

Online Prediction of Synchronization Dynamics in Coupled Oscillators System

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1 Introduction

In nature, functional behaviors are generated by the interaction between simple oscillatory elements, where they modulate the timing (phase) and speed (frequency) of the oscillation. Synchronization Phenomenon, in which phases of all oscillatory element's motions are synchronized, is one of attractive properties of coupled oscillators system.

Synchronization in coupled oscillators system is also found in human brain[1]. Understanding the spatiotemporal patterns of brain activities as a coupled oscillators system is important to understand cognitive or motor function of brain. Although researches have been conducted thus far for understanding the theoretical aspects of the dynamics, they were oriented for analysis of the dynamics [2]. Very few studies have tried to predict and control the synchronization. We here thought it is important to work on real prediction than its analysis for real life application.

In this study, we proposed a method for online prediction of synchronization in coupled oscillators system. First, we focus on synchronization phenomena in mechanical metronomes as a simple representative example of coupled oscillators system. By using prediction method based on Kuramoto model, we predict the future time evolution of the parameter that characterized the synchronization. The results indicate that our method predicts synchronization 30 s before with 10% error and 50s before with 15% error, suggesting that the proposed method would be applicable for real world applications such as predicting the dynamics of synchronization phenomenon in real life.

2 Materials and methods

2.1 Experimental setting

To reproduce the synchronous dynamics of the coupled oscillators system, we prepared an experimental setting with three mechanical metronomes (ammoon Inc.) on a plastic board (400 mm × 200 mm) hanging with 4 strings (Fig.1).

We recorded oscillating behaviors of the metronomes by using a web camera (Logicool, C922 Pro Stream Webcam, 320 × 240, 30 fps) and tracked the center horizontal positions of the weight part (black marker) of the metronomes by using OpenCV libraries in Python.

2.2 Mathematical models for synchronization

To predict the synchronous dynamics of the target coupled oscillators system, we used Kuramoto model [3], which is a simple coupled oscillators system that consists of ab-



Figure 1: Coupled Oscillators Consisting of Metronomes

stracted phase oscillator model, successfully describing the synchronization dynamics of the system. The dynamics of Kuramoto model is described by the following equation:

$$\frac{d\theta_i}{dt} = \omega_i + \frac{K}{N} \sum_{j=0}^N \sin(\theta_i - \theta_j) \quad (1)$$

where θ_i and ω_i represent the phase and angular velocity of i th oscillator, respectively. The second term in the right hand side represents the interactions between the corresponding oscillator and the other oscillators. The parameters K and N denote the strength of interaction between oscillators and the number of interacting oscillators, respectively.

In order to quantitatively evaluate synchronization in the coupled oscillators system, we used order parameter r described by the following equation (2). The order parameter is regarded as synchronization rate of the system.:

$$r e^{i\psi} = \frac{1}{N} \sum_{j=0}^N e^{i\theta_j} \quad (2)$$

where ψ represents the phase average and i represents the imaginary unit, respectively. As shown in Fig.2, when the phase differences of the system have large variance, the value of r becomes smaller (Fig. 2 left), whereas the variance of phase differences is small, the value of r becomes larger (Fig. 2 right).

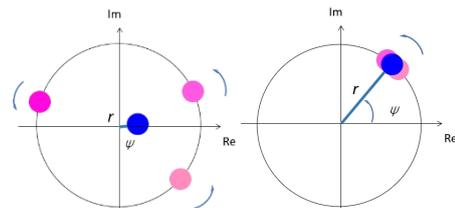


Figure 2: r value change for Non-Sync(left) and Sync(right)

2.3 Online prediction method

The estimation of the frequency and phase of each metronome is essential for the online prediction of synchronization in the system. First, we applied FFT(Fast Fourier Transform) to data to estimate frequency and performed sine curve fitting to get the phase of each metronomes.

Then, we performed numerical calculations of Kuramoto model by using the estimated frequency and phase. Numerical calculation was performed with the time step $dt = 1/30[s]$. Finally, to predict the synchronous dynamics by using Kuramoto model, we determined the optimal interaction strength K for the system in a moving time window using the least squares error method. Error function $E(k)$ is defined by following equation:

$$E(k) = \sum_{n=0}^N (r_{est} - r_{obs})^2 \quad (3)$$

where r_{obs} and r_{est} denote the observed and estimated order parameters, respectively. By repeating this calculation in moving time window, online prediction of the synchronous dynamics can be achieved.

3 Result

We set the frequency of metronomes 1 Hz and the length of moving time window 50[s]. Figure 3 shows the result of online prediction. The horizontal axis is time [s] and the vertical axis is the order parameter $r(0 \leq r \leq 1)$. The gray solid line represents the observed value of the order parameter r_{obs} , and the other colored solid lines represent the predicted value of the order parameter r_{pred} . The different colors represent the difference of the start time of predictions. The dotted black line represents the defined threshold for synchronization, where order parameter $r = 0.95$. We evaluated the prediction error μ as the following equation 4:

$$\mu = |r_{pre} - r_{obs}| \quad (4)$$

By using this index, we compared the accuracy of the predictions for all trials as shown in Table.1.

Table 1: Prediction error for each horizons

Prediction horizon : p [s]	+0	+10	+20	+30	+40	+50
μ [%]	0.0%	4.5%	8.2%	10.3%	13.6%	14.3%

4 Discussion

The proposed method successfully predicted the synchronous dynamics 30 seconds before with 10% of prediction error and 50 seconds before with 15% on the three different environmental conditions with different string length of supporting board of metronomes. If we imagine that how difficult for us to guess whether the metronomes tend to synchronize more than 10 seconds before or more, the results suggest that our method has potentials for predicting synchronous dynamics of coupled oscillators systems in real world applications. Moreover, the number of metronomes could be increased by using the proposed method. On the other hand, the prediction result more than 60 seconds before has a large prediction error. The error was caused by

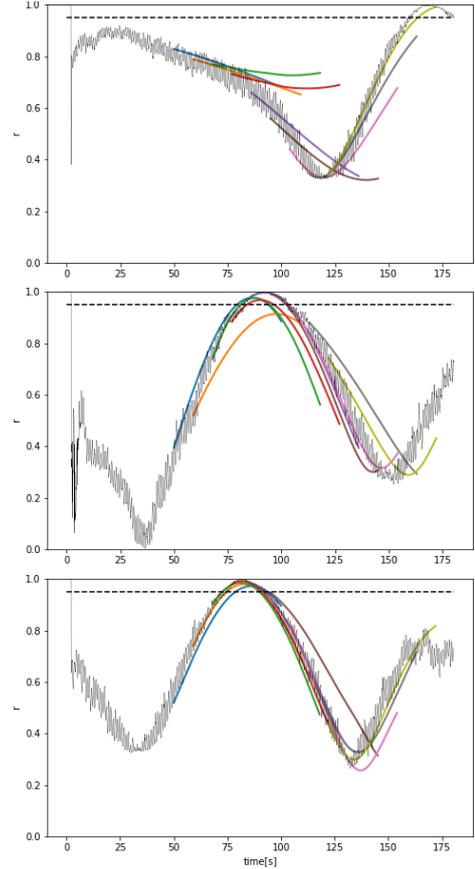


Figure 3: Results of online prediction for synchronization rate on deferent conditions. $l = 20\text{cm}$ (top), 15cm (middle), 10cm (bottom)

numerical calculation with the use of Kuramoto model, as it makes rough approximation for its simplicity. Furthermore, the oscillation of the metronome did not strictly follow the simple sinusoidal wave, depending on the complicated mechanical property of the system.

5 Conclusion

In this study, we proposed an online prediction method for synchronous dynamics of a coupled oscillator system consisting of metronomes. Synchronization prediction was successful 30 seconds before with 10% of prediction error and 50 seconds before with 15%. In order to predict the synchronous dynamics of more complicated coupled oscillator systems like nervous system, it is necessary to use biological signal processing skills and to apply proper coupled oscillator system theory for the systems.

References

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