# Thomson scattering in inhomogeneous plasma: The Role of the Fluctuation-Dissipation Theorem

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***Abstract*—A self-consistent kinetic theory of Thomson scattering of an electromagnetic field by a non-uniform plasma is derived. We show that not only the imaginary part, but also the time and space derivatives of the real part of the dielectric susceptibility determine the amplitude and the width of the Thomson scattering spectral lines. As a result of inhomogeneity, these properties become asymmetric with respect to inversion of the sign of the frequency. Our theory provides a novel and unique method of a remote probing and measurement of electron density gradients in plasma; this is based on the demonstrated asymmetry of the Thomson scattering lines.**

***Index Terms*—non-equilibrium fluctuations, FDT, Thomson scattering**

## I. Introduction

When an electromagnetic wave propagates in a plasma, its interaction with fluctuation oscillations of the plasma may result in scattering of the wave, which can be accompanied by a change in its frequency and wave vector. The intensity of scattered waves depends on both the intensity of the incident wave and the level of plasma fluctuations. Since the spectrum of plasma fluctuations exhibits sharp maxima at proper plasma frequencies, the spectrum of scattered waves will also exhibit sharp maxima at frequencies differing from the frequency of the incident wave by the according frequencies of the plasma fluctuations. The shift, width and shape of spectral lines carry information on such parameters of the plasma as its density, temperature, mean velocity, ion composition etc. A method of remote probing of a plasma, termed Thomson scattering, is a powerful plasma diagnostic tool that is widely employed in measurements of plasma parameters over a fairly broad range of plasma densities from the ionosphere to strongly coupled plasma. In such measurements the plasma must be transparent to the probe electromagnetic radiation. This may be microwave [1], laser [2] or X-ray radiation [3].

The differential Thomson scattering cross section, within an elementary solid angle $d\vartheta$ and for a frequency interval $d\omega'$ is described by the expression [4]:

$$d\Xi=\frac{1}{4\pi}(\frac{e^2}{m_e c^2})^2\frac{\omega'^2}{\omega_0^2}\sqrt{\frac{\varepsilon(\omega')}{\varepsilon(\omega_0)}}(1{+}\cos^2\theta)(\delta n_e\delta n_e)_{\mathbf{k},\omega}d\vartheta d\omega', \tag{1}$$

where $\mathbf{k}=\mathbf{k}'-\mathbf{k_0}$, $\omega=\omega'-\omega_0$; $\mathbf{k_0}$, $\mathbf{k}'$, $\omega_0$, $\omega'$ are the wave vectors and the frequencies of the incident and scattered electromagnetic fields, $\theta$ - is the scattering angle between $\mathbf{k}'$ and $\mathbf{k_0}$. Thus, the problem reduces to finding the spectral characteristics of electron density fluctuations $(\delta n_e\delta n_e)_{\mathbf{k},\omega}$ = $S(\mathbf{k},\omega)$ - the dynamic electron structure factor (structure factor). The theory of equilibrium and nonequilibrium plasma fluctuations was successfully developed in the second half of the past century [5]–[8]. In accordance with the Poisson equation, the structure factor in a spatially homogeneous system is directly linked to the electrostatic field fluctuations. In thermodynamic equilibrium, the electrostatic field fluctuations satisfy the famous Callen-Welton Fluctuation-Dissipation Theorem (FDT) [9], linking their intensity to the *imaginary* part of the response function and to the temperature T. A first comprehensive exposition of the state-of-the-art and application of the FDT and response functions to plasma is presented by Golden and Kalman [10]. In the case of plasma physics, if the electric field $\delta\mathbf{E}$ is considered to be the fluctuation variable $x$, then the response function $\alpha(\omega)$ is inversely proportional to the dielectric permittivity of the plasma $\varepsilon(\omega,\mathbf{k})$. In this case the Callen-Welton formula assumes the form:

$$(\delta\mathbf{E}\delta\mathbf{E})_{\mathbf{k},\omega}=\frac{8\pi\ T\mathrm{Im}\varepsilon(\mathbf{k},\omega)}{\omega\left|\varepsilon(\mathbf{k},\omega)\right|^2}. \tag{2}$$

The matter becomes more delicate even in the local equilibrium case. We have indeed shown [11], that in the *collisional regime* the Callen-Welton formula should be revised. There then appear new terms explicitly displaying dissipative non-equilibrium contributions and containing the interparticle collision frequency, the differences in the temperatures and the velocities, and also functions of the *real* parts of the dielectric susceptibilities. Taking into account this additional correlation for non-isothermic plasma may result in corrections to the measured temperatures amounting to tens of percents.

Eq. (2) refers to the steady state, for a space uniform system. However, it is not evident that the plasma parameters can be kept *constant* in both space and time. Inhomogeneities in space and time of these quantities will certainly also contribute to the fluctuations. Hence it is challenging to formulate the generalization of the FDT for inhomogeneous plasma and reformulate accordingly the results for the Thomson scattering.

The FDT for a local equilibrium state was proved by Balescu [12]. The parameters of a system in a local equilibrium state can be changed adiabatically on a scale greater than the particle mean free path. Inhomogeneity and nonstationarity of plasma fluctuations are manifested via a non-local dependence upon time [13] and coordinates [14]. The FDT for a non-local plasma was given in our paper [15]. A generalization of the Callen-Welton formula for systems with slowly varying parameters presented in [16].

In the present paper, applying the Klimontovich-Langevin approach [17] and the time-space multiscale technique, we show that not only the *imaginary* part but also the derivatives of the *real* part of the dielectric susceptibility determine the amplitude and width of spectral lines of the electrostatic field fluctuations and of the structure factor, as well. As a result of the inhomogeneity, these properties become asymmetric with respect to inversion of the sign of the frequency. In the kinetic regime the dynamic electron structure factor is more sensitive to space gradients than the spectral function of the electrostatic field fluctuations. Note that for simple fluids and gases a general theory of hydrodynamic fluctuations for nonequilibrium stationary inhomogeneous states has been developed in [18], [19]. In particular, it has been found that there exists an asymmetry of the spectrum for Brillouin scattering from a fluid in a shear flow or in a temperature gradient. The situation for the plasma problem we are considering is, however, quite different.

## II. Results

To treat the problem, a kinetic approach is required, especially when the wavelength of the fluctuations is larger than the Debye wavelength. To derive nonlocal expressions for the spectral function of the electrostatic field fluctuation and for the structure factor we adopt the Klimontovich-Langevin approach to describe kinetic fluctuations [17]. A kinetic equation for the fluctuation $\delta f_a$ of the one-particle distribution function (DF) with respect to the reference state $f_a$ is considered. In the general case the reference state is a nonequilibrium DF which varies in space and time both on the kinetic scale ( mean free path $l_{ei}$ and interparticle collision time ${\nu_{ei}}^{-1}$) and, also, on the larger hydrodynamic scales. These scales are much larger than the characteristic fluctuation time $\omega^{-1}$. In the nonequilibrium case we can, therefore, introduce a small parameter $\mu = \nu_{ei}/\omega$, which allows us to describe fluctuations on the basis of a multiple space and time scale analysis. Obviously, the fluctuations vary on both the "fast" $(\mathbf{r}, t)$ and the "slow" $(\mu\mathbf{r}, \mu t)$ time and space scales: $\delta f_a(\mathbf{x},t) = \delta f_a(\mathbf{x}, t, \mu t, \mu\mathbf{r})$ and $f_a(\mathbf{x},t) = f_a(\mathbf{p}, \mu t, \mu\mathbf{r})$. Here $\mathbf{x}$ stands for the phase-space coordinates $(\mathbf{r}, \mathbf{p})$. The Langevin kinetic equation for $\delta f_a$ has the form [17]

$$\widehat{L}_{a\mathbf{x}t}(\delta f_a(\mathbf{x},t) - \delta f_a^S(\mathbf{x},t)) = -e_a\delta\mathbf{E}(\mathbf{r}, t)\cdot\frac{\partial f_a(\mathbf{x},t)}{\partial\mathbf{p}}, \quad (3)$$

where $e_a$ is the charge of the particle of species $a$, $\delta\mathbf{E}$ is the electrostatic field fluctuation, and the operator $\widehat{L}_{a\mathbf{x}t}$ is defined by

$$\widehat{L}_{a\mathbf{x}t} = \frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial\mathbf{r}} + \widehat{\Gamma}_a(\mathbf{x},t); \widehat{\Gamma}_a(\mathbf{x},t) = e_a\mathbf{E}\cdot\frac{\partial}{\partial\mathbf{p}} - \delta\widehat{I}_a, \quad (4)$$

and $\delta\widehat{I}_a$ is the linearized collision operator. A model collision operators for plasmas is presented in [20].

The Langevin source $\delta f_a^S$ in Eq. (3) is determined by the following equation [17]:

$$\widehat{L}_{a\mathbf{x}t}\overline{\delta f_a(\mathbf{x},t)\delta f_b(\mathbf{x}',t')}^S = \delta_{ab}\delta(t-t')\delta(\mathbf{x}-\mathbf{x}')f_b(\mathbf{x}',t'). \quad (5)$$

The solution of Eq. (3) has the form

$$\delta f_a(\mathbf{x},t) = \delta f_a^S(\mathbf{x},t)$$

$$-\sum_b \int d\mathbf{x}' \int_{-\infty}^{t} dt' G_{ab}(\mathbf{x},t,\mathbf{x}',t')e_b\delta\mathbf{E}(\mathbf{r}', t')\cdot\frac{\partial f_b(\mathbf{x}',t')}{\partial\mathbf{p}'}, \quad (6)$$

where the Green function $G_{ab}(\mathbf{x},t,\mathbf{x}',t')$ of the operator $\widehat{L}_{a\mathbf{x}t}$ is determined by

$$\widehat{L}_{a\mathbf{x}t}G_{ab}(\mathbf{x},t,\mathbf{x}',t') = \delta_{ab}\delta(\mathbf{x}-\mathbf{x}')\delta(t-t') \quad (7)$$

with the causality condition:

$$G_{ab}(\mathbf{x},t,\mathbf{x}',t') = 0, \quad (8)$$

when $t < t'$.

Thus, $\overline{\delta f_a(\mathbf{x},t)\delta f_b(\mathbf{x}',t')}^S$ and $G_{ab}(\mathbf{x},t,\mathbf{x}',t')$ are connected by the relation:

$$\overline{\delta f_a(\mathbf{x},t)\delta f_b(\mathbf{x}',t')}^S = G_{ab}(\mathbf{x},t,\mathbf{x}',t')f_b(\mathbf{x}',t'), t > t'. \quad (9)$$

For stationary and spatially uniform systems the DF $f_a$ and the operator $\widehat{\Gamma}_a$ do not depend on time and space. In this case, the dependence on time and space of the Green function $G_{ab}(\mathbf{x},t,\mathbf{x}',t')$ is manifested only through the difference $t - t'$ and $\mathbf{r} - \mathbf{r}'$. However, when the DF $f_a(\mathbf{p},\mu\mathbf{r},\mu t)$ and $\widehat{\Gamma}_a(\mathbf{p},\mu\mathbf{r},\mu t)$ are slowly varying quantities in time and space, and when nonlocal effects are considered, the time and space dependence of $G_{ab}(\mathbf{x},t,\mathbf{x}',t')$ is more subtle:

$$G_{ab}(\mathbf{x},t,\mathbf{x}',t') = G_{ab}(\mathbf{p},\mathbf{p}',\mathbf{r}-\mathbf{r}',t-t',\mu\mathbf{r}',\mu t'). \quad (10)$$

For the homogeneous case this non-trivial result was obtained for the first time in our previous work [13]. This result was extended to inhomogeneous systems [14]. Here we want to stress that the nonlocal effects appear due to the slow time and space dependencies $\mu\mathbf{r}'$ and $\mu t'$.

In the first order expansion with respect to $\mu$ from Eq. (6) and the Poisson equation

$$\delta\mathbf{E}(\mathbf{r}, t) = -\frac{\partial}{\partial\mathbf{r}}\sum_b e_b\int\frac{1}{|\mathbf{r}-\mathbf{r}'|}\delta f_b(\mathbf{x}', t)d\mathbf{x}', \quad (11)$$

follows the generalized Fluctuation-Dissipation relation [15]

$$(\delta\mathbf{E}\delta\mathbf{E})_{\mathbf{k},\omega}$$

$$= \sum_a \frac{8\pi\ T_a Im(1 + i\mu\frac{\partial}{\partial\omega}\frac{\partial}{\partial\mu t} - i\mu\frac{\partial}{\partial\mu\mathbf{r}}\cdot\frac{\partial}{\partial\mathbf{k}})\chi_a(\mathbf{k},\omega,\mu\mathbf{r},\mu t)}{\omega_a\left|(1 + i\mu\frac{\partial}{\partial\omega}\frac{\partial}{\partial\mu t} - i\mu\frac{\partial}{\partial\mu\mathbf{r}}\cdot\frac{\partial}{\partial\mathbf{k}})\varepsilon(\mathbf{k},\omega)\right|^2} \quad (12)$$

where $\omega_a = \omega - \mathbf{k}\mathbf{V}_a$, and $\chi_a(\mathbf{k}, \omega, \mu\mathbf{r}, \mu t)$ is the susceptibility for a local-equilibrium plasma.

The effective dielectric function $\varepsilon(\omega, \mathbf{k})$ in the denominator of Eq. (12) defines the spectral properties of the electrostatic field fluctuations and imaginary part of the susceptibility $\chi(\mathbf{k},\omega)$ determines the half-width of the spectral line $(\delta\mathbf{E}\delta\mathbf{E})_{\mathbf{k},\omega}$ near the resonance:

$$\gamma = (Im\chi + \mu\frac{\partial}{\partial\omega}\frac{\partial}{\partial\mu t}Re\chi - \mu\frac{\partial}{\partial\mu\mathbf{r}} \cdot \frac{\partial}{\partial\mathbf{k}}Re\chi)/\frac{\partial}{\partial\omega}Re\chi. \quad (13)$$

In Eq. (13) there appear additional first-order terms of the small parameter $\mu$. It is important to note that the $imaginary$ part of the dielectric susceptibility is now replaced by the $real$ part, which in the plasma resonance may be greater than the $imaginary$ part by the same factor $\mu^{-1}$. Therefore, the second and third terms in Eq. (13) in the kinetic regime have an effect comparable to that of the first term. Second-order corrections in the expansion in $\mu$ only appear in the *imaginary* part of the susceptibility, and they can be reasonably neglected. It is therefore sufficient to retain the first-order corrections to resolve the problem. The half-width of the spectral lines Eq. (13) is affected by new nonlocal terms. They are not related to Joule dissipation and appear because of an additional phase shift between the induction vector and the electric field. This phase shift results from the finite time needed to set the polarization in the plasma with dispersion [21]. Such a phase shift in the plasma with space dispersion appears due to the medium inhomogeneity. For the case when the system parameters are homogeneous in space but vary in time, the correction to the half-width of the spectral lines in Eq. (13) is still symmetric with respect to the change in sign of $\omega$. However, when the plasma parameters are space dependent this symmetry is lost. The real part of the susceptibility $\chi(\mathbf{k},\omega)$ in Eq. (13) is an even function of $\omega$. This property implies that the contribution of the space derivative to the expression for the width of the spectral lines is odd function of $\omega$. Moreover, this term gives rise to an anisotropy in $\mathbf{k}$ space.

For the spatially homogeneous case there is no difference between the spectral properties of the longitudinal electric field and of the electron density, because they are related by the Poisson equation. This statement is no longer valid when an inhomogeneous plasma is considered. Indeed the longitudinal electric field is linked to the particle density by the nonlocal relation (11). In the latter case, an analysis similar to that made above can also be performed for the particle density.

$$S^e(\mathbf{k},\omega) = (\delta n_e \delta n_e)_{\omega,\mathbf{k}} = \frac{2n_e k^2}{\omega_e k_D^2}\left|\frac{1+\widetilde{\chi}_i(\mathbf{k},\omega)}{\widetilde{\epsilon}(\mathbf{k},\omega)}\right|^2 \mathrm{Im}\,\widetilde{\chi}_e(\mathbf{k},\omega) + \left|\frac{\widetilde{\chi}_e(\mathbf{k},\omega)}{\widetilde{\epsilon}(\mathbf{k},\omega)}\right|^2 \frac{T_i}{T_e}\frac{2n_e k^2}{\omega_i k_D^2}\mathrm{Im}\,\widetilde{\chi}_i(\mathbf{k},\omega), \quad (14)$$

where $k_D$ is the inverse Debye length,

$$\widetilde{\epsilon}(\mathbf{k},\omega) = 1 + \sum_a \widetilde{\chi}_a(\mathbf{k},\omega), \quad (15)$$

$$\widetilde{\chi}_a(\mathbf{k},\omega) = (1 + i\mu\frac{\partial}{\partial\omega}\frac{\partial}{\partial\mu t} - i\mu\frac{1}{k^2}\frac{\partial}{\partial\mu r_i}k_j\frac{\partial}{\partial k_i}k_j)\chi_a(\mathbf{k},\omega,\mu\mathbf{r},\mu t). \quad (16)$$

The inhomogeneous corrections $\frac{1}{k^2}\frac{\partial}{\partial\mu r_i}k_j\frac{\partial}{\partial k_i}k_j Re\chi_a$ in Eq. (15,16) is not the same as in Eq. (13) $(\frac{\partial}{\partial\mu\mathbf{r}} \cdot \frac{\partial}{\partial\mathbf{k}}Re\chi)$. The origin of this difference is that the Green functions for the electrostatic field fluctuations and density particle fluctuations are not the same in an inhomogeneous situation. As above, we can expand $\widetilde{\varepsilon}(\omega,\mathbf{k})$ near the plasma resonance $\omega = \omega_L$. Thus, for the Langmuir line,

$$S^e(\mathbf{k},\omega) = \frac{\widetilde{\gamma}}{(\omega-\omega_L sgn\omega)^2 + \widetilde{\gamma}^2}\frac{2n_e k^2}{\omega k_D^2 \partial Re\varepsilon/\partial\omega}\lfloor_{\omega=\omega_L}, \quad (17)$$

where

$$\widetilde{\gamma} = [Im\varepsilon + \mu\frac{\partial^2 Re\varepsilon}{\partial\mu t\partial\omega} - \mu\frac{1}{k^2}\frac{\partial}{\partial\mu r_i}k_j\frac{\partial}{\partial k_i}k_j Re\varepsilon]/\frac{\partial Re\varepsilon}{\partial\omega}\lfloor_{\omega=\omega_L sgn\omega} \quad (18)$$

is the half-width of the structure factor. An estimate for the plasma mode is then:

$$\widetilde{\gamma} = [\nu_{ei} + \frac{2}{n}\frac{\partial n}{\partial t} + \frac{\omega_L}{nk^2}\mathbf{k}\cdot\frac{\partial n}{\partial\mathbf{r}}(1+\frac{9k^2}{k_D^2})sgn\omega]/2. \quad (19)$$

From this equation we see that the inhomogeneous correction in Eq. (19) is greater than the one in Eq. (13) by the factor $(1 + k_D^2/9k^2)3/2$. For the same inhomogeneity; i.e., the same gradient of the density, we plot $S^e(\mathbf{k},\omega)$ together with the $(\delta\mathbf{E}\delta\mathbf{E})_{\mathbf{k},\omega}$ as functions of frequency (Fig.1).

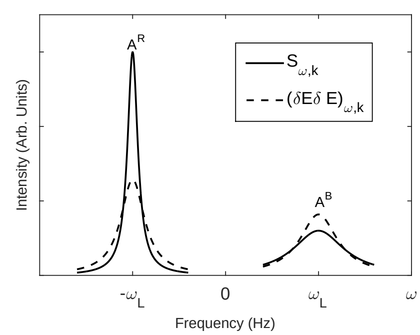


Fig. 1. The electron structure factor $S^e(\mathbf{k},\omega)$ ( solid line) and the spectral function of electrostatic field fluctuations $(\delta\mathbf{E}\delta\mathbf{E})_{\mathbf{k},\omega}$, (dashed line) as a function of frequency. $k_D/k = 3$; $\mathbf{k}\cdot\frac{\partial n}{n\partial\mathbf{r}} = \nu_{ei}k_D/27v_T$ .

This figure shows that the asymmetry of the spectral lines is present both for $S^e(\mathbf{k},\omega)$ and $(\delta\mathbf{E}\delta\mathbf{E})_{\mathbf{k},\omega}$. However, this effect is more pronounced in $S^e(\mathbf{k},\omega)$ than in $(\delta\mathbf{E}\delta\mathbf{E})_{\mathbf{k},\omega}$. Such asymmetry has been indeed detected in inhomogeneous plasma [22], [23]. The asymmetry of lines $S^e(\mathbf{k},\omega)$ can be used as a new diagnostic tool to measure local gradients in the plasma by Thomson scattering.

The Langmuir line (17) takes the Lorentz form and the amplitude of the spectral line $A$ is inversely proportional to its width

$$A = \frac{n_e k^2}{\widetilde{\gamma}k_D^2}. \quad (20)$$

The amplitude of the Langmuir line is seen to be more sensitive to the electron density gradient, than the line width. For example, in the case of a density gradient equal to $\partial n/n\partial r$=$\nu_{ei}/9v_T$ and $k_D/k = 3$, the red line width decreases by 67 percents, while at the same time the amplitude becomes 3 times larger (see Fig 1). From Eqs. (19) and (20) quite a simple formula for calculation of the electron density gradient from the Thomson scattering spectrum follows [24], [25].:

$$\mathbf{k}\cdot\frac{\partial n}{n\partial \mathbf{r}} = \frac{\nu_{ei}}{v_T}\frac{A^R - A^B}{A^R + A^B}\frac{k_D}{k_D^2/k^2+9}$$
$$= \frac{(\gamma^R+\gamma^B)}{\omega_L}\frac{A^R - A^B}{A^R + A^B}\frac{k_D^2}{k_D^2/k^2+9}, \qquad (21)$$

here $A^R$, $A^B$ and $\gamma^R$, $\gamma^B$ are the amplitudes and the half-widths of the red and blue Langmuir satellites, respectively (Fig. 1).

Thus, intensity and width measurements of the red and blue lines of the spectrum allow to determine the scalar product of the electron density gradient and the scattering vector at a given point. To determine the vector $\partial n/n\partial \mathbf{r}$ it is sufficient to measure the radiation scattered in three directions simultaneously.

From (21) it follows that the Knudsen number Kn=$\frac{\partial n}{n\partial \mathbf{r}}\frac{v_T}{\nu_{ei}}$ is equal to

$$Kn = \frac{A^R - A^B}{A^R + A^B}\frac{k_D/k}{k_D^2/k^2+9}/\cos\beta, \qquad (22)$$

where the amplitudes $A^R$ and $A^B$ are in arbitrary units, $\beta$ is the angle between the electron density gradient and the scattering vector $\mathbf{k}$.

## III. Conclusion

A first-principle kinetic theory of Thomson scattering in a non-uniform plasma is constructed, which agrees with the basic FDT and provides quantitatively correct results. Moreover, our theory provides a novel and unique method for remote probing and measurement of electron density gradients in plasma; this is based on the demonstrated asymmetry of the Thomson scattering lines in an inhomogeneous plasma. Such asymmetry has been indeed detected in spectroscopic studies of inhomogeneous plasma flows in magnetic traps [22], [23]. This method may be important for numerous technological applications, e.g. for the tokamak [26], for x-ray Thomson scattering on inhomogeneous targets [27] etc.

## References


[1] V. Heald, C.Wharton, Plasma diagnostics with microwaves, New York: Wiley, 1965.

[2] D Evans, J. Katzenstein, "Laser light scattering in laboratory plasma," Rep. Progr. Phys., vol. 32, pp. 207-271, 1969.

[3] S. Glenzer, R. Redmer, "X-ray Thomson scattering in high energy density plasma," Rev. Mod. Phys., vol. 81, pp. 1625-1663, 2009.

[4] J. Dougherty, D. Farley, "A theory of incoherent scattering of radio waves by a plasma," Proc. Roy. Soc., vol. A259, pp. 79-99, 1960.

[5] M.N. Rosenbluth, N. Rostoker, "Scattering of Electromagnetic Waves by a Nonequilibrium Plasma," Phys. Fluids, vol. 5, p. 776, 1962.

[6] Yu.L. Klimontovich, The Statistical Theory of Non-Equilibrium Processes in a Plasma, Oxford: Pergamon, 1967.

[7] S. Ichimaru, Basic Principles of Plasma Physics, Ch.7 Fluctuations, MA: Addison-Wesley, 1973.

[8] S.V. Gantsevich, V.L. Gurevich, and R. Katilus, "Theory of fluctuations in nonequilibrium electron gas," Riv. Nouvo Cimento, vol.2, p. 1, 1979.

[9] H.B. Callen, T.A. Welton, "Irreversibility and Generalized Noise," Phys. Rev., vol. 83, p. 34, 1951.

[10] K.I. Goldman and G. Kalman," The electrodynamics and Statistical Mechanics of Linear Plasma Response Functions," J. of Stat. Phys., vol.1, p. 415, 1969.

[11] V.V. Belyi, I. Paiva-Veretennicoff, "Electrostatic-field fluctuations and form factors in multi-component non-equilibrium plasmas," J. of Plasma Physics, vol. 43, p. 1, 1990.

[12] R. Balescu, Equilibrium and Nonequilibrium Statistical Mechanics, New York: Wiley, 1975.

[13] V.V. Belyi, Yu.A. Kukharenko, and J. Wallenborn, "Pair correlation function and nonlinear kinetic equation for a spatially uniform polarizable nonideal plasma," Phys. Rev. Lett., vol. 76, p. 3554, 1996.

[14] V.V. Belyi, Yu.A. Kukharenko, and J. Wallenborn, "Kinetic theory of a weakly-nonideal spatially nonuniform polarizable plasma," Contrib. Plasma Phys., vol. 42, pp. 3-26, 2002.

[15] V.V. Belyi, "Fluctuation-dissipation relation for a nonlocal plasma," Phys. Rev. Lett., vol. 88, 255001, 2002.

[16] V.V. Belyi, "Fluctuation-dissipation relation and quality factor for slow processes," Phys. Rev.E, vol.69, 017104, 2004.

[17] Yu. L. Klimontovich, Statistical Physics, Chur, Switzerland: Harwood, 1986.

[18] T.R. Kirkpatrick, E.G.D. Cohen, J.R. Dorfman, "Light scattering by a fluid in a nonequilibrium steady state, I. Small gradients," Phys. Rev.A, vol. 26, 972, 1982.

[19] V. Belyi, "Hydrodynamic fluctuations in inhomogeneously heated gas," Theor. Math. Phys., vol. 58, p. 275, 1984.

[20] V. Belyi, "Derivation of model kinetic equation," Europhys. Lett. vol. 111, 40011, (2015).

[21] M. Bornatici, Yu. Kravtsov, "Comparative analysis of two formulations of geometrical optics. The effective dielectric tensor," Plasma Phys. Controlled Fusion, vol. 42, p. 255, 2000.

[22] V. Strunnikov, Spectroscopic Study of Plasma Flows in Magnetic Traps, Ph.D. thesis (unpublished), Moscow: Kurchatov Institute, 1986.

[23] V. Belyi, V. Strunnikov, "Diagnostics of plasma inhomogeneity by Thomson scattering," Physics of Atomic Nuclei, (in press), 2019.

[24] V. V. Belyi, "Fluctuations out of equilibrium, " Philosophical Transactions R. Soc. A 376: 20170383. 2018, http://dx.doi.og/10.1098/RSTA-2017-0383.

[25] V. V. Belyi, "Thomson scattering in inhomogeneous plasma: The Role of the Fluctuation-Dissipation Theorem,", Scientific Reports, 8:7946, 2018 — DOI:10.1038/s41598-018-25319-6 SREP-16-46572B

[26] EquipeTFR, "Tokamak plasma diagnostics," Nuclear Fusion, vol. 18, p. 647, 1978.

[27] R. Thiele, P. Sperling, M. Chen, Th. Bornath, R.R. Faustlin, C. Fortmann, S.H. Glenzer, W.-D. Kraeft, A. Pukhov, S. Toleikis, Th. Tschentscher, and R. Redmer, "Thomson scattering on inhomogeneous targets, " Phys. Rev. E, vol. 82, 056404, 2010.