A Novel CPG for Smooth and Bounded Trajectory Generation from Motion Library

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1 INTRODUCTION

Cyclic motions are still fundamental patterns of robot motion. To generate such motions autonomously and reactively, we need to endow the robot with an online trajectory planner guaranteeing (i) smooth transitions from the current state to the desired motion, (ii) smooth transitions from one motion to another, (iii) the physical limits of the robot, and (iv) the capability of integrating sensory feedback [1]. The central pattern generator (CPG) is an approach promising all the above requirements. A CPG comprises coupled non-linear dynamical systems generating synchronized rhythmic signals. Modulation of CPG parameters, including coupling weights and coefficients of the dynamical systems, produces a variety of cyclic motions [2]. The main restriction for on-line implementation of CPG is the fact that tuning the parameters for a desired motion is often complex and time consuming. In this paper, we propose a CPG architecture generating a synchronized trajectory which converges smoothly to the desired one, under the assumption that a library of cyclic trajectories is given. Compared to the existing CPGs, the proposed one does not require parameter tuning and, also preserves the predefined limits both on the trajectory itself and its first time derivative. Experiments on the humanoid robot iCub, verified the soundness of the proposed CPG in generating synchronized trajectories.

2 CPG ARCHITECTURE

The proposed CPG architecture is based on a Data Driven Vector Field Oscillator (DVO). A DVO converges to any predetermined 1-dimensional non-self-intersecting periodic function in the state space while providing the possibility of generating a bounded output. In this section, first we explain the DVO dynamic structure. Then we propose an approach to provide phase synchronization of n DVOs to create a CPG generating a synchronized multi-dimensional bounded output.

2.1 Dynamic Structure of DVO

Given a predetermined T-periodic function $f(t)$: $[0,\infty) \to \mathbb{R}$ depicting a non-self-intersecting curve in the plane $(f,\dot{f})$, the differential equation of the DVO is

$$\begin{cases} \dot{s}_1 = \frac{\delta_{\dot{y}} \tanh(s_2)}{J} \\ \dot{s}_2 = \frac{\delta_{\dot{y}} \tanh(s_2)}{J s_2} \big(g_a(\varphi) - \alpha(s_2 - g_v(\varphi)) - \beta(s_1 - g_p(\varphi))\big), \end{cases} \tag{1}$$

where

- $s = (s_1, s_2)$ are the states,
- $J = \delta_y(1 - \tanh^2(s_1))$,
- $g_p(\varphi) = \tanh^{-1}\left(\left(f(\varphi) - y_{avg}\right)/\delta_y\right)$,
- $g_v(\varphi) = \tanh^{-1}\left(f'(\varphi)/\delta_{\dot{y}}\right)$,
- $g_a(\varphi) = \frac{\delta_y\left(1-\tanh^2(g_p)\right)}{\delta_{\dot{y}}^2(1-\tanh^2(g_v))} \frac{g_v}{\tanh(g_v)} f''(\varphi)$,
- $f(\varphi) : [0,T) \to \mathbb{R}$ is one period of $f(t)$, and
- $\alpha, \beta, \delta_y, \delta_{\dot{y}} \in \mathbb{R}^+$ and $y_{avg} \in \mathbb{R}$ are constant coefficients.

Superscripts $\cdot$ and $\prime$ denote the first derivative with respect to t and φ, respectively and φ is the phase variable defined based on the states s as

$$\varphi(s) = \begin{cases} g_p^{-1}(\underline{g}) & s_1 \leq \underline{g} \\ \{\varphi | g_p(\varphi) = s_1, \, g_v(\varphi) s_2 \geq 0\} & \underline{g} \leq s_1 \leq \overline{g} \\ g_p^{-1}(\overline{g}) & s_1 \geq \overline{g}, \end{cases} \tag{2}$$

where $\underline{g}$ and $\overline{g}$ are the lower and upper bounds of g_p.

The output of DVO is defined as $y = y_{avg} + \delta_y \tanh(s_1)$.

Proposition 1. Given α satisfying $(g_a + \alpha g_v)|_{s_2=0} \neq 0$ and $g_v\,(g_a + \alpha g_v) \geq 0$, then g_p is the global stable limit cycle of the DVO. This implies that the output y tracks the trajectory $f(\varphi)$ while being bounded ($|y - y_{avg}| < \delta_y$ and $|\dot{y}| < \delta_{\dot{y}}$).

Remark. A DVO can also converge to constant functions besides periodic ones.

2.2 Phase Synchronization of Multiple DVOs

The DVO dynamics can be written in a compact form as $\dot{s} = \mathbf{F}(s)$. Assuming n DVOs with different stable limit cycles g_{p_i}, $i = 1, ..., n$, the coupled dynamics that comprises phase regulation is as

$$\dot{s}_i = \frac{\mathbf{F}_i(s_i)}{\lambda(e_{\varphi_i})}, \tag{3}$$

where $\lambda(e_{\varphi_i})$ is a multiplier function defined as

$$\lambda(e_{\varphi_i}) = 1 + k_1 \tanh(k_2 e_{\varphi_i}), \; k_1, k_2 \in \mathbb{R}^+ \tag{4}$$

and $e_{\varphi_i} = \hat{\varphi}_i - \bar{\varphi}$ is the error between $\hat{\varphi}_i$ which is the phase of the i^{th} DVO, and the average phase $\bar{\varphi} = \sum_{i=1}^{n} \hat{\varphi}_i / n$. Let us define $\hat{\varphi}_i(t)$ as $\varphi_i(s_i)$ that increases monotonically as the states move along the limit cycle, *i.e.* $\hat{\varphi}_i = kT + \varphi_i$ where

$k \in \mathbb{R}^+$ is the number of times that the oscillator's states move along the limit cycle. Consequently, $\dot{\hat{\varphi}}_i = \dot{\varphi}_i$ and the time derivative of (2) gives the phase dynamics

$$\dot{\hat{\varphi}}_i = \begin{cases} \dfrac{\left(1-\tanh^2(g_{p_i})\right)\tanh(s_{i_2})}{\tanh(g_{v_i})\lambda(e_{\varphi_i})\left(1-\tanh^2(s_{i_1})\right)} & \underline{g} < s_{i_1} < \overline{g} \\ 0 & otherwise. \end{cases} \tag{5}$$

Hence, one can measure the oscillator's phase instantaneously by evolving the states,

Proposition 2. Given the coupled dynamical system (3) satisfying the following assumptions,

(A1) For $i = 1,..,n$, the periodic function $g_{p_i}(\varphi_i)$ is the stable limit cycle of the uncoupled i^{th} DVO.

(A2) The coefficients $k_1 < 1$ and k_2 are positive constants.

Then, for $i = 1,..,n$, we have:

- g_{p_i} is the stable limit cycle of the i^{th} DVO in Eq. (3).
- the phase error e_{φ_i} converges to zero.

Roughly speaking, the multiplier function affects the time evolution of DVOs. All DVOs with $\hat{\varphi}_i < \bar{\varphi}$ speed up while the DVOs with $\hat{\varphi}_i > \bar{\varphi}$ slow down. The amount of speed variation is directly related to the absolute value of the phase error. Whereas, the sign of the phase error specifies which oscillator must speeds up or slows down.

3 EXPERIMENTS AND RESULTS

The effectiveness of the proposed CPG has been validated through experiments on the humanoid robot iCub [3]. Given a motion library composed by three periodic motions with different periods, the proposed CPG takes one of these motions as input and generates the corresponding output which in turn is the reference trajectory for controlling the joints of the robot in position. For the sake of simplicity, we chose to control only two joints of the robot: the right arm's shoulder roll (`R_SH_R`) and the elbow pitch (`R_EL_P`). The other joints were kept at a constant joint position. Therefore, the CPG considered only the `R_SH_R` and `R_EL_P` in the phase synchronization. Note that, as the desired trajectory is changed by the user, the phase dynamics (5) must be reinitialized. Thus, switching from one desired trajectory to another one causes a phase jump which results in a spike in velocity of the reference trajectory. Then, the velocity spike is reflected as a spike in demanded control torque, causing potential damage in the robot actuators. Hence, we used a minimum jerk trajectory generation algorithm [4] on the phase error to guarantee a smooth phase transition. From the reference trajectories resulting from the experiments (see Figure 1), we infer that: (i) the coupled system guarantees the convergence of both the joint positions toward the desired trajectories (dashed black curves or stars in the phase plots); (ii) the joints trajectories are smooth and preserve the position and velocity limits (dashed red lines); (iii) the phase error between the two joints converges to zero, and thus the proposed coupling technique succeed at phase synchronization of CPG; (iv) the phase variable is defined appropriately since the coupled system tracks the desired motion in the joint space (black dashed lines and star in joint space).

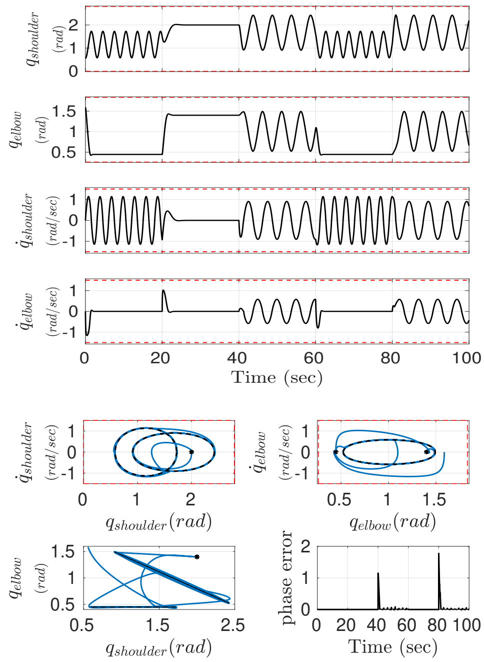

Figure 1: Position and velocity trajectories, motions in state space and joint space, and phase error, while controlling the iCub's right arm through the proposed DVO-based CPG.

4 CONCLUSION

We proposed the DVO, a dynamical system that (i) tracks any periodic function depicting a non-self-intersecting curve in the state space, and (ii) preserves predefined limits on the output and its first time derivative. Then, we presented an approach to synchronize multiple DVOs, which can be also used for synchronizing other kind of oscillators. Finally, we created a novel CPG made of synchronized DVOs that is specifically designed for robotic applications where motion modulation is required. The proposed CPG generates smooth, synchronized and bounded reference trajectory tracking the desired one, without having to tune the parameters for the desired motion. The desired trajectory is a multi dimensional function whose components are either non-self-intersecting periodic functions or constants. In a typical control architecture for robotic systems, the desired trajectory planner is immediately followed by the controller. However, inserting a CPG in between these two elements of the architecture is going to provide features prominent in the animal motions (*e.g.* limit cycle tracking, as the result of oscillatory essence of CPG, and adaptation, enabled through integration with sensory feedback.)

References

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