# Curvature-induced stiffness in mechanical mimics of the human foot

Ali Yawar [a], Shreyas Mandre [b], and Madhusudhan Venkadesan [a]
[a] Mechanical Engineering and Materials Science, Yale University, New Haven, CT, USA
[b] School of Engineering, Brown University, Providence, RI, USA
*ali.yawar@yale.edu*, *m.venkadesan@yale.edu*, *shreyas_mandre@brown.edu*

## 1 Summary

We push-off using the ball of the foot at the end of stance in walking and running. Unlike humans, the feet of other primates severely bend at the midfoot when they attempt to push-off using the ball. The longitudinal bending stiffness of the human foot is higher than other primates, thus enabling the push-off style we use. In particular, the arched structure of the human foot is thought to underlie its higher stiffness compared to non-human primates. Recent work has shown that the transverse arch of the foot may also significantly contribute to its stiffness, much like a currency note or a pizza slice that stiffens upon curving it transversally. To test this principle, we studied foot-like mechanical structures with variable transverse curvatures. Here we show that the stiffness due to the transverse arch depends on a single dimensionless parameter associated with the curvature of the transverse arch.

## 2 Introduction

Three-point bending tests are used to quantify the static stiffness of elastic structures, including feet [1]. These tests are carried out by clamping one end of the structure and applying loads at the other end. This arrangement resembles the loading experienced by the foot at push-off. When the foot is loaded in this manner, the midfoot experiences bending torques that are resisted by a combination of the soft-tissues that hold the foot together. The structural elements in the foot that stiffen the midfoot are the topic of this paper. In particular, we examine how the arched structure of the midfoot influences its stiffness (fig. 1a).

Recent work [2] has used continuum models to propose the hypothesis that curvature of the transverse arch contributes to this stiffness. However, whether the principle of curvature-induced stiffness that is evident in continuum elastic structures applies to foot-like discrete structures is unknown. The underlying principle in continuum shells is that out-of-plane bending is coupled to in-plane stretching because of the transverse curvature. To test this idea in discrete structures, we abstracted the anatomical complexity of the foot into three metatarsals in a transversally curved arrangement.

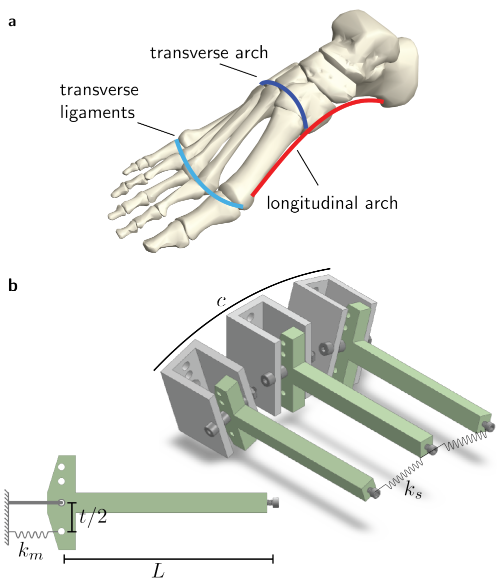


**Figure 1:** **a**, Human foot skeleton showing the medial longitudinal and transverse arches, and the location of the distal transverse ligaments that restrict metatarsal splay under load. **b**, CAD model of a foot-mimic, and an individual metatarsal.

## 3 Methods

We fabricated mechanical mimics of the foot that consisted of three rigid metatarsals hinged at their bases (fig. 1b). This abstraction of the foot lumps all the midfoot degrees of freedom into hinge joints at the base of the metatarsals and the net effect of midfoot elastic tissues into torsional stiffnesses of the hinges. Extension springs at the bases of the metatarsals provided the torsional stiffness. In order to capture the hypothesized effect of the transverse arch, namely a bending-stretching coupling, we include purely transversally oriented springs at the distal end of the metatarsals. These springs would resist splaying of the metatarsals.

The foot-mimics were attached to a 6-axis load cell (JR3 Inc., USA) (fig. 2). Quasi-static vertical displacements were applied at the distal end of the foot-mimics and the reaction force measured. Stiffness is the slope of the measured load-displacement curve. Three lengths (75 - 125 mm), six curvatures (0 - 0.025 $\mathrm{mm}^{-1}$) and three thicknesses (18.5 - 26.8 mm) were tested. Data were collected using Nexus 1.8 (Vicon Inc., UK) and processed in Python.

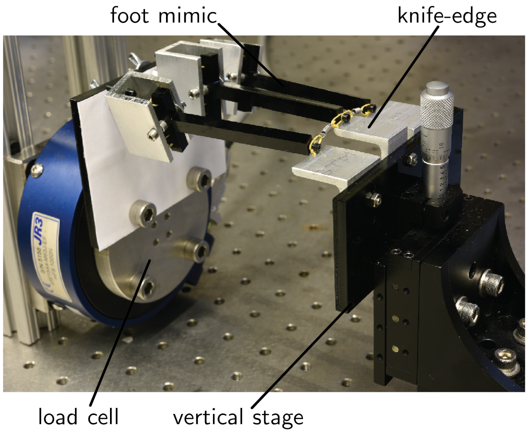


**Figure 2:** The experimental setup used to measure the stiffness of the foot-mimics.

## 4 Results and discussion

The moment arm of the extension springs is half the thickness of the foot-mimic (fig. 1b). For a flat foot-mimic of thickness $t$ and length $L$, the torsional stiffness at each hinge due to the extension spring at the metatarsal base is $k_m(t/2)^2$, leading to a total foot stiffness $k_{\text{flat}} = 3k_m(t/2)^2/L^2$ as measured at the tip. Using a previously published model of the foot-mimic [3], the stiffness of such a foot-mimic of length $L$, transverse curvature $c$, and distal transverse stiffness $k_s$ is given by

$$k = k_{\text{flat}} + \frac{2}{3}k_s c^2 L^2. \tag{1}$$

The stiffness $k_{\text{flat}}$ of a flat mimic of length $L$ and thickness $t$ is expressed relative to the measured stiffness $k_0$ of another foot-mimic of length $L_0$ and thickness $t_0$ as $k_{\text{flat}} = k_0(L_0/L)^2(t/t_0)^2$. The normalized stiffness $\hat{k} = k/k_{\text{flat}}$ of a curved foot-mimic of length $L$ and thickness $t$ is therefore,

$$\hat{k} = \frac{k}{k_0}\left(\frac{L}{L_0}\right)^2\left(\frac{t_0}{t}\right)^2. \tag{2}$$

This normalized stiffness quantifies the stiffness of a transversally curved mimic relative to a flat one. From equation (1), $\hat{k}$ is found as a function of the normalized curvature $\hat{c}$ as,

$$\hat{c} = \frac{cL^2}{t}, \qquad \hat{k} = 1 + \left(\frac{\hat{c}}{\hat{c}_t}\right)^2. \tag{3}$$

The material properties are encapsulated by $\hat{c}_t = \sqrt{9k_m/8k_s}$. For small curvatures, i.e. $\hat{c}/\hat{c}_t \ll 1$, the normalized stiffness is nearly independent of curvature and given by $\hat{k} \approx 1$. In other words, a foot with a shallow curvature behaves like a flat foot. On the other hand, when the transverse curvature is pronounced, i.e. $\hat{c}/\hat{c}_t \gg 1$, the normalized stiffness increases nonlinearly with curvature according to $\hat{k} \sim (\hat{c}/\hat{c}_t)^2$.

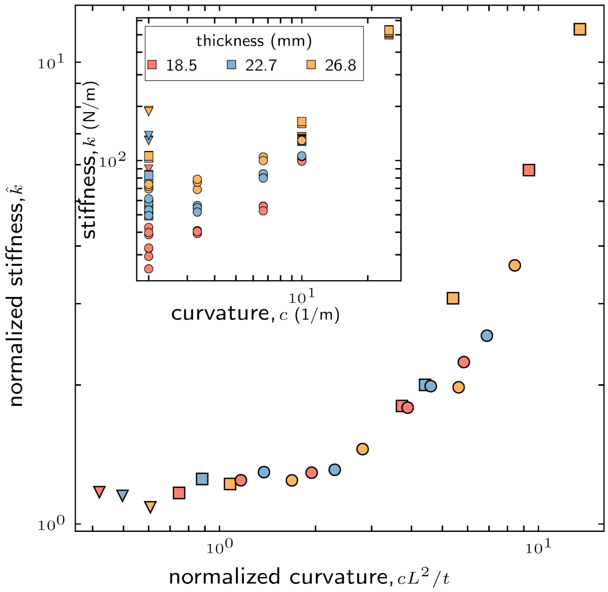


**Figure 3:** Stiffness $\hat{k}$ of each tested foot-mimic normalized by a single flat foot-mimic of $L_0 = 75$ mm and $t_0 = 18.6$ mm (equation (2)). Normalized stiffnesses all collapse to a single curve when plotted against the normalized curvature $\hat{c} = cL^2/t$. Lengths: $\bigcirc = 125$ mm, $\square = 100$ mm, $\triangledown = 75$ mm. (inset) dimensional stiffness measured for all tested mimics shown as individual data points.

We find that the normalized curvature $\hat{c}$ accurately predicts the normalized stiffness of our foot-mimics across the entire range of tested geometries (fig. 3). The normalized stiffnesses transition from having weak dependence on curvature to strong dependence at $\hat{c} \approx \hat{c}_t$.

We conclude that the dimensionless curvature parameter derived from continuum theory is valid for discrete, foot-like structures. Furthermore, the parameter provides an analysis tool for skeletal samples of extant and extinct hominin feet. However, the validity of the parameter in predicting the stiffness assumes approximate isometry with the reference foot used to normalize other feet. Future work will examine the derivation of generalized versions of the curvature parameter. Finally, curvature-induced stiffness and its encapsulation by a dimensionless parameter provide design tools for robotic and prosthetic feet.

## 5 Acknowledgment

This work was supported by the Human Frontier Science Program.

### References

[1] R.F. Ker, M.B. Bennett, S.R. Bibby, R.C. Kester, and R. McN. Alexander. The spring in the arch of the human foot. *Nature*, 325(6100):147, 1987.

[2] M. Venkadesan, M.A. Dias, D.K. Singh, M.M. Bandi, and S. Mandre. Stiffness of the human foot and evolution of the transverse arch. *arXiv preprint arXiv:1705.10371*, 2017.

[3] A. Yawar, L. Korpas, M. Lugo-Bolanos, S. Mandre, and M. Venkadesan. Contribution of the transverse arch to foot stiffness in humans. *arXiv preprint arXiv:1706.04610*, 2017.