Nonstationary Low Frequency Noise in Switched MOSFET Circuits & Circuit Simulation

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***Abstract*—Modeling and analysis of low frequency noise under strongly time-varying bias conditions is a long-standing open problem in circuit simulation. In this paper, we present the background on the nonstationary low frequency noise modeling problem in circuit simulation, the legacy noise models, and our solution based on a recently proposed computational modeling and analysis framework. Our model and simulation techniques were developed based on an analogy that relates low frequency noise in transistors and electronic circuits to stochastic behavior of ion channels in biological neurons and stochastic chemical kinetics. Results on electronic circuit examples, namely switched MOSFET circuits and oscillators, show that our computational models implemented in an electronic circuit simulator correctly predict the impact of nonstationary low frequency noise that match experimental measurement data reported in the literature, whereas the legacy noise models produce erroneous results.**

I. Introduction

Noise in electronic circuits sets the limit on the minimum detectable signal and adversely affects performance. Intrinsic noise in electronic devices, such as thermal, shot and $1/f$ noise, is usually characterized with a spectrum. Thermal and shot noise have a white spectrum, whereas there are noise sources where the power is concentrated at low frequencies. For such colored noise sources, samples in time are correlated, whereas for white noise there is no correlation. Low frequency noise has an impact on circuit performance even if operating at high frequencies, due to frequency translation, i.e., up-conversion, via circuits such as oscillators and mixers. Low frequency noise phenomena are not unique to electronic circuits, in fact, ubiquitous [1].

Noise modeling involves the representation of relevant physical properties of noise with mathematical and computational models, e.g., power spectral density (PSD). In *noise simulation*, one tries to mimic the behavior of a noisy circuit on a computer using random number generators, e.g. via transient, time-domain stochastic simulation. *Noise analysis* [2], on the other hand, is the semi-analytical assessment of the impact of noise on a circuit through computational models and numerical analysis, in a non Monte Carlo manner, without the use of random number generators, e.g., as in SPICE AC noise analysis [3], [4]. Noise simulation and analysis are crucial tools for analog, RF and mixed-signal circuits, in designing them to be robust against noise effects and in meeting stringent noise specifications with few design iterations.

In conventional noise models and AC noise analysis, it is assumed that the circuit operates in small-signal mode (no significant change in bias point), and small-signal equivalent device models are used. This essentially means that stationary noise statistics is assumed, and *Linear(ized) Time Invariant (LTI)* models are used [2]. Noise sources are modeled with PSDs in the frequency domain. Noise analysis then entails computing transfer functions from the noise sources to the output, and the output noise spectrum [3], [4].

Noise modeling and analysis for circuits under time-varying bias conditions, such as mixers and oscillators, requires a more elaborate approach. Time-varying bias implies time-varying, i.e., nonstationary, statistics for the noise sources. The nonstationary nature of thermal and shot noise can be captured by *modulating* a white and stationary noise source [2], believed to be correct and accurate. This is due to the fact that a white noise generating mechanism has no internal dynamics with memory (uncorrelated samples), and hence instantaneous modulation is appropriate. On the other hand, it is not quite obvious how one would model low frequency, colored noise, such as $1/f$ noise, with time-varying bias conditions. The legacy model used in this case is based on, again, modulating a stationary colored noise source modeled with a PSD [2]. However, this type of model was shown to be incorrect and inaccurate, overestimating the effect of low frequency noise on circuits [5]–[7]. This is partially due to the fact that the internal dynamics of low frequency, colored noise sources are not captured correctly with an LTI model. A proper model has to take into account the fact that low frequency noise exhibits correlation in time, therefore internal dynamics (not LTI) with memory.

Considerable amount of work was done and published in the devices and circuits literature, on the modeling and characterization of nonstationary low frequency noise, e.g., [5]–[12] and many others that we cannot list here. We have greatly benefited from this body of literature in order to develop a circuit simulator compatible nonstationary low frequency

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noise model, that can be used in conjunction with advanced noise analysis algorithms which can handle circuits with time-varying bias conditions.

II. Modeling of Nonstationary Low Frequency Noise for Circuit Simulators

The computational modeling and simulation framework we review in this paper [13]–[19] is founded on techniques from stochastic chemical kinetics and computational neuroscience [20], [21]. The model we have developed captures the internal random dynamics of noise sources with mathematical models, that are nonstationary, nonlinear, coupled with the device and circuit models, and compatible with circuit simulators. Based on this model, we have developed carefully crafted, stochastically correct Monte Carlo simulation algorithms [17], which are needed for the verification of the noise analysis algorithms, and that can be used for simulating transient noise phenomena. We have also implemented the models into non Monte Carlo noise analysis techniques [19].

Charge carrier traps in the gate oxides of MOSFETs are a source of low frequency noise [10], [22]. These traps randomly capture and emit charge carriers, with an impact on the current flow through the device. On the other hand, rates of the capture and emission events depend on the voltages and currents of the device. The noise source associated with such a trap can be modeled with a *random telegraph signal* (RTS), hence referred to as RTS noise [10], [22]. The noise associated with a single, stationary trap (with time-invariant capture and emission rates) can be modeled in the frequency domain with a Lorentzian PSD. It can be shown that independent multiple traps with a certain distribution of capture/emission rates can collectively generate power law noise, such as $1/f$ noise over a frequency range [7]. However, the PSD description for trap noise is appropriate only when the traps are stationary with constant capture/emission rates.

We model nonstationary low frequency noise through "traps", that are either actual or *abstract*. A notional trap can be thought of as a pseudonym for an elemental noise generation mechanism with time-constant(s) that depend on the device bias point. We capture the nonstationary trap dynamics with a Markov Chain (MC) based model, that is discrete and fine-grained, with state transition rates as functions of device voltages/currents [13], [14], [16]. Then, starting with the MC model, we derive a continuous model in the form of *Stochastic Differential Equations* (SDEs). The SDE based trap model is approximate and coarse-grained. In the fined grained MC model, the trap is either full or empty, modeled with a 0-1 variable. On the other hand, in the coarse-grained SDE model, trap state can assume fractional values. At first thought, the link between these two models may be puzzling. However, one can reach a resolution by noting that the correspondence between them is not on a sample-path basis, but rather in a second-order statistical sense. That is, the models concur with each other for means, variances, correlation functions and spectral densities, but not for individual sample paths. We have verified with practical circuit examples that the two models indeed agree with each other for these second-order stochastic characterizations [17]. In almost all circuit design scenarios, such a second-order stochastic characterization is adequate. However, in the case of very sensitive circuits where individual trap events that involve single charge carriers are of significance, one has to use the fine-grained MC based model.

The fine-grained model can be used in only transient, stochastic noise simulations, whereas the coarse-grained model can be used in non Monte Carlo noise analyses as well [17], [19]. With the discrete, fine-grained model, MC based traps are coupled with the circuit. If simulated with stochastically correct algorithms, this model yields accurate statistics. We have expended quite a bit of effort in developing numerical techniques for hybrid discrete-continuous simulations that do not result in any subtle, statistical bias [13], [17]. However, the MC model is computationally expensive due to the need to handle jump events in an otherwise continuous stochastic simulation. Hence, it can be used only for small circuits. Moreover, it is not easily integrated into existing simulation frameworks and circuit simulators. In deriving the coarse-grained SDE model from the fine-grained MC model, we use techniques that were developed in stochastic chemical kinetics [20], [21]. This derivation involves essentially the approximation of (differences of) Poisson random variables with corresponding Gaussian random variables, but has other nontrivial steps [14], [17]. The coarse-grained SDE model can also serve as the basis for stochastic transient simulations. However, there are several complications one has to deal with. We have developed a technique we call *smooth diffusion* for running robust transient noise simulations based on the SDE model [17].

The SDE based model is compatible, by construction, with circuit simulators. We can easily embed trap dynamics into circuit equations by augmenting modified nodal analysis (MNA) equations with the trap SDEs [19]. The circuit variables (e.g., node voltages, branch currents) are also augmented with the trap variables. These trap variables can be considered as additional, internal "pseudo" nodes in the device. There is of course an overhead associated with the introduction of these extra variables in a circuit simulator. However, the interactions of the trap variables with the other circuit variables are local, confined to within a device. The device model equations need to be augmented with equations that model the *two-way coupling* between the traps and the charges, currents and voltages in the device. On one hand, the trap event rates are a function of circuit variables. That is why the traps exhibit nonstationary behavior with time-varying bias conditions. On the other hand, the trap events have an impact on the state of the device, causing noise in voltages and currents. We have augmented a MOSFET model [23] with these extensions [15].

The trap enhanced MOSFET model can be used in conjunction with previously developed noise analysis techniques, such as cyclo-stationary noise analysis [24], time-domain non Monte Carlo noise analysis [2], phase noise analysis for oscillators [25]. We have implemented all of these techniques with a trap enhanced MOSFET model [19] in a MATLAB® [26] based circuit simulator called CIRSIUM [15].

In the case when the trap variables represent actual, physical charge carrier traps in a transistor, the physics-based or empirical compact model equations need to be augmented with equations that model the behaviors and interactions of the charge carrier traps with other dynamics in the device. This is a challenging task and requires extensions and enhancements in compact model development. We have addressed this issue only in a rudimentary manner [15]. The difficulty is compounded by the fact that low frequency noise arising from fabrication defects and imperfections has a statistical nature [14]. That is, the trap configuration and placement in a certain device is randomly determined at fabrication, and in a different manner in every device. Thus, the stochasticity of low frequency noise is both temporal and spatial. The modeling and analysis framework reviewed in this paper does not address the spatial statistical aspect of the problem.

III. Results

To summarize, the low frequency modeling, simulation and analysis framework we have developed features:

- Transient, stochastic simulation based on the fine-grained, discrete, MC based trap model, that can capture the effect of individual charge carrier events.
- Transient, stochastic simulation based on the coarse-grained, continuous-state, SDE based trap model.
- non Monte Carlo noise analysis, both in time and frequency domain, based on the SDE based trap model augmented with circuit equations.

We have extensively tested the techniques summarized above on a variety of circuits in a number of scenarios, comparing them against each other, as well as with standard AC noise analysis for time-invariant bias, and with the legacy, simplistic low frequency noise models for time-varying bias, e.g., for switching MOSFET circuits [13]–[19]. The test circuits and scenarios are as follows:

- MOSFET transistor in common source configuration, with both constant and switching input, with a single trap and also with multiple traps that produce $1/f$ noise.
- Ring-oscillator (where the transistors experience large time-varying bias) phase noise and timing jitter characterization, both in time and frequency domain.
- Digital memory circuits.
- RF mixer circuit.
- Sawtooth oscillator circuit [8], [9].

We have arrived at the following conclusions:

- The SDE based continuous-state model agrees with the MC based fine-grained model in the sense of second-order stochastic characterizations, i.e., correlation functions and spectral densities [17]. Except for rare scenarios where individual charge carrier events are of significance, the SDE based model is adequate for modeling nonstationary low frequency noise.
- While the use of the SDE based model in non Monte Carlo, analysis-based noise characterizations involves further approximations (in addition to the discrete-to-continuous conversion in moving from the MC model to the SDE model), results obtained with non Monte Carlo schemes match the ones obtained from (computationally expensive) transient stochastic simulations with the SDE based model [19]. Except for rare scenarios where noise magnitude is large enough to invalidate the approximations the non Monte Carlo schemes rely on, efficient near-linear-complexity analysis schemes for cyclo-stationary noise [24], phase noise [25], time-domain noise [2] may be used in conjunction with the SDE based low frequency noise model. However, there is an overhead involved, since one needs to introduce the trap states as additional circuit variables (as pseudo-nodes) into the device model.
- The legacy nonstationary low frequency model, based on the simple modulation of a stationary noise source, overestimates the noise impact in switching MOSFET circuits. On the other hand, the results obtained with the proposed model are in accordance with the measurement results reported in the literature [19].

IV. Conclusions

We believe that the computational modeling and analysis framework outlined and reviewed in this paper solves an open problem in circuit simulation, namely modeling and analysis of low frequency noise under strongly time-varying bias conditions. This modeling framework was built on, and subsumes, the immensely useful work that has been done in the devices and circuits communities over many years. The circuit simulator compatible low frequency noise model, and the analysis and simulation techniques were developed by exploiting an analogy that relates low frequency noise in transistors and electronic circuits to stochastic behavior of ion channels in biological neurons and stochastic chemical kinetics. The adaptation of the stochastic chemical kinetics formalism made it possible to develop a systematic methodology for both fine-grained and coarse-grained modeling of low frequency noise. The techniques developed were applied to practical circuits, and were shown to predict intricate, nonstationary noise effects in switching MOSFET circuits that match measurement data reported in the literature. A complete exposition of the work reviewed in this paper can be found in [13]–[19].

Furthermore, by building on the work reviewed in this paper, we have developed a noise simulator for biological neurons and neuronal circuits [27], [28], which may be used in understanding the noise control and exploitation mechanisms in neuronal circuits [29], and in designing the neuro-morphic and hybrid neuro-electronic circuits of the future.

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