# Finite frequency noise: an original probe for topological superconductors

Jerome Rech
*Centre de Physique Théorique*
*Aix-Marseille Université*
Marseille, France
jerome.rech@cpt.univ-mrs.fr

Didier Bathellier
*Centre de Physique Théorique*
*Aix-Marseille Université*
Marseille, France
didier.bathellier@gmail.com

Laurent Raymond
*Centre de Physique Théorique*
*Aix-Marseille Université*
Marseille, France
laurent.raymond@univ-amu.fr

Thibaut Jonckheere
*Centre de Physique Théorique*
*Aix-Marseille Université*
Marseille, France
thibaut.jonckheere@cpt.univ-mrs.fr

Alex Zazunov
*Institut für Theoretische Physik*
*Heinrich Heine Universität*
Düsseldorf, Germany
zazunov@hhu.de

Thierry Martin
*Centre de Physique Théorique*
*Aix-Marseille Université*
Marseille, France
thierry.martin@cpt.univ-mrs.fr

***Abstract*—Topological superconductor nanowires constitute a strong candidate for the observation of Majorana bound states, which are expected to lie at each of its ends. Here, we suggest that current-current correlations probed at finite frequency offer a promising and original tool for the further characterization of the presence of such states in condensed matter systems, complementary to properties studied thus far. Focusing on a voltage-biased junction between a normal metal and a topological superconductor nanowire, we use the nonequilibrium Keldysh formalism to compute the finite frequency emission and absorption noise. Our results suggest that the presence of a Majorana bound state leads to a characteristic behavior of the noise spectrum at low frequency. While more work is still required to ensure that this constitutes an unambiguous signature, we could already check that different features arise for a nontopological system with a resonant level, exhibiting a zero-energy Andreev bound state.**

***Index Terms*—finite frequency noise, topological superconductors, Majorana fermions**

## I. Introduction

Majorana fermions [1], a concept which originally emerged in particle physics, now constitute a very active field of study in modern condensed matter physics. In this context, it is thought to arise from the collective behavior of a many-body electronic system, rather than being an elementary particle.

What brought these peculiar excitations to the forefront is the pioneering work of Kitaev [2]. Studying a tight-binding chain of electrons equipped with $p-$wave superconducting pairing between neighboring sites, he could show that for a whole region of the parameter space (the so-called topological phase) there exist spatially localized Majorana bound states (MBSs) emerging at the boundaries of this one-dimensional system.

The practical realization of such a toy model, now known as a "Kitaev chain", has since been an ongoing experimental effort. One strong candidate for a real-life version of the Kitaev chain are topological superconductor (TS) nanowires. The latter consist of semiconducting nanowires with strong Rashba

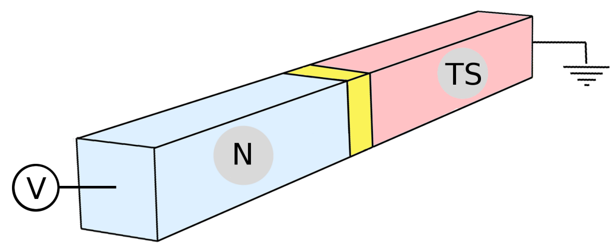


Fig. 1. Schematic view of the setup: a junction between a voltage-biased normal metal (light blue) and a grounded topological superconductor nanowire (light red).

spin-orbit coupling, proximity induced s-wave superconductivity, and a Zeeman magnetic field [3]–[8]. This triggered many attempts at finding experimental evidence of the presence of MBSs in such realistic devices. In addition to an obvious fundamental interest for such an observation, Majorana fermions also represent a promising quantum information platform with a built-in topological protection, which constitute a very strong motivation for firm and robust realizations of MBSs.

However, despite the significant experimental progress, an unequivocal signature of these MBSs is still lacking. Promising experiments were conducted [9], but the features they reveal are ubiquitous and could be associated with other sources. This lead to many more theoretical proposals [10]–[15] for the detection of Majorana fermions, usually relying on the quantum transport properties of more elaborate devices. The search for such a "smoking gun", a signature that would unequivocally signal the presence of Majorana fermions, is still quite active as it has not reached a definite answer so far.

Here, we propose to consider the finite-frequency noise (i.e. current-current correlations) of a voltage-biased normal metal-topological superconductor (NTS) junction. This transport property offers an original probe for the study of TS, as it provides valuable information, complementary with, say, the differential conductance, opening the way for a different approach consisting in combining different resources in order to rule out alternative physical mechanisms.

## II. Model

The setup we consider is a junction between a normal metallic lead and a semi-infinite topological superconductor wire. The latter corresponds to the continuous version of the Kitaev chain in the low-energy limit, and is described as an effectively spinless single-channel $p-$wave superconductor with a Hamiltonian of the form

$$H_j = \int_0^{+\infty} dx\ \psi_j^\dagger(x)\left(-iv_F\partial_x\sigma_3 + \Delta_j\sigma_2\right)\psi_j(x). \quad (1)$$

The leads are labeled as $j=0$ and 1, for the TS nanowire and the normal metal lead respectively. Here $\sigma_{1,2,3}$ are Pauli matrices in Nambu space, while $v_F$ stands for the Fermi velocity and $\Delta_0$ is the superconducting gap. The Hamiltonian for the normal lead in the absence of voltage is given by a similar form in the limit of vanishing gap parameter $\Delta_1$. The Nambu spinor $\psi_j^\dagger(x) = \left(c_{Rj}^\dagger(x), c_{Lj}(x)\right)$ introduced here combines right- and left-moving fermion operators $c_{R,L}(x)$.

The coupling between the two leads is then described by the following tunneling Hamiltonian

$$H_\mathrm{T} = \lambda c_0^\dagger c_1 + \text{H.c.} = \Psi_1^\dagger W_{10}\Psi_0, \quad (2)$$

where $c_j$ are boundary fermions given by $c_j = c_{Rj}(0) + c_{Lj}(0)$, combined under the Nambu notation $\Psi_j^\dagger = \left(c_j^\dagger, c_j\right)$, and we introduced the tunneling matrix $W_{10} = \lambda\sigma_3$.

The current operator through the normal lead is readily obtained from this tunneling Hamiltonian as

$$I_1 = ie\Psi_1^\dagger\sigma_3 W_{10}\Psi_0 = ie\lambda\Psi_1^\dagger\Psi_0. \quad (3)$$

This then allows us to define the current-current correlations in real time, which reads

$$S_{11}(t,t') = \langle I_1(t)I_1(t')\rangle - \langle I_1(t)\rangle\langle I_1(t')\rangle. \quad (4)$$

In order to compute the physical properties of the junction, an essential tool is the Keldysh Green's functions for the boundary fermions. For each lead, they are given by a set of 4 matrices in Nambu space defined as

$$G_{j_1j_2}^{\eta_1\eta_2}(t_1,t_2) = -i\left\langle T_K\Psi_{j_1}\left(t_1^{\eta_1}\right)\Psi_{j_2}^\dagger\left(t_2^{\eta_2}\right)\right\rangle \quad (5)$$

where $\eta_j = \pm$ corresponds to the Keldysh time index specifying the position of time along the Keldysh contour, and $T_K$ denotes the Keldysh time-ordering prescription.

The average current can then be reexpressed in the form of a trace in Nambu space involving the Keldysh Green's function

$$\langle I_1\rangle = e\lambda\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\mathrm{Tr_N}\left[G_{01}^{-+}(\omega)\right] \quad (6)$$

and similarly, the real-time noise correlator reads

$$S_{11}(t,t') = e^2\lambda^2\mathrm{Tr_N}\left[G_{00}^{-+}(t,t')G_{11}^{+-}(t',t) - G_{01}^{-+}(t,t')G_{01}^{+-}(t',t)\right]. \quad (7)$$

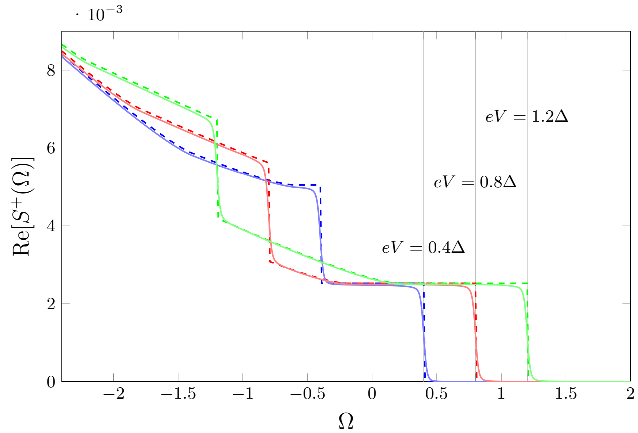


Fig. 2. Emission noise $S^+(\Omega)$ (in units of $e^2\Delta$) as a function of frequency $\Omega$ (in units of $\Delta$) in the tunneling regime (transparency $\tau = 0.02$) and low temperature ($\theta = 0.01\Delta$) for different values of the voltage bias. The full numerical result (full lines) is compared with the analytic form of Eqs. (15)-(18) (dashed lines).

Now the full Keldysh Green's functions entering these expressions are readily obtained from the Dyson's equation expressed in Keldysh-Nambu-lead space, involving the tunneling matrix $W$ along with the bare Green's functions associated with the two leads [16]:

$$g_{00}^{R/A}(\omega) = \frac{\sqrt{\Delta^2-(\omega\pm i0^+)^2}\sigma_0 + \Delta\sigma_1}{\omega\pm i0^+} \quad (8)$$

$$g_{00}^K(\omega) = \left[g_{00}^R(\omega) - g_{00}^A(\omega)\right]\tanh\left(\frac{\omega}{2\theta}\right) \quad (9)$$

$$g_{11}^{R/A}(\omega) = \mp i\sigma_0 \quad (10)$$

$$g_{11}^K(\omega) = -2i\tanh\left(\frac{\omega\sigma_0 - eV\sigma_3}{2\theta}\right), \quad (11)$$

where $eV$ is the voltage applied to the normal lead, and $\theta$ is the temperature of both leads.

## III. Finite frequency noise

Since we apply a constant voltage, the NTS junction under study is in a stationary situation. It follows that the current-current correlations $S_{11}(t,t')$ introduced in Eq; (4) actually only depend on the time difference $t-t'$. This allows us to introduce two distinct correlators, and subsequently Fourier transform them, to define the emission and absorption noise as

$$S^+(\Omega) = \int_{-\infty}^{+\infty} d\tau S_{11}(0,\tau)e^{i\Omega\tau} \quad (12)$$

$$S^-(\Omega) = \int_{-\infty}^{+\infty} d\tau S_{11}(\tau,0)e^{i\Omega\tau}. \quad (13)$$

These two quantities are not independent, as they satisfy $S^-(\Omega) = S^+(-\Omega)$, so that it is enough to consider the emission noise for all frequencies on the real axis to fully describe the whole range of physical parameters.

As it turns out, the structures arising in the emission noise tend to get smoothed out by increasing either temperature or transparency, so that the most promising setups to observe characteristic signatures of the MBS are low temperature tunnel junctions.

Interestingly, this regime allows for a tractable analytic derivation, as the emission noise then reduces to

$$S^{+}(\Omega) = e^2\lambda^2 \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \mathrm{Tr_N}\left[g_{00}^{-+}(\omega) g_{11}^{+-}(\omega)\right]. \quad (14)$$

Working out the integral explicitly in the zero temperature limit, one is left with 4 contributions of the form

$$S_a^{+}(\Omega) = e^2\lambda^2\Delta\ \Theta\left(eV - \Omega\right) \quad (15)$$

$$S_b^{+}(\Omega) = e^2\lambda^2 \frac{2}{\pi} \Theta\left(eV - \Delta - \Omega\right) \left[\frac{\sqrt{(eV-\Omega)^2 - \Delta^2}}{\Delta} - \arctan\left(\frac{\sqrt{(eV-\Omega)^2-\Delta^2}}{\Delta}\right)\right] \quad (16)$$

$$S_c^{+}(\Omega) = e^2\lambda^2\Delta\ \Theta\left(-eV - \Omega\right) \quad (17)$$

$$S_d^{+}(\Omega) = e^2\lambda^2 \frac{2}{\pi} \Theta\left(-eV - \Delta - \Omega\right) \left[\frac{\sqrt{(eV+\Omega)^2 - \Delta^2}}{\Delta} - \arctan\left(\frac{\sqrt{(eV+\Omega)^2-\Delta^2}}{\Delta}\right)\right] \quad (18)$$

where $\Theta(x)$ is the Heaviside distribution. As can be seen from Fig. 2, this result agrees very well with the emission noise obtained from a full numerical solution of the Dyson's equations in the tunneling regime at low temperature.

The finite frequency noise is related to fluctuations of the current. It is therefore crucial to understand the processes that contribute to the current, by transferring electrons between leads along with the emission or absorption of a photon. They are pictured qualitatively in Fig. 3. As it turns out, based on their frequency and voltage dependence, along with the frequency range over which they contribute, one can find a one-to-one correspondence between these basic processes and the contributions arising in our analytic derivation of the finite frequency noise (which is why we used matching labels).

From this, one readily sees that the presence of a MBS is directly probed via the emission and absorption processes of Fig. 3(a) and (c). The process shown in Fig. 3(a) corresponds to electrons hopping from the normal metal to the Majorana state either by emitting or absorbing a photon (depending on the sign of the electron energy), therefore covering the whole range of frequency $\Omega \in [-\infty, +eV]$. Similarly, the process in Fig. 3(c) represents an electron transmitted from the Majorana state of the topological superconductor at zero energy to the empty states of the normal metal, accompanied by the absorption of a photon, leading to a finite result for frequencies $\Omega \in [-\infty, -eV]$. While the process from Fig. 3(c) is hard to resolve as it is dominated by other terms (typically $S_b^{+}(\Omega)$), the one from Fig. 3(a) is the leading contribution at low frequency. Furthermore, this contribution is frequency-independent, leading to a wide plateau in the finite frequency noise, a feature which arises from the sharp peak in the density of states (DOS) of the TS lead associated with the MBS.

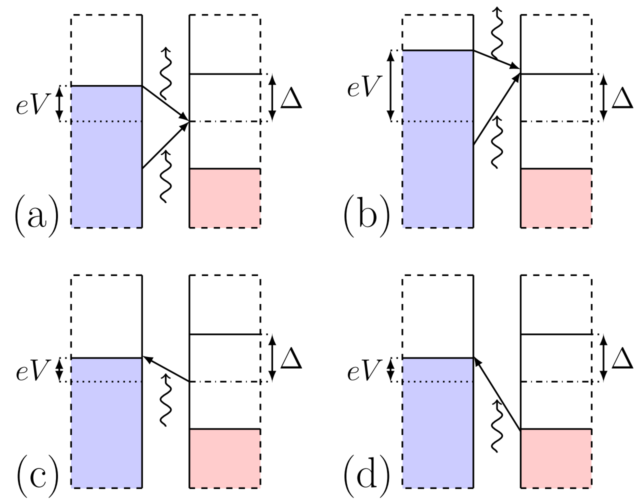


Fig. 3. Energy diagram of the junction illustrating emission [top part of (a) and (b)] and absorption [bottom part of (a) and (b), (c) and (d)] processes. Occupied electronic states are in light blue for the normal lead, and in light red for the TS one.

## IV. Comparing with an N-dot-S setup

To illustrate the added value of finite frequency noise over other physical quantities, we now consider a junction involving a quantum dot between a normal metal and a conventional BCS superconductor. This device corresponds to a nontopological normal metal/superconductor junction, which bears Andreev bound states located at zero energy.

More importantly, this system can be tuned to yield a differential conductance which looks qualitatively similar to that of a low transparency NTS junction, with a conductance peak reminiscent of the signature expected for a MBS despite the absence of such excitations. This is achieved by considering symmetrical couplings $\Gamma_N = \Gamma_S$ from the leads to the dot, and setting the energy $\epsilon$ of the dot to zero.

We focus on this particular regime, and compute the finite frequency current correlations, using a Keldysh Green function formalism [17], [18]. Unlike the NTS case, the final result for the emission and absorption noise turns out to be non-perturbative in the coupling strength, preventing a simple analytical treatment in the regime of low transparency of the junction, even in the regime of large gap. We therefore rely on a full numerical approach which treats the tunneling between dot and leads at all orders. The resulting finite frequency noise is displayed in Fig. 4.

As in the NTS junction, the emission noise shows a clear plateau which drops sharply to zero at frequency $\Omega = eV$. This plateau arises from the presence of a narrow peak in the DOS, associated with weakly coupled discrete energy level of the dot. However, not only this plateau does not extend to the same energy scale for negative frequencies, it also shows a very different behavior at low frequency. Indeed, close to zero frequency, the emission noise suddenly dips down, reaching a value that corresponds to half that of the plateau, a known result in double-barrier symmetric junctions [19].

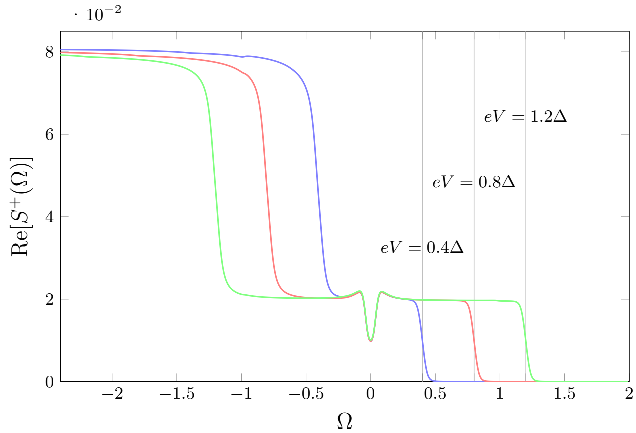


Fig. 4. Emission noise $S^+(\Omega)$ (in units of $e^2\Delta$) as a function of frequency $\Omega$ (in units of $\Delta$) for a symmetric N-dot-S junction, with coupling $\Gamma = 0.02\Delta$ and low temperature ($\theta = 0.01\Delta$) for different values of the voltage bias.

It follows that, while we cannot claim that the finite frequency noise signatures uniquely identify the presence of a MBS, our results clearly show that it is able to discriminate between the NTS and N-dot-S system when the differential conductance could not.

## V. Conclusions

Focusing on a voltage biased NTS junction, we showed that the finite frequency noise at low transparency could be understood in terms of basic processes, and lead to distinctive features arising from the presence of a MBS. Considering a N-dot-S system, we argued that unlike the differential conductance, the finite frequency noise could be used to discriminate between a real MBS and an accidental Andreev bound state at zero energy.

Finally, this feature should be measurable in actual experiments at low temperature. In practice, in order to measure the finite-frequency noise, one needs a noise detector which, in principle, should be described within a quantum mechanical framework [20]–[22], just as the nanodevice it is connected to. Considering for the detection an LC resonant circuit inductively coupled to the junction, and following [20] and [23], one can predict the result of such a measurement and obtain an expression of the so-called measurable noise. In the regime of low detector temperature, $T_{LC} \ll \Omega$, this actually reduces to the emission noise, up to a constant prefactor [18], thus showing the same low-frequency behavior which we could associate with the presence of a MBS.

## Acknowledgment

The authors gratefully acknowledge discussions with R. Egger and A. Levy-Yeyayi. The project leading to this publication has received funding from Excellence Initiative of Aix-Marseille University – A$*$MIDEX, a French Investissements d'Avenir program.

## References

[1] E. Majorana, "Teoria simmetrica dellelettrone e del positrone," *Nuovo Cim*, vol. 14, p. 171, 1937.

[2] A. Y. Kitaev, "Unpaired majorana fermions in quantum wires," *Physics-Uspekhi*, vol. 44, no. 10S, pp. 131–136, oct 2001.

[3] R. M. Lutchyn, J. D. Sau, and S. Das Sarma, "Majorana fermions and a topological phase transition in semiconductor-superconductor heterostructures," *Phys. Rev. Lett.*, vol. 105, p. 077001, Aug 2010.

[4] Y. Oreg, G. Refael, and F. von Oppen, "Helical liquids and majorana bound states in quantum wires," *Phys. Rev. Lett.*, vol. 105, p. 177002, Oct 2010.

[5] J. Alicea, Y. Oreg, G. Refael, F. von Oppen, and M. P. A. Fisher, "Non-abelian statistics and topological quantum information processing in 1d wire networks," *Nature Physics*, vol. 7, p. 412, 2011.

[6] V. Mourik, K. Zuo, S. M. Frolov, S. R. Plissard, E. P. A. M. Bakkers, and L. P. Kouwenhoven, "Signatures of majorana fermions in hybrid superconductor-semiconductor nanowire devices," vol. 336, no. 6084, pp. 1003–1007, 2012.

[7] A. Das, Y. Ronen, Y. Most, Y. Oreg, M. Heiblum, and H. Shtrikman, "Zero-bias peaks and splitting in an alinas nanowire topological superconductor as a signature of majorana fermions," *Nature Physics*, vol. 8, p. 887, 2012.

[8] R. M. Lutchyn, E. P. A. M. Bakkers, L. P. Kouwenhoven, P. Krogstrup, C. M. Marcus, and Y. Oreg, "Majorana zero modes in superconductorsemiconductor heterostructures," *Nature Reviews Materials*, vol. 3, p. 52, may 2018.

[9] H. Zhang, C.-X. Liu, S. Gazibegovic, D. Xu, J. A. Logan, G. Wang, N. van Loo, J. D. S. Bommer, M. W. A. de Moor, D. Car, R. L. M. Op het Veld, P. J. van Veldhoven, S. Koelling, M. A. Verheijen, M. Pendharkar, D. J. Pennachio, B. Shojaei, J. S. Lee, C. J. Palmstrm, E. P. A. M. Bakkers, S. D. Sarma, and L. P. Kouwenhoven, "Quantized majorana conductance," *Nature*, vol. 556, p. 74, 2018.

[10] J. Alicea, "New directions in the pursuit of majorana fermions in solid state systems," *Reports on Progress in Physics*, vol. 75, no. 7, p. 076501, jun 2012.

[11] M. Leijnse and K. Flensberg, "Introduction to topological superconductivity and majorana fermions," *Semiconductor Science and Technology*, vol. 27, no. 12, p. 124003, nov 2012.

[12] C. Beenakker, "Search for majorana fermions in superconductors," *Annual Review of Condensed Matter Physics*, vol. 4, no. 1, pp. 113–136, 2013.

[13] T. D. Stanescu and S. Tewari, "Majorana fermions in semiconductor nanowires: fundamentals, modeling, and experiment," *Journal of Physics: Condensed Matter*, vol. 25, no. 23, p. 233201, may 2013.

[14] S. R. Elliott and M. Franz, "Colloquium: Majorana fermions in nuclear, particle, and solid-state physics," *Rev. Mod. Phys.*, vol. 87, pp. 137–163, Feb 2015.

[15] R. Aguado, "Majorana quasiparticles in condensed matter," *Riv. Nuovo Cimento*, vol. 40, p. 523, 2017.

[16] A. Zazunov, R. Egger, and A. Levy Yeyati, "Low-energy theory of transport in majorana wire junctions," *Phys. Rev. B*, vol. 94, p. 014502, Jul 2016.

[17] D. Chevallier, J. Rech, T. Jonckheere, and T. Martin, "Current and noise correlations in a double-dot cooper-pair beam splitter," *Phys. Rev. B*, vol. 83, p. 125421, Mar 2011.

[18] D. Bathellier, L. Raymond, T. Jonckheere, J. Rech, A. Zazunov, and T. Martin, "Finite frequency noise in a normal metal–topological superconductor junction," *Phys. Rev. B*, vol. 99, p. 104502, 2019.

[19] T. Martin, "Course 5 noise in mesoscopic physics," in *Nanophysics: Coherence and Transport*, ser. Les Houches, H. Bouchiat, Y. Gefen, S. Guron, G. Montambaux, and J. Dalibard, Eds. Elsevier, 2005, vol. 81, pp. 283 – 359.

[20] G. B. Lesovik and R. Loosen, "On the detection of finite-frequency current fluctuations," *Journal of Experimental and Theoretical Physics Letters*, vol. 65, no. 3, pp. 295–299, Feb 1997.

[21] R. Aguado and L. P. Kouwenhoven, "Double quantum dots as detectors of high-frequency quantum noise in mesoscopic conductors," *Phys. Rev. Lett.*, vol. 84, pp. 1986–1989, Feb 2000.

[22] U. Gavish, Y. Levinson, and Y. Imry, "Detection of quantum noise," *Phys. Rev. B*, vol. 62, pp. R10 637–R10 640, Oct 2000.

[23] A. Zazunov, M. Creux, E. Paladino, A. Crépieux, and T. Martin, "Detection of finite-frequency current moments with a dissipative resonant circuit," *Phys. Rev. Lett.*, vol. 99, p. 066601, Aug 2007.