

Generation-recombination noise of magnetic monopoles in spin ice

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Abstract—We theoretically investigate the fluctuations in the number of emergent magnetic monopoles in spin ice. The Langevin equation for these fluctuations is deduced. That allows us to calculate the spectrum of relative fluctuations, which can be measured experimentally.

Keywords—magnetic monopoles; spin ice; fluctuations; ice rules; generation-recombination.

I. INTRODUCTION

In the last time there was a lot of works, due to a great interest in the study of spin ice [1]. Such an interest was caused by the theoretical prediction of unusual magnetic defects in spin ice, which were called as emergent magnetic monopoles [2, 3].

Originally, the term “spin ice” was applied to the compounds such as $\text{Ho}_2\text{Ti}_2\text{O}_7$. Magnetic ions in them are located at the vertices of regular tetrahedra linked into a three dimensional pyrochlore lattice. Due to the strong anisotropy, the magnetic moments can be directed either toward the center of each tetrahedron, or away from this center. Note that dual lattice formed by the centers of the tetrahedra is a usual diamond lattice. The ground state of spin system is characterized by the ice rule: two spins of each tetrahedron are directed toward its center and two other ones away from its center (Fig.1 a). This corresponds to a distribution of electrical dipole moments hydrogen bonds in hexagonal ice, solid phase of water in natural conditions [1]. That justifies the term “spin ice”. Subsequently, this term began to apply to any two-dimensional and three-dimensional systems in which the magnetic moments (spins) are distributed in accordance with the ice rules.

The flipping of an arbitrary spin in ground state configuration, which takes place at finite temperatures, breaks the ice rule in two neighboring tetrahedra. One tetrahedron has three spins directed “in”, and one – “out”, while its neighbor has one “in” and three – “out” (Fig. 1 b). These tetrahedra can be considered as positive and negative emergent magnetic monopoles, respectively. The positive magnetic monopole is a magnetic analogue of a positive ion in water ice. Accordingly, the negative magnetic monopole is a magnetic analogue of a negative ionic defect in water ice [2]. As one can see from Fig. 1, the flipping of any of three identical magnetic spins of the defect is equivalent to the defect movement to another site. This movement is not accompanied by the formation of new defect and by the increase of energy. By means of further spin flips the magnetic monopoles can move through the lattice and thereby change the spin configuration. Monopoles play the role of classical quasiparticles, which movement through the

lattice change the spin configuration on all the distance traveled, resulting in the formation of a “spin spaghetti” [3]. Therefore, in comparison with defects in semiconductors, which may negatively influence on characteristics of devices [4–9], magnetic defects in spin ice [10–15] play the positive role of carriers of magnetic charge. But, in spin ice (in difference with ordinary ice [16–21]) a direct current of magnetic monopoles is impossible.

The purpose of our work is the investigation of the fluctuations in the number of magnetic monopoles. To implement this we derive the Langevin equation for these fluctuations by the Einstein–Fokker–Planck equation method. That allows us to calculate the spectrum of relative fluctuations in the number of magnetic monopoles. Then we discuss a possible relation of fluctuations in the number of pairs of magnetic monopoles with measurable characteristic of spin ice and an experimental display of obtained results.

II. GENERATION-RECOMBINATION NOISE OF MAGNETIC MONOPOLES

To analyze fluctuations in the number of magnetic monopoles, we consider the processes of generation and recombination of magnetic monopoles pairs [22]. It is the simplest stochastic analysis, which shows that these fluctuations can be described by the Langevin equation. Thus, these fluctuations can be considered as generation–recombination noise of quasiparticles (magnetic monopoles). The spectrum of this noise is determined.

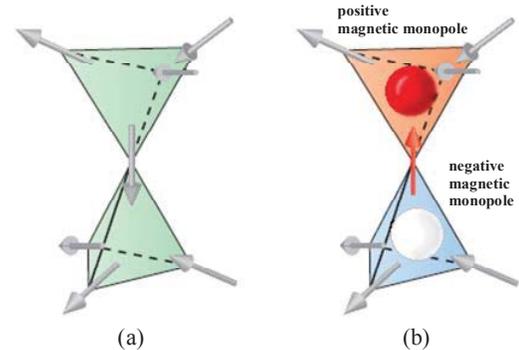


Fig. 1. (a): two neighboring tetrahedra obeying the ice rule, two spins are directed “in” and two – “out”. (b): inverting the shared spin generates a pair of magnetic monopoles.

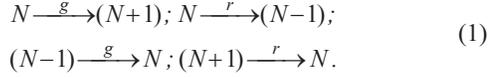
To derive the basic equation we make the following approximations and assumptions.

(a) We assume that monopoles are created and annihilated in pairs. Thus, we introduce the concentration of monopole pairs instead of the concentrations of monopoles.

(b) We assume that only one pair can be created or annihilated over a sufficiently small time dt .

(c) We discuss relatively small temperatures at which the concentration of monopoles is low. In this case, we can consider the monopoles as non-interacting quasiparticles.

At the first step we use standard treatment for generation–recombination noise, see, e.g., [23]. We consider probability $P(N, t)$ of the event that N pairs of quasiparticles (magnetic monopoles) are present at time t . This probability is changed on value dP during time dt due to following transitions:



Here transitions, which are specified in the first line, reduce the probability, i.e. $dP < 0$. On the contrary, remaining two transitions give $dP > 0$. Transitions, marked by symbol “g” above the arrow, are caused by the generation of a pair; respectively, the symbol “r” means recombination of the pair.

The rates of generation, $g = g(N)$, and recombination, $r = r(N)$, depend only on number N of the pairs at time t . These rates determine the probability $g \cdot dt$ of generation and $r \cdot dt$ of recombination of one pair over an arbitrarily small time dt . As a result, the analysis of transitions (1) leads to the Einstein–Fokker–Planck equation [24] for $P(N, t)$:

$$\frac{\partial P}{\partial t} = -\frac{\partial}{\partial N} [K_1(N) \cdot P] + \frac{1}{2} \frac{\partial^2}{\partial N^2} [K_2(N) \cdot P]. \quad (2)$$

Kinetic coefficients in this equation are determined by the generation and recombination rates of monopoles pairs:

$$K_1(N) = g(N) - r(N), \quad K_2(N) = g(N) + r(N). \quad (3)$$

At this stage, we restrict the analysis by the stationary case. We consider the relative fluctuations in the number of pairs $\delta N = \delta N(t)$ near the stationary state N_0 . Thus, the total number of pairs is $N = (1 + \delta N) \cdot N_0$. The variance $\sigma_{\delta N}^2$ of these fluctuations is assumed to be sufficiently small, $\sigma_{\delta N}^2 \ll 1$.

Eq. (2) corresponds to the following equation for fluctuations in $N(t)$:

$$\frac{d}{dt} \delta N = -\frac{1}{\tau_0} \delta N + 2 \sqrt{\frac{\sigma_{\delta N}^2}{\tau_0}} \cdot \zeta(t). \quad (4)$$

Here $\zeta(t)$ is stationary white noise with normalized spectrum $S_{\zeta}(f) = 1$ [Hz/Hz]. Note that eq. (4) coincides in form with the Langevin equation, see, e.g. [23]. Here τ_0 is the relaxation time, defined via the difference of derivatives from velocities of recombination and generation, taken in point N_0 :

$$\tau_0 = \left[\frac{d(r-g)}{dN} \Big|_{N_0} \right]^{-1}. \quad (5)$$

Eq. (4) allows determining of the spectrum $S_{\delta N}(f)$ of relative fluctuations in the number of pairs:

$$S_{\delta N}(f) = \frac{4\tau_0}{1 + (2\pi f\tau_0)^2} \cdot \sigma_{\delta N}^2. \quad (6)$$

Now we can investigate the dependence of this spectrum on the magnitude τ_0 (see Fig. 2), assuming the variance of the fluctuations constant, $\sigma_{\delta N}^2 = \text{const}$.

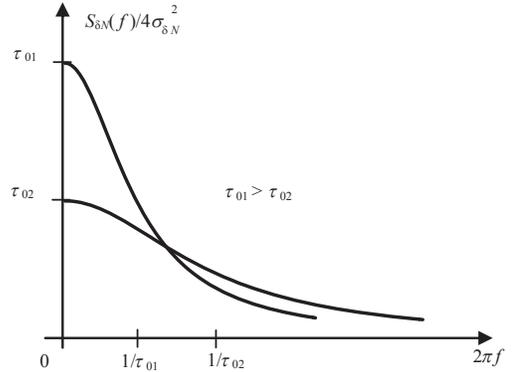


Fig. 2. Dependence of the spectrum on the magnitude τ_0 .

While relaxation time τ_0 is increased the height of the spectrum is increased proportionally to τ_0 , and the width is decreased as τ_0^{-1} . Thus, an increase of the lifetime of quasiparticles τ_0 leads to increased low-frequency component of fluctuations of generation–recombination of quasiparticles and to attenuation of high frequency components.

III. RELATION OF FLUCTUATIONS IN THE NUMBER OF PAIRS OF MAGNETIC MONOPOLES WITH MAGNETIC MOMENT

Very often in literature pairs of magnetic monopoles, which are formed by inverting the shared spin (Fig.1 b), are called bound. On the other hand, the magnetic monopoles, which move through the lattice, are called unbound or free. But, the “unbound” or “free” monopoles are bounded by the “spin spaghetti”, which is formed by changing of the spin configuration along the way of monopole.

Following [10], we define number n_b of bound, and unbound (free) n_u pairs, $N_0 = n_b + n_u$ – the total number of pairs, and $\alpha = n_u/N_0$ – the degree of dissociation.

We now assume that the vast majority of pairs of quasiparticles remain in a bound state, i.e. $\alpha \ll 1$, and one pair of bound quasiparticles interacts weakly with another pair. Thus, $N_0 \approx n_b$ – the total number of pairs is approximately constant at a given temperature.

A small change, caused by small perturbation in concentration of unbound (free) pairs Δn_u , yields small change $\Delta \alpha$ in degree of dissociation. The relaxation of $\Delta \alpha$ is determined by recombination of unbound pairs. The authors of paper [10], in terms of theory of magnetic Wien effect in the weak field limit, conclude that fluctuations $\Delta \alpha$ cause proportional fluctuations ΔM in magnetic moment. In our case of a small change in concentration n_u we have $N = n_b + n_u + \Delta n_u$, or $N = N_0 + \Delta n_u$. That is $N = N_0 \cdot (1 + \Delta n_u/N_0) =$

$N_0 \cdot (1 + \Delta\alpha)$. Thus, $\Delta\alpha = \delta N$ – fluctuations in N through the fluctuations in α (eq. (8) in [10]) lead to fluctuations in magnetic moment:

$$\Delta M \propto \frac{\mu_0 Q^3}{8\pi kT} \cdot \delta N. \quad (7)$$

Here μ_0 is the permeability of the vacuum, Q is the elementary magnetic charge, k is the Boltzmann's constant, T is the temperature.

For spectra we can write following equation:

$$S_{\Delta M}(f) \propto \left(\frac{\mu_0 Q^3}{8\pi kT} \right)^2 \cdot S_{\delta N}(f). \quad (8)$$

Here $S_{\Delta M}(f)$ is the spectrum of absolute fluctuations of magnetic moment. It can be expected, that measurement of fluctuations in the magnetic moment may give access to information about fluctuations in the number of pairs of magnetic monopoles in spin ice.

Finally we note that in a spin ice a few relaxation times may exist. As a result, the total spectrum $S_{\delta N}(f)$ in eq. (8) of relative fluctuations in the number of pairs of monopoles may be a superposition of spectra (6) with different relaxation times.

Experimental proof of this result is given in [25]. The authors reported development of a high-sensitivity superconducting quantum interference device (SQUID) based flux-noise spectrometer, and consequent measurements of the frequency and temperature dependence of spectral density of magnetic-flux noise due to generation-recombination fluctuations of magnetic monopoles for $\text{Dy}_2\text{Ti}_2\text{O}_7$ samples. Virtually all the elements predicted in [22] for the fluctuations in the number of magnetic monopoles, including the existence of intense magnetization noise and its characteristic frequency, were detected experimentally.

IV. CONCLUSIONS

We investigated the fluctuations in the number of pairs of magnetic monopoles. It is shown that these fluctuations can be described by equation of Langevin type. The spectrum of the fluctuations is determined and analyzed. The way how to detect these fluctuations by the measurement of the magnetic moment fluctuations is suggested. Our theoretical results are in agreement with experimental results of other authors.

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