Optimizing the periodic motion of a foil at high-Reynolds number

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1 Introduction

The flapping of animal and robotic appendages in a surrounding fluid, such as wings [1–4], tails [5], etc, to generate propulsion is a common strategy in the biological realm. Engineering applications have also sought to mimic biology for robotic propulsion [6, 7]. Other engineering applications of flapping wings include kinetic energy extraction from a flowing fluid for wind and hydrokinetic energy generation [8–14]. Periodic motion of appendages or the whole body is also used by bacteria [15] and nematodes [16, 17], resulting in a surround flow with a small Reynolds number. For all these cases, the choice of the periodic kinematics is critical to the resulting mechanical performance of the engineering design or biological behaviour. However, the space of all possible periodic kinematics is infinite-dimensional and the response of the fluid is governed by partial differential equations, which makes searching for candidate kinematics formidable even if the continuum deformation of these appendages is replaced by rigid-body linkages. When inertial forces in the surrounding flow resulting from the flapping are overwhelmed by viscous forces, the underlying partial differential equations acquire a time-reversal kinematic symmetry, which simplifies the identifiaction of optimum periodic kinematics in some cases [18–21]. However, for many applications such as bird flight, fish swimming and energy extraction, the inertial forces dominate over viscous ones, i.e., when the Reynolds number associated with the resulting flow is large. While many such cases are motivated by practical applications, the absence of time-reversal kinematic symmetry renders the problem of determining optimum kinematics fundamentally challenging. Numerical and experimental investigations of flapping foil fluid dynamics proceeded by parametrizing the kinematic space, thereby reducing it to a finite-dimensional one [9–11, 13, 22–28]. However, it remains unclear how such a parameterization should be chosen or whether the optimum within the finite-dimensional space approximates the infinite-dimensional optimum. The latter is especially significant because, as we show in this work, a small deviation away from parameterized kinematics has the potential significantly alter the performance landscape. Therefore, an exploration of the infinite-dimensional kinematic space is critical to understanding flapping locomotion in the biological context and designing it in the engineering context.

Here we present a method for systematically exploring the infinite-dimensional kinematic space. As a representative example, we apply the method to improve the efficiency of power extraction by an oscillating foil capable of pitching and heaving; it can also be applied for other applications, such as locomotion The experimental setup used to evaluate the energy extraction performance for different kinematics, shown in Fig. 1, consists of a foil that can pitch by an angle $\theta(t)$ and heave according to $h(t)$. The fraction of the incident kinetic energy flux on the swept area of the foil that is extracted is the efficiency, η, of energy extraction. Parameterizing the foil kinematics to be purely sinusoidal, our experiments show that the maximum possible efficiency is 39.7% (kinematics B in Tab. 1). We further show that extending the kinematics to include non-sinusoidal motion yields a maximum efficiency of 42.1% and likely no more (kinematics D in Tab. 1). Remarkably, this increase in efficiency does not arise from a small change to the optimal sinusoidal kinematics; removing the higher harmonic content from kinematics D reduces the efficiency to 27.7% (kinematics E). This observation indicates the presence of pockets of high performance in the kinematics landscape, which could be missed by incomplete or poor parameterization of the kinematic space.

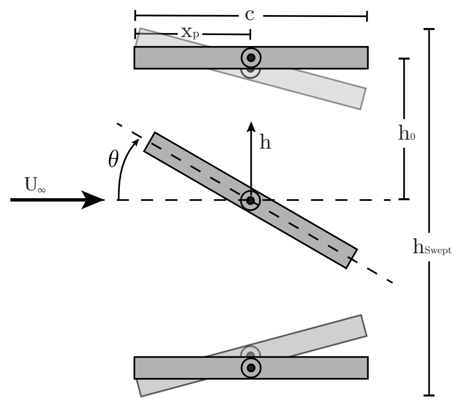

Figure 1: Schematic setup of the oscillating foil for extracting power from oncoming flow.

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Table 1: Comparison of total efficiency η for three kinematics labeled B, D and E. Here f^* is the reduced frequency $\omega c/2\pi U$, and the kinematics are expanded as $\theta(t) = \sum_{n=1}^{\infty} \theta_{n,S}\sin(n\omega t) + \theta_{n,C}\cos(n\omega t)$, $h(t) = \sum_{n=1}^{\infty} h_{n,S}\sin(n\omega t) + h_{n,C}\cos(n\omega t)$, where $\theta_{n,C}$, $\theta_{n,S}$ and $h_{n,C}$, $h_{n,S}$ are the amplitudes of the n^{th} Fourier mode in pitch and heave, respectively. Without loss of generality, we take $\theta_{1,C} = 0$ and represent the fundamental heave kinematic as $h_1\sin(\omega t + \Phi)$, so that $h_{1,C} = h_1\sin\Phi$ and $h_{1,S} = h_1\cos\Phi$. B and D are determined through an optimization devised for this work and E is obtained by removing the higher harmonics from D.

Label	B	D	E
f^*	0.136	0.136	0.136
θ_1	74°	79°	79°
h_1	$0.70c$	$0.39c$	$0.39c$
Φ	96°	100°	100°
$\theta_{3,S}$	0	5.0°	0
$\theta_{3,C}$	0	4.7°	0
$h_{3,S}$	0	$-0.06c$	0
$h_{3,C}$	0	$-0.08c$	0
$\eta_{\max}$ (%)	39.7 ±0.4	42.1 ±0.5	27.7 ±0.5

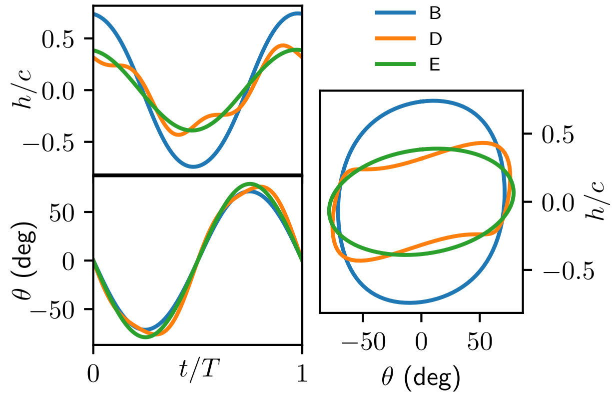

Figure 2: Comparison of kinematics B, D and E.

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