}=\frac{\rho}{2 \pi L}\left(\ln \left(\frac{4 L}{a}\right)-1\right) \tag{1}
\end{equation*}
$$

In which $\rho$ is the resistivity of the ground, $L$ is the length of the grounding rod, and $a$ is the radius of the grounding rod. In (4), we can observe a weak dependence on the radius $a$. For example, increasing the radius of a $10-\mathrm{m}$ long rod from 2 cm to 10 cm (five times) will reduce the resistance by only about $20 \%$.

The results for the potential to remote earth for an apex angle of $30^{\circ}$ are shown in Figure 2. A cone-shaped terrain results in an increase of the potential to remote earth, compared to a flat ground, since the effective conducting volume is decreased. In the case of flat terrain, the current will propagate both in the horizontal and the vertical directions. On the other hand, in the case of the elevated terrain, most of the current flows only in the vertical direction.

[^0]

Fig. 1. Geometry of the problem. Vertical rod buried in a cone-shaped ground. $\varphi=45^{\circ}$.


Figure 2. Potential to remote earth for the case of a vertical grounding rod with $\mathrm{L}=10 \mathrm{~m}, \sigma=0.001 \mathrm{~S} / \mathrm{m}$, and $\varphi=30^{\circ}$. The applied current source is 1 A . Simulations obtained using COMSOL [8].

The coefficient expressing the increase of the grounding resistance with respect to the flat-ground case in terms of the apex angle, derived in [7] for the case of a hemispheric grounding system, is given by:

$$
\begin{equation*}
k=\frac{R_{\text {cone }}}{R_{\text {flat }}}=\frac{1}{(1-\cos (\varphi))(1+\operatorname{ctg}(\varphi))} \tag{2}
\end{equation*}
$$

This coefficient can be also be calculated for the case of a vertical grounding rod as the numerical ratio of simulated values. Table 1 presents the results of the grounding resistance versus apex angle. In the third and fourth columns, we present, respectively, the coefficients for the simulated vertical rod and for the hemispheric grounding. The last column presents the relative error of Eq. (5) if used in the case of a vertical rod.

TABLE 1. Grounding resistance for $\sigma=0.001 \mathrm{~S} / \mathrm{m}, \mathrm{L}=10 \mathrm{~m}, \mathrm{a}=10 \mathrm{~cm}$ and $\mathrm{R}_{\text {flat }}=79.44 \Omega$ eq. (4)

| Apex <br> angle $(\phi)$ | $\mathrm{R}(\Omega)$ | $\mathrm{k}_{\text {simulation }}$ | $\mathrm{k}_{\text {hemisphere }}$ | Relative <br> Error eq. (5) |
| :---: | :---: | :---: | :---: | :---: |
| 60 | 98 | 1.24 | 1.27 | $2.78 \%$ |
| 45 | 125 | 1.58 | 1.71 | $8.34 \%$ |
| 30 | 204 | 2.57 | 2.73 | $6.49 \%$ |
| 15 | 576 | 7.25 | 6.2 | $-14.46 \%$ |
| 10 | 733 | 14.38 | 9.87 | $-31.37 \%$ |

Figure 3 presents the plot of the ratio of the hemispheric electrode to the flat-ground grounding resistances calculated using (5) and the one for rods of different lengths obtained using numerical simulations. We can observe that, for steepness levels not exceeding 15 degrees, Equation (5) provides a reasonable estimate of the increase of the resistance. In the case of apex angles smaller than about $20^{\circ}$, the longer the rod is, the more similar the grounding resistance coefficient is to the case of hemispheric grounding. Fig 3 suggests that Eq. (5) could be used for geometries other than hemispheric as a first approximation to estimate the influence of an elevated terrain in the worst case scenario of a grounding system located in the apex of a cone-shaped mountain.


Figure 3. Coefficient of increase of grounding resistance as a function of the apex angle for different geometries.

## B. Variation of the Cone Top Radius

In this section, we will make numerical simulations for the case of a more realistic scenario in which the elevated terrain has A flat region around the grounding system (truncated cone). The geometry of the truncated cone with a top radius $r_{\text {top }}$ analyzed here is presented in Fig. 4.


Fig. 4. Vertical rod buried on top of a truncated cone-shaped ground.
Table 2 summarizes the results for different values of the apex angle and the top radius. The results with $r_{\text {top }}=0.1 \mathrm{~m}$ correspond to the case presented in the previous section. We can observe how the results converge toward the values of a flat ground when the apex angle and/or the top radius increase. For the case of an apex angle of $45^{\circ}$, the resistance is within $3 \%$ as that in the case of a flat ground for a top radius of 40 m and the difference is only about $10 \%$ for a top radius of 10 m . However, in the case of very steep terrain, the effect is still not negligible even for a top radius as high as 40 m .

| TABLE 2. Grounding resistance for $\sigma=0.001 \mathrm{~S} / \mathrm{m}, \mathrm{L}=10 \mathrm{~m}, \mathrm{a}=10 \mathrm{~cm}$ and $\mathrm{R}_{\text {flat }}=79.44 \Omega(4)$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Apex | Resistance ( $\Omega$ ) |  |  |  |
| ( $\theta$ ) | $\mathrm{r}_{\text {top }}=0.1 \mathrm{~m}$ | $\mathrm{r}_{\text {top }}=5 \mathrm{~m}$ | $\mathrm{r}_{\text {top }}=10 \mathrm{~m}$ | $\mathrm{r}_{\text {top }}=40 \mathrm{~m}$ |
| 45 | 125.17 | 97.27 | 88.06 | 81.38 |
| 30 | 203.81 | 126.07 | 104.67 | 82.67 |
| 15 | 575.97 | 224.46 | 155.57 | 93.64 |
| 10 | 1142 | 326.16 | 204.02 | 101.28 |

## C. Variation of the Cone Height

In the previous sections, we implicitly assumed that the height of the cone was infinite. Here, we will discuss a finiteheight cone attached to a flat ground. Figure 5 shows the geometry used in this case study.

We present two different cases. In the first case, we will analyze the grounding resistance of a grounding rod of $2-\mathrm{m}$ length buried at the top of a relatively small mound of a few meters height in the shape of a truncated cone with a fixed 1m top radius. In a second case, we will analyze the influence of a truncated cone of finite height in the case of a 5-m rod with a $10-\mathrm{m}$ top radius.


Fig. 5. Vertical rod buried on the top of a truncated, finite-height, coneshaped ground.

Fig. 6 presents the distribution of the potential to remote earth for the case of an apex angle of $30^{\circ}, L=2 \mathrm{~m}$ and $H=5 \mathrm{~m}$. Table 3 summarizes the results for different apex angles ranging from $10^{\circ}$ to $45^{\circ}$, and different terrain heights, assuming $r_{\text {top }}=1$ m . We can observe that when the height of the terrain is about 50 m or larger, the results are very similar to those of an infinite cone. More importantly, we can observe that even for small mounds, the grounding resistance can be significantly increased. For example, in the case of a $15^{\circ}$ apex angle and a $5-\mathrm{m}$ tall hill, the resistance is increased by more than a factor of two.


Fig. 6. Potential to remote earth for the case of $\mathrm{L}=2 \mathrm{~m}, \mathrm{H}=5 \mathrm{~m}, \sigma=0.001 \mathrm{~S} / \mathrm{m}$, $\varphi=30^{\circ}$ and 1 A current source applied to the vertical rod. Simulated in commercial tool COMSOL [8].

TABLE 3. Grounding resistance for $\sigma=0.001 \mathrm{~S} / \mathrm{m}, \mathrm{L}=2 \mathrm{~m}, \mathrm{a}=10 \mathrm{c} \mathrm{m}, \mathrm{r}_{\text {top }}=1$
m and $\mathrm{R}_{\text {flat }}=269.13 \Omega$ (4)

| m and $\mathrm{R}_{\text {flat }}=269.13 \Omega(4)$ |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Apex <br> angle | $\operatorname{Resistance~}(\Omega)$ |  |  |  |  |
| $(\varphi)$ | $\mathrm{H}=1 \mathrm{~m}$ | $\mathrm{H}=5 \mathrm{~m}$ | $\mathrm{H}=20$ <br> m | $\mathrm{H}=50$ <br> m | $\mathrm{H}=\infty$ |
| 45 | 295 | 345 | 366 | 370 | 375 |
| 30 | 306 | 424 | 488 | 505 | 519 |
| 15 | 318 | 615 | 875 | 960 | 1028 |
| 10 | 323 | 752 | 1239 | 1425 | 1583 |

Table 4 presents similar results but, in this case, the radius $r_{\text {top }}$ is assumed to be 5 m . We can observe that for a height of about 100 m , the resistance becomes similar to the case of an infinite cone. Again, even for small values of H , one can observe a significant increase of the resistance.

| Apex | Resistance ( $\Omega$ ) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| ( $\varphi$ ) | $\mathrm{H}=10 \mathrm{~m}$ | $\mathrm{H}=25 \mathrm{~m}$ | $\begin{gathered} \mathrm{H}=100 \\ \mathrm{~m} \end{gathered}$ | $\begin{gathered} \mathrm{H}=250 \\ \mathrm{~m} \end{gathered}$ | $\mathrm{H}=\infty$ |
| 45 | 151 | 157 | 161 | 162 | 164 |
| 30 | 162 | 177 | 190 | 193 | 196 |
| 15 | 181 | 224 | 276 | 293 | 307 |
| 10 | 192 | 257 | 355 | 392 | 426 |

## IV. Conclusion

We analyzed the influence of an elevated terrain on the grounding resistance of a vertical rod, using a cone and a truncated cone geometry. We showed that the grounding resistance can be significantly increased compared to the case of a flat terrain, both in the case of small mounds with heights of a few meters and in the case of high hills or mountain tops. The increase is essentially due to a reduced conducting volume compared to a flat ground.

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