Torsional Body Flexibility Effect on Stability in Trot and Pace based on a Simple Model

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1 Introduction

Quadrupeds use various gaits, depending on their locomotion speed [1]. At the middle speed range, they selectively use two gaits: trot and pace. In trot, quadrupeds synchronize their diagonal legs. On the other hand, in pace, they use ipsilateral pairs of legs [2, 3]. Most quadrupeds generally use trot gait, and a few quadrupeds inherently use pace gait [1–4]. However, the reason remains unclear.

It is well known that body flexibility of quadrupeds highly affects on their gaits [5, 6]. In this research, the authors assumed that torsional flexibility of torso plays an important role in selections of trot or pace in animals. To investigate effects of torsional flexibility from the viewpoint of dynamics, the authors constructed a simple model with a torsional spring and derived periodic solutions analytically corresponding to trot and pace gait. Stability of periodic solutions was evaluated with eigenvalue analysis of linearized Poincaré maps.

2 Model

As Figure 1 shows, the model consists of two rigid body segments and four spring legs, projected onto transverse plane. The position of the center of mass is (y,z), and the angles of forward and backward segments are ϕ_1 and ϕ_2, respectively. Table. 1 shows parameters of the model. Body segments are connected with a torsional spring k_t that represents body flexibility. For convenience, all of following expressions are formulated in dimensionless quantities using the total mass m, a time scale $\tau := \sqrt{l_0/g}$, nominal leg length l_0, and the gravitational constant g.

Let right fore leg, right hind leg, left fore leg and left hind leg be leg 11, leg 12, leg 21, leg 22, respectively. Events of touchdown and liftoff for each leg switch equations of motion. Linearized equations of motion are follows:

$$\ddot{y} = 0, \tag{1}$$

$$\ddot{z} + \sum_{i,j} f_{ij} + 1 = 0, \tag{2}$$

$$j\ddot{\phi}_1 + \sum_i (-1)^{i+1} d f_{i1} + k_t(\phi_1 - \phi_2) = 0, \tag{3}$$

$$j\ddot{\phi}_2 + \sum_i (-1)^{i+1} d f_{i2} + k_t(\phi_2 - \phi_1) = 0, \tag{4}$$

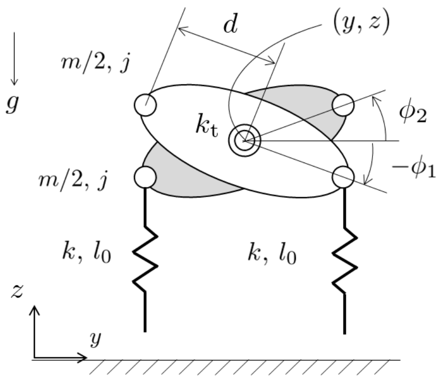

Figure 1: The simple model with a torsional joint. In the figure, display of swing legs is omitted.

Table 1: Lists of mechanical parameters. All values are normalized with respect to total mass, nominal leg length, and the gravitational constant.

Param.	Unit	Description
m	$[\cdot]$	total mass
l_0	$[\cdot]$	nominal leg length
g	$[\cdot]$	gravitational constant
d	$[l_0]$	half of body width
j	$[ml_0^2]$	inertia of half segment
k	$[mg/l_0]$	leg spring constant
k_t	$[mgl_0/\mathrm{rad}]$	torsional spring constant

where, f_{ij} corresponds to the reaction force from leg ij $(i, j = 1, 2)$, i.e.

$$f_{ij} = \begin{cases} k(z + (-1)^{i+1} d\phi_j - 1) & \text{in stance} \\ 0 & \text{in flight} \end{cases}$$

A touchdown or liftoff event of leg ij occurs when the following boundary condition(BC) is satisfied.

$$r_{ij}(z, \phi_1, \phi_2) = z + (-1)^{i+1} d\phi_j - 1 = 0 \tag{5}$$

Gait is a periodic motion. Therefore, the authors only focused on periodic motions representing pace and trot. Each periodic motion was defined with its boundary conditions corresponding to its foot patterns. Trot and pace have bilateral symmetry. Hence half of periodic motions were consid-

Table 2: Boundary conditions(BC) of trot and pace for a half cycle. "Simul. TD" and "Simul. LO" indicate simultaneous touchdown and liftoff, respectively.

	BC0	BC1	BC2
Pace	Leg 11 & 12 Simul. TD	Leg 11 & 12 Simul. LO	Leg 21 & 22 Simul. TD
Trot	Leg 11 & 22 Simul. TD	Leg 11 & 22 Simul. LO	Leg 12 & 21 Simul. TD

ered as Table 2 shows, and the number of BC is three. Periodic solutions were derived analytically from their governing equations and BCs. Solutions were classified into two categories: feasible solutions, which have adequate stride frequency for animal, and not-feasible solutions, which have too high stride frequency or include physically impossible states such that a stance leg length becomes negative.

3 Stability analysis

The authors investigated stability of fixed points using a Poincaré map [7]. A Poincaré section was defined as the timing when leg 11 touches down. A half map consists of a stance phase, a flight phase, and variable conversions. Therefore, the half map $P_{\rm h}$ is represented as a composition of factored maps [8, 9];

$$P_{\rm h} = B_{\rm LR} \circ P_{\rm f} \circ P_{\rm s} \tag{6}$$

Where $P_{\rm s}$ and $P_{\rm f}$ are the stance and flight map respectively, and $B_{\rm LR}$ is a mapping to flip horizontally. With linearization of each factored map, the Jacobian of $P_{\rm h}$ is obtained as follows:

$$DP_{\rm h} = B_{\rm LR} \cdot DP_{\rm f} \cdot DP_{\rm s} \tag{7}$$

By calculating the maximum magnitude of eigenvalues of $DP_{\rm h}$ ($|\lambda|_{\max}$), the authors defined a stable fixed point when $|\lambda|_{\max} \leq 1$ is satisfied. The Jacobian matrices of trot and pace were derived analytically.

4 Result & discussion

The authors calculated each $|\lambda|_{\max}$ numerically using $DP_{\rm h}$ which was derived with analytical processes. With fixed leg spring constant $k = 5.0$ and torsional spring constant $k_t = 0.01k$, the authors investigated stability of trot and pace for various body width and inertia.

Figure 2 shows the relationship between stability of trot and pace. In the figure, solutions in gray regions are stable, and those in black regions are unstable. Regions of non-feasible solutions were displayed with white. From the figure, both periodic solutions of trot and pace include stable and unstable area. However, trot has the broader area of stable regions than that of pace. The difference partly explains the reason why many quadrupeds prefer trot to pace from the viewpoint of stability.

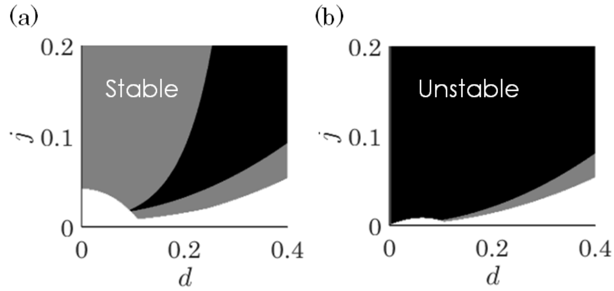

Figure 2: Stability of (a) Trot and (b) Pace for various d and j. Gray and black regions respectively represent stable and unstable regions. No feasible solutions exist in white regions.

In this research, parameters of real quadrupedal animals were not applied. Hence, in the future, the authors would like to investigate the effects of torsional flexibility by applying estimated parameters of animals.

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