Non-reciprocal manipulation of subwavelength fields in locally-resonant metamaterial crystals

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***Abstract*—Locally-resonant metamaterial crystals are artificial materials built from small spatially-local resonant inclusions arranged periodically at subwavelength scale. Unlike conventional continuous metamaterials, for which spatial dispersion originates mostly (but not exclusively) from the non-locality of their inclusions, they exhibit large spatially non-local effects that emerge solely at the array level because of the periodic structuration of simple spatially-local scatterers, often allowing for an intrinsically subwavelength granularity. Here, we demonstrate the unique relevance of metamaterial crystals to induce non-reciprocal electromagnetic propagation at deep subwavelength scales. This is obtained by combining the breaking of time-reversal symmetry, using an externally biased magnetic material, with appropriate spatial-dispersion engineering, via subwavelength structural modification of the metamaterial crystal. Interestingly, the material unit cell can be scaled down without affecting this functionality, leading to the exciting possibility of largely enhanced wave-matter interaction at deep subwavelength scales. Altogether, our proposal provides an interesting route for transposing the rich physics of non-reciprocal systems down to the subwavelength scale.**

***Index Terms*— Metamaterials, non-reciprocity, parity-time symmetry, subwavelength wave manipulation.**

I. Introduction

Electromagnetic media typically support two kinds of dispersive effects. The first one, temporal dispersion [1], describes the fact that polarization always occurs with some delay with respect to an applied external field [2]. The second one, spatial dispersion [1,3], is related to the fact that the polarization state at a given point may not only depend on the external field at that location, but also on the incident field at distant points, due to multiple scattering effects. Together, these two different dispersive effects capture the vast majority of electromagnetic properties of natural and artificial materials.

While spatial dispersion can be observed in natural substances, explaining the optical activity of quartz or the bright colors of butterfly wings [4], it only reaches its full applicative potential in artificially engineered media, such as metamaterials [5] or photonic crystals [6]. Spatially-dispersive metamaterials can be built, for instance, from bi-anisotropic [7] inclusions that support significant magneto-electric coupling [8]. Designing the inclusion can allow some control over the spatial dispersion up to the second order, leading to various unconventional effects. For instance, giant optical activity can be observed in metamaterials designed to support strong first order spatial dispersion [9–11]. On the other hand, metamaterials exhibiting artificial magnetism [12–15] and negative refraction [16,17] require spatial dispersion of second order [18], which cannot be found in natural materials. To engineer spatial dispersion beyond second order, a relevant approach consists in directly tailoring the dependency of the dispersion relation on the wave number, as it is done in photonic crystals [4,6,19]. In the latter, dispersion results mostly from multiple scattering effects on the periodic wavelength-scaled structure, a phenomenon called Bragg scattering [6,20-23]. Both of these conventional approaches, however, share a common limitation: they do not allow scaling down the crystal to extremely small electrical scales without affecting its functionality, strictly prohibiting field manipulation and focusing at *deep* subwavelength scales.

Recently however, a very unconventional solution to tackle this limitation has been proposed, allowing for strong spatial dispersion even at the subwavelength scale, by using the concept of locally-resonant metamaterial crystals (LRMCs) [24–26]. Such crystals, which are built from simple, spatially local, resonant subwavelength inclusions, indeed support strong spatially dispersive effects that are due exclusively to their locally resonant subwavelength structure and array characteristics [25]. Unlike photonic crystals, their operating frequency depends mostly on the resonant behavior of the inclusion, and is relatively decoupled from the periodic spatial scale. When the local resonators are electrically small, deep subwavelength structuration is possible, allowing for subwavelength focusing, wave-guiding [23–25], and a scaling of the structure without strong shifts of the operation frequency [26]. Besides, LRMCs exhibit strong multiple scattering at the level of their subwavelength array, allowing for a different way of controlling spatial dispersion at extremely small spatial scales.

In a seemingly unrelated field of research, namely non-reciprocal physics [28-30], there has been an intense surge of interest to achieve non-reciprocal wave propagation at deep subwavelength scales, paving the way for miniaturizing conventional microwave non-reciprocal devices such as circulators, isolators, etc. While there exist some reports on such a salient behavior in regular metamaterials, here we

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explore this in LRMCs. In particular, we numerically demonstrate that combining subwavelength structural modifications in a metamaterial crystal with a broken time-reversal symmetry, via an externally biased magnetic material, enables the design of a novel class of non-reciprocal bulk electromagnetic waves based on subwavelength spatial dispersion. We furthermore evidence that a spatial scaling operation, as a unique prerogative of LRMCs, leads to a large increase of the wave-matter interaction. This allows for a drastic reduction of the magnetic bias strength required to obtain a given level of non-reciprocity. This property paves the way towards engineering ultra-compact non-reciprocal devices and efficient wave-matter interaction platforms.

II. Breaking the Fundamental Symmetries of the Metamaterial

We use, as the elementary brick of the LRMC, one of the simplest electromagnetic resonators, that is an infinite high dielectric cylinder with a radius of $r = 0.3\ mm$ and an index of $n = 80$. Note that the specific implementation of the subwavelength resonance is unimportant for the purpose of this work, and other types of resonators, like finite metallic rods, are also possible [26,27]. We first highlight the resonant behavior of this single scatterer by evaluating, using full-wave finite-element calculations, its transmission coefficient T when embedded in a single-mode waveguide (with boundaries corresponding to "perfectly magnetic conductors" which impose cancellation of the tangential magnetic field) and excited by a transverse electric (TE) polarized wave (Fig. 1a). As a signature of a magnetic resonance, we observe a sudden drop of transmission at $f_0 = 2.33\ GHz$ ($\lambda_0 = 129\ mm$), revealing the fact that this inclusion is of deep subwavelength ($r = \lambda_0/430$) in the propagation plane. We can then build a LRMC by forming a triangular array of such resonators, with a unit cell of size $b = a\sqrt{3}$ in which $a = 1\ mm$ (Figure 1a). Remarkably, although the resonators and the lattice constant are of small electrical length, a strong spatial dispersion exists in this LRMC, due to multiple scattering effects. The strong dependency of the phase velocity on the wave number, equivalent to a **k**-dependent refractive index, symptomatic of spatial dispersion, is evidenced by the band structure of the TE modes shown in the inset. Note that the lowest band lies well below the light cone (dashed orange line), confirming the subwavelength nature of the non-local effects.

To induce non-reciprocal behavior, we now consider a more complex crystal, namely a honeycomb lattice, obtained by adding a second resonator to the previous unit cell as shown in Fig. 1b, 1. The dispersion bands of this metamaterial crystal are shown in Fig. 1c. It is interesting to note two things regarding the obtained band structure. First, it evidences that spatially dispersive effects well beyond second order can be obtained using the subwavelength structuration of the metamaterial [25,26]. Moreover, the fact that the cone frequency is very close to the resonance frequency of the inclusion reveals that the latter sets the frequency of operation, contrarily to non-resonant photonic crystals, which are wavelength scaled. Naturally, the LRMC under study is reciprocal, and its band-structure is symmetric with respect to Γ point. Starting from the reciprocal

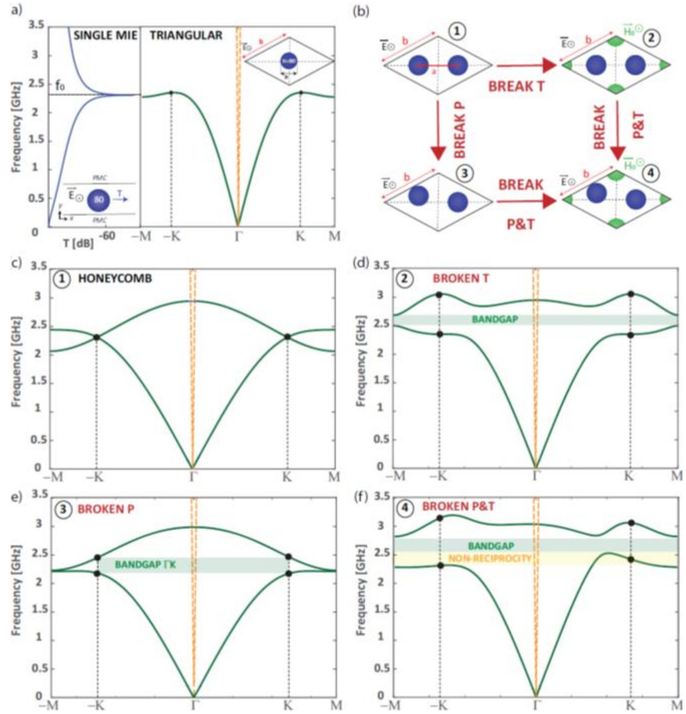

Fig. 1. Design of a non-reciprocal LRMC: a, (left) Transmission coefficient (log-scale) of a single cylindrical Mie particle (blue disk) excited by a TE wave and (right) dispersion relation of a subwavelength triangular lattice of resonators (green) shown together with the light cone (orange). b, A single unit cells of the honeycomb medium (1), with additional ferrite cylinders (green disks) biased by $H_0 = 1.62e^5\ A/m$ to break *T*-symmetry (2), with a slight spatial shift of one of resonators inside the unit cell ($\delta y = 0.217\ mm$), breaking *P*-symmetry (3), and with both modifications, breaking both *P* and *T* symmetries (4). **c-f,** Band structures (green) corresponding to the cases 1-4 in panel b, with band-gaps opening at K points (shaded green rectangle, black dots) for symmetry broken media. The non-reciprocal frequency range is shaded in yellow.

lattice, our goal is now to induce strong non-reciprocal wave propagation at this subwavelength scale. To do so, we have to make its band structure asymmetric with respect to Γ point, so as to create a frequency range in which propagation is allowed in one direction (bulk modes can propagate in +ΓK direction for instance), whereas it is prohibited in the opposite direction (the crystal exhibits a band gap for the modes travelling in -ΓK direction). This requires first opening a band-gap by lifting the degeneracy at K, which can be obtained by breaking one of the metamaterials fundamental symmetries, be it time-reversal (*T*-symmetry) or spatial inversion (*P*-symmetry). As presented in the inset of Fig. 1b, the *T*-symmetry can be broken by adding magnetically biased ferrite rods ($r = 0.3\ mm$, $H_0 = 1.62e^5\ A/m$ biased along z) to the unit cell (note that the direction of the external magnetic bias is chosen in such a way that it preserves the TE polarization of the mode). The dispersion relation of the corresponding crystal, displayed in Figure 1d, confirms the opening of a complete band-gap around the Dirac cone, due to the ferrite magnetic effect (this band-gap is of topological nature, however this is not crucial to the effect reported here). Breaking the *P*-symmetry, on the other hand, requires modifying the structure of the honeycomb unit cell, which can be accomplished by, for instance, slightly shifting ($\delta y = \lambda_0/600$) the position of one of the resonators (Fig. 1b, 3).

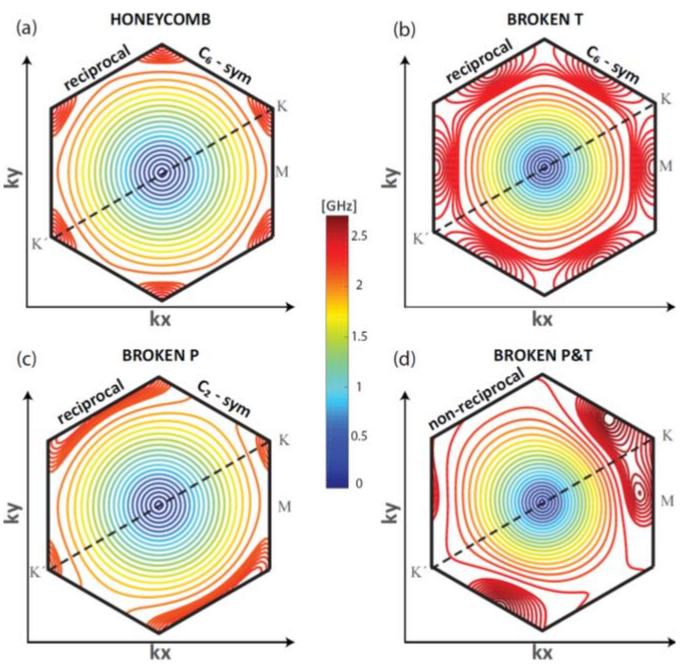

Fig. 2. Numerical isofrequency contours of the first band for a, the honeycomb b, the *T*-broken, c, the *P*-broken symmetry and d, the *P&T*-broken symmetry metamaterial crystals. The direction K'ΓK is highlighted (dotted black line).

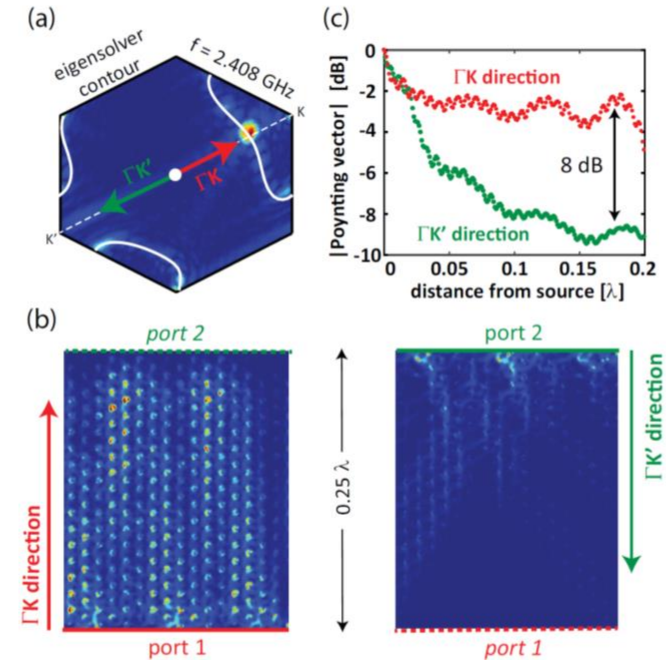

Fig. 3. Demonstration of bulk non-reciprocity in the metamaterial crystal. a, Isofrequency contour at $f = 2.408$ *GHz*, together with with the Fourier transform of E_z field in a crystal consisting of 43*43 unit cells. b, Profile of the norm of Poynting vector when the crystal is excited from port 1 (left) and port 2 (right). Evolution of the norm of Poynting vector when moving away from the source along ΓK (red dots) and ΓK' (green dots) direction.

As seen in Fig. 1e, such modification has a profound impact on the band structure of the crystal although it does not lead to a complete band-gap opening.

Both of the latter band structures are perfectly symmetric with respect to Γ. This result was somehow expected [28,29] since both time reversal and spatial inversion symmetries map $+\mathbf{k}$ into $-\mathbf{k}$. Hence, the relation $\omega(+\mathbf{k}) = \omega(-\mathbf{k})$ holds true as long as the crystal has either *T* or *P* symmetry. To make the band gap asymmetric with respect to Γ, one has to break both symmetries, simultaneously, as schemed in Figure 1b, 4. The dispersion associated with this *P*- and *T*- broken symmetry metamaterial crystal (Fig. 1f) evidences a non-reciprocal behavior, since now $\omega(+\mathbf{k}) \neq \omega(-\mathbf{k})$ for the first two bands. In particular, an extreme asymmetry (i.e. a one-way propagation) is achieved in the yellow shaded region for which electromagnetic waves are allowed to propagate along +ΓK, while their propagation is completely prohibited along the opposite direction (ΓK', referred to as -ΓK in what follows). In other words, the *P&T* symmetry breaking imposes that the interferences due to multiple scattering are constructive in one direction, allowing for propagation, and destructive in the opposite one, creating a band gap. This is an exceptional feature as spatial dispersion at subwavelength scales is often considered as a weak effect and left unexploited at small electrical scales, except in rare cases [31–33].

It is now instructive to study the effects of *PT* symmetry breaking on wave propagation in all possible directions in the *xoy* plane. To that aim, we plot in Figure 2, for each of the four previous media, a collection of isofrequency contours corresponding to the lowest band, in the first Brillouin zone. As observed, the isofrequency contours of the honeycomb lattice exhibit a C_6-rotational symmetry, which largely fulfills the reciprocity criteria (Fig. 2a). Indeed, a consequence of reciprocity is the C_2 symmetry of the isofrequency contours, that, strictly speaking, corresponds to a mapping of the modes at $\mathbf{k} = [k_x, k_y]$ into $-\mathbf{k} = [-k_x, -k_y]$.

Let us now evaluate how breaking the fundamental symmetries of the lattice affects the isofrequency contours. We start with breaking *T*-symmetry (Fig. 2b). While breaking *T*-symmetry substantially changes the shape of the contours, we see that the C_6 symmetry, and therefore reciprocity, still holds. On the other hand, breaking the *P*-symmetry breaks the C_6 folding of the dispersion (Fig. 2c). The C_2 symmetry, however, still holds due to the time-reversal symmetry, leading to a reciprocal behavior. Figure 2d displays the interesting case of the broken *P* and *T* symmetries. In this case, non-reciprocity can be observed, especially for frequencies closely below the Mie resonance corresponding to contours near the Brillouin zone edges. In particular, we unambiguously observe the non-reciprocal behavior along K'ΓK direction, consistent with our findings in Figure 1 (black dotted line). For the sake of the clearest possible demonstration, we will from now on focus on wave propagation along K'ΓK direction in order to study the subwavelength non-reciprocal wave propagation.

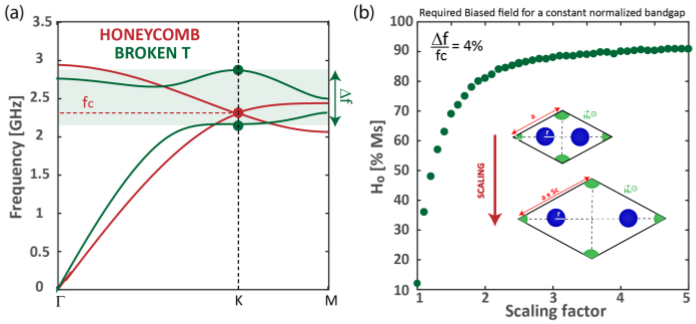

Fig. 4. Effect of scaling on the *T*-broken metamaterial crystal. a, dispersion relations of the Honeycomb (red) and *T*-Broken metamaterial (green). The cone frequency f_c and the band-gap width Δf at K point are highlighted. b, Evolution of the H_0 bias field (in % of the saturation magnetization of the ferrite M_s) required to open a band-gap of 4% of the cone frequency as a function of the scaling factor. Principle of the scaling operation (inset).

III. Non-reciprocal wave propagation

To evidence the asymmetric wave propagation in both ΓK and ΓK' directions, we now move on to a large-scale full-wave simulation using a metamaterial crystal composed of 43×43 unit cells. The TE excitation is implemented using a linear set of out of plane current-line sources (Port 1). The sources frequency, $f = 2.408\,GHz$ ($\lambda = 124.6\,mm$), is chosen such that it allows wave propagation in ΓK direction (red arrow) but not in ΓK' direction (green arrow). At this frequency, the line source length is $\lambda/5$ while the medium side length reaches $0.7\,\lambda$. To prevent unwanted reflections, progressive dielectric losses are added to the last ten unit cells. To confirm the non-reciprocal behavior, the E_z field map at f is extracted from our finite-element simulations and is Fourier transformed (Fig. 3a). The profile of the excited modes in the first Brillouin zone confirms that there is no propagation in ΓK', a result that is perfectly consistent with our isofrequency contour prediction. This can be further verified by plotting the Poynting vector amplitude ($\|\vec{\pi}\|$) in the medium. In Figure 3b, we display profile of $\|\vec{\pi}\|$ when the crystal is excited from port 1 (left), and port 2 (right), respectively. The obtained energy maps clearly demonstrate the expected non-reciprocal behavior.

To get a more quantitative estimation of the efficiency of non-reciprocity in this system, we plot in Fig. 3c the averaged amplitude of the Poynting vector as we move away from the source position along ΓK (red dots) and ΓK' (green dots) directions (note that the oscillations observed in the figure are due to the transverse averaging of the Poynting vector). We observe a clear difference of transmission between the two cases, that reaches almost 8 dB after a propagation length of around $\lambda/6$.

Finally, we stress that, in stark contrast to photonic crystals, the proposed metamaterial crystal can be scaled down in the (xOy) plane without affecting its original functionality. Such behavior is of particular interest since it enables a deeper subwavelength confinement of the field, which necessarily improves the interaction between the Bloch waves and the biased ferrites. To demonstrate such a salient behavior, we focus on the *T* broken metamaterial crystal. We then study the effect of scaling on the strength of the magnetization effect, namely the strength of the lifting of the modes degeneracy at K point. In order to build a dimensionless figure of merit, we focus on the normalized band-gap, defined by $FOM = \Delta f / f_c$ where f_c is the cone frequency in the honeycomb ferrite free medium (Fig. 4a). We then assume that the higher the FOM is, the better the wave interacts with the ferrite. For both the initial honeycomb resonator array and the *T*-broken metamaterial crystal, the scaling operation was implemented by keeping the size of resonators and ferrite cylinders constant, and scaling the lattice dimensions by a given factor S_c in the (xOy) plane (Fig. 4b, inset). For each value of S_c, we then calculated the required magnetic bias field amplitude H_0 so that the value of FOM remains constant. This amounts to assessing the required magnetic bias field amplitude H_0, for each scaling factor, to maintain the strength of the wave interaction with ferrites constant. In Figure 4b, we plot H_0, normalized by the ferrite magnetization saturation M_s, as a function of S_c, considering a constant FOM of 4%. Interestingly, this curve exhibits a rapid falling of the required bias for the contracted metamaterial crystals. It thus clearly evidences the significant impact of the scaling since the required H_0 expands from around 10% of M_s when $S_c = 1$ to 90% of M_s for a five times larger crystal.

IV. Conclusion

In summary, we have demonstrated the unique relevance of locally-resonant metamaterial crystals as a platform for non-reciprocal wave manipulation at the subwavelength scale, highlighting the key role of spatial dispersion in these complex subwavelength media. As a unique prerogative of metamaterial crystals, we further demonstrated that scaling down the unit cell enables a significant improvement of the wave/matter interaction, especially the interaction with ferrite. This paves the way for realizing ultra-compact, non-reciprocal microwave devices using accessible biased magnetic fields. Note that, while we neglected material losses in our study, they can become important at higher frequencies and when metallic resonators are considered. Besides, we note that non-reciprocity is not exclusively brought via magnetic effects [34-36], and other solutions based on material non-linearity [37–53], or time-modulation [54–66], may be equivalently used as a source of non-reciprocity at the subwavelength scale. We hence believe that our study may inspire some new developments in nonreciprocal subwavelength wave propagation.

Appendix

Ferrite model: Throughout all of the numerical simulations, the ferrite magnets were assumed to be lossless and modeled with the permeability tensor of the form

$$\bar{\bar{\mu}} = \mu_0 \begin{vmatrix} \mu_r & j\kappa_r & 0 \\ -j\kappa_r & \mu_r & 0 \\ 0 & 0 & 1 \end{vmatrix} \tag{1}$$

in which μ_r and κ_r are of the form

$$\mu_r = 1 + \frac{\omega_0 \omega_m}{\omega_0^2 - \omega^2}$$
$$\kappa_r = \frac{\omega \omega_m}{\omega_0^2 - \omega^2} \tag{2}$$

where is the angular frequency of the microwave field and and are defined as $\omega_0 = \mu_0 \gamma H_0$, and $\omega_m = \mu_0 \gamma M_s$, in which γ, H_0 and M_s are respectively the gyromagnetic ratio of the electron, the external magnetic bias field and saturation magnetization.

Numerical methods: The band structures of all considered media were calculated using the eigensolver mode of the finite element simulation tool COMSOL Multiphysics. To that aim, Floquet periodic boundary conditions (PCB) were used for the lateral sides of the unit cell. On each of these boundaries, a phase relation was applied corresponding to a given wavenumber in the medium's first Brillouin zone. The dispersion relation in ΓKM (resp. ΓK'M) direction plotted in Figure 1 by varying the wavenumber in reciprocal space. The isofrequency contours of Figure 2 were calculated while spanning the wavenumber over the entire Brillouin zone.

Geometrical details: We provide here details regarding geometries of the proposed P and T symmetry broken metamaterial crystal. The resonators were first separated by a distance $a = 1\ mm$, with coordinates $(a, 0)$ and $(2a, 0)$ in the (xOy) plane. The unit cell sides are of length $b = a\sqrt{3} \ll \lambda$. Magnetically biased ferrites rods (green disks) are employed to break T-symmetry, using a H_0 field biased along z (out-of-plane). To obtain the final design, we optimized both the position of the first resonator and the H_0 biased magnetic field in order to get the largest possible frequency range for the non-reciprocal behavior. The optimized geometry corresponds to a vertical displacement of $\delta y = \lambda_0/600$ for the first resonator and a bias field $H_0 = 1.62e5\ A/m$.

Large scale simulations: Large-scale simulations of Figure 3 were performed in a diamond-shape medium of side dimension $L = 8.64\ cm = 0.7\lambda$. The medium is composed of 86 (resp. 57) unit cells along x (resp. y), corresponding to approximately 43 unit cells along each sides of the medium. To prevent unwanted reflections, we numerically imposed tapered losses for the last ten unit cell layers, with progressive losses ranging from $\tan\delta = 0.1$ to $\tan\delta = 1$. The source port was defined by a set of out-of-plane line current points forming a line of length $l_s = 7.48\ cm = 0.6\lambda$. To calculate the field map, the complex E_z field at the frequency of operation was exported from COMSOL. A two-dimensional Fourier transform was then applied to this complex field. The real part of the result is plotted in Fig. 5, along with the position of the source (red line).

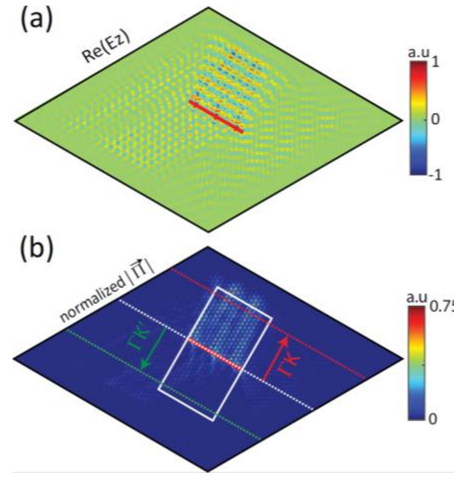

Fig. 5: (a) Map of the real part of the Ez field when the medium is excited at $f = 2.408\ GHz$. (b) Map of the normalized amplitude of the Poynting vector.

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