Three Problems of Liquidity under Asymmetric Information

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Abstract

This thesis develops models for three problems of liquidity under asymmetric information.

In the chapter "Disclosures, Rollover Risk, and Debt Runs", I build a model of dynamic debt runs without perfect information in order to understand the impact of asset opacity and disclosure policies on run likelihood and economic efficiency. I find that opacity is desirable with respect to both these metrics if and only if fundamentals are strong enough; and that a bank should commit to disclose truthfully any information it has unless the level of opacity is large. The model also uncovers rich interactions between debt dynamics, beliefs dynamics and equilibrium outcomes: short-term yields may remain low while risk builds up in the background, and therefore may not contain a warning sign of an upcoming crisis; and a disclosure regime might produce higher beliefs about collateral quality but nevertheless imply larger financing costs and thus lead to a bank failure.

In the chapter "Short-term Bank Leverage and the Value of Liquid Reserves", we extend the modelling toolbox of the global games literature by providing a fully rational setup where liquid reserves are modeled explicitly. The banks' balance sheet decisions and the prices of all securities are endogenous in the model. A bank possesses two instruments to manage illiquidity risk: its funding policy and the size of its liquid asset holdings, modeled as government bonds. Higher short-term indebtness allows the bank to better capture the liquidity benefits priced into deposits but increases illiquidity risk. Holding more bonds makes the bank more robust to withdrawals but it reduces the bank's asset returns. The impact of an increase in bond supply on bank leverage depends on the general-equilibrium change in the cost of absorbing the supply. The model also illustrates how considering an endogenous leverage decision is key to predict the impact of the economic environment on the liquidity premium.

In the chapter "Insider Trading under Penalties", we establish existence and uniqueness of equilibrium in a generalised one-period Kyle (1985) model where insider trades can be subject to a size-dependent penalty. The result is obtained by considering uniform instead of Gaussian noise and holds for virtually any penalty function. We apply this result to regulation issues. We show that the penalty functions maximising price informativeness for given noise traders' losses eliminate small rather than large trades. We generalise this result to cases where a budget constraint distorts the set of penalties available to the regulator.

Keywords: dynamic debt runs, opacity, disclosure policy, global games, bank runs, leverage, liquid reserves, Kyle equilibrium, insider trading.

Résumé

Cette thèse développe des modèles économiques afin d'étudier trois problèmes de liquidité dans un contexte d'information asymétrique.

Dans le chapitre "Annonces, risque de refinancement et crises de dette", je construis un modèle de crise de dette dynamique sans information parfaite afin de comprendre l'impact de l'opacité des actifs et des politiques d'annonce sur la probabilité des crises bancaires et l'efficacité économique. Je conclus que l'opacité est désirable pour l'une ou l'autre de ces mesures si et seulement si les fondamentaux sont suffisamment solides ; et que les banques devraient s'engager à révéler systématiquement leur information, à moins que le niveau d'opacité ne soit élevé. Mon modèle met également à jour de riches interactions entre la dynamique de la dette, la dynamique des croyances et les issues dictées par l'équilibre. Les taux de la dette à court terme peuvent rester bas alors qu'en arrière-plan, le risque augmente ; ainsi, les taux ne contiennent pas nécessairement de signe précurseur d'une crise. De plus, une politique d'annonce peut produire des meilleures croyances quant à la qualité du collatéral de la banque mais cependant impliquer des coûts de financement plus élevés, et donc mener à une faillite.

Dans le chapitre "Levier à court terme des banques et valeur des réserves liquides", nous construisons un nouvel outil pour la littérature utilisant les jeux globaux en mettant au point un modèle entièrement rationnel où les réserves liquides sont modélisées explicitement. Les décisions d'actif et de passif des banques ainsi que le prix de tous les instruments financiers du modèle sont endogènes. Une banque peut gérer son risque de devenir illiquide de deux façons : par sa politique de financement et par la taille de ses réserves liquides, modélisées ici par des obligations souveraines. Émettre plus de dépôts à vue permet de profiter davantage de la prime de liquidité que les investisseurs sont prêts à payer, mais augmente le risque d'une panique bancaire. Posséder davantage d'obligations rend la banque plus solide en cas de retraits, mais réduit le rendement moyen de ses actifs. L'impact d'une augmentation de l'offre d'obligations sur le levier des banques dépend du changement à l'équilibre général du coût d'absorber cette demande. Le modèle illustre également à quel point il est important de prendre en compte le caractère endogène de la décision de levier afin de prédire correctement l'impact de l'environnement économique sur la prime de liquidité des obligations souveraines.

Dans le chapitre "Opérations d'initié répréhensibles", nous prouvons l'existence et l'unicité de l'équilibre dans une version généralisée du modèle à une période de Kyle (1985), où l'agent initié peut être soumis à une pénalité qui est fonction de la taille de son ordre. Pour obtenir ce résultat, qui est valide pour essentiellement toute fonction de pénalité, nous considérons

des bruits uniformes et non normaux. Nous appliquons notre résultat à des problèmes de régulation. Nous démontrons que les fonctions de pénalité qui maximisent l'information contenue dans les prix pour un niveau donné de pertes des agents non informés éliminent les petits et non les grands ordres de l'agent initié. Nous généralisons ensuite ce résultat à des situations où le régulateur est soumis à une contrainte de budget qui change l'ensemble des pénalités qu'il peut utiliser.

Mots-clés : crises de dette dynamiques, opacité, politiques d'annonces, jeux globaux, paniques bancaires, levier, réserves liquides, équilibre de Kyle, opérations d'initié.

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Introduction

Asymmetric information can generate liquidity problems due to strategic concerns. This general economic phenomenon manifests itself in the three chapters of this thesis, under different forms.

In the first chapter, I study a problem of dynamic debt runs in order to understand how asset opacity and disclosure policies impact the likelihood of bank failures and economic efficiency. When banks cannot or do not commit to reveal truthfully all their information, their optimal policy is to disclose good news and conceal bad news. Investors rationally anticipate this strategy. This creates situations where the bank is genuinely uninformed but creditors believe that bad news are withheld. In this case, a rollover crisis can hit a bank which could have continued to operate under symmetric information. However, not committing to communicate news has a silver lining: the bank can in some cases hide bad news and subsequently recover, yielding a better outcome than under symmetric information. Comparing the relative merits of the commitment regime—voluntary disclosure policy— and the no-commitment regime—mandatory disclosure policy— is one of the main goals of the chapter. I show how different disclosure policies can lead to dramatically different outcomes, even along an identical path of fundamentals. At the aggregate level, the model indicates that mandatory disclosure is more efficient than voluntary disclosure, unless the level of opacity is large. The second main goal of the chapter is to discuss whether opacity is a desirable feature of the financial sector, as argued by some academics. To answer this question, the model compares probabilities of a bank failure and the economic efficiency across two polar cases: full opacity and full transparency. I find that opacity is more efficient and generates less bank runs if and only if fundamentals are sufficiently strong. This chapter also uncovers a rich interaction between debt dynamics, beliefs dynamics, and equilibrium outcomes. I illustrate how short-term yields can remain low while risk builds up in the background, and therefore not contain a warning sign of an upcoming crisis; and how a disclosure regime can produce consistently higher beliefs about collateral quality but nevertheless imply larger financing costs and therefore lead to a bank failure.

In the second chapter, Damien Klossner and I study the banks' balance sheet decisions in a model which captures explicitly a key trade off in the banking literature: the fact that investors value the demand deposits issued by banks, making them an attractive source of funding, but that these deposits also increase the likelihood of bank runs. We extend the modelling

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toolbox of the global games literature by providing a fully rational setup where liquid reserves are modeled explicitly. Global games theory turns out to be very convenient for our purposes. This branch of game theory allows to pin down a unique equilibrium in games that would a priori invite multiple equilibria; this equilibrium moves continuously with the fundamentals, which is a key property for the economist. To obtain such a result, the global games framework posits that players have asymmetric information about a relevant underlying state. Strikingly, even if the asymmetry is arbitrarily small—but positive—the desired equilibrium obtains. In the context of bank runs, this information asymmetry leads to liquidity crises in states where the bank could have survived, had the underlying state been common knowledge. I was struck by Hyun Song Shin abstract and powerful description of asymmetric information-driven liquidity crises during the 13th Swiss Finance Institute Annual Meeting in Zürich: "consensus is strong, but not strong enough for action". With very low information asymmetry, almost all creditors of the bank can assess that it is (extremely likely to be) solvent: there is consensus. Nevertheless, because of strategic concerns—the uncertainty about actions taken by other players, not about the fundamentals—creditors do not manage to coordinate on the most efficient action—rolling their debt over—and run on the bank. The framework developed in Chapter 2 allows to obtain endogenously the banks' leverage and liquid holdings decisions and to price all the securities of the model. The bank's optimal funding policy involves a mix of equity and deposit funding, as soon as the likelihood of a liquidity crisis is sensitive to the bank's short-term indebtness. The model also captures a dual value of liquid reserves—modeled by government bonds: they allow to reduce the risk that early withdrawals by impatient investors lead to inefficient liquidations: a direct value. But they also decrease the propensity of patient investors to run on the bank: a strategic value. The willingness of banks to hoard liquid reserves is then a function of these two values and of the government bond price. When the supply of bonds is scarce, banks bid up their price above the valuation of investors and this valuation wedge can be identified as a liquidity premium. Our model thus features a pecking order of government debt ownership, in the sense that investors hold bonds in equilibrium only when banks do, but the reverse does not hold. This creates a rich interaction between bank leverage and the liquidity premium. The model captures two possible reactions of bank's leverage to an increase in government bond supply. It also illustrates how considering an endogenous leverage decision is key to predict the impact of the economic environment on the liquidity premium.

In the third chapter, Pierre Collin-Dufresne, Franck Gabriel and I examine a problem of insider trading in a variation of the Kyle (1985) model. In this context, asymmetric information adversely impacts liquidity in the following sense. There is a market for a financial asset where some agents are uninformed and trade for reasons that are not related to the fundamental value of the asset, and can be interpreted as liquidity traders. There is also an agent with superior information—the insider—who knows what the asset is worth; she will be willing to buy the asset when its value is large and sell it when its value is low. Nevertheless, by posting an order, she is conveying information to the market. A market maker correctly anticipates the insider's strategy and absorbs the aggregate demand at a price such that he does not

make losses on average. But because of the presence of liquidity traders, the market maker cannot exactly infer the true asset value from the insider's order. Hence, the insider retains a positive expected profit in equilibrium, at the expense of the liquidity traders who undergo the corresponding loss. The chapter establishes a novel result in the Kyle one-period framework: by considering uniform—instead of Gaussian—noise, we are able to obtain existence and uniqueness even in case where insider trading is penalised, *i.e.* the insider can be subject to a punishment if she trades. We then apply this result to a regulation problem. We show that the penalty functions maximising price informativeness for given liquidity traders' losses eliminate small rather than large trades. We generalise this result to cases where a budget constraint distorts the set of penalties available to the regulator.

1 Disclosures, Rollover Risk, and Debt Runs

Sylvain Carré[1]

Procedures such as stress tests increase the transparency of the financial system by dampening information asymmetries between banks and the regulator. Is this transparency desirable? How should the regulator disclose information to the public? To answer these questions, I construct a model of dynamic debt runs where debt yields are endogenous and mapped explicitly to the degree of transparency, the regulatory disclosure regime and the state of the economy. I find that: in terms of run likelihood and efficiency, opacity is desirable if and only if fundamentals are strong; transparency can be more efficient even when it entails more runs; the regulator should commit to disclosure except at large levels of opacity. The model also uncovers a rich interaction between debt dynamics, beliefs dynamics, and equilibrium outcomes: short-term yields may remain low while risk builds up in the background, and therefore may not contain a warning sign of an upcoming crisis; and a disclosure regime might produce consistently higher beliefs about collateral quality but nevertheless imply larger financing costs and potentially lead to a bank failure for that reason.

Keywords: dynamic debt runs, opacity, disclosure policy.
JEL Classification Numbers: G21, G28.

1.1 Introduction

Financial institutions issuing short-term debt collateralised by long-term assets are exposed to bank run phenomena: creditors may demand to withdraw their funds and trigger costly liquidations. Debt runs are prominent features of financial crises: during the turmoil of 2007-2008, runs hit the asset-backed commercial paper market, the repo market, money market mutual funds and banks such as Northern Rock and Bear Stearns.[2]

Many institutions managing opaque assets struggled during the crisis (Gorton (2008)). This ignited a debate among both academics and policy makers about the impact of opacity on

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[1] Swiss Finance Institute and École Polytechnique Fédérale de Lausanne.

[2] Gorton and Metrick (2012) document the run on the repo market, and Covitz et al. (2013) investigate the run on the ABCP market.

financial fragility. One line of thinking, represented by Gorton and Ordoñez (2014) and Dang et al. (2017), advocates that opacity is actually a desirable characteristic of the financial sector and should be fostered. On the other hand, the policy responses to the crisis seemed to go in a different direction: regulators engaged in a considerable effort to both gather and disclose more information about banks. This was evidenced by the start of the Supervisory Capital Assessment Program (SCAP) in February 2009, a massive effort to submit all major banking institutions in the United States to thorough stress tests. Gathering and disclosing information are two distinct decisions: the regulator may collect information to have the option to reduce opacity by releasing it to the public, but prefer ex post not to do so. In fact, there were concerns that fully releasing the results of the SCAP stress tests might have a destabilising effect.[3]

The present chapter aims at answering the following questions: how does the accessibility of information impact the resilience of financial institutions to debt runs? under which circumstances should the regulator strive to collect information regularly? if the regulator has incentives to withhold information in some states, should he commit ex ante to a policy of full disclosure?

To do so, I modify the discrete time dynamic debt runs model of Acharya et al. (2011) by allowing the bank's assets to be opaque and information release to be strategic. My model features an uninsured financial institution ("bank") trying to roll over its short-term debt until its assets mature. The bank cannot communicate credibly. Instead, creditors have to rely on regulatory disclosures when deciding whether to renew their credit to the bank. Because the bank assets can be complex and investigation is costly, the regulator may not be able to constantly assess the soundness of the bank: the frequency at which he can and wishes to obtain bank-specific information defines the degree of transparency in the model. Opacity is defined as the opposite of transparency. In a regime of commitment (*mandatory* disclosure), the regulator conveys truthfully any information he has to the bank's creditors. Absent commitment (*voluntary* disclosure), the regulator finds it optimal to only release good news.

Because of the simple structure chosen for the bank's asset process, I am able to characterise analytically the interest rates demanded by creditors to roll over the bank's debt, and the states in which they instead decide to run. Runs are assumed to entail deadweight liquidation costs proportional to the fundamental value of the asset at the run time. Inefficiency is then defined as the expected liquidation costs.

Constructing a dynamic framework where the cost of debt is endogenous allows to uncover two channels that would not be apparent in a model with a single rollover date. First, I capture a *funding cost channel*: a signal provided to a creditor has a contemporaneous effect (it will

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[3]Bernanke (2010) mentions these concerns; see also Goldstein and Sapra (2014).

trigger a run with some probability today), but also impacts the required interest rate. Hence, it affects future debt levels and thereby future incentives to run. The efficiency of an opacity level and a disclosure policy depend on both the direct and the indirect effect. Second, the model recognises that the disclosure policy of the regulator impacts the beliefs dynamics, which in turn impact future rollover decisions. When the regulator does not commit to disclosure, short-term funding costs are lower in good times. However, the lack of commitment generates depressed beliefs as long as the bad state does not realise, potentially leading to a larger probability of bank failure at longer horizons. A model with a single rollover date would obliterate this *beliefs channel* and the costs it entails; when in fact, all the costs associated with non-commitment are due to the fact that it generates worse beliefs.

The interaction between the information structure, debt dynamics and beliefs dynamics is rich. Short-term debt yields are determined by the number of default states tomorrow under the given information structure, not by the expected value of the collateral computed under the beliefs generated by this structure: yields do not primarily reflect the current expected collateral quality. Two results of the chapter relate to this fact. First, there need not necessarily be a warning sign of a run in the time-series of short-term returns: yields may remain low while risk builds in the background. Second, there are situations in which the expected collateral value is always larger under one disclosure regime, but the bank nevertheless faces larger financing costs under this regime, and therefore fails only in the seemingly better scenario.

At the policy level, the main results are the following. First, I find that opacity reduces run probability and inefficiency only when fundamentals are strong enough: in situations where the regulator believes that the economy is healthy and likely to remain so for a long time, collecting and releasing information about banks is detrimental; in other situations, the regulator wants to implement transparency. Second, opacity may decrease run probability but increase inefficiency: the objective of the regulator is not to minimise the probability of a bank failure, but rather the expected costs associated with liquidation. Under transparency, runs may occur more frequently but they are concentrated on bad banks, for which liquidation is less inefficient. Third, voluntary disclosure is more efficient than mandatory disclosure except at large levels of opacity: this implies that the regulator should commit to disclose stress test results as soon as his access to information is relatively easy. Thus, my model shows that whether stress test results should be systematically disclosed depends on the degree of asset opacity.

Relation to the literature. The game-theoretic study of bank runs traces back to the seminal paper of Diamond and Dybvig (1983): in the bad equilibrium, agents "panic" about the run decision of others, leading to an outcome where all creditors run on the bank and force an inefficient liquidation. Building on the global games literature pioneered by Carlsson and

van Damme (1993) and Morris and Shin (1998), Rochet and Vives (2004) and Goldstein and Pauzner (2005) provide bank run models where the equilibrium is unique and runs arise as the result of both a coordination failure and concerns about the fundamentals. In these models, the coordination problem comes from the fact that creditors are dispersed and must decide simultaneously whether to withdraw their funds. Models of dynamic debt runs provide a related but distinct approach. There, the coordination problem is intertemporal in the sense that an agent may withdraw his funds because of concerns about future rollover decisions of other creditors. He and Xiong (2012) and Schroth et al. (2014) provide such models and use them to quantify the impact of factors such as maturity mismatch, leverage and liquidation costs on run likelihood, with a focus on the 2007 run on ABCP.

As Acharya et al. (2011), this chapter highlights the importance of the specific nature of the information structure to the outcome of the rollover problem. In a broader framework, Kamenica and Gentzkow (2011) show how one can optimally design information structures (*i.e.* select signals[4]) to maximise non-linear functions of some agent's beliefs, what they call Bayesian persuasion. Finding the optimal opacity level and disclosure policy in the present model can be seen as a Bayesian persuasion problem, because it means choosing ex ante which signals about the fundamental to show to investors, and the non-linear function of their belief is the rollover decision. Papers linking explicitly the Bayesian persuasion approach to the research on stress tests include Goldstein and Leitner (2017), Inostraza and Pavan (2017) and Quigley and Walther (2017).

While the models of dynamic debt runs mentioned above assume full information, there is also a significant body of literature on banking under opacity. Alvarez and Barlevy (2014) develop a network model of banking where imposing mandatory disclosure of losses can only improve welfare when contagion concerns are strong. de Faria e Castro et al. (2016) study the interaction between the fiscal capacity of the government and optimal disclosure policies. When deposit insurance can be provided at a low social cost, a disclosure policy that would be suboptimal absent insurance because of the run risk it implies may become desirable. In a model of coordination failures *à la* Goldstein and Pauzner (2005), Bouvard et al. (2015) investigate how a regulator endowed with perfect information about aggregate and idiosyncratic shocks on the banking sector should communicate with the public. My model does not distinguish between these shocks, but introduces the possibility that the regulator herself has no information: this generates a different commitment problem. Additionally, their model features a single rollover date and therefore does not capture the funding cost channel and the beliefs channel described above. Finally, Monnet and Quintin (2017) map the need for transparency to the degree of a bank's asset liquidity and show that opacity is

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[4]By "selecting signals" one means of course selecting *ex ante* a random variable, rather than being able to show or conceal the realisation of a given signal.

preferable when secondary markets are shallow.

This chapter also bears a connection with the series of papers by Gorton and Pennacchi (1990), Dang et al. (2013), Dang et al. (2015), Gorton and Ordoñez (2014) and Dang et al. (2017). These authors focus on the notion of *information sensitivity*. A security is information insensitive when agents have no incentive to acquire costly signals about it. Because of their capped payoff, debt contracts are natural candidates for information insensitivity, and more so if collateral is opaque. If, in addition, the expected value of collateral is large enough, debt is risk-free and of constant value: it can be used as money. Therefore bank should be "secret keepers" (Dang et al. (2017)). Deterring information acquisition with opaque collateral also ensures that information is always symmetrical. This prevents market freezes due to adverse selection issues (Dang et al. (2015)). One can similarly define the information sensitivity status of debt in my model and map this status to the current state of the world, the degree of opacity, and the disclosure regime.

1.2 The Model

Time is discrete: $t = 0, 1, 2, ...$. The model features an uninsured financial institution ("bank") whose short-term debt must be refinanced by successive creditors until its asset reaches maturity.[5]

1.2.1 The Bank

Asset side

The bank holds a long-term asset. For tractability purposes, its maturity is modelled as a random time ζ_ϕ. ζ_ϕ is assumed to be independent of all other variables and geometrically distributed with parameter $\phi \in (0, 1)$: the expected maturity is $\mathbb{E}[\zeta_\phi] = \frac{1}{\phi}$. At time ζ_ϕ, the asset delivers its payoff, agents receive their payments, and the world ends. The asset does not pay anything before maturity.

The asset side of the bank is modelled by a Markov chain $(y_t)_{t \geq 0}$ with two states: $y^G > y^B$. The meaning of y_t is the following: if maturity occurs at time t ($\zeta_\phi = t$), the asset payoff is y_{ζ_ϕ}.

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[5]A significant part of the short-term debt of financial institutions is not insured, and even bank deposits are typically insured only up to some limit. Moreover, ex-post liquidity assistance may not be systematical but contingent to some criteria (see for instance Santos and Suarez (2019)). For simplicity, I consider uninsured debt, but it is straightforward to amend the model solution to the case where the institution is bailed out with some exogenous probability when a run occurs.

Assume that the asset is initially in the good state: $y_0 = y^G$. The transition matrix of (y_t) is

$$\Lambda = \begin{pmatrix} \lambda^{GG} & 1 - \lambda^{GG} \\ \lambda^{BG} & 1 - \lambda^{BG} \end{pmatrix}. \tag{1.1}$$

λ^{GG} represents the probability to stay in the good state from one period to the next, while λ^{BG} can be interpreted as a recovery probability. Under the conditions $\lambda^{GG} > \frac{1}{2}$ and $\lambda^{BG} < \frac{1}{2}$, we have

$$V^G \equiv \mathbb{E}[y_{\zeta_\phi} | y_t = y^G, t < \zeta_\phi] > \mathbb{E}[y_{\zeta_\phi} | y_t = y^B, t < \zeta_\phi] \equiv V^B. \tag{1.2}$$

(1.2) means that being in the state y^G before maturity signals a high expected payoff at maturity, so y^G is indeed the "good state".

Liability side

The initial capital structure of the bank is taken as given. The bank has raised an amount D_0 of short-term (*i.e.* one-period) debt D_0.[6] Equity is the residual claim and is owned by the banker. Since the asset does not pay anything before maturity, short-term debt must be refinanced: to do so, the bank has access to a pool of potential short-term creditors (see section 1.2.2). No other sources of financing are available.

Short-term debt can stop being rolled over in two cases. (i) (*strategic default*) The bank can decide to default on the debt, in which case its asset is liquidated at a fraction of its current expected value. The strategic default time is denoted ζ_s. (ii) (*rollover freeze*) If debt is too high, there is no short-term debt contract that compensates adequately for default risk. No creditor accepts to roll over the debt, forcing the bank into liquidation. The time at which this happens is denoted ζ_z.

Important details on the liquidation procedure are given in section 1.2.4. Let $\zeta_\ell = \min\{\zeta_s, \zeta_z\}$ be the liquidation time. I will use the convention $\zeta_l = \infty$ when liquidation does not occur prior to maturity. Finally, define the end date as

$$\zeta_f = \min\{\zeta_\ell, \zeta_\phi\}. \tag{1.3}$$

It is convenient to introduce the following assumption.

Assumption 1 $D_0 > V^B$.

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[6]Explicit motivations for short-term debt include Calomiris and Kahn (1991) and Diamond and Rajan (2001). Brunnermeier and Oehmke (2013) show how debt maturities can endogenously shorten in response to dilution concerns. The second chapter of this thesis provides a global games model for the short-term leverage choice of a bank whose debt provides liquidity but creates rollover risk.

This condition ensures that the bank is insolvent when the bad state is revealed, which triggers liquidation.

1.2.2 Creditors

The bank has access to an unlimited pool of risk-neutral and competitive creditors.

I assume that all the short-term debt is held by a single investor at each period, and that the investor entering the debt contract at date t exits forever the pool of creditors after receiving his payment at $t + 1$.

Given an amount of debt to roll over at time t, the bank offers a contract with a promised repayment at time $t + 1$, the face value F. The risk-free rate is normalised to zero. Hence, since creditors compete to obtain the debt contract, the equilibrium face value is such that a creditor makes zero profit on average. If no face value satisfies the zero profit condition, liquidation occurs (*i.e.* ζ_z is reached). I use the convention $F = \emptyset$ in that case, since the bank cannot offer any acceptable face value.

1.2.3 The Regulator

A regulator who may obtain information about the bank's asset and can disclose them to creditors. When the regulator does not commit to reveal all its information, he selects his disclosure policy to minimise inefficiency. Note that since creditors make zero expected profit in equilibrium, the regulator's objective is in fact to maximise the banker's equity value: see the equilibrium definition in section 1.2.5. The next section describes the information structure and provides details about the constraints under which the regulator operates.

1.2.4 Information Structure

Asset Opacity

I make the following assumptions. First, the bank observes (y_t) but cannot credibly communicate any information to investors. Second, at each time t, the regulator observes the current state of the chain, y_t, with probability p, independently of everything else.

It will be convenient to define the dummy variables

$$\omega_t = \begin{cases} 1 & \text{if the regulator observes } y_t \\ 0 & \text{otherwise.} \end{cases} \tag{1.4}$$

By assumption, $(\omega_t)_{t \geq 0}$ is an i.i.d. sequence of Bernoulli variables with parameter p. p characterises the degree of opacity of the asset. When $p = 1$, there is full information, while $p = 0$ corresponds to the case of a fully opaque asset.

Agents in the pool of creditors cannot make any direct observation and rely on the regulator's disclosures.

The motivation for this particular modelling of opacity is the following. One wants to capture the fact that it is not feasible for the regulator to monitor the bank at all times, because of the excessive costs this would imply. As Bernanke (2010) noted, "The SCAP represented an extraordinary effort on the part of the Federal Reserve staff and the staff of other banking agencies. In a relatively short time, the supervisors had to gather and evaluate an enormous amount of information".

Considering an exogenous ω_t allows to maintain tractability; and letting $p < 1$ incorporates the regulator's constraints into the model as desired.

Disclosure Regimes

At each time t, the regulator has the opportunity to disclose information to the pool of creditors after the realisation of ω_t. Disclosure takes the form of an announcement δ_t:

$$\delta_t = \begin{cases} \varnothing & \text{"I did not observe the asset value "} \\ y^G & \text{"I observed the asset value and } y_t = y^G \text{"} \\ y^B & \text{"I observed the asset value and } y_t = y^B \text{".} \end{cases} \tag{1.5}$$

I compare two disclosure regimes: *voluntary* and *mandatory.*

Under mandatory disclosure, the regulator is compelled by law to announce the truth. That is, he has been able to credibly commit to communicate any information he has. In that case, disclosure is mechanical:

$$\delta_t = \begin{cases} y_t & \text{if } \omega_t = 1 \\ \varnothing & \text{if } \omega_t = 0. \end{cases} \tag{1.6}$$

Under voluntary disclosure, the regulator can conceal news. That is, he can claim to be uninformed while he is. Formally, it means he can play $\delta_t = \varnothing$ when $\omega_t = 1$. However, if a state is announced, is must be accompanied with evidence. Hence, it is impossible to announce that a state has been observed when it is not the case. Formally, it means that $\delta_t = y^i$ implies $\omega_t = 1$ and $y_t = y^i$ for $i = G, B$. These assumptions on the voluntary disclosure regime are

borrowed from Dye (1985).

The equilibrium under voluntary disclosure will feature a *sanitisation strategy*:[7] the regulator discloses the good state and conceals the bad state. That is, he plays $\delta = \delta^S$, where

$$
\delta_t^S \equiv \begin{cases} y^G & \text{if } \omega_t = 1 \text{ and } y_t = y^G \\ \emptyset & \text{otherwise} \end{cases}
\tag{1.7}
$$

is the sanitisation strategy.

Denote $(\mathscr{F}_t^I)_{t \geq 0}$ the filtration of the investors:

$$
\mathscr{F}_t^I = \sigma \left((\delta_s)_{s \leq t}, \zeta_\ell \mathbb{1}_{\{\zeta_\ell \leq t\}}, \zeta_\phi \mathbb{1}_{\{\zeta_\phi \leq t\}} \right),
\tag{1.8}
$$

$(\mathscr{F}_t^R)_{t \geq 0}$ the filtration of the regulator:

$$
\mathscr{F}_t^R = \sigma \left((\omega_s, y_s \mathbb{1}_{\{\omega_s = 1\}})_{s \leq t}, \zeta_\ell \mathbb{1}_{\{\zeta_\ell \leq t\}}, \zeta_\phi \mathbb{1}_{\{\zeta_\phi \leq t\}} \right),
\tag{1.9}
$$

and $(\mathscr{F}_t^B)_{t \geq 0}$ the filtration of the bank, which observes everything but cannot communicate information credibly.

A belief system of the investors $\mathscr{B}$ is a $(\mathscr{F}_t^I)$-adapted process $(q_t^{\mathscr{B}})$. $q_t^{\mathscr{B}} = \mathbb{P}(y_t = y^G | \mathscr{F}_t^I, \mathscr{B})$ is the probability to be in the good state at time t from the perspective of the investors.

Liquidation

If liquidation occurs at time t ($t < \zeta_\phi$), the value αV is recovered, where $\alpha \in [0, 1]$ and $V = \mathbb{E}[y_{\zeta_\phi} | \mathscr{F}_t^I]$ is the fundamental value of the asset computed under the outsiders' information set at time t.

In case of a strategic liquidation under asymmetric information, the liquidation decision has a signalling content and I need to specify the beliefs of outsiders. For simplicity, I focus on equilibria where the bank's decision to liquidate at t when $\delta_t = \emptyset$ is interpreted as the fact that the bank has observed the bad state ($y_t = y^B$):

I restrict the attention to equilibria where the belief system $\mathscr{B}$ is such that

$$
\zeta_s = t, \, \delta_t = \emptyset \implies q_t^{\mathscr{B}} = 0.
$$

When $\delta_t \neq \emptyset$, payoff-relevant information is symmetric because announcements of states are

---

[7]See Shin (2003).

trustworthy. Hence, there is no signalling problem in that case.

$1 - \alpha$ is a measure of illiquidity as it represents the fraction of asset value destroyed due to premature liquidation.[8] Because of the deadweight cost $(1 - \alpha)V$, liquidation is never efficient in this model; the inefficiency is large for good banks (*i.e. V* high) and small for bad banks (*i.e. V* low).[9]

1.2.5 Equilibrium

Debt Dynamics

Assume we are at time $t < \zeta_f$ with a level of debt D_t. The bank has promised the face value D_{t+1} to the current creditor. The actual payment, $\tilde{D}_{t+1}$, satisfies:

$$\tilde{D}_{t+1} = \begin{cases} \min\{y_{t+1}, D_{t+1}\} & \text{if } \zeta_\phi = t+1 \\ \min\{\alpha V_{t+1}, D_{t+1}\} & \text{if } \zeta_\phi > t+1 \text{ and } \zeta_\ell = t+1 \\ D_{t+1} & \text{otherwise.} \end{cases} \tag{1.10}$$

In the first case, maturity occurs at time $t+1$ and the asset delivers the payoff y_{t+1}. In the second case, the bank is liquidated at the value αV_{t+1} where $V_{t+1} = \mathbb{E}[y_{\zeta_\phi}|\mathscr{F}_{t+1}^I]$. In the third case, the banker is able to roll its debt over. That is, she obtains the financing necessary to repay D_{t+1} in full.

Lemma 1 *The break-even condition of lenders is equivalent to the property that $(\tilde{D}_{t \wedge \zeta_f})_{t \geq 0}$ is a $(\mathscr{F}_t^I)$-martingale.*

(All proofs are relegated to Appendix A.1).

Quantities of Interest

My goal is to understand how asset opacity and disclosure regimes impact the likelihood of debt runs and their inefficiency. In this section, I explain how these quantities are measured in the model. I then define formally the equilibrium.

The probability of a run is

$$\mathscr{P} \equiv \mathbb{P}(\zeta_\ell < \zeta_\phi). \tag{1.11}$$

---

[8] If the bank is the first-best user of the asset, transferring its control rights to another party reduces its value (Shleifer and Vishny (1992)).

[9] The typical situation in 3-dates models of runs is that premature liquidations are efficient when the fundamental is very low and inefficient otherwise. The common conclusion is that the deadweight cost of liquidating good banks is larger.

Because lenders make zero profit on average, the bank bears the costs of inefficient runs. Optimality for the bank coincides with a social planner's optimality in the model, that is, maximising the expectation of the payoff U of the asset. This quantity is given by

$$U \equiv \alpha V_{\zeta_l} \mathbb{I}_{\{\zeta_\ell < \zeta_\phi\}} + y_{\zeta_\phi} \mathbb{I}_{\{\zeta_\phi \le \zeta_\ell\}}. \tag{1.12}$$

Equivalently, the measure of *inefficiency* is the expected deadweight cost

$$\mathscr{I} \equiv \mathbb{E}[(1-\alpha) y_{\zeta_\ell} \mathbb{I}_{\zeta_\ell < \zeta_\phi}] = V^G - \mathbb{E}[U]. \tag{1.13}$$

Saying that the banker maximises equity value is equivalent to saying that she maximises $\mathbb{E}[U]$ or minimises $\mathscr{I}$.

I am now ready to define the equilibrium.

Equilibrium concept

Definition 1 *Given a belief system $\mathscr{B}$, a consistent bank policy is a promised face value schedule F and a time of strategic liquidation ζ_s such that*

i) *F_t is Markov in $(D_t, q_t^{\mathscr{B}})$; $D_{t+1} = F_t$ and the process $(\tilde{D}_t)$ associated with (D_t) is a $(\mathscr{F}_t^I)$-martingale. F is required to satisfy*

 (M) *F_t is non-decreasing in D_t and non-increasing in q_t,[10]*

 (NP) *$F \le K$ for some constant $K > y^G$.*

ii) *ζ_s is $\mathscr{F}_t^B$-adapted, and, given F, it minimises $\mathscr{I}$.*

An equilibrium is $(F, \zeta_s, \delta, \mathscr{B})$ such that

i) *given $\mathscr{B}$, (F, ζ_s) is a consistent bank policy that minimises $\mathscr{I}$.*

ii) *δ is $\mathscr{F}_t^R$-adapted and given $(F, \zeta_s, \mathscr{B})$ it minimises $\mathscr{I}$.[11]*

iii) *the belief system $\mathscr{B}$ is compatible with the disclosure policy δ.[12]*

The implicit assumption here is that the banker commits to an interest rate schedule at date 0. Otherwise, the banker would convey signalling information when offering a face value

---

[10]Recall the convention $F = \emptyset$ when there is no acceptable face value. The meaning of the monotonicity condition is then that if $F(D_1) = \emptyset$ and $F(D_2) \in \mathbb{R}$, $D_1 > D_2$.

[11]Under mandatory disclosure, δ and $\mathscr{B}$ are mechanical and are *de facto* not equilibrium objects.

[12]The unique belief system compatible with a given disclosure policy δ will be denoted $\mathscr{B}(\delta)$.

to creditors. In particular due to the specification of out-of-equilibrium beliefs, this would complicate significantly the formalisation of the game without bringing additional insights. With the formulation of the text, all the signalling is contained in the disclosure decision.

Requiring that F_t is Markov in (D_t, q_t) is to simplify the exposition. I could just demand that F_t is $(\mathscr{F}_t^I)$-adapted; but since q_t encapsulates all the relevant information about the asset payoff, the bank has nothing to gain to condition its face value to other $\mathscr{F}_t^I$-measurable variables.

Condition (NP) rules out Ponzi schemes, and, as usual, the constraint $F \leq K$ is never binding in equilibrium.[13] This is a consequence of the following useful lemma:

Lemma 2 *In a consistent bank policy, an insolvent bank is necessarily forced into liquidation.*

This is the standard result that insolvency implies illiquidity (of course, the converse is not true). Hence, since $K > y^G$, the bank would be ran upon before debt can reach K, so the constraint $F \geq K$ does not bind.

Figure A.1 sums up the model setup graphically. The baseline parameters I use are $y^G = 100$, $y^B = 0$, $p = 50\%$, $\lambda^{GG} = 1 - \lambda^{BG} = 97\%$, $\phi = 15\%$ and $\alpha = 85\%$. The set of parameters is denoted Θ and Θ_{-x} denotes this set without the variable x.

1.3 Model Solution

The first step towards solving the model is to establish that the bank never wishes to force liquidation:

Lemma 3 *The bank never liquidates strategically in a consistent bank policy for $\alpha \in [0, 1)$:* $\zeta_s = \infty$.

(In the extreme case $\alpha = 1$, there is no cost associated with liquidation. Thus, when $y_t = y^G$ is observed, the bank is indifferent between holding the asset or liquidating it.) The intuition behind this result is the following. Since debt comes at a zero expected cost for the bank, the banker has no incentive to incur the deadweight liquidation cost today: she is always better off waiting.

---

[13] In the sense that the banker never actually sets F at K. But of course the constraint binds in a dynamic sense since it rules out Ponzi schemes. Also note that absent requirement (NP), there is a Ponzi equilibrium where each lender is simply betting against maturity, *i.e.* hoping he is not the last in line (this is made possible by the random maturity assumption). Of course, the actual asset value is irrelevant in that case. See *e.g.* Blanchard and Watson (1982).

1.3.1 Voluntary Disclosure

I now characterise the policy of the regulator in the voluntary disclosure case.

Lemma 4 *The regulator follows the sanitisation strategy δ^S (defined in (1.7)) in equilibrium.*

This result is very intuitive. When the regulator observes the good state, it is clearly in his best interest to communicate it to investors. When the regulator observes the bad state, it is always best to conceal it. Even if creditors understand that the regulator may be hiding information, their updated belief about the probability of the good state cannot be worse than if the regulator had revealed the bad state.[14]

State Variables

Suppose we are at time $t < \zeta_\phi$ and current debt is D. From Lemma 4, we know that the regulator discloses only $\emptyset$ or y^G in equilibrium. Let τ be the time elapsed since the last disclosure of y^G:

$$\delta_{t-\tau} = y^G, \delta_{t-\tau+1} = \emptyset, \delta_{t-\tau+2} = \emptyset, \ldots, \delta_t = \emptyset. \tag{1.14}$$

Given the stationarity of the problem, the data of (D, τ) contains all the relevant information for decision making and I can select (D, τ) as the state variable:

Remark 1 *Any face value schedule F in a consistent bank policy is Markov in (D, τ). Due to Lemmas 3 and 4, what remains to be determined in order to find the equilibrium is which $F(D, \tau)$ are compatible with a consistent bank policy, and which one maximises the banker's equity value.*

The full characterisation of the equilibrium is in section 1.3.2. The next sections explain how to get there.

Beliefs Dynamics

I begin by determining the beliefs dynamics under δ^S, *i.e.* the compatible belief system $\mathscr{B}(\delta^S)$. The probability to be in state y^G, under $\mathscr{F}_t^I$, sums up the outsiders' beliefs: denote it q. Initially we have $q = 1$, and immediately after any disclosure $q = 1$ as well, because disclosure only occurs when the regulator observes y^G. Now assume no disclosure at $t = 1$. Either the state was bad and observed (probability $p(1 - \lambda^{GG})$) or the state was not observed (probability $1 - p$).

---

[14]I simplify the strategic disclosure problem to the maximum in order to focus on the comparison between voluntary and mandatory disclosure regimes.

So non-disclosure happens with probability $1 - p + p(1 - \lambda^{GG})$. And non-disclosure in the good state happens with probability $(1 - p)\lambda^{GG}$. Hence, the probability to be in state y^G after one non-disclosure period is

$$q_1 = \frac{(1 - p)\lambda^{GG}}{1 - p + p(1 - \lambda^{GG})}. \tag{1.15}$$

And the probability to be in state y^G at $t = 2$ is

$$\gamma_1 = q_1 \lambda^{GG} + (1 - q_1)\lambda^{BG}. \tag{1.16}$$

Recall that τ is the time elapsed since the last disclosure. Let

$$q_k(t) = \mathbb{P}(y_t = y^G | \tau = k, \zeta_\phi > t) \tag{1.17}$$

be the value of q after k periods of non-disclosure and

$$\gamma_k(t) = \mathbb{P}(y_{t+1} = y^G | \tau = k, \zeta_\phi > t) \tag{1.18}$$

be the probability to be in state y^G tomorrow after k periods of non-disclosure. These quantities only depend on t to the extent that t must be smaller than the maturity time. Hence, I can drop the dependency in t. Also for notational simplicity, the subscript k will be denoted τ. Using Bayesian updating, as in the case $k = 1$ detailed above, I obtain recursively:

$$q_{\tau+1} = \frac{(1 - p)\gamma_\tau}{1 - p + p(1 - \gamma_\tau)}, \tag{1.19}$$

$$\gamma_\tau = q_\tau \lambda^{GG} + (1 - q_\tau)\lambda^{BG}. \tag{1.20}$$

To each τ corresponds one q_τ; Figure A.2 provides a graphical representation. Note that q_τ decreases to a limit weight q_V^*, which bears an economic interpretation, discussed in section 1.3.2.

Fundamental Value

Let $V(q)$ be the fundamental value of the asset when the probability to be in state y^G is q. Let $\mathbf{y} = (y^G \ y^B)^{\mathbf{T}}$ be the vector of states, and $\mathbf{q} = (q \ 1 - q)$ be the vector of weights on the two states. By assumption the asset has not matured at time $t = 0$, and the probability of the maturity being $\zeta_\phi = t + 1$ for $t \geq 0$ is $(1 - \phi)^t \phi$. At time $t + 1$, the weights on the 2 states are given by the

vector $\mathbf{q}\Lambda^{t+1}$, so the expected asset value conditional on $t+1 = \zeta_\phi$ is $\mathbf{q}\Lambda^{t+1}\mathbf{y}$. Therefore

$$
\begin{aligned}
V(q) &= \sum_{t\geq 0} \mathbb{E}[y_t | \zeta_\phi = t+1]\mathbb{P}(t+1 = \zeta_\phi) \\
&= \sum_{t\geq 0} (1-\phi)^t \phi \mathbf{q}\Lambda^{t+1}\mathbf{y} \\
&= \phi\mathbf{q}\Lambda(\mathrm{Id}_2 - (1-\phi)\Lambda)^{-1}\mathbf{y}.
\end{aligned}
\tag{1.21}
$$

Note that V is affine in q:

$$
V(q) = qV^G + (1-q)V^B.
\tag{1.22}
$$

V can also be expressed as a function of τ, the time since last disclosure:

$$
V_\tau \equiv V(q_\tau).
\tag{1.23}
$$

Debt Capacity

Definition 2 *The debt capacity is the maximal amount of debt financing that can be obtained by pledging the assets under management as collateral. Under a consistent bank policy, it depends on the state τ and is defined by*

$$
C(\tau) = \inf\{D \geq 0, F(D, \tau) = \varnothing\}.
\tag{1.24}
$$

By definition, if debt exceeds debt capacity during the lifespan of the asset, it is no longer possible to find investors to roll debt over. In my model, this forces a premature liquidation, because no other sources of financing are available: a run occurs. Hence, debt capacity coincides here with a *run threshold*.

Definition 3 *The fair pricing function in state τ, m_τ, is the mapping that associates to any promise F the expectation of the actual payment, under the creditors' information. In a consistent bank policy,*

$$
m_\tau(F(D, \tau)) = D
\tag{1.25}
$$

holds in state (D, τ).

Of course, m_τ, an inverse of F, is also an equilibrium object and remains to be determined, jointly with the debt capacities. In general, the following relationship holds:

$$
C(\tau) = \sup_{F\geq 0} m_\tau(F).
\tag{1.26}
$$

That is, today's debt capacity is the maximum amount of financing that a promise of F tomorrow can buy.

The first key observation towards the analytical characterisation of debt capacities is the following:

Lemma 5 *Assume we are in state τ and let $\chi_1,\ldots,\chi_k$ be the possible states of the world tomorrow, and $C(\chi_i)$ the maximum available financing in state χ_i. Then today's debt capacity satisfies*

$$C(\tau) = \max\{m_\tau(C(\chi_1)),\ldots,m_\tau(C(\chi_k))\}. \tag{1.27}$$

This means that I do not need to consider *all* promises, as suggested by equation (1.26), but only the maximal viable promises in tomorrow's states of the world. The intuition is the following. When the banker increases the face value from F to $F + dF$, two cases are possible. If the states χ_i in which there is default are unchanged, then the expected repayment under $F + dF$ must be larger: $m_\tau(F + dF) > m_\tau(F)$. By contrast, if the increase in face value creates an additional default state, the expected repayment *decreases* because of the deadweight liquidation cost. Hence, as F increases, $m_\tau(F)$ increases, except when a new default state is created, in which case it jumps downwards. When is χ_i a default state? It is precisely when the face value is larger than $C(\chi_i)$. Hence Lemma 5.

I now proceed and describe tomorrow's states of the world in my model (from the point of view of outsiders). There are always four:

 - χ_1: the asset has just matured ($\zeta_\phi = t + 1$), in the good state y^G.

 - χ_2: the asset has just matured ($\zeta_\phi = t + 1$), in the bad state y^B.

 - χ_3: the asset has not matured ($\zeta_\phi > t + 1$), and a disclosure was made ($\tau = 0$).

 - χ_4: the asset has not matured ($\zeta_\phi > t + 1$), no disclosure was made ($\tau \to \tau + 1$).

From Lemma 5, I obtain

$$C(\tau) = \max\{m_\tau(C(0)), m_\tau(C(\tau + 1)), m_\tau(y^G), m_\tau(y^B)\}. \tag{1.28}$$

The second key observation in determining the debt capacities is that there is only "one kind of good news": the observation of y^G. In order to sustain today's debt capacity, one must promise a face value that will be paid in better states of the world, because in worst states, less financing is available than today. Hence, I can directly map the $C(\tau)$ to $C(0)$, and $C(0)$ to y^G:

through a simple choice of asset process, I have been able to obtain an analytically tractable functional equation for debt capacity. The following derivations make these intuitions formal.

From condition (M), $C(\tau + 1) \leq C(\tau)$, and since $m_\tau(F) \leq F$ always holds, equation (1.28) reduces to:

$$C(\tau) = \max\{m_\tau(C(0)), m_\tau(y^G), m_\tau(y^B)\}, \tag{1.29}$$

for $\tau \geq 1$ and

$$C(0) = \max\{m_0(y^G), m_0(y^B)\}. \tag{1.30}$$

y^B is the worst state of the world, so in equilibrium the banker can make a risk-free promise:

$$m_\tau(y^B) = y^B. \tag{1.31}$$

Let us now deal with the pricing of bonds with face value y^G and $C(0)$, respectively.

– In case the asset matures tomorrow, there will be full payment in the good state (state χ_1) and payment of y^B in state χ_2. Otherwise, there will be liquidation, since $C(\tau) < y^G$. The liquidation value will be either V_0 (in state χ_3) or $V_{\tau+1}$ (in state χ_4). So

$$m_\tau(y^G) = \phi(\gamma_\tau y^G + (1 - \gamma_\tau)y^B) + \alpha(1 - \phi)(p\gamma_\tau V_0 + (1 - p\gamma_\tau)V_{\tau+1}). \tag{1.32}$$

– Payments in states χ_1 to χ_4 are respectively $C(0)$, y^B, $C(0)$, $\alpha V_{\tau+1}$. So

$$m_\tau(C(0)) = \phi(\gamma_\tau C(0) + (1 - \gamma_\tau)y^B) + (1 - \phi)(p\gamma_\tau C(0) + \alpha(1 - p\gamma_\tau)V_{\tau+1}). \tag{1.33}$$

Notice that m_τ was *a priori* unknown. But equation (1.32) provides a necessary expression for $m_\tau(y^G)$, which determines $C(0)$ thanks to (1.30) and (1.31). In turn, $m_\tau(C(0))$ is determined by equation (1.33). (1.29) concludes the characterisation of the debt capacities in all states. Hence, the following is proven:

Proposition 1 *The equilibrium debt capacities in the voluntary disclosure case are characterised analytically by equations* (1.29) *to* (1.33).

Endogenous Bond Yields

Proposition 1 says that we know the debt capacities in all states, and so we also know the default states, which means that the equilibrium fair pricing functions m_τ are determined. We are then in a position to obtain the following.

Proposition 2 *The equilibrium face value schedule is characterised by*

$$F(D, \tau) = \min\{F \geq 0, m_\tau(F) = D\}. \tag{1.34}$$

(Details and the explicit expression for m_τ are in Appendix A.1.6). Notice that the gross bond yield in state τ is $R(D, \tau) = F(D, \tau)/D$.

1.3.2 Mandatory Disclosure

The model solution under mandatory disclosure is both similar and simpler: there is no disclosure policy and information is symmetric. I quickly repeat the analysis above in order to obtain the debt capacities and bond yields under mandatory disclosure.

Beliefs Dynamics

Let again denote q the probability of the asset being in state y^G (now the same for the bank and outsiders) and τ be the time since the last disclosure of y^G. As in the voluntary disclosure case, there is a correspondence between τ and q. The updating rule is modified. Here, $q(\tau = 0) = 1$ and

$$q_{\tau+1} = q_\tau \lambda^{GG} + (1 - q_\tau)\lambda^{BG}. \tag{1.35}$$

Figure A.2 provides a graphical representation.

Since asset observability is now independent from asset value, the weights on states after τ periods without observation are simply given by the iterated transition matrix, $\Lambda^{\tau+1}$. The qualitative behaviour of q_τ is the same as in the voluntary disclosure case. Here, it decreases to the stationary weight

$$q_M^* = \frac{\lambda^{BG}}{1 + \lambda^{BG} - \lambda^{GG}}, \tag{1.36}$$

which is above the limit q_V^* of q_τ in the voluntary disclosure case. The intuition is that with mandatory disclosure, no information does not mean a higher chance of bad news being concealed. Under voluntary disclosure, a protracted lack of disclosure indicates that the state is likely to be y^B.

I again define γ_τ as the probability to be in state y^G tomorrow given τ periods of non-disclosure. Here, we simply have

$$\gamma_\tau = q_{\tau+1}. \tag{1.37}$$

The Stationary Weights

There is an economic intuition behind q_M^*, which represents (up to an affine transformation) the asymptotic expected value of collateral, as the economy becomes information-less. If even in the information-less economy, agents accept to roll over debt because the expected value of collateral is high enough—q_M^* large enough—it is pointless to gather information. It would even be inefficient, since the (rare) bad banks would be inefficiently closed. This is one message of Gorton and Ordoñez (2014). But even if q_M^* is large enough, the expected value of the bond collateral in the information-less economy can be insufficient to ensure information insensitivity when disclosure is strategic. Indeed, the absence of information under voluntary disclosure is worse news than under mandatory disclosure. Formally, we have the following:

Lemma 6 *For any opacity parameter $p \in (0, 1)$, the stationary weights (the probability to be in the good state when the time since the last disclosure becomes large) satisfy $q_V^* < q_M^*$. As a consequence, the expected value of the bond collateral in the information-less economy is smaller in the voluntary disclosure case.*

Fundamental Value

The formula (1.22) for $V(q)$ still holds, using the probability q to be in the good state that obtains under mandatory disclosure. I still denote $V_\tau \equiv V(q = q_\tau)$ the fundamental value of the asset after τ periods without disclosure.

Debt Capacity and Endogenous Bond Yields

Using the same method as before, I obtain the parallel of Propositions 1 and 2:

Proposition 3 *The equilibrium debt capacities in the mandatory disclosure case are characterised analytically by equations* (1.29) *to* (1.31), *where the operator m_τ is modified (see Appendix A.1.7 for its explicit expression). Again, the equilibrium face value schedule satisfies*

$$F(D, \tau) = \min\{F \geq 0, m_\tau(F) = D\}. \tag{1.38}$$

Figure A.3 provides a graphical representation.

Equilibrium characterisation

Collecting the results obtained so far, I can exhibit the equilibrium:

In the voluntary disclosure case, the equilibrium is $(F, \zeta_s = \infty, \delta^S, \mathscr{B}(\delta^S))$ where F is given in Proposition 2. In the mandatory disclosure case, the equilibrium is $(F, \zeta_s = \infty)$ where F is given in Proposition 3.

Appendix A.2.1 reports the model solution in the limit of continuous-time, *i.e.* when debt has vanishing maturity.

1.4 Results

1.4.1 Opacity, Information Sensitivity and Rollover Risk

Notions of Information Sensitivity

The notion of *information sensitivity* is at the heart of a series of papers: Gorton and Pennacchi (1990), Dang et al. (2013), Dang et al. (2015), and Gorton and Ordoñez (2014). A security is information-insensitive when agents accept to trade it without paying to obtain a costly signal about it, and has a high information sensitivity when agents are ready to spend a lot to obtain a signal. Debt is a natural candidate to information insensitivity because its payoff is constant over all the range of non-default states.

Adverse selection. In the papers of Dang et al., this property is desirable mainly because it allows to sidestep adverse selection issues. Debt is liquid because agents are not concerned that the next buyer knows more about the collateral than they do. In this context, opacity is efficient since it makes debt information-insensitive in more states of the world.

Pooling. In Gorton and Ordoñez (2014), opacity permits the pooling of firms with good collateral with firms with bad collateral. If the average quality of collateral is high enough, firms obtain credit from lenders who do not verify firm-specific collateral quality. This financing is invested in positive NPV projects, and opacity is therefore desirable: it provides insurance to banks in terms of their access to financing. To the contrary, when information about a firm's collateral is cheap, debt becomes information-sensitive: lenders verify collateral quality and lend only conditional on good news. Firms with bad collateral are deprived of credit and welfare is lower.

One can also define the notion of information sensitivity in my model:

Definition 4 Let (D, τ) be the state today, and $F(D, \tau)$ the promised face value due tomorrow. I say that debt is *information-insensitive* if the full repayment of $F(D, \tau)$ does not imply disclosure tomorrow. To the contrary, debt is *information-sensitive* if the absence of disclosure

tomorrow entails a run.

Endowed with this definition, it will be easier to understand how the information structure—the degree of transparency and the disclosure policy—impact rollover risk and the price of debt.

Rollover Risk, Funding Costs and the Information Structure

In this section, I back up formally the following claims.

- Transparency increases funding costs in good times; the reverse holds in bad times. As long as debt remains information-insensitive, there are less default states under opacity. This can backfire as conditions deteriorate: when debt becomes information-sensitive, the release of good news is required to avoid a bank failure, but this release is unlikely under opacity.

- Voluntary disclosure implies lower funding costs than mandatory disclosure as long as debt remains information-insensitive. However, we will see (Lemma 7) that voluntary disclosure also induces more pessimistic beliefs that mandatory disclosure as long as the bad state does not realise.

Figure A.4 plots the gross yields $R(D, \tau) = F(D, \tau)/D$ after $\tau = 1$ period of non-disclosure in the voluntary disclosure case. The plot is qualitatively similar for other values of τ. $R(., \tau)$ exhibits upwards jumps, which correspond the the creation of an additional default state, as explained in section 1.3.1. The jump points define regions, labelled II, IS, P and L in the Figure, with the following economic interpretation.

In the information-insensitive region (II), debt is safe: the face value satisfies $F(D, \tau) \leq C(\tau + 1)$: it is below tomorrow's debt capacity if there is no disclosure. Hence, unless the asset matures tomorrow in the bad state, debt will necessarily be rolled over. In the II region, debt is money-like.

The information-sensitive (IS) region corresponds to face values $F(D, \tau)$ between $C(\tau + 1)$ and $C(0)$: those are higher than tomorrow's debt capacity if there is no disclosure. Hence, a run will occur tomorrow in the case no disclosure is made. Since $F(D, \tau) \leq C(0)$, however, a run will not occur if the bank discloses y^G tomorrow. Avoiding liquidation is contingent on the disclosure of good news.

The region P (for "pre-liquidation") corresponds to face values $F(D, \tau)$ above $C(0)$: liquidation will happen tomorrow unless the project matures in state y^G. This means that the bank can

survive for one more period but not more. In order to provide incentives for lenders to stay in the game, the bank has to offer very high yields.

Finally, the liquidation region L corresponds to levels of debt where a run occurs today, for lack of an admissible face value to roll debt over: $D > C(\tau)$.

As Figure A.5 shows, the behaviour of bond yields in the mandatory disclosure case is qualitatively similar. Note that now, the bank can survive long periods of non-disclosure (here $\tau = 7$) because investors know that the regulator is genuinely uninformed. When the probability to fall into the bad state is low, the asset still has a good chance to be in state y^G after several non-disclosure periods.

I now present a series of analytical results implied by the expression of yields found in section 1.3. In turn, a first set of economic conclusions are derived from these results.

Proposition 4 *Let the superscript $[p]$ designate a variable relative to the model solution for opacity parameter p. The following holds:*

(a) (safer information-insensitive debt) *For a given D, if regions II and IS both exist, short-term yields are lower in the II region.*

(b) (opacity and information sensitivity) *For high opacity, i.e. small values of p, debt cannot be information-sensitive. In the voluntary disclosure case, when $p \to 1$, the information-insensitive region shrinks: for any τ, (D, τ) can not be in the II zone for p close enough to 1.*

(c) (bond yield discontinuity) *Bond yields are discontinuous in the value of debt for a given τ. As debt reaches the information-sensitivity threshold, yields jump upward.*

(d) (opacity component of short-term spreads) For a given (D, q), short-term spreads can vary with the opacity level:
(d1) *If (D, q) is at the right of the information-sensitive region for opacity parameters p_1 and p_2 with $p_1 < p_2$ then $F^{[p_1]}(D, q) > F^{[p_2]}(D, q)$.*
(d2) *If (D, q) is in the information-insensitive region for opacity parameters p_1 and p_2 with $p_1 < p_2$ then $F^{[p_1]}(D, q) \leq F^{[p_2]}(D, q)$, with equality if and only if disclosure is voluntary.*

(e) (disclosure component of short-term spreads) *For a given (D, q), voluntary disclosure provides lower yields in the V-II region: $F^V(D, q) < F^M(D, q)$.*

Points **(a)** and **(b)** confirm that information-insensitive debt is safer, and that opacity can increase the size of the information-insensitive region. This does not mean, however, that

opacity is always desirable. Indeed, when $p = \mathbb{P}(\omega_t = 1)$ is small, disclosures are rare, and τ is large on average, meaning that q_τ often takes low values, and the debt level may escape the II zone. By contrast, under transparency (p close to 1), the II region is tiny and a single period of non-disclosure can trigger default; but non-disclosure is very rare.

Point **(d)** contains the prediction that there is an opacity component in short-term spreads. Spreads are primarily linked to future rollover decisions, not to the asset fundamental value. But rollover decisions occur at each node of the asset tree, whose structure depends on disclosures. Therefore opacity matters: the model predicts that in good times (D low, or q high) transparency increase spreads, while in crisis (at the right of the IS zone) transparency decrease spreads.

Point **(e)** contains the prediction that there is a disclosure component in short-term spreads and states that for a given belief about the current state of the world, voluntary disclosure allows the bank to borrow at better terms as long as debt is information-insensitive under this disclosure regime. This does not mean, however, that voluntary disclosure is always desirable. Indeed, voluntary disclosure produces more pessimistic beliefs than mandatory disclosure as long as the bad state does not realise: see Lemma 7 in the next section, where I formalise the comparison between the two disclosure regimes.

As Figures A.4 and A.5 show, the IS zone is in general tiny, and can also not exist. In that case, the debt directly switches from being information-insensitive to being defaulted upon, making the trade off between short-term protection and long-term exposure even clearer. This occurs typically when the maturity of short-term debt is small compared to the expected time before the next observation of the asset. In this situation, disclosure is unlikely. An information-sensitive debt would therefore be defaulted upon with such a high probability that no information-sensitive contract is feasible.

The analysis so far indicates that in good times, opacity makes debt safer in the short term, at the cost of increasing exposure to runs at longer horizons; and that a similar tension exists regarding the comparison of the disclosure policies: voluntary disclosure allows to lower financing costs in the short term but can lead to significantly worse beliefs at longer horizons. In section 1.4.3, I study how these effects play out at the aggregate level, *i.e.* which ones affect the most efficiency and run probabilities.

1.4.2 Impact of the Disclosure Regime on Equilibrium Outcomes

Methodology

In order to understand how the nature of disclosure affects the dynamics of debt and our quantities of interest—run probability and our measure of inefficiency—I need to compare them all other things being equal. This is achieved in the following way. Consider some

$$\psi \equiv \left(\zeta_\phi, (y_t)_{t \leq \zeta_\phi}, (\omega_t)_{t \leq \zeta_\phi}\right). \tag{1.39}$$

ψ is the data of a maturity date ζ_ϕ, all the positions of the asset in the Markov chain before ζ_ϕ, and all the observability shocks before ζ_ϕ. I can now compute, for the same ψ, the equilibrium paths of debt $(D_t^V(\psi))$, $(D_t^M(\psi))$ and liquidation times (if any) $\zeta_\ell^V(\psi)$, $\zeta_\ell^M(\psi)$ in the cases of voluntary and mandatory disclosure, respectively.

The fundamental value of the asset is identical at all times across both scenarios. The same holds true for the information collected by the regulator. Moreover, if along ψ, $y_t = y^G$ for all t, the signals received by the creditors are also the same at all times across both scenarios (at any t, they received either $\delta_t = y^G$, announcement of the good state, or $\delta_t = \emptyset$, announcement that the asset has not been observed by the regulator). Even in that case, the debt and beliefs dynamics will be different across the two disclosure regimes considered. This can lead to dramatically different outcomes, as shown in the next section.

Hence, I am able to isolate effects due the disclosure policy by fixing a history ψ and computing the debt and beliefs dynamics along ψ in both disclosure regimes. Having defined formally the comparison between regimes, the following result, announced in section 1.4.1, is now clear:

Lemma 7 *Along any ψ, $q_t^V \leq q_t^M$ for any $t < \min\{\tau_f^V, \tau_f^M\}$ with equality only when $\omega_t = 1$ and $y_t = y^G$.*

The lemma states that voluntary disclosure consistently produces depressed beliefs as long as the bad state does not realise—in which case the bank fails under mandatory disclosure—because investors anticipate the possibility that the bank may conceal bad news. The only case where the beliefs are the same under both disclosure regimes is when the bank has just announced the good state.

I am also in a position to define the following:

Definition 4 *A voluntary disclosure-induced run (or credibility run) is a fundamental path ψ that produces a run when disclosure is voluntary but no run under mandatory disclosure. A mandatory disclosure-induced run is defined similarly.*

The alternative name "credibility run" for a voluntary disclosure-induced run comes from the fact that under voluntary disclosure, the regulator lacks credibility when she announces no observation of the asset, even when this is actually the case. Because she has not taken any commitment, the absence of news release is interpreted as very bad news by the creditors. In situations where no observations are made for a protracted period of time ($\omega_t = 0$ for several consecutive t), creditors rationally downgrade a lot their beliefs about the asset quality. This potentially leads to a run that would have been avoided under mandatory disclosure. Indeed, under mandatory disclosure, creditors are safe in the knowledge that the regulator is genuinely uninformed, and not trying to conceal bad news.

Voluntary disclosure-induced and Mandatory disclosure-induced Runs

Figures A.7 and A.8 (reported in Appendix A.3) plot two sample paths of debt for both disclosure regimes, and the associated beliefs dynamics: $q_t = \mathbb{P}(y_t = y^G | \mathscr{F}_t^I)$ is the probability to be in the good state under the creditors' information set. A black dot at time t indicates that the regulator has observed the asset at time t: $\omega_t = 1$. Along both sample paths, the asset actually always was in the good state: $y_t = y^G$ for all t. As mentioned in the previous section, this means that the fundamentals, the regulator's information, and the signals received by the creditors are identical in each example across the two disclosure regimes. All differences in outcomes are explained by the difference in the commitment decision of the regulator, which leads to different information structures and therefore different beliefs and debt dynamics.

Figure A.7 depicts a credibility run. In the beginning, interest rates are lower under voluntary disclosure. This is because the bad state, should it occur, will not be revealed under voluntary disclosure, but will be revealed under mandatory disclosure. Hence, voluntary disclosure produces less default states and reduces the bank's cost of financing, leading to a slower growth of the stock of debt. But a run suddenly occurs: this is because news have not been released for a protracted period of time, leading to a sharp decline in the creditors' beliefs, as illustrated by the bottom panel: observe the plunge of q_t between periods $t = 10$ and $t = 14$. In turn, this strong decline in beliefs leads to a strong decline in debt capacities. By contrast, under mandatory disclosure, the bank is resilient to long non-disclosure periods because creditors know that the regulator would be forced to reveal the bad state, had it been observed.

Prior to $\tau_\ell^V - 1$, the yields $\frac{D_{t+1}}{D_t}$ are lower under voluntary disclosure, but *it is under this disclosure regime that the bank undergoes a run.* This means that one cannot unconditionally map the current value of the short-term yield to the health of a financial institution: yields are to a large extent determined by the opacity of the collateral and the disclosure policy; and because they only reflect next period's rollover risk, they can remain low even when the probability of a run at a small horizon is very large.

Figure A.8 shows a mandatory disclosure-induced run.

At $\tau_\ell^M = 52$, the bank undergoes a run under mandatory disclosure. Prior to that date, good news were regularly released, producing consistently large values of q_t and maintaining the information-insensitive status of debt under voluntary disclosure. Similar to point **(e)** in Proposition 4, the bank was therefore able to borrow at better terms under voluntary disclosure.

Across both disclosure regimes, the fundamentals and the signals are identical at all times, and the probability to be in the good state is in fact *always weakly larger under mandatory disclosure*, but it is nevertheless under this disclosure regime that the bank undergoes a run. *The critical channel here is the endogenous refinancing cost:* the funding cost channel. Mandatory disclosure produces an information structure that generates more default states in good times (even though it produces better *average* beliefs). This implies larger financing costs, and the stock of debt grows faster. This can lead to a run that only occurs under mandatory disclosure.

Finally, note that debt became information-sensitive at $t = 79$ under voluntary disclosure, consistent with a sharp decrease in q_t (see bottom panel of Figure A.8). This corresponds to an upward "jump" in the stock of debt. A run was nevertheless avoided, because the required good news were indeed announced: $\omega_{80} = 1$.

The discussion in this section evidences that the two considered disclosure regimes can lead to dramatically different outcomes, but that none of them is uniformly better that the other. The question of which regime is more efficient on aggregate is studied in section 1.4.3.

1.4.3   Global Results

Impact of opacity on run probability and efficiency

In this section only, I abstract from the disclosure regime and ask whether opacity is efficient. I compare the polar cases $p = 0$ and $p = 1$, where disclosure regimes are equivalent.

When $p = 0$, the only random variable actually observed is maturity. Hence, before maturity, the paths of debt and beliefs about the current state are deterministic. Given an initial level of debt D_0, there is a deterministic $t_0(D_0)$ such that liquidation always occurs at t_0 if maturity is not reached yet.

t_0 can be interpreted as a time-to-crisis. Until time $t_0 - 1$, the bond is money-like. The key point here is that the quasi-absence of risk in the beginning is only due to the possibility to

liquidate the asset in the future. The bond is not risky because it will always be possible to run when the liquidation value approaches the debt level. Short-term spreads are not informative about the longer-term risk of the project and are low precisely because of the option to run.

When $p = 1$, conditional on $y_t = y^G$ for all t, there is a deterministic time $t_1(D)$ such that liquidation occurs at t_1 as soon as $\zeta_\phi > t_1$. This is because debt grows while the states remain the same.

In these two polar cases, the quantities of interest—our measure of inefficiency and run probability— can be expressed in quasi-closed-form as functions of t_0 and t_1 in both the discrete-time model and its continuous-time limit. In the latter case, one obtains simple formulas, presented in Appendix A.2.1. I use those to get the main result of this section:

Numerical Result 1 *For any parameter set Θ, there are $\left(\lambda^{GG}\right)^* \left(\Theta_{-\lambda^{GG}}\right), \left(\lambda^{BG}\right)^* \left(\Theta_{-\lambda^{BG}}\right), \phi^*(\Theta_{-\phi})$, $\alpha^*(\Theta_{-\alpha})$ and $D_0^*(\Theta_{-D_0})$ such that $\mathscr{I}(p=0) < \mathscr{I}(p=1)$ if and only if*

- $\lambda^{GG} > \left(\lambda^{GG}\right)^*$

- $\lambda^{BG} > \left(\lambda^{BG}\right)^*$

- $\phi > \phi^*$

- $\alpha > \alpha^*$

- $D_0 < D_0^*.$

The same result holds for the comparison of the run probabilities $\mathscr{P}(p=0)$ and $\mathscr{P}(p=1)$.

(The result is only numerical in the sense that is verified on a discrete version of the parameter space; but for each parameter set, the computations are exact thanks to the analytical expressions given in Appendix A.2.1. It is therefore possible to consider a very fine discretisation of the full parameter space.)

The intuition is the following. A large λ^{GG}, a large λ^{BG}, a large ϕ, a large α or a small D_0 all correspond to situation with good fundamentals: the probability that the asset matures in the good state is large, liquidation costs are low, or the initial stock of debt is a lot below debt capacity. In those cases, debt is more likely to be information-insensitive, so the drawbacks associated with opacity matter less.

Figure A.6 provides a graphical illustration: $\tilde{\mathscr{I}} = \frac{1}{1-\alpha}\mathscr{I}$ designates the expected value of the asset conditional on premature liquidation and preserves the ordering between opacity and

transparency given by $\mathscr{I}$. Note that under $\mathscr{I}$ both curves agree at 0 when $\alpha = 1$, in which case there is no loss of value upon liquidation. For low α, the short-term protection of opacity lasts less because debt capacities are low. Moreover runs, when they occur on good banks, are particularly harmful in terms of efficiency. The reverse holds for α close to 1.

Maximising efficiency does not imply minimising the likelihood of runs: one can have $\mathscr{I}(p = 0) > \mathscr{I}(p = 1)$ even if $\mathscr{P}(p = 0) < \mathscr{P}(p = 1)$. This is well illustrated by the following analytical result obtained in a special case:

Proposition 5 *Consider the continuous-time limit of the model and assume $V^B = 0$.*

When $\phi > 1 - \lambda^{GG}$, there exists α^* such that for $\alpha_{min} := \frac{D_0}{V^G} < \alpha < \alpha^*$, $\mathscr{P}(\alpha; p = 1) < \mathscr{P}(\alpha; p = 0)$ and for $\alpha > \alpha^*$, $\mathscr{P}(\alpha; p = 1) > \mathscr{P}(\alpha; p = 0)$.

When $\phi < 1 - \lambda^{GG}$, we always have $\mathscr{P}(\alpha; p = 1) > \mathscr{P}(\alpha; p = 0)$.

However, $\mathscr{I}(p = 1; \alpha) < \mathscr{I}(p = 0; \alpha)$ for any interior α.

According to Proposition 5, there are parameters such that a bank undergoes more runs on average under transparency, but where the expected costs of premature liquidation are nevertheless larger under opacity. The intuition is that under opacity, a bank can be hit by a runs and nevertheless be healthy. With our assumption that runs on good banks are more costly, it follows that the average cost of a run under opacity is larger.

It is interesting to link this result to section 1.4.1, where I discussed the argument of Gorton and Ordoñez (2014) that opacity provides insurance to banks in their access to funding and may therefore be desirable. Proposition 5 says that opacity may improve access to financing in the sense that it lowers the probability that creditors refuse to refinance the debt, but be less efficient. Intuitively, the pooling of good and bad banks backfires, as some healthy institutions, for which credit is the most valuable, can be denied credit.

Comparison between mandatory and voluntary disclosure

I now return to the general case of interior levels of opacity, $p \in (0, 1)$. The main result of this section is the following; it is obtained by simulation of the continuous-time model (see details in Appendix B.2).

Numerical Result 2 *Mandatory disclosure is more efficient than voluntary disclosure for $p \in \left(p^*\left(\Theta_{-p}\right), 1\right)$, with $\frac{p^*}{\phi} < 20\%$.*

This result can be made formal in the limit $p \to 1$:

Proposition 6 *For any parameter set Θ, there exists $p^*\left(\Theta_{-p}\right) < 1$ such that for any $p \in \left(p^*\left(\Theta_{-p}\right), 1\right)$ mandatory disclosure dominates voluntary disclosure in terms of efficiency and run probability: $\mathcal{I}^M < \mathcal{I}^V$ and $\mathcal{P}^M < \mathcal{P}^V$.*

I begin by providing the intuition behind Proposition 6. When p is very close to 1, creditors know that the bank is likely to be informed, and the bank's option to conceal bad news is effectively worthless: any absence of disclosure is interpreted as a very negative signal and triggers a run. There are, however, some rare instances in which the bank was genuinely uninformed: in these cases, the run would have been avoided under mandatory disclosure. Hence, when p approaches 1, there are no advantages to voluntary disclosure, only drawbacks. Nevertheless, one should note that the two disclosure regimes are very close to each other in this situation: both are nearly identical to the full opacity regime. This implies that the levels of inefficiency only differ marginally.

We have a mirror situation when p is very small: because the bank is so unlikely to be informed, beliefs dynamics are virtually the same in both regimes, but under voluntary disclosure, the bank can successfully conceal the bad state, should it realise and be observed. This can render voluntary disclosure more efficient; again, the levels of inefficiency only differ marginally.

For levels of opacity away from the bounds 0 and 1, the two disclosure regimes stand in much sharper contrast: the disclosure regime matters economically. Numerical Result 2 states that in these cases, mandatory disclosure improves efficiency. This means that the beliefs channel outweighs the funding cost channel. Being able to borrow at better terms in good times and having the option to conceal bad news does not compensate for the fact that voluntary disclosure depresses investor's beliefs and therefore the debt capacities.

The value of $\frac{p}{\phi}$ at which mandatory disclosure begins to be more efficient is striking. At this level, the banker only has approximately a one out of five chance to observe the asset once before maturity: the asset is very opaque. Numerical Result 2 says that unless opacity is at such an extreme level, mandatory disclosure is preferable.

To conclude the section, I present two particular examples (results obtained with $D_0 = 40$ and the baseline parameters). The main additional take-away is that while the disclosure regime does matter in terms of efficiency—as evidenced by Numerical Result 2—it has an even larger effect on realised outcomes along an individual path: the situations where the bank survives under one disclosure regime but not the other, as studied in section 1.4.2 represent a significant fraction of the failure scenarios under that regime.

p	0.2	0.5	0.95	1
$\mathbb{E}$[Residual Claim] - Voluntary	43.8	43.8	44.2	44.7
$\mathbb{E}$[Residual Claim] - Mandatory	44.3	44.6	44.7	44.7
$\mathbb{P}$(Run under V and M)	0.096	0.103	0.130	0.151
$\mathbb{P}$(Run only under V)	0.040	0.059	0.034	0
$\mathbb{P}$(Run only under M)	0.033	0.037	0.020	0
$\mathbb{P}$(Run under V)	0.136	0.162	0.164	0.151
$\mathbb{P}$(Run under M)	0.129	0.141	0.150	0.151

α	0.7	0.8	0.9	0.95
$\mathbb{E}$[Residual Claim] - Voluntary	41.3	43.1	44.4	44.8
$\mathbb{E}$[Residual Claim] - Mandatory	43.2	44.3	44.8	45.0
$\mathbb{P}$(Run under V and M)	0.142	0.118	0.096	0.091
$\mathbb{P}$(Run only under V)	0.074	0.068	0.050	0.043
$\mathbb{P}$(Run only under M)	0.029	0.030	0.040	0.042
$\mathbb{P}$(Run under V)	0.216	0.186	0.146	0.135
$\mathbb{P}$(Run under M)	0.172	0.147	0.137	0.133

Rows 1 and 2 give the expected residual claim under both voluntary and mandatory disclosure. Given that debt payment is equal to D_0 in expectation, this quantity is equal to $\mathbb{E}[U] - D_0$.

Row 3 gives the probabilities that a run occurs under both disclosure regimes.

Row 4 gives the probabilities that a run occurs only under mandatory disclosure. This can happen in a situation where the bank can successfully weather a crisis under voluntary disclosure: $\omega_t = 1, y_t = y^B, \delta_t = \emptyset$ for some t and $\omega_{t'} = 1, y_{t'} = y^G, \delta_{t'} = y^G$ for some $\zeta_\phi \geq t' > t$. By contrast, a run occurs at t under mandatory disclosure. This can also happen if $\omega_t = 1$ sufficiently often so that debt grows at a slower rate under voluntary disclosure, as illustrated in section 1.4.2.

Row 5 gives the probabilities of credibility runs, *i.e.* runs that occur only under voluntary disclosure.

Rows 6 and 7 are the sum of rows 3 and 4, and 3 and 5 respectively, and show the total run probability under each regime.

Impact of varying the degree of opacity. The first table shows that the efficiency under full transparency (column $p = 1$) is higher, although this regime may feature more runs. The

intuition is similar to the discussion following Proposition 5: it is under full transparency that the bank risks the most to be forced to reveal the bad state. This creates more bank failures; however, all these failures concern bad banks and are their inefficiency is therefore low. By contrast, when transparency is not full, fewer failures may occur, but the expected liquidation costs conditional on failure are larger.

Another conclusion that can be drawn from the first table is that when p is away from the bounds 0 and 1 (columns $p \in \{0.2, 0.5\}$), the disclosure regime matters significantly to the realised outcome of the model: a significant fraction of runs are regime-specific, *i.e.* do not happen under the same scenario for the other disclosure regimes. This sharp contrast at the path-wise level gets dampened to some extent at the aggregate level: the differences in the expected efficiency of both regimes are sizeable but remain quantitatively moderate—and mandatory disclosure is more efficient, consistent with Numerical Result 2. The whole of the second table also supports these conclusions (the baseline value of p: 0.5, is used for that table).

Impact of the liquidity parameter. A larger α increases debt capacities and decreases run likelihood. Additionally, it reduces the deadweight loss upon liquidation for any given fundamental value of the asset at the liquidation time. Hence, as illustrated by the second table, low liquidation costs imply both less runs and less inefficiency.

It is a somewhat woeful consequence of the intertemporal coordination problem that *runs are the most likely precisely when they are the most harmful.*

Conclusion

Opacity and disclosure regimes matter to the outcome of the rollover game because they shape the information tree, and therefore the short-term yields and the beliefs dynamics. Starting from the good state, opacity provides protection in the short run, but is likely to increase exposure at longer horizons—a tension which is amplified under voluntary disclosure. At the aggregate level, the model predicts that opacity reduces run probability and inefficiency only in situations where the fundamentals are strong anyways; that opacity may decrease run probability but increase inefficiency; and indicates that mandatory disclosure is more efficient than voluntary disclosure except at large levels of opacity.

Several extensions of the model appear interesting. First, relaxing the rigid structure of the bank's balance sheet should provide valuable additional insights; for instance, the bank may also have long-term debt, or use cash reserves to manage its risk of run. Second, one could introduce state-contingent regulation rather than fixing ex ante the disclosure regime. Third,

the information structure could be refined by considering a richer set of signals about the current asset value, in order to hone the modelling of the regulator's strategy set. One could then reformulate the model as an explicit Bayesian persuasion problem and compare it to the existing Bayesian persuasion literature on stress tests. Finally, the bank could have access to several investment opportunities, and may have moral hazard incentives to engage into inefficient projects. Clearly, the bank's portfolio decision and the regulator's opacity and disclosure choices would affect each other, giving rise to a potentially rich interaction.

2 Short-term Bank Leverage and the Value of Liquid Reserves

Sylvain Carré[1] and Damien Klossner[2]

We extend the modelling toolbox of the global games literature by providing a fully rational setup where liquid reserves are modeled explicitly. The banks' balance sheet decisions and the prices of all securities are endogenous in the model. A bank possesses two instruments to manage illiquidity risk: its funding policy and the size of its liquid asset holdings, modeled as government bonds. Higher short-term indebtness allows the bank to better capture the liquidity benefits priced into deposits but increases illiquidity risk. Holding more bonds makes the bank more robust to withdrawals but it reduces the bank's asset returns. When the supply is scarce, banks are the natural buyers of bonds, which creates a rich interaction between bank leverage and the liquidity premium of these bonds. The impact of an increase in bond supply on bank leverage depends on the general-equilibrium change in the cost of absorbing the supply. The model also illustrates how considering an endogenous leverage decision is key to predict the impact of the economic environment on the liquidity premium.

2.1 Introduction

The efforts of academics and regulators to identify the causes of the 2007-09 financial crisis have led to a general agreement regarding the role of excessive short-term leverage as an important source of instability, since it made financial institutions vulnerable to a run by short-term creditors. Run-like phenomena arose in multiple contexts during the crisis, for instance on the repo market (Gorton and Metrick, 2012), as well as on the asset-backed commercial paper market (Covitz et al., 2013). Federal Reserve Chairman Ben S. Bernanke famously described the crisis as "a classic financial panic transposed into the novel institutional context

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[2]Swiss Finance Institute and École Polytechnique Fédérale de Lausanne.

of the 21st century financial system."[3]

In the aftermath of the crisis, academics have put forward several proposals to improve financial regulation. For instance, Admati et al. (2011) have forcefully argued for higher equity requirements, by analysing how banks have incentives to take on too much leverage due to various subsidies of debt. Notably, they discuss how more equity funding can stabilise bank liabilities, in additional to its traditional role in mitigating agency problems between shareholders and creditors. Morris and Shin (2008) stress the importance of this alternative role of equity funding in a system context, where the claims of financial institutions are interwoven. They also point out that, in addition to equity requirements, other balance sheet indicators should be considered when assessing funding stability, including the ratio of liquid assets to total assets and the ratio of short-term liabilities to total liabilities. These proposals are now largely part of the international regulatory framework for banks, since the Basel III reforms have introduced a minimum leverage ratio requirement to constrain excess leverage and two liquidity requirements, the Liquidity Coverage Ratio (LCR) and Net Stable Funding Ratio (NSFR), to mitigate excessive liquidity risk and maturity transformation.

On the contrary, some authors have cautioned against the risk of imposing too high equity requirements, by emphasising the role of banks as "producers of safe/liquid debt". In the words of DeAngelo and Stulz (2015), "mainstream theory focuses on operating firms and ignores financial intermediation, and so it views debt as a source of funds for issuers, not as a good produced to meet a specific demand". The trade-off between liquidity provision and the risk of liquidity crises lies at the core of the banking literature and goes back, at least, to the classic contributions of Diamond and Dybvig (1983) and Allen and Gale (1998).

A natural question, then, is to understand how the trade-off between liquidity provision and the risk of liquidity crises shapes the joint choice of leverage and liquid reserves by banks. In turn, what are the implications of the banks' incentives to build liquid reserves for the pricing of liquid assets, such as short-term government debt? These are the questions we set out to explore in the present chapter.

Our three-dates model features banks with access to a profitable but illiquid project, and to a perfectly liquid government bond that can used to build liquid reserves. Bankers own all of the equity of a single bank, but they initially have no funds. They must therefore raise the initial funds by issuing debt or selling equity to investors. Debt takes the form of uninsured, demand-deposit contracts.

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[3]Closer to the present time, in the summer of 2017 Spanish bank Banco Popular underwent "eurozone's first large-scale bank run," with the vice-president of the ECB commenting: "There was a bank run. It was not a matter of assessing the developments of solvency as such, but the liquidity issue." (Source: ft.com, *"Banco Popular faced eurozone's first large-scale bank run, ECB says"*, June 8, 2017.)

Deposits can be an attractive source of funding for banks because they help meet the investors' demand for liquidity. Following Diamond and Dybvig (1983), liquidity preference is modeled as uncertainty about the time preference for consumption. Each investor faces an uninsurable liquidity shock in the interim period: when he is of type "patient", he values consumption in both the interim and final periods, but he is of type "impatient" he only values consumption in the interim period. Because investors do not know ex-ante whether they will be patient or impatient, they have an incentive to buy assets that can provide consumption at the interim date. Demand-deposit contracts have this property.

On the other hand, high short-term indebtness increases the risk that the bank is unable to roll over its deposits and fails, forcing a liquidation of its assets. We refer to this event as a *run* or a *liquidity crisis*. A run occurs when sufficient depositors withdraw their funds in the interim period that the bank is forced to liquidate the project below its continuation value. The resulting destruction of value makes it more likely that the remaining deposits will not be paid in full, and validates the withdrawal of funds as the depositors' individually optimal course of action. As is well-known, this generates strategic complementarities between the actions of depositors, and invites multiplicity of equilibria. To resolve the multiplicity of equilibria, we model the uncoordinated behavior of investors using global games techniques (Carlsson and van Damme (1993) and Morris and Shin (1998)). This allows us to obtain a unique probability of runs, and to link it to the bank's balance sheet parameters, such as the level of short-term leverage or the size of liquid reserves. Additionally, this approach has the desirable feature that liquidity crises are driven by bad expectations: there is an intermediate range of fundamentals where fear about the actions of other depositors precipitates a default of the bank, even though the value of its assets is sufficient to repay in expectation all depositors in full. In this region the bank is said to be 'solvent but illiquid' (Bagehot, 1873).

Banks possess two instruments to manage illiquidity risk: a reduction in leverage, or an increase in liquid asset holdings. We model liquid assets through the inelastic supply of short-term, risk-free government bonds. This allows us to link the liquidity value of government bonds to the demand of banks for liquid reserves.

Our main results are as follows. First, a key contribution of the paper is to enrich the modelling toolkit of the literature by providing a fully rational global games setup with explicit liquid reserves. So far, to preserve the key property that an equilibrium in threshold strategy exists, the trade off in the literature had been the following. Either an explicit liquidity buffer was considered at the cost of introducing a behavioral assumption, as in Rochet and Vives (2004). Or an exogenous liquidity-return menu was considered, as in Carletti et al. (2018). In this reduced-form approach, the bank has a single asset, which is risky. Its interim liquidation value is used as a proxy for the liquidity of a whole bank portfolio—that would include both

risky and safe/liquid assets—and one posits a decreasing relationship between this variable and the final payoff of the bank's asset. While this choice is convenient, it effectively does not model liquid reserves.

Second, our model allows to obtain the optimal funding response of a bank to its environment. As long as the likelihood of a liquidity crisis is sensitive to the bank's short-term indebtness, the optimal funding policy involves a mix of equity and deposit funding. Consistent with Admati et al. (2011) and Hellwig (2015), pure deposit funding is only optimal if the bank holds very safe assets.

Third, government bond holdings serves a dual role for banks. They reduce the risk that early withdrawals by impatient investors lead to inefficient liquidations (a *direct value*). Moreover, they stabilise deposits by decreasing the propensity of patient investors to run on the bank (a *strategic value*). When the supply of government bonds is scarce, banks bid up their price above the valuation of investors and this valuation wedge can be identified as a *liquidity premium*. Our model thus features a pecking order of government debt ownership, in the sense that investors hold bonds in equilibrium only when banks do, but the reverse does not hold. Regarding welfare, even when the bond supply is scarce, an increase in the supply does not necessarily engineer a Pareto improvement. The bond liquidity premium reduces the government's costs of funding, and this may indirectly benefit the bank's creditors in their role as taxpayers. Since an increase in the bond supply reduces the liquidity premium, it may thus be detrimental to creditors.

Finally, because banks are natural buyers of the government debt, a rich interaction between bank leverage and the liquidity premium of this debt arises. The impact of an increase in bond supply on bank leverage depends on the general-equilibrium change in the cost of absorbing the supply. The model also illustrates how considering an endogenous leverage decision is key to predict the impact of the economic environment on the liquidity premium.

Relation to the literature. The original articles which explain bank runs as the result of self-fulfilling prophecies in the strategic interaction among depositors are Bryant (1980) and Diamond and Dybvig (1983).

Global game methods were introduced by Carlsson and van Damme (1993) and used by Morris and Shin (1998) to study currency crises. Since then, several papers have modelled the roll-over decisions of short-term creditors using this approach, as it allows the modeler to pin down a unique equilibrium and reconciles the view that liquidity crises are the result of a coordination failure among creditors with the observation that firms with weak fundamentals are far more likely to experience a run (Morris and Shin, 2000).

Goldstein and Pauzner (2005) analyse whether banks should hedge the liquidity risk of investors when it makes them vulnerable to runs. On the methodological side, they show that, in games without global strategic complementarities between the actions of players, a much weaker single-crossing property is sufficient to guarantee existence of an equilibrium in threshold strategies. Their approach is one of the building blocks of this chapter, and there are important parallels and differences between both models. In their model, banks issue a single contract, which is akin to debt in the interim period and to equity in the final period. Like Diamond and Dybvig (1983), they analyse how bank deposits enable risk-averse investors to hedge the risk steming from the uncertainty about their time preference for consumption. The (interim) deposit rate is the key variable, as it determines both the extent of risk-sharing achieved by the deposit contract and the ex-ante probability of runs. By contrast, banks in our model fund with deposits and equity. The key variable is the quantity of deposits issued by the bank, and our focus on the implications of the demand for liquid assets by investors for bank leverage and the pricing of liquid assets.

Morris and Shin (2009) decompose the total credit risk of a financial institution into 'illiquidity risk' (a run by creditors) and 'solvency risk' (asset insolvency).

Schilling (2016) introduces a standard debt contract in Goldstein and Pauzner (2005), and studies the comparative statics of the probability of runs. She shows that, if the liquidity of bank assets is very low, the probability of runs may *de*crease with short-term indebtness. In her approach, the pricing of deposits is exogenous, and their issuance is not motivated by the liquidity demand of investors.

Rochet and Vives (2004) analyse the effects of regulatory solvency and liquidity ratios, as well as the rationale for the provision of emergency liquidity assistance. In their paper, the decision to run is delegated to fund managers who follow a "behavorial" decision rule. This ensures global strategic complementarity of investors' actions. Two papers which build on their framework are Vives (2014) and Koenig (2015). Vives (2014) shows that degree of strategic complementarity of investors' actions is the key quantity for characterising the sensitivity of equilibrium to changes in parameters and for analysing policy effectiveness. Koenig (2015) shows that liquidity requirements may increase the default risk of banks. The intuition is that, while an increase in cash holdings makes the bank more robust to withdrawals, it also raises solvency risk by reducing the bank's portfolio returns.

Rochet and Vives (2004), Morris and Shin (2009), Vives (2014), and Koenig (2015) all link the solvency and illiquidity risk of a financial institution to its balance sheet composition. In particular, they demonstrate that liquid requirements might reduce the potential for runs. However, these papers do not endogenise the prices at which financial institutions can issue their securities, nor the bank's optimal choice of capital structure and liquid asset holdings.

Tourre (2015) extends the continous-time model of debt runs of He and Xiong (2012) and shows that liquid reserves are also effective in stabilising bank funding when runs are driven by an intertemporal coordination failure between creditors. He does not motivate explicitly the use of short-term debt and does not characterise the optimal leverage.

Szkup (2013) analyzes the debt maturity choice of firms in a model with coordination risk and endogenous debt yields. His information structure provides another building block to this chapter. But the main motivation for issuing short-term debt in his model differs from ours, and he does not consider liquid reserves.

Outside of the global games literature this chapter connects to the paper of Hellwig (2015), who builds a general equilibrium model to analyse the effect of equity funding on the ability of banks to provide liquidity. His approach, however, differs strongly from ours as he models liquidity benefits in reduced-form and does not consider debt runs or liquid reserves.

Also related to our work is the contribution of Hugonnier and Morellec (2017), who study the interactions between financing decisions and liquid reserves policy of banks in a dynamic structural framework. They show that a solvency and a liquidity ratio are required to control the likelihood of default and the magnitude of bank losses in default. By contrast, imposing liquidity requirements as a standalone regulatory tool reduces the magnitude of bank losses in default, but at the cost of increasing the likelihood of default.

2.2 The Model

There are three dates $t = 0, 1, 2$, a single, perishable good, and two groups of agents, each of mass 1: banking entrepreneurs ("bankers") and investors.

2.2.1 Bankers

There is a continuum of identical bankers, with utility function

$$u^{\mathrm{B}}(c_0, c_1, c_2) = c_0 + c_1 + c_2. \tag{2.1}$$

Each banker $j \in [0, 1]$ initially owns all of the equity in a single bank. We normalise a bank's outstanding amount of equity to 1.

Risky Project

As in Goldstein and Pauzner (2005), each bank has the opportunity to invest in a project that requires $I = 1$ unit of funds at time $t = 0$. The project matures at $t = 2$ and is risky. At $t = 1$, the project can be scaled down, but it is illiquid. For each unit that has been liquidated early, only an amount $\ell < 1$ can be recovered. In the final period $t = 2$ the project yields a random return Z equal to $z > 0$ with probability $p(\theta)$ and to 0 with probability $1 - p(\theta)$. θ is the state of the economy. $p(\cdot)$ is strictly increasing: a higher realisation of θ increases the probability of success.

For simplicity, we also assume that projects are perfectly correlated across banks: If the project is successful, every bank receives the return z. Thus, all banks are identical and when convenient, we drop the bank index j.

At the time of investment $t = 0$, there is uncertainty about the state of the economy, which is drawn from a Gaussian distribution with mean μ and variance τ^{-1}, $\theta \sim N\left(\mu, \tau^{-1}\right)$. θ is realised at the beginning of the interim period $t = 1$, but is unobserved before period 2. Instead, bankers and investors observe noisy signals at $t = 1$, which convey additional information about the state (see section 2.2.4).

Liquid Reserves

In addition to the risky project, banks have the opportunity to acquire liquid assets in the form of risk-free government bonds. Bonds are issued in the initial period $t = 0$ at a price q_b and pay 1 unit of the single good in the interim period $t = 1$. The quantity of bonds held by a bank is denoted with b, which we will refer to interchangeably as the bank's *bond holdings*, *liquid reserves* or *liquidity buffer*.

Financing

Each banker initially has no funds and has to raise the entire amount necessary to finance the risky project and acquire liquid reserves by offering uninsured demand-deposit contracts and/or selling equity to investors.[4]

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[4]We consider uninsured deposits for the following reasons. First, although since 2008 many governments have extended their guarantees on both retail deposits and the wholesale liabilities of financial institutions, insurance remains in most cases partial. Second, our "banks" can be interpreted more generally as financial institutions, and our "deposits" as short-term debt, a significant part of which remains uninsured. Third, government guarantees may not be credible due to feasibility or time-consistency issues. These two factors — partial insurance and credibility concerns — imply that agents include the possibility of credit and liquidity events into their decision making process (Allen et al., 2015). Finally, government guarantees "would not unambiguously improve outcomes [...] because of risk-taking incentives" (see Kashyap et al. (2017) (p. 28) and the references therein).

Deposit contracts are issued at $t = 0$ at a price q_d, and promise to pay a period-1 gross interest rate r_1 (set at $t = 0$) and a period-2 gross interest rate r_2 (set at $t = 1$). Deposit contracts are liquid, in the sense that—instead of rolling over at $t = 1$—depositors have the right to demand payment of r_1.[5] When pricing the deposit contract, it will be convenient to fix the value of $r_1 = 1$ once and for all, and to determine endogenously the time-0 price of the contract, q_d, as well as the period-2 interest rate, $R = r_2$.[6]

The payoff to equity holders consist of all dividends Y_t paid by the bank at $t = 0, 1, 2$. In the final period, equity holders are residual claimants, so Y_2 is the liquidating dividend: the payoff of the project net of debt repayments.

We now describe in greater details the flows to debt and equity holders in each period, starting with the initial period $t = 0$. Let δ denote the quantity of deposit contracts issued, $\psi \in [0, 1]$ the fraction of the equity sold by the banker and q_e the time-0 price of equity. When solving the model, we will focus on the case where the banker does not find it optimal to pay an initial dividend.[7] Thus, $Y_0 = 0$ and the banker's initial budget constraint reads:

$$q_d \delta + q_e \psi = I + q_b b. \tag{2.2}$$

The left-hand side gives the proceeds from the sale of deposits and equity claims, while the right-hand side is the amount required to finance the risky project and acquire a quantity b of government bonds. We call (δ, ψ) the bank's *funding policy*.

To capture the illiquidity risk to which a bank is exposed if it relies heavily on (uninsured) deposit funding, we assume the following financing friction:

Assumption 2 *The bank cannot raise additional funds at the interim date.*

If the bank cannot roll over all of its deposits, and if its liquid reserves are insufficient to cover early withdrawals, the bank may be forced into liquidation. When raising funds to repay withdrawing depositors, the bank is assumed to follow a liquidation *pecking order*:

Assumption 3 *To satisfy early withdrawals, the bank uses the payout of its bond portfolio. If these funds are insufficient, then it rescales the risky project.*[8]

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[5]Due to risk-neutrality at $t = 1, 2$, a depositor always finds it optimal to withdraw either none or all of its funds. Therefore we do not need to consider partial withdrawals.

[6]The model is invariant to the normalisation $r_1 \to 1$, $r_2 \to \frac{r_2}{r_1}$, and $\delta \to \delta r_1$. It simply redefines what a unit of debt is. We thus set $r_1 = 1$ without loss of generality.

[7]A sufficient condition is given in the Appendix.

[8]Under our assumptions, the project is always fully liquidated when liquidation is efficient, i.e. when ℓ exceeds the (time-1) expected payoff of the project. The order in which assets are liquidated is therefore irrelevant in that

Therefore, if a proportion n of depositors withdraw at $t = 1$, the bank has to liquidate a fraction of the project, given by $(n\delta - b)^+ / \ell$. Three cases are possible:

1. $\ell + b < n\delta$: Early withdrawals exceed the sum of liquid reserves and liquidation value of the entire project, so the bank defaults at $t = 1$.

2. $b < n\delta \leq \ell + b$: Liquid reserves are insufficient to cover the early withdrawals, so the bank is forced to liquidate a fraction $(n\delta - b)/\ell$ of the project.

3. $n\delta \leq b$: The quantity b of government bonds held by the bank is sufficient to repay early withdrawals in full. There is no liquidation of the project.

If bankruptcy occurs at $t = 1$ (case (i)), the bank follows a *sequential service constraint*: it pays 1 unit of the single good to withdrawing investors until liquid reserves and proceeds from the liquidation are exhausted. We assume that the position of depositors in the queue is random. Then, the payment to an individual withdrawing investor is 1 with probability $(\ell + b)/(n\delta)$ (the fraction of total available resources to the total claim), and 0 with probability $1 - (\ell + b)/(n\delta)$. Equity holders as well as investors who rolled over, get nothing.

Absent default at $t = 1$ (cases (ii) and (iii)), the bank repays all withdrawing depositors, pays an interim dividend $Y_1 \geq 0$, and continues to operate until $t = 2$. Because government bonds mature at $t = 1$ and the bank follows a pecking order to satisfy early withdrawals, the interim dividend is given by

$$Y_1(n, \delta, b) = (b - \delta n)^+, \tag{2.3}$$

which is strictly positive in case (iii). In the final period $t = 2$ the bank receives the payoff

$$\left(1 - \frac{(\delta n - b)^+}{\ell}\right) Z, \quad Z \in \{0, z\}$$

(the return of the project scaled down due to early withdrawals). If this payoff is smaller than the bank's remaining credit obligations, $(1 - n)\delta R$, equity holders exercise their limited liability option. Thus, the equity payoff at $t = 2$ is given by

$$E_2(n, \delta, b, R; Z) = Y_2(n, \delta, b, R; Z) = \left[\left(1 - \frac{(\delta n - b)^+}{\ell}\right) Z - \delta R(1 - n)\right]^+. \tag{2.4}$$

If the bank is insolvent at $t = 2$, it follows a sequential service constraint. The probability of being served for a given depositor is equal to the fraction of total available resources to total

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case. If liquidation is inefficient, our assumed pecking order maximises expected output. Note, however, that the banker may want to follow another liquidation strategy due to limited liability. The pecking order of Assumption 3 can be interpreted as a debt covenant preventing the banker from expropriating debt holders at $t = 1$.

credit obligations:

$$\frac{1 - (\delta n - b)^+ / \ell}{\delta R (1 - n)} Z. \tag{2.5}$$

Clearly, this probability is equal to zero when $Z = 0$. Assume to the contrary that the project is successful, an increase in the mass of running creditors n has two opposite effects on the probability of being served. On the one hand, the bank has to liquidate a larger fraction of the project (a decrease in the numerator), which reduces the pool of assets backing deposits at $t = 2$.[9] On the other hand, as more depositors withdraw early, the bank's debt obligations are reduced, so the remaining depositors can rely on a greater "share of the pie" (a decrease in the denominator): the second effect is dubbed the *congestion effect* by Karp et al. (2007). We introduce two assumptions to ensure that the first effect is sufficiently strong. Under these assumptions, a run is bad news for the remaining depositors, in the sense that the probability of being served, conditional on the project being successful, is non-increasing in the mass of running depositors. *This, in turn, guarantees the existence of an equilibrium in threshold strategies* for the coordination game played by investors in the interim period, and this is why dealing with the congestion effect is so important.

Define a bank's *liquidity ratio* as the quantity of the numéraire good that can be realised at $t = 1$ relative to short-term liabilities,

$$\overline{n}(\delta, b) = \frac{\ell + b}{\delta}. \tag{2.6}$$

$\overline{n}(\delta, b)$ also corresponds to the critical mass of running depositors above which the asset is fully liquidated at time 1.

Assumption 4 $\overline{n}(\delta, b) < 1$

Assumption 4 is standard in the literature—similar to *e.g.* Goldstein and Pauzner (2005)—and states that the bank is not able to raise sufficient resources in the short run to repay *all* of its deposits.

The next assumption is specific to our framework and caps the deposit rate that can be promised by the bank in the interim period. It excludes cases where the bank "requires" that some depositors withdraw to have a chance of avoiding default in the final period.

Assumption 5 $R \leq z / \delta$

Imposing this bound on the admissible deposit rates allows us to consider explicitly a liquidity buffer in a fully rational global games model of bank runs. As explained in the introduction,

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[9]The numerator decreases in n if $n\delta > b$. No liquidation takes place when $n\delta \leq b$.

the trade off in the literature had so far been the following. To preserve the existence of an equilibrium in threshold strategies, either (i) an explicit buffer was considered at the cost of introducing a behavioral assumption or (ii) an exogenous "liquidity-return" menu was considered. In this approach, the bank has a single, risky, asset. Its interim liquidation value ℓ is used as a proxy for the liquidity of a bank portfolio that would also include a safe/liquid asset and considered as a control variable—with the assumption that increasing ℓ decreases the final payoff z. The next paragraph details how our setup reconciles approaches (i) and (ii).

Liquid reserves and the congestion effect. When the bank follows the liquidation pecking order prescribed by Assumption 3, the presence of a liquidity buffer $b > 0$ introduces a region, where *more creditors running is simply good news*: for $0 \leq n < \frac{b}{\delta}$, the bank does not have to rescale the risky project, hence the main effect of early withdrawals is to decrease the number of debt claims on the assets at $t = 2$.

Introducing Assumption 5 and assigning zero to the value of the project in case of failure at $t = 2$ allows us to recover the property of one-sided strategic complementarities and in turn to obtain an equilibrium in threshold strategies, as illustrated by Figure B.3. Indeed: (1) Since $\delta R \leq z$, when $n = 0$ the probability of being served, conditional on the project being successful, equals one for deposit holders who rolled over; thus, an increase of n in cannot further increase their payoff, which is capped by R. (2) Since the project yields 0 when unsuccessful, the congestion effect does not play any role in this case.

It is the combination of (1) and (2) that allows us to consider explicitly liquid reserves. We now return to our exposition. We have obtained a unique cut-off level $n^{\mathrm{def}}(\delta, b, R)$; computations show that

$$n^{\mathrm{def}}(\delta, b, R) = \left(\overline{n}(\delta, b) \frac{z}{\ell} \frac{1}{R} - 1 \right) \Big/ \left(\frac{z}{\ell} \frac{1}{R} - 1 \right). \tag{2.7}$$

Finally, for all $n > \overline{n}(\delta, b)$ the probability of being served is zero since the asset is fully liquidated at $t = 1$. Summarising, the probability of being served at $t = 2$ conditional upon the project being successful is given by

$$Q(n, \delta, b, R) = \begin{cases} 1 & \text{if} \quad n \leq n^{\mathrm{def}}(\delta, b, R) \\ \frac{1 - (\delta n - b)/\ell}{\delta R(1-n)} z & \text{if} \quad n^{\mathrm{def}}(\delta, b, R) < n \leq \overline{n}(\delta, b) \\ 0 & \text{if} \quad \overline{n}(\delta, b) < n, \end{cases} \tag{2.8}$$

and the expected payoff of the deposit contract held by investor i if he decides to roll over is

$$D_2(n, \delta, b, R; Z) = RQ(n, \delta, b, R)\mathbb{1}_{\{Z=z\}}. \tag{2.9}$$

Strategy

A banker's *strategy* consists of choosing: (i) a funding policy (δ, ψ) and bond holdings b (set at $t = 0$); and (ii) a deposit rate R (set at $t = 1$). An optimal strategy maximises her expected utility $\mathbb{E}\left[u^{\mathrm{B}}(c_0, c_1, c_2)\right]$, subject to her budget constraint (2.2) and given the strategy of investors and all other bankers.

2.2.2   Investors

Types and Preferences

Investors are ex-ante identical. Following Diamond and Dybvig (1983), each investor faces a privately observed and uninsurable liquidity shock: with probability λ, the investor is impatient, and with probability $1 - \lambda$ he is patient. Impatient investors only care about consumption at $t = 0$ or $t = 1$, while patient investors value consumption in any period. Specifically, each investor i has the state-dependent utility

$$u^{\mathrm{I}}(c_0^i, c_1^i, c_2^i) = \begin{cases} v(c_0^i) + c_1^i & \text{if } i \text{ is of type ``impatient"} \\ v(c_0^i) + \beta(c_1^i + c_2^i) & \text{if } i \text{ is of type ``patient"} \end{cases} \tag{2.10}$$

where c_t^i denotes his date-t consumption, v is increasing and concave, and $0 < \beta \le 1$ is the discount factor of the patient type. The linearity of utility at dates 1 and 2 makes the investors' run decisions independent of their equity holdings (see section 2.3.1), while the concavity of date-0 utility pins down an interior choice of savings. We assume that investors have endowments ω_0 in period 0 and ω_1 in period 1.

Investors' types are i.i.d. Thus, assuming a law of large numbers, λ is also equal to the fraction of impatient types in the investor population. We also allow for *aggregate uncertainty* about the level of liquidity demand by assuming that λ is itself random. At the beginning of the interim period $t = 1$, nature draws a realisation λ from a continuous distribution with support $[\underline{\lambda}, \overline{\lambda}] \subset [0, 1]$ p.d.f. g, and mean μ_λ. This realisation determines an investor's probability of being of a given type—which they learn at the same time that λ realises—and, assuming a law of large numbers, the fraction of impatient investors in the continuum. We will refer to λ as either the *probability of being impatient* for any individual investor, or as the *aggregate liquidity needs* of investors, depending on the context.

Strategy

To highlight how the potential liquidity benefits of deposits interact with illiquidity risk to determine the bank's optimal funding policy, we assume that investors can only obtain liquidity from the demand-deposit contracts they own at the beginning of period 1. We make the following assumption:

Assumption 6 *No trading takes place at the interim date.*

Therefore, the only action that investors need to take at $t = 1$ is to decide whether to roll over their deposits ($a_i = 1$) or not ($a_i = 0$).

Let b^i, d^i and e^i denote the number of government bonds, deposits and equity contracts, respectively, purchased by investor i at $t = 0$.[10] His initial budget constraint is:

$$c_0^i = \omega_0 + T_0 - q_b b^i - q_d d^i - q_e e^i,\tag{2.11}$$

where T_0 is an initial lump-sum transfer paid by the government to every investor.

A *strategy* for investor i is: (i) a portfolio choice $(b^i, d^i, e^i) \in \mathbb{R}_+^3$ (made at $t = 0$); and (ii) a rollover decision $a_i \in \{0, 1\}$ (taken at $t = 1$). An optimal strategy maximises his expected utility $\mathbb{E}\left[u^{\mathrm{I}}(c_0^i, c_1^i, c_2^i)\right]$, subject to his budget constraint (2.11) and given the strategy of bankers and all other investors.

2.2.3 Government Bonds

Government bonds are issued in the initial period $t = 0$ and retired in the interim period $t = 1$ using lump-sum taxes levied on investors. Proceeds from the issue are transferred in a lump-sum fashion to investors at $t = 0$. We denote the date-0 price with q_b and the total supply with B. The date-0 transfer to investors from bond issuance, T_0, and the date-1 transfer (tax) from retiring the bonds, T_1, satisfy

$$T_0 = q_b B \tag{2.12}$$
$$T_1 = -B. \tag{2.13}$$

We assume that the date-1 endowment of investors is large enough to always cover the lump-sum taxes T_1: $\omega_1 \geq B$.

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[10]In a symmetric equilibrium, each investor spreads his deposit and equity holdings equally across banks. Thus, d^i (e^i) is the number of deposit (equity) contracts held by investor i in each of the continuum of banks.

2.2.4 Information Structure

The information structure in our model follows Szkup (2013). Both investors and bankers receives additional information about the state by observing a *public* signal θ_p. This captures the fact that short-term debt can be a cheap source of financing partly because its pricing can be adjusted to changing circumstances. In addition, investors observe *private* signals. This ensures that investors are uncertain about each other's behavior in equilibrium, which will, in turn, allow us to derive a unique equilibrium of the coordination game played among patient investors (see Morris and Shin (2006)).

The state θ of the economy is random, with $\theta \sim N\left(\mu, \tau^{-1}\right)$. The ex-ante distribution is common knowledge among all the agents. θ is realised at the start of the interim period $t = 1$, but it is not publicly revealed before period 2. Instead, at $t = 1$, bankers and investors observe a public signal θ_p of precision τ_p,

$$\theta_p = \theta + \tau_p^{-1/2}\varepsilon_p, \tag{2.14}$$

with Gaussian noise $\varepsilon_p \sim N(0, 1)$ independent of θ. Once the public signal θ_p is observed, bankers and investors share the common posterior belief

$$\theta|\theta_p \sim N(m(\theta_p), \tau_y^{-1}), \tag{2.15}$$

where $\tau_y^{-1} = (\tau + \tau_p)^{-1}$ and $m(\theta_p) = \tau_y^{-1}(\tau\mu + \tau_p\theta_p)$. At this point, bankers choose the deposit rate R. Subsequently, each investor i observes a private signal x_i of precision τ_x,

$$x_i = \theta + \tau_x^{-1/2}\varepsilon_i \tag{2.16}$$

where $(\varepsilon_i)_{i \in [0,1]}$ are Gaussian noise terms that are independent and identically distributed according to $N(0, 1)$, and are independent of θ and ε_p. After observing x_i, the updated belief about θ of investor i is

$$\theta|x_i, \theta_p \sim N(m(x_i, \theta_p), (\tau_x + \tau_y)^{-1}) \tag{2.17}$$

where

$$m(x_i, \theta_p) = (\tau_x + \tau_y)^{-1}(\tau_y m(\theta_p) + \tau_x x_i). \tag{2.18}$$

To recapitulate, the mean of the distribution of θ is initially $m(\emptyset) = \mu$, is updated to $m(\theta_p)$ after the public signal θ_p is realised and to $m(x_i, \theta_p)$ after the private signal x_i is realised. with the latter being given by (2.18). In parallel, the precision is initially τ, is updated to τ_y after θ_p is realised and to $\tau_x + \tau_y$ after x_i is realised.

Following much of the global games literature, we will solve the model in the limit case where of arbitrarily precise signals. More precisely, we will focus on the case where the noise in private and public signals vanishes, and where private signals become arbitrarily precise

compared to the public signal. We summarise these conditions in the next definition.

Definition 5 *We say that* noise vanishes *in the following cases:*

- *when we study the run decision of investors, if $\tau_x \to \infty$*

- *when we characterise the period-2 deposit rate set by banks and compute asset prices in the initial period, if $\tau_x, \tau_y \to \infty$ with $\frac{\tau_x}{\tau_y} \to \infty$.*

Studying the model in the limiting case of vanishing noise presents several benefits. First, the coordination game played by investors in the interim period has a unique equilibrium in threshold strategies. Second, the equilibrium threshold can be expressed in closed-form. Third, a result due to Szkup (2013) allows us to characterise analytically the state-dependent deposit rate set by banks at $t = 1$. Fourth, the set of realisations of the fundamental θ which results in "partial liquidations" (i.e. banks are forced to rescale the risky project but are still able to avoid an early default) has measure zero. This greatly simplifies the expressions of the payoffs of the banks' debt and equity.

One may wonder, however, if considering the case of vanishing uncertainty about θ at $t = 1$ does not suppress *strategic uncertainty*, i.e. the uncertainty of investors about the actions and beliefs of other investors. It is not so. In fact, in the limit case of $\tau_x \to \infty$, patient investors face "maximal strategic uncertainty" (Vives (2014), p. 3558), as the belief of the pivotal agent — the patient investor who is indifferent between running or not — regarding the proportion of patient investors who will choose to run, becomes uniform over the unit interval. Morris and Shin (2006) refer to such beliefs as being *Laplacian*.

Before moving to the model solution, we recapitulate the timeline (also represented graphically in Figure B.2). In the initial period $t = 0$, each banker raises funds by issuing δ deposit contracts and selling a fraction ψ of equity. These funds are used to purchase government bonds and to finance investment in a risky project.

At the beginning of the interim period $t = 1$, the state of the world θ, as well as the probability of the preference shock λ, realise. Both bankers and investors observe λ, but not the individual realisations of the preference shocks to investors, which are private information. Moreover, instead of observing θ exactly, bankers and investors observe a noisy public signal θ_p. Given the realisation θ_p, bankers set the period-2 deposit rate R promised to investors who do not withdraw. In addition, before making their run decision but after bankers set R, investor i observes a private signal x_i. Then, given R and the information conveyed by the signals θ_p and x_i, each investor decides whether or not to run on the banks. Banks satisfy early withdrawals by using the payout from their maturing bond portfolio and, if this buffer proves

to be insufficient, by rescaling the project. If early withdrawals exceed the sum of the bond portfolio payout and the proceeds of the project's liquidation, banks default at $t = 1$. Otherwise, they potentially distribute a dividend and we move to $t = 2$.

In the final period $t = 2$, the return of the project realises. If the payoff collected on the remaining assets is sufficient to meet the banks' debt obligations, depositors are paid R and equity holders receive the residual, if any. Otherwise, the banks default.

2.3 Model Solution

We solve the model by backward induction. We have already described the period-2 payoffs of the debt and equity claims issued by banks, so the first step is to analyze the run decisions of depositors in the interim period $t = 1$. This decision takes place after both the realisation of the probability of being impatient λ and the draw of the public signal θ_p. We then move to the beginning of the interim period and analyze the choice of deposit rate by bankers. Finally, we move to the initial period $t = 0$ and compute the bankers' optimal funding policy and choice of bond holdings, as well as equilibrium asset prices.

2.3.1 The Investors' Game at $t = 1$

Assume, as will be the case in our symmetric equilibrium, that at $t = 0$ all banks follow identical funding policies (δ, ψ) and hold the same liquidity buffer b, and that they set the period-2 deposit rate equal to some level $R > 1$ (identical across banks) at the beginning of the interim period $t = 1$. Finally, assume that every investor holds deposits in every bank. We now analyze the run decisions of an individual bank's depositors.

Since λ realises at the *start* of the interim period and is observed by everyone, the choice of deposit rate by bankers and the run decisions of investors are all made conditional on λ. However, for ease of notation, throughout this section and the next we do not make the dependence on λ explicit.

Utility Differential

Impatient investors always withdraw, because they do not value consumption in period 2. For patient investors, however, the incentive to run depends on the utility differential between withdrawing in period 2 versus period 1.

Conditional on θ, the probability of success of the risky project equals $p(\theta)$, so the expected

payoff from staying ($a_i = 1$, "stay") is

$$D_1^s(\theta, n, \delta, b, R) = p(\theta) D_2(n, \delta, b, R; Z), \tag{2.19}$$

where D_2 is defined by (2.9). The payoff from withdrawing early ($a_i = 0$, "run") is

$$D_1^r(n, \delta, b) = \min\left\{1, \frac{\ell + b}{\delta n}\right\}. \tag{2.20}$$

Thus, the utility differential from staying rather withdrawing early when the fundamental realises at θ and a mass n of investors chooses to run, is given by[11]

$$\xi(\theta, n) := D_1^s(\theta, n, \delta, b, R) - D_1^r(n, \delta, b). \tag{2.21}$$

Since the payoff gain from withdrawing in period 1 versus period 2 depends on both the fundamental θ as well as the fraction n of depositors withdrawing early, each patient investor needs to form beliefs about the actions and beliefs of other patient investors. This results in a coordination game among patient investors.

Deriving the Equilibrium Threshold

We restrict our attention to symmetric equilibria in *threshold strategies*: equilibria in which all patient investors run if they observe a private signal below some common cut-off point, and do not run otherwise. Suppose patient investor i has observed signal x_i and believes that all other patient investors are following a threshold strategy with cut-off point x^*. When the fundamental realises at θ, the probability that any investor observes a private signal less than x^* is $\Phi\left(\frac{x^* - \theta}{\tau_x^{-1/2}}\right)$, where $\Phi(\cdot)$ is the c.d.f. of the standard normal distribution. Since there is a continuum of investors and the realisations of the signals are independent conditional on θ, we can define a deterministic function, $n(\theta, x^*)$, that specifies the proportion of investors who run, when the fundamental realises θ

$$n(\theta, x^*) = \lambda + (1 - \lambda)\Phi\left(\frac{x^* - \theta}{\tau_x^{-1/2}}\right). \tag{2.22}$$

Inserting (2.22) in (2.21) the utility gain from staying when the true fundamental is θ, is given by $\xi(\theta, n(\theta, x^*))$. Investor i does not observe the true fundamental θ. Instead, he chooses his best response by weighting the payoff gains from choosing one action rather than the other, according to his posterior belief on the distribution of θ given by (2.17). Denote by $\Delta(x_i, x^*)$

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[11]Our function ξ corresponds to the function ν of Goldstein and Pauzner (2005). In our case, given the linearity of investors' utility function in period 1 and 2, the utility differential is simply the payoff gain from staying rather than demanding early repayment.

the expected payoff gain from rolling over rather than withdrawing. We have:

$$\Delta(x_i, x^*) = \int_{-\infty}^{\infty} (\tau_x + \tau_y)^{1/2} \xi(u, n(u, x^*))) \phi \left(\frac{u - m(x_i, \theta_p)}{(\tau_x + \tau_y)^{-1/2}} \right) \mathrm{d}u, \tag{2.23}$$

where ϕ is the p.d.f. of a standard normal variable and $m(x_i, \theta_p)$ is defined in (2.18).

Upon reception of the private signal x_i, investor i will play $a_i = 1$ if $\Delta(x_i, x^*) \geq 0$, and $a_i = 0$ if $\Delta(x_i, x^*) < 0$.[12] But, in a threshold equilibrium, a patient investor must prefer to stay if he observes a signal above the threshold, and run if he observes a signal below it,

$$\Delta(x_i, x^*) \quad > \quad 0 \quad \text{for} \quad x_i > x^* \tag{2.24}$$

$$\Delta(x_i, x^*) \quad < \quad 0 \quad \text{for} \quad x_i < x^*. \tag{2.25}$$

By continuity, for a patient investor exactly at the switching point the expected payoff from rolling over has to equal the expected payoff from withdrawing

$$\Delta(x^*, x^*) = 0. \tag{2.26}$$

The continuity of the function $x \mapsto \Delta(x, x)$ combined with one additional assumption on the banks' technology which ensures the existence of *lower and upper dominance regions* — regions where staying or withdrawing early is the dominant action for an investor regardless his belief regarding the action of other investors — ensures that there is at least one value x^* that satisfies (2.26). Moreover, Assumptions 4 and 5 together ensure that the game features *one-sided strategic complementarities*: $\xi(\theta, \cdot)$ is non-increasing when it takes positive values. This, in turn, implies that the function $\xi(\cdot, n(\cdot, x^*))$ satisfies a single-crossing property.[13]

Finally, a result due to Athey (2002) states that, if the utility differential $\xi(\cdot, n(\cdot, x^*))$ satisfies a single-crossing property and if the noise distribution of the private signal satisfies a super-modularity condition, then the expected utility differential $\Delta(\cdot, x^*)$ satisfies conditions (2.24) and (2.25), so x^* is indeed an equilibrium of the game. Furthermore, in the limit of vanishing noise there is a unique equilibrium in threshold strategies. These results are summarised in the following proposition:

Proposition 7 *The global game defined above has an equilibrium in threshold strategies, which is unique in the limit $\tau_x \to \infty$.*

(All proofs are relegated to Appendix B.1.)

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[12] We suppose that, when indifferent, a depositor chooses to roll over. This assumption is immaterial as being indifferent occurs with probability 0.

[13] Suppose $\xi(\theta', n(\theta', x^*)) = 0$. For $\theta > \theta'$, fundamentals are better and fewer investors run. Both effects increase the payoff gain from rolling over, so $\xi(\theta, n(\theta, x^*)) > 0$.

When noise vanishes, uncertainty about the fundamental θ becomes arbitrarily small, while the belief of the pivotal agent — the patient investor receiving signal x^* — regarding the proportion of patient investors who choose to run, becomes uniform over the unit interval. Therefore, in this case, the indifference condition (2.26) can be derived simply by averaging over n the utility gain from rolling rather than withdrawing, eq. (2.21), evaluated at $\theta = x^*$ This yields the functional form for the equilibrium run threshold.

Proposition 8 *When $\tau_x \to \infty$, the unique equilibrium run threshold satisfies*

$$x^*(\delta, b, R) = p^{-1}\left(\frac{\int_\lambda^1 D_1^r(n, \delta, b)\,\mathrm{d}n}{R \int_\lambda^1 Q(n, \delta, b, R)\,\mathrm{d}n}\right), \tag{2.27}$$

where

$$\int_\lambda^1 D_1^r(n, \delta, b)\,\mathrm{d}n = \overline{n} - \lambda - \overline{n}\log\overline{n}$$

$$\int_\lambda^1 Q(n, \delta, b, R)\,\mathrm{d}n = (n^{\mathrm{def}} - \lambda)^+ + \frac{z}{\ell}\frac{1}{R}\left[\overline{n} - \max\{\lambda, n^{\mathrm{def}}\} + (1 - \overline{n})\log\left(\frac{1 - \overline{n}}{1 - \max\{\lambda, n^{\mathrm{def}}\}}\right)\right]$$

with $\overline{n} \equiv \overline{n}(\delta, b)$ defined by (2.6) and $n^{\mathrm{def}} \equiv n^{\mathrm{def}}(\delta, b, R)$ defined by (2.7). For realisations of θ above x^*, only impatient investors withdraw their deposits early. For realisations of θ below x^*, all investors withdraw their deposits early and the bank defaults at $t = 1$.

Comparative Statics of Equilibrium Threshold

Proposition 9 states the comparative statics of the equilibrium threshold x^* in the limit case of vanishing noise when the banker's choice variables δ, b, and R are given exogenously.[14]

Proposition 9 *In the limit $\tau_x \to \infty$, the unique equilibrium threshold x^* is:*

1. *increasing in the aggregate liquidity needs λ.*

2. *increasing in the quantity of deposits δ;*

3. *decreasing in the liquidity buffer b;*

4. *decreasing in the deposit rate R as long as $R < \min\{R^M, z/\delta\}$, where*

$$R^M = \frac{1 - (\lambda\delta - b)^+/\ell}{1 - \max\{\lambda, b/\delta\}}\frac{z}{\delta} \tag{2.28}$$

---

[14]Schilling (2016) performs a similar comparative statics exercise, but her model does not feature an explicit liquidity buffer. We thus amend her results to include the case where banks can hold liquid reserves and follow the liquidation pecking order of Assumption 3.

Result 4 is intuitive because a higher R increases the expected payoff of staying but does not affect the payoff of withdrawing, thereby reducing the net incentive to run. However, promising a higher R does not increase the expected payoff of deposits once the probability of being served at $t = 2$, conditional upon the project being successful, drops below one: in this case, for any depositor, the increase in the date-2 payoff conditional on being served is exactly offset by the decline in the probability that he will be served. This yields the following bound on admissible deposit rates (see the proof of Proposition 9 for details)

$$\overline{R} := \min\{R^M, z/\delta\}. \tag{2.29}$$

The first term is the level above which the expected payoff of staying, computed under the uniform belief, does not increase. The second term is due to Assumption 5.

2.3.2 Choice of the Period-2 Deposit Rate

At the beginning of the interim period, bankers set the period-2 deposit rate, R, so as to maximise their expected utility. Since by that time the fraction ψ of bank equity sold to investors is fixed and since bankers are risk-neutral, this is equivalent to maximising the equity's expected payoff at $t = 1$. Bankers face the following trade-off. On the one hand, promising a lower R reduces the debt servicing costs at $t = 2$, and thus increases the payoff to equity holders, conditional on no default. On the other hand, reducing R lowers the depositors' incentives to stay which may trigger a run.

In this section, we show how trading off higher financing costs against a higher probability of bank runs leads to a tractable characterisation of the banker's choice of deposit rate in the limiting case of vanishing noise.

Characterisation of Optimal Deposit Rate

Given a realisation of the public signal θ_p, let $x^*(\theta_p, \delta, b, R)$ be an equilibrium threshold, i.e. a root of the function $x_i \mapsto \Delta(x_i, x_i)$ defined by (2.23).[15] If the banker sets the deposit rate to R and if a mass n of investors chooses to run, then the date-1 expected value of equity, conditional on the true state θ, is

$$E_1(\theta, n, \delta, b, R) = p(\theta) E_2(n, \delta, b, R; z) + Y_1(n, \delta, b), \tag{2.30}$$

---

[15]When the precision of private signals is finite, the equilibrium threshold strategy depends on the public signal θ_p, as can be seen from the definition (2.23). We indicate this dependency by including θ_p as an argument of the function x^* whenever we consider the case of finite private signal precision. By Proposition 8, in the limit $\tau_x \to \infty$, the equilibrium threshold is unique, independent of θ_p and given by (2.27).

where Y_1 and E_2 were defined in (2.3) and (2.4).

Upon observing the realisation of θ_p, the banker anticipates that patient investors will play a threshold strategy around $x^*(\theta_p, \delta, b, R)$, and hence that the mass of running depositors in state θ will be $n(\theta, x^*(\theta_p, \delta, b, R))$. Inserting this expression into (2.30), and weighting the resulting payoff according to his posterior belief on the distribution of θ yields the expected equity payoff at $t = 1$,

$$\mathscr{E}_1(\theta_p, \delta, b, R) = \mathbb{E}_{m(\theta_p), \tau_y^{-1}} \left[E_1\left(\theta, \underbrace{n(\theta, \overbrace{x^*(\theta_p, \delta, b, R)}^{\text{equilibrium threshold}})}_{\text{mass of running investors}}, \delta, b, R\right)\right]. \tag{2.31}$$

where, for ease of notation, we introduce the following notation:

Notation 1 *For any function f, we denote:* $\mathbb{E}_{m, \sigma^2}[f(\theta)] = \int_{-\infty}^{+\infty} \sigma^{-1} f(u) \phi\left(\frac{u - m}{\sigma}\right) du.$

The optimal period-2 deposit rate is then given by[16]

$$\hat{R}(\theta_p; \delta, b) = \operatorname*{arg\,max}_{R \in [1, \overline{R}]} \mathscr{E}_1(\theta_p, \delta, b, R) \tag{2.32}$$

Notice that R enters (2.31) in two ways: (i) indirectly, through the equilibrium threshold x^*; and (ii) directly, as the promised payment to depositors. The first channel captures the fact that increasing R provides incentives for depositors to stay, thereby reducing the probability of suffering a run. The second channel captures the fact that promising a higher R increases the bank's financing costs.

The next proposition, first given in Szkup (2013) without a formal proof, provides a useful characterisation of the banker's choice of deposit rate as the public signal noise vanishes.

Proposition 10 *For all $\theta > x^*\left(\delta, b, \overline{R}\right)$, let $R^*(\theta; \delta, b)$ denote the unique solution to*

$$\theta = x^*(\delta, b, R). \tag{2.33}$$

Then, as $\tau_x, \tau_y \to \infty$ with $\frac{\tau_x}{\tau_y} \to \infty$, we have the following convergence in probability of the optimal deposit rate:

$$\hat{R}(\theta_p; \delta, b)\mathbb{I}_{\{\theta > x^*(\theta_p, \delta, b, \overline{R})\}} \xrightarrow{p} R^*(\theta; \delta, b)\mathbb{I}_{\{\theta > x^*(\delta, b, \overline{R})\}}. \tag{2.34}$$

---

[16]Setting $R < 1$ is never optimal, since this would trigger a run with probability one: if the probability of being served at $t = 1$ is equal to one, the payoff from withdrawing early is higher than the expected payoff from staying; but if this probability is smaller than one, the date-2 payoff is null (since the asset is fully liquidated at $t = 1$). Because the expected payoff from withdrawing early is higher than the expected payoff from staying in all states, withdrawing early is the dominant action.

Since the function $R \to x^*(\delta, b, R)$ is strictly decreasing for $R < \overline{R}$ (see Proposition 9), equation (2.33) indeed defines uniquely $R^*(\theta; \delta, b)$.

The intuition behind Proposition 10 is as follows. When the *private* signal noise vanishes, the mass of running investors is arbitrarily close to 1 for all $\theta < x^*(\delta, b, R)$, and arbitrarily close to λ for all $\theta > x^*(\delta, b, R)$. Moreover, when the *public* signal noise vanishes — but the public signal remains infinitely noisy relative to private signals — then the banker's uncertainty about the true state becomes vanishly small. In this case, it is optimal to set the deposit rate arbitrarily close to (but higher than) the value of R which solves (2.33). Indeed, if the banker were to set R below this level, then with probability one the bank would experience a full run, and her equity would be worthless. But increasing R beyond this level is not optimal either: This would raise financing costs while having a negligible impact on the probabability of runs, since the banker knows — with a high degree of certainty — that n is arbitrarily close to its lower bound, λ.

Note that the optimal strategy $\hat{R}(\theta_p; \delta, b)$ (the left-hand side of (2.34)) is a function which specifies the banker's choice of deposit rate for each possible *public signal* θ_p about the true state θ. By contrast, the limit object $R^*(\theta; \delta, b)$ (the right-hand side of (2.34)) is the model's prediction regarding the deposit rate set in the *true state θ*. In the limiting case of vanishing noise, we do not need to specify the realisations of the public signal observed by bankers nor to derive the exact form of their strategies to predict the deposit rate set in state θ: for any optimal strategy $\hat{R}$ and for all signal realisations θ_p, the banker's choice of R in state θ is arbitrarily close to the level defined by Proposition 10.

Probability Density of Deposit Rate

Proposition 10 induces a random variable $\theta \mapsto R^*(\theta; \delta, b)$ with domain $\left[x^*(\delta, b, \overline{R}), \infty \right)$, which is the equillibrium deposit rate observed in state θ, *conditional upon there not being run.* The following result is an immediate consequence of Proposition 10.

Corollary 1 *As noise vanishes ($\tau_x, \tau_y \to \infty$ with $\frac{\tau_x}{\tau_y} \to \infty$), the equilibrium deposit rate R^*,* conditional on there not being a run*, is:*

1. *decreasing in the fundamental θ, for fixed δ and b;*

2. *increasing in the quantity of deposits δ and decreasing in the liquidity buffer b, for a given fundamental θ.*

The intuition behind 1 is clear. When fundamentals are weak (low θ), then with high probability the banker observes a low public signal θ_p. The public signal provides information to the banker about the likely beliefs of investors. Upon observing a low public signal the banker

thinks it is likely that investors will receive low private signals, and hence assigns a high probability to the possibility of a run. She responds by promising higher interest rates in order to deter patient investors from withdrawing early.

Result 2 follows from the fact that a higher amount of deposits δ or a lower quantity of liquid reserves b increases the propensity of patient investors to run in any given state θ. Thus, for every realisation θ the bank must promise a higher interest rate to prevent a run.

2.3.3 Portfolio and Financing Decisions at $t = 0$

We now solve for the optimal decisions of investors and bankers at $t = 0$. We look for a symmetric equilibrium in which all bankers select the same funding policy and bond holdings and all investors choose identical portfolios. All derivations are made in the limiting case of vanishing noise as defined by Definition 5. From now one, we make the dependence on probability of the liquidity shock, λ, explicit.

Ex-ante Probability of Runs

In the previous section, we have seen that $\forall \theta \geq x^*(\delta, b, \overline{R}; \lambda)$ the bank is able to avoid a run (with probability one) by setting the period-2 deposit rate judiciously. What happens when $\theta < x^*(\delta, b, \overline{R}; \lambda)$? In this case, the fundamental is so weak that a run cannot be prevented. Indeed, using an argument similar to the one in the proof of Proposition 10, one obtains the following result (the proof is omitted).

Proposition 11 *When $\tau_x, \tau_y \to \infty$ with $\frac{\tau_x}{\tau_y} \to \infty$, the equilibrium mass of running investors satisfies following convergence in probability*

$$n\Big(\theta, x^*\big(\theta_p, \delta, b, \hat{R}\big(\theta_p; \delta, b, \lambda\big); \lambda\big)\Big) \xrightarrow{p} \lambda \mathbb{1}_{\{\theta > x^*(\delta, b, \overline{R}; \lambda)\}} + \mathbb{1}_{\{\theta < x^*(\delta, b, \overline{R}; \lambda)\}}.$$

For all $\theta < x^*(\delta, b, \overline{R})$, even if the bank sets R equal to the maximal admissible value $\overline{R}$ the proportion of running investors converges to 1 in the limit of vanishing noise. We say that a "run" occurs in this case, and that "no run" occurs otherwise.

Recalling that $\theta \sim N(\mu, \tau^{-1})$, we can now compute the ex-ante *conditional* (on λ) and *unconditional* probability of runs in equilibrium:

Corollary 2 *As noise vanishes ($\tau_x, \tau_y \to \infty$ with $\frac{\tau_x}{\tau_y} \to \infty$), the ex-ante* conditional *(on λ) prob-*

ability of runs is given by

$$\pi(\delta, b; \lambda) = \Phi\left(\frac{x^*(\delta, b, \overline{R}; \lambda) - \mu}{\tau^{-1/2}}\right). \tag{2.35}$$

The ex-ante unconditional *probability runs is obtained by averaging the conditional probability over all possible realisations of* λ,

$$\pi(\delta, b) := \int_{\underline{\lambda}}^{\overline{\lambda}} \pi(\delta, b; \lambda) g(\lambda) \, d\lambda. \tag{2.36}$$

It is increasing in the quantity of deposits δ *and decreasing in the liquidity buffer* b. *It is decreasing in the expected value* μ *and increasing (decreasing) in the volatility* $\tau^{-1/2}$ *of the fundamental as long as the probability of runs is below (above)* $1/2$.

Expected Payoffs of Bank Claims

Consider the date-0 expected payoff of one unit of debt held to maturity, given a draw of λ. When the private and public signal precision is finite, the expected payoff can be written as

$$\mathbb{E}_{\mu,\tau^{-1}}\left[\mathbb{E}_{\theta,\tau_p^{-1}}\left[D_1^s\left(\theta, n(\theta, x^*(\theta_p, \delta, b, \hat{R}(\theta_p; \delta, b, \lambda); \lambda)), \delta, b, \hat{R}(\theta_p; \delta, b, \lambda)\right)\right]\right],$$

where x^* is an equilibrium threshold and $\hat{R}$ is the banker's choice of deposit rate at $t = 1$. The first expectation is with respect to θ; the second expectation is with respect to θ_p, conditional on θ (and on λ). Using the results of Proposition 10 and 11, in the limiting case of vanishing noise this expression reduces to the following expectation over θ only:

$$\mathbb{E}_{\mu,\tau^{-1}}\left[D_1^s\left(\theta, n = \lambda, \delta, b, R^*(\theta; \delta, b, \lambda)\right) \times \mathbb{1}_{\{\theta > x^*(\delta, b, \overline{R}; \lambda)\}} + 0 \times \mathbb{1}_{\{\theta < x^*(\delta, b, \overline{R}; \lambda)\}}\right], \tag{2.37}$$

where the second term uses the fact that, when $n = 1$, the asset is fully liquidated at $t = 1$ and the payoff of the deposit contract (if held to maturity) is 0. Only two cases matter for the calculation of expected payoffs at $t = 0$: either (almost) all investors withdraw, or (almost) only impatient investors withdraw.

Dividing (2.37) by the conditional probability of runs $\pi(\delta, b; \lambda)$, and repeating the same argument for the equity payoff, we obtain:

$$D_0^s(\delta, b; \lambda) = \frac{\mathbb{E}_{\mu,\tau^{-1}}\left[D_1^s\left(\theta, n = \lambda, \delta, b, R^*(\theta; \delta, b, \lambda)\right) \mathbb{1}_{\{\theta > x^*(\delta, b, \overline{R}; \lambda)\}}\right]}{1 - \pi(\delta, b; \lambda)} \tag{2.38}$$

$$E_0(\delta, b; \lambda) = \frac{\mathbb{E}_{\mu,\tau^{-1}}\left[E_1\left(\theta, n = \lambda, \delta, b, R^*(\theta; \delta, b, \lambda)\right) \mathbb{1}_{\{\theta > x^*(\delta, b, \overline{R}; \lambda)\}}\right]}{1 - \pi(\delta, b; \lambda)}. \tag{2.39}$$

D_0^s is the date-0 expected payoff of one unit of deposit held to maturity, i.e the investor chooses action $a_i = 1$ ("stay"), *conditional on no run and on λ*. E_0 is interpreted similarly in the case of equity.

Investors' Portfolio Choice

In a symmetric equilibrium, deposit and equity contracts are identical across banks. Denote with $q_d^j = q_d$ and $q_e^j = q_e$ the date-0 price of the deposit and equity contract, respectively. Since the returns of the risky projects are perfectly correlated, the way that an investor spreads his holdings across banks does not affect his portfolio payoff. We assume that he buys the same quantities at each bank: $d^{ij} = d^i$, $e^{ij} = e^i$.

Let $\bar{c}_{tP}^i(\lambda)$ denote investor i's expected consumption level at date $t = 1, 2$, conditional on being patient *and on the realisation of λ*. $\bar{c}_{tNP}^i(\lambda)$ is defined similarly for the case of an impatient investor.

$$\bar{c}_{tP}^i(\lambda) = \mathbb{E}_{\mu,\tau^{-1}}[c_t^i | \lambda, \text{patient}]$$
$$\bar{c}_{tNP}^i(\lambda) = \mathbb{E}_{\mu,\tau^{-1}}[c_t^i | \lambda, \text{impatient}],$$

where the expectation with respect to θ. At $t = 0$, each investor i chooses his holdings of government bond, equity and deposit (b^i, e^i, d^i) so as to maximise his expected utility, taking prices as given:

$$\max_{(b^i,d^i,e^i)\in\mathbb{R}_+^3} U^I = v(c_0^i) + \int_{\underline{\lambda}}^{\overline{\lambda}} d\lambda g(\lambda)\Big(\lambda \bar{c}_{1NP}^i(\lambda) + (1-\lambda)\beta\Big(\bar{c}_{1P}^i(\lambda) + \bar{c}_{2P}^i(\lambda)\Big)\Big) \quad (2.40)$$

subject to

$$c_0^i = \omega_0 + T_0 - q_b b^i - q_d d^i - q_e e^i \quad (2.41)$$
$$\bar{c}_{1NP}^i(\lambda) = \omega_1 + T_1 + b^i + \pi(\delta,b)\frac{\ell+b}{\delta}d^i + (1-\pi(\delta,b))d^i \quad (2.42)$$
$$\bar{c}_{2NP}^i(\lambda) = (1-\pi(\delta,b))E_0(\delta,b)e^i \quad (2.43)$$
$$\bar{c}_{1P}^i(\lambda) = \omega_1 + T_1 + b^i + \pi(\delta,b)\frac{\ell+b}{\delta}d^i \quad (2.44)$$
$$\bar{c}_{2P}^i(\lambda) = (1-\pi(\delta,b;\lambda))\Big(E_0(\delta,b)e^i + D_0^s(\delta,b)d^i\Big). \quad (2.45)$$

(2.41) says that the date-0 consumption equals the investor's initial endowment plus the date-0 transfer, minus portfolio investment. When the investor is of type impatient he always chooses to withdraw early. Since he is unable to monetise his equity holdings, his date-1 consumption level is equal to date-1 endowment, net of the date-1 transfer (a tax), plus the payoff of the

government bond and his bank deposits. He is entitled to withdraw d^i from his deposit. But if a run occurs, he is only served with probability $\frac{\ell+b}{\delta}$. This yields (2.42).

Impatient investors still get the equity payoff, see (2.43). However, since they do not care about date-2 consumption, it does not enter the expected utility function. We want to interpret this as the limiting case of small *positive* utility from date-2 consumption, so that impatient investors have no interest in forfeiting their shares.

Patient investors only withdraw early when there is a run on the bank. Accordingly, comparing (2.42) and (2.43) with (2.44) and (2.45), the term $(1 - \pi(\delta, b))d^i$ does not appear in (2.44). Instead, the term $D_0^s(\delta, b)d^i$ now appears in (2.45), which is the expected payoff of deposits when held to maturity. The first-order conditions for problem (2.40) are

$$v'(c_0^i)q_b \geq \alpha, \tag{2.46}$$

$$v'(c_0^i)q_d \geq \int_{\underline{\lambda}}^{\overline{\lambda}} d\lambda\, g(\lambda) \left\{ \pi(\delta, b; \lambda)\left(\lambda + (1-\lambda)\beta\right)\frac{\ell+b}{\delta} + (1 - \pi(\delta, b; \lambda))\left(\lambda + (1-\lambda)\beta D_0^s(\delta, b; \lambda)\right) \right\}, \tag{2.47}$$

$$v'(c_0^i)q_e \geq \int_{\underline{\lambda}}^{\overline{\lambda}} d\lambda\, g(\lambda)(1-\lambda)\beta\left(1 - \pi(\delta, b; \lambda)\right)E_0(\delta, b; \lambda), \tag{2.48}$$

where, for each asset, the equality is strict if the investor holds this asset in equilibrium, and

$$\alpha = \beta + (1-\beta)\mu_\lambda. \tag{2.49}$$

is the expected utility from one unit of date-1 consumption.

Bankers' Decisions

Bankers are perfectly competitive and take prices as given.[17] Price-taking has a clear meaning when considering the behavior of bankers on the government bond market. However, when banker j decides to issue a quantity δ of (uninsured) deposits or to hold an amount b of government bonds, this decision impacts the payoff profile (and hence the market valuation) of the equity and deposit claims issued by her bank. Since there is no quoted price for these contracts for every possible choice of δ and b, the banker must form *price conjectures* in order to select its funding policy and bond holdings. Let $\hat{q}_d(\delta, b)$ and $\hat{q}_e(\delta, b)$ denote the banker's anticipation about the the market valuation of the deposit and equity claims for every possible choice of δ and b.

At $t = 0$ a banker's problem is to choose a funding policy (δ, ψ) and bond holdings b, so as to maximise her expected utility, taking the government bond price as given.

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[17]This paragraph follows closely the description of price conjectures by Bisin et al. (2014) and Hellwig (2015). A seminal paper is Grossman and Hart (1979).

The banker's choices need to be consistent, in the sense that — given the price conjectures $\hat{q}_d$ and $\hat{q}_e$ and the government bond price q_b — the funding policy (δ, ψ) fully finances investment $I + q_b b$. Using this last constraint to eliminate ψ, the banker's problem reduces to a two-dimensional constrained optimisation problem in δ and b. It will prove convenient to write the banker's expected utility directly as a function U^B of these two choice variables. Given a level of deposits δ and bond holdings b, let $\psi(\delta, b)$ denote the fraction of bank equity that needs to be sold to investors in order to finance total investment

$$\psi(\delta, b) = \frac{I + q_b b - \hat{q}_d(\delta, b)\delta}{\hat{q}_e(\delta, b)}. \tag{2.50}$$

The problem of the banker can be written as

$$\max_{\delta, b} U^B(\delta, b) = (1 - \psi(\delta, b)) \int_{\underline{\lambda}}^{\overline{\lambda}} d\lambda\, g(\lambda)(1 - \pi(\delta, b; \lambda)) E_0(\delta, b; \lambda), \tag{2.51}$$

where

$$\mathscr{A} = \{(\delta, b) \in \mathbb{R}_+^2 : \ell + b < \delta \text{ and } \psi(\delta, b) \in [0, 1]\}. \tag{2.52}$$

The bank's equity profit is null in case of runs. Conditional on no run and on λ, the date-0 expected profit is $E_0(\delta, b; \lambda)$, and the banker's share of the profits is $1 - \psi$. The banker weights the conditional payoffs of her equity stake over all possible realisations of λ. In addition to nonnegativity constraints, the banker's choices (δ, b) are subject to two constraints, which are described in (2.52). The first constraint is Assumption 4. The second constraint bounds the equity stake sold to investors to the interval $[0, 1]$.

Let (δ, b) denote date-0 equilibrium decisions by banks. The price conjectures $\hat{q}_d$ and $\hat{q}_e$ are required to satisfy the following criteria for rationality:

1. They are correct in equilibrium: $\hat{q}_d(\delta, b) = q_d$, $\hat{q}_e(\delta, b) = q_e$;

2. Suppose an individual banker contemplates a deviation (δ', b') from (δ, b). The banker's conjecture regarding the market valuations of its deposit and equity contracts under the choice (δ', b') are given by the expected discounted value of their payoff, computed using the *equilibrium* discount factor of investors.

In our model, the price conjectures take a very simple form: Given the linearity of investors' utilities at $t = 1, 2$, it boils down to determining their equilibrium date-0 marginal utility. Since the banker relies solely on external financing, in equilibrium it must be the case that:

$$\begin{aligned} q_d d^i + q_e e^i &= I + q_b b \\ q_b b^i + q_d d^i + q_e e^i &= I + q_b\left(b + b^i\right) = I + q_b B = I + T_0. \end{aligned} \tag{2.53}$$

where the second line uses the bond market-clearing condition — recall that b denotes the quantity of bonds helds by banks and b^i denotes the quantity held by investors — and the government budget constraint at $t = 0$. Substituting into the first-order conditions (2.47)–(2.48) (imposing equality), we deduce the price conjectures:

$$\hat{q}_d(\delta, b) = \frac{\int_{\underline{\lambda}}^{\overline{\lambda}} d\lambda\, g(\lambda) \left\{ \pi(\delta, b; \lambda) \left(\lambda + (1 - \lambda)\beta \right) \frac{\ell + b}{\delta} + (1 - \pi(\delta, b; \lambda)) \left(\lambda + (1 - \lambda)\beta D_0^s(\delta, b; \lambda) \right) \right\}}{v'(\omega_0 - I)} \quad (2.54)$$

$$\hat{q}_e(\delta, b) = \frac{\int_{\underline{\lambda}}^{\overline{\lambda}} d\lambda\, g(\lambda)(1 - \lambda)\beta \left(1 - \pi(\delta, b; \lambda)\right) E_0(\delta, b; \lambda)}{v'(\omega_0 - I)}, \quad (2.55)$$

We can similarly define the investors' valuation of government bonds, as

$$\hat{q}_b = v'(\omega_0 - I)^{-1}\alpha. \quad (2.56)$$

If investors hold government bonds in equilibrium, then the equilibrium price must equal the investors' bond valuation, $q_b = \hat{q}_b$. However, these two quantities will differ if banks hold the entire bond supply in equilibrium, $q_b > \hat{q}_b$.

2.3.4 Equilibrium

We are now ready to formalise our equilibrium concept:

Definition 6 *Given a funding policy* (δ, ψ) *and bond holdings* b, *a* symmetric equilibrium *is a collection* $(q_b, \hat{q}_d(\cdot, \cdot), \hat{q}_e(\cdot, \cdot), x^*(\cdot, \cdot, \cdot), R_2^*(\cdot; \cdot, \cdot), T_0, T_1)$ *such that*

- *The government's budget constraint holds, i.e.* (T_0, T_1) *satisfy* (2.12) *and* (2.13);

- x^* *is an equilibrium threshold of the investors' game, i.e. it is given by* (2.27) *where* $R = R^*(\theta; \delta, b)$, $\forall \theta, \delta, b$;

- R^* *is consistent with maximisation of bankers' utility at* $t = 1$, *i.e. it solves* (2.33);

- *The portfolio* (b^i, d^i, e^i) *invested uniformly across all banks maximise the expected utility of investors at* $t = 0$, *i.e. it satisfies the first-order conditions* (2.46)–(2.48);

- $\hat{q}_d$ *and* $\hat{q}_e$ *are rational price conjectures, i.e. they are given by* (2.54) *and* (2.55);

- *Markets clear:* $b + b^i = B$, $d^i = \delta$, *and* $e^i = \psi$.

Definition 7 *A* symmetric equilibrium with endogenous funding policy and bond holdings *is a symmetric equilibrium such that the funding policy* (δ, b) *and liquidity buffer* b *maximise the expected utility of bankers at* $t = 0$, *i.e. they solve problem* (2.51) *subject to* (2.52).

The baseline parameters selected for the presentation of numerical results are displayed in Figure B.1.

2.4 Results

2.4.1 Bankers' Decisions

Decomposition of Bankers' Utility

As a preliminary to our analysis, we derive a convenient expression for the utility of bankers at equilibrium (in the sense of Definition 6), given a choice (optimal or not) of funding policy and bond holdings. This expression will prove useful when we interpret our results.

Lemma 8 *The equilibrium utility of bankers satisfies*

$$U^B = \Gamma \times \widehat{\mathbb{E}} \left[\overbrace{z\mathscr{P}(x^*(\lambda))}^{\substack{(1)\ \text{value of project} \\ \text{conditional on no run}}} - \overbrace{\frac{I}{v'(\omega_0 - I)^{-1}(1-\lambda)\beta}}^{(2)\ \text{costs of illiquid financing}} + \overbrace{\frac{\lambda}{1-\lambda}\frac{1-\pi(\lambda)(1-\ell/\delta)}{\beta}\delta}^{(3)\ \text{liquidity benefits of deposits}} \right.$$

$$\underbrace{-\pi(\lambda)\left(z\mathscr{P}(x^*(\lambda)) - \ell\right)}_{\substack{(4)\ \text{liquidation costs} \\ \text{due to runs}}} \underbrace{-(1-\pi(\lambda))\lambda\delta\left(\frac{z\mathscr{P}(x^*(\lambda))}{\ell} - D_0^s(\lambda)\right)}_{\substack{(5)\ \text{liquidation costs due to} \\ \text{withdrawals by impatient investors}}}$$

$$+\underbrace{(1-\pi(\lambda))\left(\frac{z\mathscr{P}(x^*(\lambda))}{\ell}\min\{b,\lambda\delta\} + (b-\lambda\delta)^+\right)}_{(6)\ \text{direct value of liquid reserves}}$$

$$\left. -\underbrace{\left((1-\pi(\lambda))\left(1 + \frac{\lambda}{1-\lambda}\frac{1}{\beta}\right) + \frac{q_b - \hat{q}_b}{v'(\omega_0 - I)^{-1}(1-\lambda)\beta}\right)b}_{(7)\ \text{cost of funding the liquid reserves}} \right] \tag{2.57}$$

where $\widehat{\mathbb{E}}\left[f(\lambda)\right] = \mathbb{E}\left[\frac{1-\lambda}{1-\mu_\lambda}f(\lambda)\right]$ *and, given a realisation of* λ,

$$\mathscr{P}(x^*(\lambda)) := \mathbb{E}\left[p(\theta)|\theta > x^*(\lambda)\right] \tag{2.58}$$

is the expected success probability of the project conditional on no run, $x^*(\lambda) \equiv x^*(\delta, b, \overline{R}; \lambda)$ *is the threshold strategy of investors,* $\pi(\lambda) \equiv \pi(\delta, b; \lambda)$ *is the ex-ante probability of runs, and* $D_0^s(\lambda) \equiv D_0^s(\delta, b; \lambda)$ *is the expected payoff of one unit of deposit held to maturity.* $\Gamma(\delta, b)$ *is given by expression* (B.18) *in the appendix.*

The operator $\widehat{\mathbb{E}}$ has the effect of decreasing (increasing) the valuation of date-2 (date-1) payoffs that are positively correlated with λ compared to integrating the payoff realisations over the density g. For example, if a claim's date-2 payoff is more highly correlated with λ, then it

is less valuable to investors since it is more likely that the investor is impatient — and thus does not profit from the date-2 payoff — just when its realisation is high. Γ adjusts for the difference between the valuation of investors (who have state-contingent utility (2.10)) and that of bankers (who are risk-neutral).

The first term is the expected payoff of the project, conditional on there not being a run.

The second term is the cost of financing the project entirely with equity:[18] Conditional on λ, $v'(c_0^i)^{-1}(1-\lambda)\beta$ is the investors' valuation of a state-contingent claim which pays one unit of the single good conditional upon the investor being patient, and nothing otherwise; thus, if the banker were to fund the project entirely with equity, he would need to offer an aggregate return of $I/[v'(c_0^i)^{-1}(1-\lambda)\beta]$ to investors.

The third term captures the liquidity benefits of deposits. Conditional on λ, the valuation by investors of the option to withdraw early embedded in the contract is given by $v'(c_0^i)^{-1}\lambda \times (1 - \pi(\lambda)(1-\ell/\delta))$, i.e., the investors' valuation of a state-contingent claim which pays one unit of the single good conditional upon being impatient (and nothing otherwise), multiplied by the expected payoff of withdrawing early. Hence, every unit of deposit issued by the bank reduces the required date-0 value of illiquid financing by $v'(c_0^i)^{-1}\lambda(1-\pi(\lambda)(\ell/\delta))$. Since raising one unit at $t = 0$ through illiquid financing requires offering a return of $1/[v'(c_0^i)^{-1}(1-\lambda)\beta]$, the third term in (2.57) follows.

The fourth term corresponds to the cost of runs. Since a run precipitates the failure of the bank, only the liquidation value of the project, ℓ, can be recovered, instead of its long-term expected return, $z\mathscr{P}(x^*)$. Due to the coordination failure among depositors, runs often result in an inefficient liquidation of the project, which implies: $z\mathscr{P}(x^*) > \ell$.

But even absent a run, deposit funding still entails a cost due to early withdrawals by impatient investors. For every unit of deposit held by impatient investors, the bank is forced to liquidate $1/\ell$ units of the project (each unit being worth $z\mathscr{P}(x^*)$) to satisfy early withdrawals. This loss is partially offset through a reduction in the credit obligations of the bank at $t = 2$, which have expected value D_0^s per contract. Multiplying the net unit cost by the quantity of deposits held by impatient investors, $\lambda\delta$, yields the fifth term.

We dub the sixth term the *direct value* of bond holdings. It consists of two components. The first one captures the fact that holding bonds reduces the project liquidations that are required to satisfy the withdrawals of impatient investors. The second component arises from the fact that, when the quantity of bonds is sufficient to repay all impatient investors, the banker distributes the surplus as an interim dividend. Hence, there is a discontinuity in

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[18]More precisely: with any asset which is illiquid at $t = 1$.

the *marginal* direct value of bonds. When $b < \lambda\delta$, holding one additional bond reduces the project liquidation by $1/\ell$ units, with each unit being worth $z\mathscr{P}(x^*)$. If liquidations are highly inefficient $z\mathscr{P}(x^*) \gg \ell$, bonds are very valuable to the banker in this region. When $b \geq \lambda\delta$, holding an additional bond increases the interim dividend by one unit. Bond holdings are of limited value in this region.

The seventh term is the cost of financing the liquid reserves. The component in brackets, $1 + \frac{\lambda}{1-\lambda}\frac{1}{\beta}$, corresponds to the cost of financing the *investors*' valuation of government bonds. Bankers may also bid up the price above the investors' valuation, $q_b > \hat{q}_b$. The last component captures the cost of financing the valuation wedge $q_b - \hat{q}_b$.

Direct and strategic value of liquid reserves

In addition to their *direct value*, there is a second reason why bond holdings are valuable: They make banks less vulnerable to runs. This is captured by the fact that the run threshold x^* in (2.57) is decreasing in b. We dub this effect the *strategic value* of bond holdings.

Source of illiquidity risk

It it noteworthy that in the interim period the coordination failure among depositors imposes two distinct, but related, costs on the bank. The first cost materialises when an *actual* run triggers an early default. In addition, when fundamentals are weak, the *threat* of a run forces the banker to promise higher deposit rates so as to deter depositors from withdrawing early. Thus, even if the banker survives the liquidity crisis, higher deposit rates reduce his profit at maturity. Higher short-term indebtness increases both costs: the likelihood of runs, $\pi(\delta, b)$, and the deposit rate promised in state θ, $R^*(\theta; \delta, b, \lambda)$, are both increasing in δ.

Yet, as can be seen from expression (2.57), the only impact of the period-2 deposit rate R^* is to reduce the liquidation costs due to early withdrawals by impatient investors, thereby *increasing* the value of the banker's equity stake. The interpretation is that, conditional on there not being a run, early withdrawals have a silver lining: they reduce debt servicing costs at maturity; this effect is more pronounced, the higher the period-2 deposit rate.

Why does the sensitivity of the period-2 deposit rate on the level of indebtness not negatively impact the banker's utility? Because investors anticipate that, in those states where fundamentals are sufficiently weak so as to create funding stress for the bank, but not so weak that it precipitates its failure, the banker will be forced to promise higher rates to avoid a complete loss of funding. With rational expectations, this effect is incorporated into the price of the deposit contract. Consequently, it is the risk of a *complete loss of funding*, captured by the

probability of runs, rather than the risk of an *increase in funding costs* which deter the banker from carrying high short-term debt.

Choice of Funding Policy

When choosing a funding policy, the banker faces the following trade-off. On the one hand, financing a larger fraction of the project with deposits allows the banker to capture the liquidity benefits priced into deposits. Deposits are priced at a premium relative to equity because they allow depositors to pull their money out on short notice. On the other hand, heavy use of deposit funding makes the bank vulnerable to runs.

Define a bank's *leverage ratio* as the fraction of total investment financed with debt

$$L \equiv q_d \delta / (I + q_b b).$$

Figure B.5 illustrates graphically the optimal choice of funding policy.

Liquidity-enhancing role of equity

DeAngelo and Stulz (2015) have shown how, when safe debt commands a liquidity premium, banks should be expected to select highly-levered capital structures. As pointed out by Admati et al. (2011) this benchmark relies on the assumption that banks are able to create a riskless asset portfolio through suitable hedging. They write: "With uncertainty, there is a chance that the bank will not be able to fulfill its obligations to the holders of deposits and other presumably liquid claims. In that case, these claims will cease to be liquid." (p. 39).

The model captures the liquidity-enhancing role of equity. For our baseline parameters, funding entirely with deposits would result in excessively high costs of runs. By using a mix of equity and deposit funding, the bank is able to fund investment while carrying less short-term debt. This makes the bank more robust to early withdrawals which, in turn, makes any investor less concerned about the actions of other investors. As a result, the set of fundamental realisations where investors run on the bank is reduced. In the words of Hellwig (2015), "equity funding supports the liquidity of deposits by making a default of the bank and illiquidity of deposits less likely." (p. 1)

Other equilibrium types

Throughout the chapter, we restrict our attention to equilibria which exhibit an interior choice of leverage, i.e. $\ell + b < \delta$ and $L < 1$. We now briefly describe two other equilibrium types.

First, if runs are too costly, the banker finds it optimal to issue as little deposits as possible in order to achieve the minimal run probability, obtained in the limit $\delta \to \ell$, and the bound in Assumption 4 binds. This will occur, for instance, if the liquidation costs incurred by the bank to satisfy the withdrawals of impatient investors are sufficiently large relative to the liquidity benefits priced into deposits.

Second, if the expected costs of runs is very low, the optimal funding policy involves pure deposit funding. This will occur, for instance, if the expected success probability is sufficiently high and/or the volatility of the fundamental is sufficiently low.

Figure B.6 provides one illustration of how the equilibrium type varies with the characteristics of bank assets. The line marks the expected success probability of the project that makes a banker indifferent between selling a fraction of her equity to finance investment, or relying exclusively on deposit funding. The unshaded region corresponds to the first equilibrium type. In this region, the cost of runs is sufficiently low that it does not deter the banker from selecting a highly levered capital structure. One such equilibrium is illustrated graphically in Figure B.9.

Choice of Bond Holdings

When optimising the size of the bank's liquid reserves, bankers must balance their *direct value* (a lower risk that withdrawals by impatient investors result in inefficient partial liquidations) and their *strategic value* (a decline in the likelihood of runs) against the costs of financing these holdings.

When bond holdings are low, the marginal benefit bankers get from holding an extra unit of bonds is large due to two effects. First, when b is low, it is likely that the bond proceeds will be used to reduce the size of the project liquidations required to repay impatient investors. The second effect arises from the nonlinearity of the run probability: The marginal impact of b on π is larger when runs occur more frequently. These two effects eventually reduce the marginal value of bonds. First, as b increases it becomes more likely that the bank holds more liquid reserves than necessary to repay impatient investors: we could say that the bank ex-post holds *excess liquid reserves*. Excess reserves are distributed to equity holders as an interim dividend. But if there is a cost of carry associated with government bonds, i.e. their yield $1/q_b - 1$ is low relative to the bank's financing costs, then carrying an excess inventory of government bonds is costly. Second, when the ex-ante likelihood of runs is already low, further increasing bond holdings has a small impact on the propensity of investors to run.

The bankers' choice of bond holdings is illustrated graphically in Figure B.7. Starting from a zero bond position, Figure B.7a decomposes the *change* in the banker's expected utility due to

an increase in b according to expression (2.57). The dotted line depicts the cost of funding the bond holdings (7). The dashed-dotted line plots the *direct* value of liquid reserves (6). The change in the banker's utility, $\Delta U^B = U^B(\delta, b) - U^B(\delta, 0)$, is given by the height of the shaded area. The *strategic* value of liquid reserves is equal to the fraction of the utility gain (gross of the costs of financing the bond holdings) which is not captured by the direct value of liquid reserves, i.e. the vertical distance between the dashed-dotted line and the solid line. As can be seen from Figure B.7a, the benefits of holding bonds increase more rapidly when b is low than when it is high, while the cost of financing the bond holdings is approximately linear in b. Hence, balancing the low returns of government bonds against the benefits of holding liquid assets pins down the bankers' choice of bond holdings. Figure B.7b shows the impact of varying b on the banker's expected utility U^B.

Iso-utility Curves

Figure B.8 illustrates the *joint* choice of funding policy and bond holdings. The surface depicts the bankers' objective function, U^B, as a function of leverage L and bond holdings b. It is apparent from the figure that there is a unique choice of leverage and bond holdings which maximises the bankers' expected utility.

The contour diagram drawn on the horizontal plane plots the banker's *iso-utility curves*. Pick any of the lines projected on the plane. The region in the top corner is characterised by excessive leverage and insufficient liquid reserves. In this region, an increase in leverage must be accompanied by an increase in bond holdings to keep the banker's utility unchanged. By contrast, the region on the left side of the page is characterised by aggressive leverage ratios *and* excess liquid reserves. In this region, an increase in bond holdings must be accompanied by a decrease in leverage if the banker's utility is to remain unchanged. The other regions are interpreted similarly.

Previous papers in the literature have instead focused on *iso-risk curves*: combinations of leverage and liquid reserves which yield the same run probability and/or default rates (see Vives (2014), Koenig (2015), Tourre (2015)). These papers draw policy implications by solving for combinations of leverage and liquidity requirements that achieve a given level of illiquidity and/or solvency risk. By endogenising the prices of the financial claims issued by banks our model allows us to characterise the bankers' endogenous choice of leverage and liquid asset holdings, thus offering a complementary perspective to the earlier literature.

2.4.2 Equilibrium Price of Government Bonds

Bond Liquidity Premium

Having described the bankers' decisions, we turn to the asset pricing implications. Figure B.11a plots the equilibrium bond price q_b (dashed-dotted line) and the investors' valuation $\hat{q}_b$ (dotted line). The linearity of investors' utility means that their equilibrium bond valuation is a constant, which is equal to $\hat{q}_b = v'(\omega_0 - 1)^{-1}\alpha$.

There are two cases to consider. When the supply of government bonds is low ($B < 0.2$), banks hold the entire stock in equilibrium: $b = B, b^i = 0$.[19] As banks compete with one another for the scarce supply, they bid up the bond price above the valuation of investors, $q_b > \hat{q}_b$. We refer to the wedge $q_b - \hat{q}_b$ as the *liquidity premium.*

When the supply of government bonds is abundant ($B > 0.2$), the liquidity premium disappears. In this region, there is not shortage of public liquidity. The bond price is equal to the investors' valuation, banks' bond holdings reach their maximal level, and investors absorb the residual supply. We have: $q_b = \hat{q}_b$ and $b = b_{\max}, b^i = B - b_{\max}$, where $b_{\max}$ denotes the maximal bond holdings of banks.

Our model features a negative relationship between liquidity premium and bond supply. This prediction is consistent with the study by Krishnamurthy and Vissing-Jorgensen (2012), which exhibits a negative relationship between the spread of corporate bond yields over Treasury bond yields and the supply of Treasury bonds. They interpret this finding as suggestive that the liquidity and safety of U.S. Treasuries are priced attributes.[20] When the supply of Treasuries is low, investors assign a high value to the liquidity and safety attributes of Treasuries. As a result, the Treasury bond yield is low relative to the Aaa rated corporate bond yield. The opposite holds true when the supply of Treasuries is high.[21]

Krishnamurthy and Vissing-Jorgensen (2012) set up an asset-pricing model to guide their empirical analysis. Since their model relies on the representative agent assumption and captures the liquidity and safety benefits of Treasuries in reduced-form, it is not explicit about the class of agents that values the liquidity benefits of Treasuries. In our model, when the bond supply is low, *banks* are the natural buyers. This is because bond holdings serve a dual

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[19]Investors would like to short government bonds, but this is excluded by the short-selling constraints.

[20] Nagel (2016), however, shows that liquidity premia on near-money assets such as Treasuries are best explained by the opportunity cost of money (the level of short-term interest rate), and that the Treasury supply loses most of its explanatory power once this cost is taken into account.

[21]Holmström and Tirole (2011), which builds on Holmström and Tirole (1998), build a model with an illiquid technology and a demand for funds in the interim period which generates a government bond demand which resembles Figure 1 of Krishnamurthy and Vissing-Jorgensen (2012)'s paper. Our contribution here is to link the bankers' demand for liquid assets to both the liquidity demand of impatient investors and the coordination failure among patient investors.

role. First, they lower the risk that early withdrawals by impatient investors lead to inefficient liquidations (direct value). Second, they reduce the propensity of patient investors to run on the bank (strategic value).

Moreover, we have described above how an increase in bond holdings will lower their *marginal* benefits due to the higher likelihood that the bank ends up with excess reserves and due to the nonlinearity of the probability of runs. As a result, the liquidity premium slopes downwards as long as banks are the marginal bond investors.

Pecking Order of Bond Ownership

Our model features a *pecking order* of public debt ownership, since investors hold bonds directly only if bankers hold bonds, but the reverse does not hold: when public liquidity is scarce bankers outbid investors and hold the entire stock of bonds. This arrangement strictly dominates from a welfare perspective the alternative where bonds are held directly by investors.[22] The reason is that an investor only requires liquidity in the interim period when he is impatient. Without frictions impeding trade in the interim period, a patient investor could use the proceeds from his bond position to purchase deposit and equity contracts from patient investors. If investors are unable to execute such a transaction (Assumption 6), then a proportion $1 - \lambda$ of the liquidity benefits of government bonds is wasted.

By contrast, when government bonds sit on the banks' balance sheets rather than directly in the hands of investors, their proceeds are transferred in priority to impatient investors, who ex-post value date-1 consumption more. By using government bonds to back deposits banks ensure an efficient ex-post allocation of public liquidity in the presence of frictions which impair trading among investors.

Management of Illiquidity Risk

When leverage is endogenous, illiquidity risk can be managed through two levers: the size of liquid reserves, and the bank's funding policy. How do bankers choose which lever to use, and how does this joint choice varies with the economic environment?

No aggregate uncertainty about liquidity needs. Assume that the banker can perfectly forecast the fraction λ of impatient agents. Then, an analytical characterisation of the joint choice of debt issuance and liquid reserves is available under the assumption that the cap on the deposit rate, $\overline{R}$, is constant:

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[22]Investors are weakly worse off while bankers are strictly worse off when investors directly hold the bonds.

Proposition 12 *Consider the choice* (δ^*, b^*) *of a banker facing a bond price* q_b*, with a fixed cap on deposit rates* $\overline{R}$*. Then it is optimal to have either: (i)* $b^* = 0$ *or (ii)* $b^* = \lambda\delta^*$ *or (iii)* $b^* = \infty$*.*

When q_b is low, the bond demand is infinite. When q_b is high, bankers do not want to hold any bond. Neither case if consistent with equilibrium given a finite bond supply B. Therefore, in equilibrium it must be the case that $b^* = \lambda\delta^*$ at the optimal funding policy δ^* and bond holdings b^*. When there is no aggregate uncertainty about liquidity needs, the optimal funding policy and bond holdings are such that the bank exactly covers withdrawals by impatient agents.

The reason behind this result is that the value of liquid reserves exhibits a kink at $b = \lambda\delta$. For $b < \lambda\delta$, holding an extra unit of liquid reserves is very valuable to the banker, because she knows with certainty that liquid reserves will be used to avoid costly liquidations of the project. By contrast, for $b > \lambda\delta$, an extra unit of liquid reserves is a lot less valuable: in case of run, the payoff to the banker is anyway null; in the absence of a run, the banker knows with certainty that the bank will hold excess liquid reserves (to be paid as dividend).

However, it is important to note that Proposition 12 is an *equilibrium result*: it holds at the joint optimal choice of (δ, b), *not* for any fixed δ. If δ is exogenous, the marginal (strategic) value of an unit of liquid reserves above $b = \lambda\delta$ may well be above the bond price. By contrast, when δ is endogenous, the banker can adjust simultaneously her debt issuance and her liquid holdings. The proof of the Proposition then obtains by considering joint adjustments that maintain the liquidity ratio $\overline{n}$ constant.

The result of Proposition 12 captures the willingness of bankers to avoid costly liquidations that are certain to occur. But it driven by the assumption that banks can perfectly forecast the fraction of impatient agents. Introducing uncertainty about aggregate liquidity needs allows us to capture the incentives of bankers to acquire liquid reserves in a more gradual manner. Since the banker does not know a priori the fraction of impatient agents, she has to decide on the level of liquid reserves following a probabilistic approach: How likely it is that the bank will run out of liquid reserves, or on the contrary hold excess reserves? In this way, the direct and strategic value of bond holdings get incorporated smoothly into prices, which yields a continuous bond demand (see Figure B.10) and ensures that the market-clearing bond price moves continuously with the fundamentals.

Impact of the volatility parameter. Figure B.12a illustrates the impact of the volatility of the fundamental, $\tau^{-1/2}$, on bank leverage and the bond liquidity premium.

If leverage were held constant, more uncertainty in the interim period would increase the

liquidity premium. With a higher volatility, there is a higher chance of a bad realisation of the fundamental, which, everything else being equal, increases the run threshold played by patient investors. This provides an incentive for bankers to increase their bond holdings. As bankers compete for the fixed supply of government bonds, it has the general equilibrium effect of driving up the liquidity premium $q_b - \hat{q}_b$.

When leverage is endogenous, however, a banker can also respond to the higher volatility by issuing less short-term debt. Increasing bond holdings reduces the likelihood of runs, but it also makes it more likely that the bank ends up with excess reserves in the states where a run is avoided. Taking on lower leverage reduces the need for bond holdings due to both the direct and the strategic effects, but also implies that the banker must finance a larger part of her investment with equity. In the situation depicted in Figure B.12a, the banker reacts to an increase in volatility by applying the second strategy—reducing leverage—and bonds become *less* valuable.

This means that in order to predict the impact of the economic environment on the liquidity premium, one must account for the fact that leverage is endogenous: not doing so can not only have a quantitative impact, but even *change the sign* of the correlation between the variable—here, the volatility of the fundamental— and the liquidity premium.

Endogenous leverage and the liquidity premium. The influence of an endogenous leverage decision on the liquidity premium is well illustrated in Figure B.10: the sensitivity of the banker's liquid holdings demand, b, as a function of the bond price q_b, is magnified compared to the case where debt issuance is exogenous. The reason is that when q_b is low, banks increase their leverage—issuing valuable demand deposits to investors— and cheaply dampen the associated illiquidity risk by buying bonds. Similarly, when q_b is large, managing illiquidity risk through bonds becomes expensive, and banks instead do so by reducing debt issuance.

2.4.3   Welfare

Increase in Government Bond Supply

Suppose the liquidity premium is strictly positive, $q_b - \hat{q}_b > 0$. A higher bond supply will increase the utility of bankers, as they are able to reduce their illiquidity risk at a lower cost. But do investors always benefit from a higher bond supply? In other words, does an increase in the supply of government bonds necessarily generate a Pareto improvement?

To answer this question, the next lemma computes the utility of investors at equilibrium (in the sense of Definition 7), given a choice of funding policy and liquidity buffer by banks.

Lemma 9 *The equilibrium utility of investors satisfies*

$$U_b^I = U_0^I + v'(\omega_0 - I)(q_b - \hat{q}_b)b \tag{2.59}$$

where $U_0^I = v(\omega_0 - I) + \alpha\omega_1 + v'(\omega_0 - I)I$ is the equilibrium investors' utility when banks do not hold any bond.

Since $q_b \geq \hat{q}_b$ and $b \geq 0$, it is immediate to see that $U_b^I \geq U_0^I$: positive government supply makes investors weakly better off compared to the benchmark case without government bonds. The gain in utility $U_b^I - U_0^I$ is equal to the spread (in units of the single good) between the price paid by bankers for their aggregate bond position and the investors' valuation of that same position, multiplied by the investors' marginal utility of date-0 consumption. Since we know that $q_b > \hat{q}_b$ if B is sufficiently low, an increase in the bond supply generates a Pareto-improvement when bonds are very scarce.

In addition, note that $U_b^I = U_0^I$ *either* if the government bond supply is zero, $b = B = 0$, *or* if the supply so high that both banks and investors hold bonds in equilibrium, $q_b = \hat{q}_b$. Thus, if the shortage of bonds is moderate, an increase in the supply does *not* generate a Pareto-improvement. Figure B.11b traces out the path of investors' and bankers' utility as the bond supply increases from 0 to 0.3.

The first units of bond holdings are very valuable to bankers, which lead them to bid up the bond price well above the valuation of investors. Since the proceeds from the debt issuance are transferred in lump-sum fashion to investors, all parties are better off. If the bond price declines sufficiently rapidly as the supply increases, however, a higher supply makes investors worse off. We have,

$$\frac{\mathrm{d}U^I}{\mathrm{d}b} > 0 \quad \text{if} \quad |\varepsilon| > 1,$$

where $\varepsilon = \frac{\mathrm{d}b/b}{\mathrm{d}(q_b - \hat{q}_b)/(q_b - \hat{q}_b)}$ is the elasticity of the bankers' bond demand with respect to the bond liquidity premium.

This result is quite dependent on our specific assumptions — in particular, the linearity of investors' utility functions, and the fact that taxes are entireld paid by (and lump-sum transfers paid to) investors. However, it illustrates well the double layer of liquidity provison which is going on in our model. The *population* of investors, in their role as taxpayers, provide *public* liquidity to banks. Banks provide *private* liquidity to each *individual* investor.

Ceiling on Short-term Leverage

When heavy use of short-term debt results in excessive illiquidity risk, it is optimal for the banker to include equity into the funding mix. But is the *privately* optimal funding policy also *socially* optimal? The next proposition answers this question.

Proposition 13 *For a given government bond supply B, the banker's equilibrium choice of funding policy (δ, ψ) (in the sense of Definition 7) achieves a Pareto optimal allocation.*

The reason behind this efficiency result is that bankers rationally anticipate the impact of the funding mix on the bank's financing costs and on the likelihood of runs.

A number of papers have analysed how illiquidity risk varies with the composition of the asset and liability side of a financial institution's balance sheet (Rochet and Vives, 2004; Morris and Shin, 2009; Vives, 2014; Tourre, 2015). These papers highlight how mandating a decrease in the ratio of short-term liabilities to total liabilities reduces the likelihood that a financial institution suffers a liquidity crisis. Our analysis points out that the presence of a coordination failure is not sufficient *per se* to justify regulations aiming at curtailing the proportion of short-term debt funding. Even though inefficient runs occur in equilibrium and impose large private and social costs, the ex-ante likelihood of runs at the equilibrium funding policy optimally balances the expected costs due to inefficient liquidations against the liquidity benefits offered by deposits.

Whilst this efficiency result provides a useful benchmark, it is also important to nuance its implication regarding the potential welfare benefits of curtailing the short-term indebtness of financial institutions. First, our model assumes that the market pricing of the deposit and equity contracts accurately reflect the risk of a liquidity-driven failure. As pointed out by Hellwig (2015), the implicit assumption is that banks are able to commit and credibly communicate their funding policy to investors. By contrast, if bankers are unable either to commit to or credibly communicate their funding policies to investors, they will not take into account the effect of their choice of funding policy on the prices of the securities issued by their bank. Hellwig (2015) builds a general equilibrium model of liquidity provision by banks, and studies the equilibrium properties in both cases, i.e. with and without commitment of banks about their funding policies. In the first case, banks choose the constrained efficient level of equity funding. In the second case, the privately optimal funding policy involves pure deposit funding. Equity requirements can then improve welfare by reducing the distortions from the banks' inability to commit to and communicate their funding choices.

Second, even if the market valuation incorporates the impact of a bank's funding policy, prices might still be distorted by various frictions giving an advantage to debt over equity.

These include tax deductibility of interest payments as well as implicit or explicit government guarantees such as deposit insurance or bailout expectations of senior debt holders.

Third, this efficiency result relies on the fact that there are no externalities among the different banks, so that the deposit and equity contracts which one bank offers to its investors does not affect the payoff of the contracts offered by other banks. The model thus does not capture systemic effects that, for instance, played a prominent role during the financial crisis of 2007-09. Eisenbach (2017) studies these effects and their implications in terms of leverage requirements.

2.4.4 Covariation of Bank Leverage and Liquidity Premium

Impact of the government bond supply. Suppose that the government decides to increase B, exerting downward pressure on the liquidity premium. How does banks' leverage react?

Two effects are at play, as illustrated by Figure B.11a.

When the general equilibrium response of q_b to a change in B is small, as is the case for small values of B, absorbing the bond supply at $q_b B$ becomes more costly. Similar to the "solvency effect" described by Koenig (2015), an increase in B has then the effect of worsening the bank's portfolio return, leading the banker to pick a more conservative debt issuance strategy in order to mitigate its illiquidity risk.

By contrast, when the response of q_b to a change in B is sufficiently large, banks respond to lower bond prices by issuing more deposits. By holding a larger stock of bonds, banks are able to create more liquid claims—and thus better capture the liquidity benefits priced into these claims—without correspondingly high illiquidity risk. This "leverage channel of monetary policy" is consistent with the findings of Drechsler et al. (2018).

Impact of other variables. We have described in Section 2.4.2 how leverage and the liquidity premium can covary negatively following a change in the volatility of the fundamental.

Figure B.12b shows the impact of varying mean aggregate liquidity needs, μ_λ, on bank leverage and the liquidity premium.

Everything else being equal, a higher μ_λ increases the likelihood that bond holdings are insufficient to cover withdrawals by impatient investors. This provides incentives for banks to hold more bonds, which increases the liquidity premium. The impact of varying μ_λ on bank leverage results from two opposite effects. On one hand, higher withdrawal risk provides

incentives for bankers to reduce leverage. On the other hand, from an individual investor's perspective, a higher μ_λ increases the risk of facing a liquidity shock, which increases the valuation of deposits relative to equity and compels bankers to pursue higher leverage. For our baseline parameter values, the interaction between these two forces leads to a non-monotonic effect on leverage. Thus, bank leverage and the liquidity premium covary positively (negatively) following an increase in mean aggregate liquidity needs from initially low (high) levels.

Our model therefore indicates an estimation of the correlation between bank leverage and the liquidity component of government bond prices must be done *conditional on the variables driving the banks' choice of funding policy and bond holdings*. Bank leverage and the liquidity premium can covary positively for a given change in the environment, and negatively for another.

2.5 Conclusion

We have built a model where liquidity provision and illiquidity risk jointly shape the optimal funding policy of banks and the liquidity value of government bonds. Liquid reserves are modeled explicitly in a fully rational setup. Optimal leverage trades off the capture of the liquidity benefits priced into deposits against the expected costs of liquidity crises. The optimal size of bond holdings balance the direct and strategic value of holding liquid assets against the costs funding these holdings. Given the assumed frictions impeding trade between investors in the interim period, the model features a pecking order of government debt ownership: when the supply is scarce, banks bid up the price of government bonds above the investors' valuation.

The fact that banks are natural buyers of government bonds but can also use their leverage decision to manage their illiquidity risk induces a rich interaction between bank leverage and the liquidity premium. The model shows how considering an endogenous bank leverage matters for predicting the covariation between fundamental variables and the liquidity premium. It also captures two possible general-equilibrium effects of an increase in government bond supply. When absorbing more bonds is costly for banks, lowering their portfolio return, they issue less debt. When, in contrast, liquidity becomes cheaper, banks can afford to issue more demand deposits.

The following extensions seem to be promising avenues for future research. First, one would want to understand the impact of introducing deposit insurance or central bank interventions into the model. The impact of deposit insurance is not a priori clear when bank leverage is endogenous and depositors face a coordination problem. On the one hand, it lowers the risk of loss of depositors, which reduces their propensity to run. On the other hand, in the

absence of actuarially fair premia, deposit insurance provides incentives for banks to take on excessive leverage, which makes them more vulnerable to runs. Is it possible then that the introduction of partial deposit insurance lowers welfare compared to the laissez-faire equilibrium without deposit insurance? Regarding the question of the scope and efficiency of central bank interventions, a chapter of the thesis of Klossner (2019) builds on the present chapter and analyses the costs of benefits of several assistance mechanisms in crises.

Second, it would be interesting to analyse a modification of the model where the supply of government bonds is also endogenous. This could be achieved, for instance, by introducing a central bank with an interest rate target which has the ability to adjust the bond supply so as to hit its target. Such a model could be well-suited to study questions of optimal regulation when the regulatory toolkit contains both a ceiling on the ratio of short-term to long-term liabilities (à la NSFR) and a floor on the ratio of cash to total assets (à la LCR). More needs to be done on this challenging but very interesting topic.

3 | Insider Trading with Penalties

Sylvain Carré [1], Pierre Collin-Dufresne [2], Franck Gabriel [3]

We establish existence and uniqueness of equilibrium in a generalised one-period Kyle (1985) model where insider trades can be subject to a size-dependent penalty. The result is obtained by considering uniform noise and holds for virtually any penalty function. Uniqueness is among *all* non-decreasing strategies. The insider demand and the price functions are in general non-linear, yet tractable.

We apply this result to regulation issues. We show analytically that the penalty functions maximising price informativeness for given noise traders' losses eliminate small rather than large trades. We generalise this result to cases where a budget constraint distorts the set of penalties available to the regulator.

Keywords: Kyle equilibrium, insider trading.
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3.1   Introduction

In his seminal 1985 contribution, Albert Kyle showed how the insider information of strategic traders incorporates into market prices. Under the assumption that both the asset fundamental value and the noise traders demand are normally distributed, he established the existence of an equilibrium where the insider trader demand and the price function are linear.

While Kyle proved that his model features only one linear equilibrium, the issue of uniqueness among all strategies was until recently an open question. Absent uniqueness, the model cannot predict unambiguously an outcome. This renders policy implications less clear-cut.

Another investigation avenue opened up by Kyle's work is the study of strategic trading based on *illegal* insider information. In Kyle's model, the insider never faces any penalty if she trades. How do insider demand and price functions change when this is no longer the case? Despite being natural, this question had so far been left unanswered, at least in a model *à la* Kyle.

Somewhat surprisingly, our approach allows to simultaneously tackle the uniqueness problem and to deal with the introduction of penalties on insider trading. Our solution involves a change in the distributional assumptions. We then obtain an intuitive lemma which holds regardless of the presence of a penalty function and is the key to the equilibrium characterisation. In particular, it is no more work in our analysis to prove existence and uniqueness when insider trading is penalised, compared to the case where it is not. By contrast, the recent uniqueness results under normally distributed noise only apply to the original Kyle model without penalties. They rely on advanced mathematical techniques which might be hard to amend to a more general framework with penalties.

We use our existence and uniqueness theorem to contribute to the long-standing debate on insider trading regulation. Proponents of legal insider trading wonder why one should ban an activity that contributes to the efficiency of prices. Opponents recall that allowing insider trading imposes costs upon the less informed agents. If these agents are sufficiently concerned by losses due to insiders, they could refuse to enter trades altogether, leading to a market breakdown. Our measure of price efficiency is given by the expected post-trade standard deviation of the asset fundamental value. For any penalty function, our theorem allows to compute unambiguously the equilibrium value of this quantity. Similarly, we can map any penalty function to a level of expected losses of the noise traders.

We consider a regulator aiming at maximising price efficiency for a given level of noise traders' losses. This maximisation problem is over the function space of all penalties which are non-decreasing in the magnitude of the insider's order. We show analytically that optimal penalties eliminate small rather than large trades. In cases where the fundamental realises at very high

or very low values, the insider finds it optimal to trade despite the high penalty. Although such trades–if they occur–are costly for noise traders, they signal extreme events and therefore incorporate a lot of information into prices.

A regulator operating under a budget constraint could select different penalty functions.

The first reason is that investigating and convicting an agent of insider trading is a costly process–and more so if the regulator wants to implement a severe sanction. With a limited amount of resources, the regulator may only be able to correctly identify a fraction of actual insider trades, and may not be able to implement every level of punishment. When penalties are non-pecuniary, the regulator cannot even use the fines she levies in order to soften this constraint. From the perspective of the insider, this means that being caught only occurs with some probability, which reduces the expected sanction associated with trading; and that even when caught, punishment cannot be arbitrarily large. In effect, the budget constraint of the regulator restrains the set of feasible penalties. We establish that in the non-pecuniary case, either (i) a level of noise traders losses is too low to be implemented, or (ii) it can be implemented and in that case, the regulator uses the same penalty function as in the unconstrained case in order to maximise price efficiency at this level of losses. Hence we cannot distinguish, based solely on the penalty function she selects, an unconstrained regulator with preferences biased towards price efficiency, and a constrained regulator imposing non-pecuniary fines.

The second reason is that–if penalties are pecuniary–the regulator may need to convict sufficiently enough agents of insider trading in order to balance her budget. This also leads to a distortion of the set of feasible penalties. In this case, we establish that the regulator selects new penalties compared to the unconstrained case. When such a penalty is observed, one can theoretically infer the extent of the regulator's constraint. New patterns emerge in the demand schedules of the insider trader and the associated price functions.

Related literature.

Models à la *Kyle.* Throughout his analysis, Kyle (1985) uses the assumption that noise is Gaussian. Bagnoli et al. (2001) show that, in the static setting, this distributional assumption is not necessary to obtain the existence of a linear equilibrium: this holds as soon as the distribution of the noise traders demand equals in law an affine transformation of the fundamental. [4]

While uniqueness among linear equilibria still holds in the generalisation of Bagnoli et al. (2001), their approach is not aimed at tackling the general uniqueness problem.

Such an approach was provided by Rochet and Vila (1994) under the new assumption that

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[4]Then, the demand of the insider has the same distribution as the demand of the noise traders, what we call a *mimicking* property. Focusing on the mimicking property sheds a new light on the results of Bagnoli et al. (2001): see Appendix C.3.

the informed trader is able to observe the noise traders demand. In that context, they show existence and uniqueness of equilibrium regardless of the distributional assumptions on the noise. Unfortunately, their approach does not allow in general to construct explicitly the equilibrium. The general uniqueness result in the original one-period Kyle model was only proven recently by Boulatov et al. (2013) and McLennan et al. (2017), using involved mathematical techniques tailored to Gaussian noise. Similarly to Rochet and Vila (1994), extending the existence and uniqueness result to the case of penalties seems very challenging.

Insider trading regulation. DeMarzo et al. (1998) study a problem where the regulator min-imises the losses of the uninformed agents due to insider trading, subject to a budget con-straint. [5] Their characterisation of optimal penalty and investigation probability schedules relies on the assumption that the regulator's strategy set is so large that he can effectively dictate state by state the insider demand. A peculiar feature of their equilibrium is that the regulator never collects any fine. In fact she investigates only because she has committed to do so; no investigation leads to a conviction.

While DeMarzo et al. (1998) do not consider the positive aspects of insider trading, Leland (1992) studies the trade-off between liquidity traders losses and information incorporation. Information is valued by outside traders–different from liquidity traders–who have to take investment decisions. Leland compares two extreme cases, with or without insiders, and abstracts from intermediary regimes where insider trading is regulated but does not disappear altogether.

In a speech of the Securities and Exchange Commission, Newkirk and Robertson (1998) gather important insights from the U.S. regulator regarding insider trading. They emphasise various constraints that regulators face in implementing their laws. In particular, they explain that increasing penalties when aiming at reducing insider trading can backfire: stronger sanctions are more difficult to implement because the burden of proof is heavier. Agents who anticipate this may have incentives to engage more easily, rather than less, in insider trading. [6] Newkirk and Robertson (1998) also discuss the implications of civil and penal prosecution. The selection of a prosecution mode impacts the behaviour of potential insider traders and the nature of legal procedures. Additionally, and importantly for our purposes, it affects differently the budget constraint of the regulator: only civil convictions lead to the collection of a fine.

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[5]They do not consider a market structure *à la* Kyle but rather borrow it from Admati and Pfleiderer (1989) and Easley and O'Hara (1992).

[6]Newkirk and Robertson (1998) mention the case of Netherlands at the end of the XXth century: "Dutch authorities heralded its new law against insider trading as the 'toughest in the world.' The statute provides for criminal penalties only. [...] Dutch prosecutors began their case against four individuals charged with insider trading [...] The judges dismissed the case against all of the defendants on the grounds that the evidence was circumstantial, and, therefore, did not satisfy the heavy burden of proof that must be met to support a criminal conviction. The outcome was reported as the latest in a series of disappointments for Dutch prosecutors, who have made only one insider trading prosecution stick in the last 10 years."

We include these various features when considering the regulator's problem under a budget constraint: see Section 3.4.3.

3.2 A One-Period Kyle Model with Penalties

As in the one-period version of Kyle (1985), the model features a risk-neutral insider trader (IT), noise traders (NT) and a market maker (MM). Agents are trading an asset with fundamental value v. The IT perfectly observes v and places an order $X(v)$. NT have a stochastic demand u independent of v. MM observes the total demand $X(v) + u$ and executes orders at a price P such that she breaks even on average.

The first difference of our model with Kyle (1985) is that we consider uniform–instead of Gaussian–noise:

$$\begin{aligned} u &\sim &U(-1,1), \\ v &\sim &U(-1,1), \\ u &\perp &v. \end{aligned}$$

The choice of $[-1,1]$ as the support is for clarity and without loss of generality; one could equivalently assume $u \sim U(-a,a)$ and $v \sim U(b,c)$ with $a > 0$ and $b < c$: see Appendix C.1.1.

The second difference is that a regulator may decide to penalise trades of size x by a cost $C(x)$. We interpret C as a product

$$C = \alpha \tilde{C}: \tag{3.1}$$

α is the exogenous probability that the regulator starts and successfully completes an investigation, while $\tilde{C}(x)$ is the cost imposed to the IT conditional on the investigation being successful and the order of the IT being x. Success of the investigation means that the regulator correctly identifies the order of the informed trader x and gathers sufficient evidence to enforce payment of the corresponding fine. In other cases, the IT can not be constrained to pay any fine. Under these assumptions, the regulator never makes type 1 errors (never convicts a trader that didn't use insider information) but can make type 2 errors (not convicting a trader that did use insider information).

3.2.1 Benchmark Equilibrium without Penalties

In the absence of penalties, the IT solves

$$\max_{x \in I} x \mathbb{E}_u[v - P(x + u)] \tag{3.2}$$

taking the price function P of the MM as given. The MM breaks even on average:

$$P(d) = \mathbb{E}[v|X(v) + u = d].$$ (3.3)

An equilibrium is a pair (X, P) that satisfies (3.2) and (3.3). For example, (X, P) defined by

$$X(v) \quad = \quad v$$ (3.4)

$$P(x + u) \quad = \quad \frac{x + u}{2},$$ (3.5)

is an equilibrium of the one-period Kyle model without penalty. We call it the *(linear) mimicking equilibrium* since $X(v)$ and u are equal in distribution. We will prove later that this equilibrium is unique among all equilibria featuring a non-decreasing demand whose image lies in $[-1, 1]$.

With penalties, the optimal demand is no longer mimicking the random demand u. One intuitive interpretation is that while mimicking u allows the IT to best conceal herself from the market maker, he can't hide from the regulator (in case investigation is open and succeeds). This leads to a lower demand than in the case without penalties. We now define formally the equilibrium with penalties.

3.2.2 Equilibrium with Penalties

The IT solves

$$\max_{x \in I} x \mathbb{E}_u[v - P(x + u)] - C(x),$$ (3.6)

taking the price function P of the MM as given. The MM breaks even on average:

$$P(d) = \mathbb{E}[v|X(v) + u = d].$$ (3.7)

This game involving the IT and the MM is denoted $\mathcal{K}(C)$. An equilibrium of $\mathcal{K}(C)$ is a pair (X, P) such that X solves (3.6) and P satisfies (3.7).

The interval $I \subset \mathbb{R}$ in the maximisation program (3.6) is the set of admissible insider's demands. We will use the following assumption:

Assumption 7 $I = [-1, 1]$.

The bounds of I are those that obtain in the linear mimicking equilibrium when there is no penalty function. They are therefore natural: a demand function X whose image is not contained in $[-1, 1]$ would imply that for some values of the fundamental v, the magnitude of the IT order is *higher* when there is a penalty, compared to the linear equilibrium without

penalty.[7]

To conclude this section, we state two remarks and introduce some notation.

(i) The data of a strategy X implies a pricing function P via equation (3.7). We denote the pricing function associated with a demand schedule X by $P(X)$.

(ii) In the IT's maximisation program (3.6), the pricing function P only intervenes through the *expected price function*, denoted $\hat{P}$ and defined by

$$\hat{P}(x) = \mathbb{E}_u[P(x + u)]. \tag{3.8}$$

$\hat{P}$ represents the price that the IT will face on average if he places an order x. The program (3.6) can be rewritten in terms of the expected price function only:

$$\max_{x \in I} x(v - \hat{P}(x)) - C(x). \tag{3.9}$$

3.2.3 Out-of-Equilibrium Pricing

The noise u we consider has bounded support. Moreover, from Assumption 7, the equilibrium demand functions X we consider satisfy $|X| \leq 1$. This means that the aggregate order, $d = X(v) + u$ belongs to a bounded set D. The conditional expectation in (3.7) is not defined for values of $d \notin D$, meaning that we must make an assumption on the out-of-equilibrium pricing of the MM:

Assumption 8 *For any equilibrium (X, P) of $\mathcal{K}(C)$ we consider, with X non-decreasing and $X([-1, 1]) \subset [-1, 1]$, we always impose the following out-of-equilibrium pricing (letting $x_M = X(1)$ and $x_m = X(-1)$):*

$$
\begin{aligned}
P(d) &= 1 &&\text{for} \quad d > 1 + x_M, \\
P(d) &= -1 &&\text{for} \quad d < -1 + x_m.
\end{aligned}
$$

This assumption states that when the MM observes an aggregate order larger than the maximal possible equilibrium order, she prices the asset as if it had realised at its maximal value, $v = 1$. Similarly, when the aggregate order is smaller than the minimal possible equilibrium order, the MM prices as if $v = -1$. When verifying that (X, P) is an equilibrium, one must not only check that $X(v)$ maximises the IT's program (3.6) among all x in the candidate support $[x_m, x_M]$, but also among values of x in $I \setminus [x_m; x_M]$. For these values of x, the aggregate order $d = x + u$

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[7]An interesting question is whether there exists an equilibrium for which Assumption 7 does not hold.

realises in the out-of-equilibrium region with positive probability, in which case Assumption 8 defines the price $P(d)$.

Finally, notice that Assumption 8 fully characterises out-of-equilibrium pricing: indeed, any $d \in [-1 + x_m, 1 + x_M]$ belongs to the support of $u + X(v)$, since u is $U(-1, 1)$ and $-1 \le x_m \le x_M \le 1$.

3.2.4 A first example

We now present an example of an equilibrium of $\mathcal{K}(C)$. More illustrations can be found in section 3.3.4, where we discuss the intuitions behind some typical behaviours of the equilibrium demand and price functions in the presence of penalties.

Let $K \in \left(0, \frac{1}{2}\right)$ and

$$C(x) = K\mathbb{1}_{x \ne 0}.$$

Under this penalty function, the insider trader undergoes an expected sanction of K if he trades. We will show that $(X, P(X))$ is an equilibrium, where

$$X(v) = v\mathbb{1}_{|v| > \sqrt{2K}}. \tag{3.10}$$

As we will see, the price function $P(X)$ is non-linear but the expected price function $\hat{P}$ satisfies $\hat{P}(x) = \frac{x}{2}$. Hence, the IT maximises under the same expected price function as in the linear mimicking equilibrium. Facing an expected price identical to the one without penalties, the IT only trades when its previously optimal strategy–the linear mimicking demand–allows her to recoup the penalty K on average. Without penalties, the profit of the IT when he observes a fundamental v is $\frac{v^2}{2}$. With a constant penalty upon trading equal to K, the IT does not trade as long as $\frac{v^2}{2} < K$. When $\frac{v^2}{2} > K$, the IT considers K as a sunk cost and optimises as if there was no penalty, thus selecting $X(v) = v$. Notice that the demand function is non-linear and exhibits a jump at $\pm\sqrt{2K}$.

3.2.5 Indistinguishable Equilibria

In the equilibrium of the example above, the IT would earn the same profit upon observation of $v = \pm\sqrt{2K}$ by selecting $X(v) = 0$ or $X(v) = v$: zero in both cases. In general, when the penalty function exhibits jumps, we should expect the existence of such indifference points. At these points, the IT can achieve a given profit by placing a small order and undergoing a small expected sanction or by placing a larger order, associated with a larger expected penalty. However, as long as the set of v such that the maximisation program of the IT (3.6) admits several solutions has measure zero, these indifference points will almost surely not be reached.

The equilibrium will therefore be independent of the choice of the maximiser $X(v)$, in the sense that any *ex post* model observable is almost surely the same–*e.g.* demand of the IT $X(v)$, observed price $P(d)$–and any *ex ante* model quantity–such as the IT expected profit or the expected penalty collected from the IT–is the same. In that case, we wish to consider that any choice of maximiser X induces the same equilibrium. We formalise this by introducing an equivalence relation between equilibria that we call indistinguishability.

Definition 8 *Let (X, P) and (X', P') be two equilibria of $\mathcal{K}(C)$. We say that (X, P) and (X', P') are* indistinguishable *if X and X' agree outside of a countable set.*

From now on, we identify an equilibrium of $\mathcal{K}(C)$ to its equivalence class. Definition 8 is useful because we will see that maximisers of (3.6) agree outside of a countable set, so the equilibria they induce belong to the same equivalence class.

3.2.6 Admissible penalty functions

We allow the regulator to select any penalty function that only depends in a non-decreasing manner on the magnitude of the order of the insider trader.

Definition 9 *$C : [-1; 1] \rightarrow \mathbb{R}_+$ is a penalty function if it is symmetric and non-decreasing, left-continuous over $[0; 1]$ and satisfies $C(0) = 0$. The set of penalty functions is denoted $\mathscr{C}$.*

The monotonicity assumption reflects the fact that it might be difficult to implement higher sanction on a smaller trade for political reasons. The left-continuity assumption makes sure that the supremum of the possible IT profits is attainable.

3.3 Existence and uniqueness of equilibrium for $\mathcal{K}(C)$

In this section, we set out to prove our main theorem:

Theorem 1 *For any $C \in \mathscr{C}$, the Kyle game $\mathcal{K}(C)$ with penalty function C admits a unique equilibrium $(X(C), P(C))$ such that X is non-decreasing. In general, $X(C)$ and $P(C)$ are non-linear.*

3.3.1 Analysis of the expected price function

The expected price function is linear regardless of the IT demand

Lemma 10 contains the key observation at the root of our analysis.

Lemma 10 *Let $X : [-1,1] \rightarrow [-1,1]$ be a non-decreasing function, $x_M = X(1)$ and $x_m = X(-1)$. The expected price function $\hat{P}$ is linear on $[x_m, x_M]$:*

$$\hat{P}(x) = \frac{x}{2}.$$

Lemma 10 is crucial because it makes the surprising statement that the expected price function that must prevail in equilibrium is $\hat{P}(x) = x/2$ for $x_m \le x \le x_M$; neither the form of C nor guesses about X or P are needed. In fact, using Assumption 8, we will prove that the equality $\hat{P}(x) = x/2$ holds for any admissible demand x.

Hence, the equilibrium demand of the IT, $X(v)$, must be a maximiser of

$$\psi_C(., v) : x \mapsto x\left(v - \frac{x}{2}\right) - C(x). \tag{3.11}$$

If X is such a maximiser, we claim that $(X, P(X))$ is the unique equilibrium of $\mathcal{K}(C)$ such that X is non-decreasing. To reach this conclusion, several issues remain to be addressed. First, we need to show that $\hat{P}(x) = x/2$ for any x as claimed above. Second, to make sure that Lemma 10 applies, we must check that any (selection of) maximiser is non-decreasing. Third, in order to obtain uniqueness, we need to show that $\psi_C(., v)$ admits a unique maximiser except for a countable number of values of v.

We now provide the proof of this lemma. Section 3.3.1 clarifies the main intuitions.

Proof of Lemma 10. We use the notation $p(.)$ for a density and $p(.|.)$ for a conditional density. Write

$$
\begin{aligned}
p(v|d) \quad &\propto \quad p(d|v)p(v) \\
&\propto \quad \mathbb{I}_{X(v) \in [d-1; d+1]} \mathbb{I}_{v \in [-1; 1]}.
\end{aligned}
$$

That is, for $-1 + x_m \le d \le 1 + x_M$, $v|d$ is uniform over

$$
\begin{aligned}
\{v \in [-1; 1] | X(v) \in [d-1; d+1]\} \quad &= \quad \{v \in [-1; 1] | X(v) \in [d-1; d+1] \cap [x_m; x_M]\} \\
&= \quad [(X_\ell^{-1}((d-1) \vee x_m); X_r^{-1}((d+1) \wedge x_M))] \tag{3.12}
\end{aligned}
$$

where $X_\ell^{-1}(x) = \inf\{v | X(v) \ge x\}$ and $X_r^{-1}(x) = \sup\{v | X(v) \le x\}$. X_ℓ^{-1} and X_r^{-1} only disagree when there is v such that $X(v) = x$ and X is locally constant at v, i.e. they agree outside of a countable set. Then, letting $P = P(X)$,

$$P(d) = \frac{1}{2}\left(X_\ell^{-1}((d-1) \vee x_m) + X_r^{-1}((d+1) \wedge x_M)\right).$$

Now since

$$\hat{P}(x) = \frac{1}{2} \int_{x-1}^{x+1} P(z)\, \mathrm{d}z,$$

it is enough to show that $P(x+1) - P(x-1) = 1$ a.e.. Using the expression of P found above, we obtain that for $x_m \le x \le x_M$:

$$
\begin{aligned}
2(P(x+1) - P(x-1)) &= X_\ell^{-1}(x \vee x_m) + X_r^{-1}((x+2) \wedge x_M) \\
&\quad - X_\ell^{-1}((x-2) \vee x_m) - X_r^{-1}(x \wedge x_M) \\
&= X_r^{-1}(x_M) - X_\ell^{-1}(x_m) \\
&= 2
\end{aligned}
$$

a.e.. This is because $X_\ell^{-1} = X_r^{-1}$ a.e., $X_r^{-1}(x_M) = 1$, and $X_\ell^{-1}(x_m) = -1$. ∎

Having identified $\hat{P}$, we know that the insider trader's problem is to maximise $\psi_C(., v)$ as defined in (3.11). Because we will use this function throughout the chapter, we repeat its definition here:

Definition 10 *The insider's expected profit (under the expected price function $\hat{P}(x) = x/2$) for a demand x when the fundamental value is v is*

$$\psi_C(x, v) = x\left(v - \frac{x}{2}\right) - C(x). \tag{3.13}$$

Notice that ψ_C is an "expected" profit because we interpret C as an average cost–an investigation may not be started or not succeed–while $\hat{P}(x)$ is an expected price because the realisation of u is random and the realised price is $P(x + u)$.

Intuition

In order to isolate the intuition behind Lemma 10, let us consider the case where X is continuous and strictly increasing.

Assume that the market maker observes an aggregate order $d > 0$. Since the demand of the noise traders u takes values in $[-1, 1]$, the possible demands of the IT $X(v)$ consistent with the observation of d are exactly the admissible demands such that $d - 1 \le X(v) \le d + 1$. Because admissible demands satisfy $X(v) \le 1$ and $d + 1 > 1$, the information obtained by the market maker when she observes d is that $X(v) \ge d - 1$. Thus, she knows that $v \ge X^{-1}(d - 1)$. Intuitively, the fact that the aggregate order is positive rules out extreme negative values of v and the MM deduces a lower bound on v, $X^{-1}(d - 1)$.

Moreover, due to the uniform noise assumption, all values of v above this lower bound are

equally likely. Therefore, the price $P(d)$ is given by the midpoint of the interval $\left[X^{-1}(d-1),1\right]$.

In a similar manner, when $d < 0$, the price $P(d)$ is given by the midpoint of the interval $\left[-1, X^{-1}(d+1)\right]$.

Now, assume that the IT wants to place an order x. The IT is only concerned by the expected price impact, $\hat{P}(x)$, which is a uniform average of the $P(d)$ over $d \in [x-1, x+1]$, the set of possible aggregate demands given an IT demand x. If, instead, the IT decides to place an order $x + \Delta x$, the set of possible aggregate demands d is $d \in [x - 1 + \Delta x, x + 1 + \Delta x]$: see Figure C.1.

Thus, the only contribution to the marginal increase in expected price $\hat{P}(x + \Delta x) - \hat{P}(x)$ is due to the fact that the weight that was attributed to the interval $[x - 1, x - 1 + \Delta x]$ is now attributed to the interval $[x + 1, x + 1 + \Delta x]$. Crucially, this weight is the same due to the uniform noise assumption. Considering a vanishing Δx, one concludes that the marginal impact of increasing demand on expected price is proportional to $P(x + 1) - P(x - 1)$.

We have seen above that $P(x + 1)$ is the midpoint of $\left[X^{-1}((x+1)-1),1\right] = \left[X^{-1}(x),1\right]$, and that $P(x - 1)$ is the midpoint of $\left[-1, X^{-1}((x-1)+1)\right] = \left[-1, X^{-1}(x)\right]$. Therefore, the marginal impact on the expected price is proportional to the distance between these two midpoints:

$$\frac{d}{dx}\hat{P}(x) \propto P(x+1) - P(x-1) = \frac{1 + X^{-1}(x)}{2} - \frac{X^{-1}(x) - 1}{2} = 1. \tag{3.14}$$

Figure C.2 provides an illustration of this result. From (3.14), we see that the expected price function is linear.

3.3.2   Candidate optimal demands are unique up to changes on a countable set

In this section, we set out to obtain an unambiguous definition of the strategy X that will be the maximiser of ψ_C defined in (3.13).

Definition 11 *Let V, I be two intervals of $\mathbb{R}$. A correspondence $\mathcal{X} : V \to \mathcal{P}(I) \setminus \emptyset$ is non-decreasing if for any $v_1 < v_2$ in V, $\sup \mathcal{X}(v_1) \le \inf \mathcal{X}(v_2)$.*

If $\mathcal{X}$ is a one-to-one mapping, we recover the usual notion of a non-decreasing function.

Lemma 11 *Let $\mathcal{X} : V \to \mathcal{P}(I) \setminus \emptyset$ be a non-decreasing correspondence. Then for all v in V except on a countable set, $\mathcal{X}(v)$ is a singleton.*

Proof. The argument is the same as for the proof that a non-decreasing function has at most a countable number of discontinuities. ∎

For a given penalty $C \in \mathscr{C}$, let $\mathscr{X}_C$ be the correspondence mapping $v \in [-1;1]$ to the set of maximisers of the insider trader's profit function when she observes a realisation v of the fundamental:

$$\mathscr{X}_C(v) = \arg\max_x \psi_C(x, v).$$

Lemma 12 *For any $v \in [-1, 1]$, $\mathscr{X}_C(v) \neq \emptyset$, and $\mathscr{X}_C$ is a non-decreasing correspondence.*

Proof. See Appendix C.1.2.

Lemma 12 reflects the fact the IT trades more aggressively for large values of the fundamental. The combination of Lemmas 11 and 12 ensures that the maximiser of the IT's expected profit is unique except for a countable number of values of v:

Lemma 13 *There exists a non-decreasing function X_C such that for all $v \in [-1, 1]$ except on a countable set, $\mathscr{X}_C(v) = \{X_C(v)\}$. All such X_C agree outside of a countable set.*

As we identify equilibria in a same equivalence class, as introduced in Definition 8, we do not need to specify which particular X_C we consider: we can unambiguously talk about *the* maximiser of the expected profit. We are now ready to prove our main result.

3.3.3 Existence and uniqueness of the equilibrium of $\mathcal{K}(C)$

We recast the statement of Theorem 1 using the expression of $\hat{P}$ obtained in Lemma 10.

Let $C \in \mathscr{C}$ and $X_C(v)$ be the maximiser of $x \mapsto x\left(v - \frac{x}{2}\right) - C(x)$. Then $(X_C, P(X_C))$ is an equilibrium of $\mathcal{K}(C)$. This is the unique equilibrium among the pairs (X, P) such that $X : [-1, 1] \to [-1, 1]$ is non-decreasing.

Proof of Theorem 1. From Lemma 10, $\hat{P}(x) = \frac{x}{2}$ for $x_m \leq x \leq x_M$. Since $X_C(v)$ is a maximiser of $x\left(v - \frac{x}{2}\right) - C(x)$, $x = X(v)$ is an optimal response to the expected price function $\hat{P}$ among all $x \in [x_m, x_M]$. To confirm that $(X_C, P(X_C))$ is an equilibrium, we need to check what happens if the IT makes a choice outside of the candidate support $[x_m, x_M]$, knowing that the out-of-equilibrium pricing is defined by Assumption 8. Consider for instance the case $x \in (x_M, 1]$, as

the case $x \in [-1, x_m]$ works identically. Then

$$
\begin{aligned}
\hat{P}(x) &= \frac{1}{2} \int_{x-1}^{x+1} P(z)\, \mathrm{d}z \\
&= \frac{1}{2}(x - x_M) + \frac{1}{2} \int_{x_M-1}^{x_M+1} P(z)\, \mathrm{d}z - \frac{1}{2} \int_{x_M-1}^{x-1} P(z)\, \mathrm{d}z \\
&= \frac{1}{2}(x - x_M) + \hat{P}(x_M) - \frac{1}{2} \int_{x_M-1}^{x-1} P(z)\, \mathrm{d}z \\
&= \frac{1}{2}(x - x_M) + \frac{x_M}{2} - \frac{1}{2} \int_{x_M-1}^{x-1} P(z)\, \mathrm{d}z \\
&= \frac{x}{2}.
\end{aligned}
\tag{3.15}
$$

This is because when $z \in [x_M - 1, x - 1]$, $z - 1 < x - 2 \le -1 \le x_m$ and $z + 1 \ge x_M$ so from (3.12), $v|z$ is uniform over $[-1, 1]$ and $P(z) = 0$.

As $X(v)$ maximises $x \mapsto x\left(v - \frac{x}{2}\right) - C(x)$, and $\hat{P}(x) = \frac{x}{2}$ for $x \in [-1, x_m) \cup (x_M, 1]$, $X(v)$ maximises $x \mapsto x(v - \hat{P}(x)) - C(x)$ over $[-1, 1]$: $(X_C, P(X_C))$ is an equilibrium.

We now prove uniqueness. Let $X' : [-1, 1] \to [x'_m, x'_M]$ be a non-decreasing strategy of the IT. By Lemma 10, the expected price $\hat{P}'$ associated with X' is $\frac{x}{2}$ for $x \in [x'_m, x'_M]$. But the computation of $\hat{P}'$ outside of $[x'_m, x'_M]$ is the same as the computation of $\hat{P}$ in (3.15). Hence, for all $x \in [-1, 1]$, $\hat{P}'(x) = \frac{x}{2}$. So, if $(X', P(X'))$ is an equilibrium of $\mathcal{K}(C)$ such that X' is non-decreasing, X_C and X' maximise the same objective ψ_C over $[-1; 1]$. By Lemma 13 the maximisers agree outside of a countable set, hence so do X_C and X'. In turn, we have $P(X') = P_C$. Therefore (X_C, P_C) and $(X', P(X'))$ are the same equilibrium, which establishes uniqueness. ∎

Since $\psi_C(x, v) = \psi_C(-x, -v)$, the Theorem implies that the equilibrium demand function of the IT must be an odd function. In particular we know that the minimal demand x_m equals $-x_M$.

3.3.4 Examples of equilibria

In this section, we use Theorem 1 in order to understand how the presence of penalties affects the trading strategy of the IT and the pricing function.

Consistent with intuition, penalties reduce the demand of the IT. By how much $X(v)$ is reduced depends on the functional form of the cost C and the realisation of v. This leads in general to a non-linear demand schedule. In the following examples, we will illustrate some important determinants of the IT demand.

The price function can be very flat in some regions and increase sharply in others. In particular,

the price impact of a marginal uninformed trade $\frac{d}{du}P(X(v) + u)$ strongly depends on both the realisations of u and v. By constrast, in the mimicking equilibrium of the model without penalties, this price impact is constant, regardless of the distributional assumptions on the noise.

We consider three examples of penalty: quadratic, linear, and constant over large trades.

Quadratic penalty

In this particular instance, X remains linear after the introduction of the penalty (but not P). [8] Imposing quadratic costs is akin to increasing the perceived expected price impact. Since this cost is in x^2 while the gross gains of trading are in x, the IT always trade as soon as $v \neq 0$, and the magnitude of the trade increases with the absolute value of v.

When $|d| \leq 1 - x_M (= 0.2)$, any demand of the IT is compatible with the observed aggregate order, so all values v remain equally likely, as explained in section 3.3.1. No information is incorporated and the price remains at the initial expected value of the asset: 0. When $d > 1 - x_M$, one knows that v has not realised at a very low value. This provides a lower bound on v and the price becomes positive. As d increases, so do the lower bound and the price, until $d = 1 + x_M (= 1.8)$. In that case, one knows for sure that the IT has placed an order x_M, which means that $v = 1$, and P reaches 1. The situation is symmetrical for values of d below $x_M - 1 (= -0.2)$.

Linear penalty

When the penalty is linear, $C(x) = \alpha |x|$, for positive values of v, the maximisation program of the IT can be rewritten as

$$\max_{x \in [0,1]} x \left((v - \alpha) - \frac{x}{2} \right).$$

If $v \geq \alpha$, one sees that a linear cost has the same effect as reducing the value of the fundamental v by an amount α, and having no cost. Therefore, the strategy of the IT for values $v \in [\alpha, 1]$ is a translation of the linear mimicking strategy over $v \in [0, 1 - \alpha]$. Similarly, the strategy of the IT for values $v \in [-1, -\alpha]$ is a translation of the linear mimicking strategy over $v \in [\alpha - 1, 0]$. This creates the two increasing linear segments in the left panel of Figure C.5. In the flat middle section, v is not sufficient to cover the expected penalty and the IT does not trade.

The price function depicted in the right panel of Figure C.5 exhibits a flat section in the center surrounded by increasing linear segments. The intuition is exactly the same as in the quadratic

---

[8] In the special case of a quadratic penalty, the uniform noise assumption is not necessary to obtain a tractable solution of the Kyle problem, as the model with Gaussian noise admits a linear equilibrium.

penalty case: when the magnitude of d is small ($|d| \leq \alpha (= 0.3)$), all values of v remain equally possible and no information is incorporated. As d grows, a lower bound on v can be deduced and the price increases. The key difference with the quadratic penalty case is that the price function jumps at $d = \pm 1$. Indeed, when $d > 1$, the market maker knows for sure that the insider has placed a positive order. But the IT only does so when $v > \alpha$. By contrast, if $d = 1^-$, $X(v) = 0$ remains possible, so we can only deduce that $v > -\alpha (= -0.3)$. In terms of information incorporation, there is a huge difference between $d = 1^+$ and $d = 1^-$.

Constant cost on trades of magnitude larger than x_0

Absent penalties, the IT picks $X(v) = v$. Hence, if he is sanctioned only for trades of magnitude larger than x_0, he will not change her demand as long a $|v| \leq x_0$: this corresponds to the increasing linear section in the middle of Figure C.7. For intermediate values of v, the IT prefers to block his demand at the value x_0 (or $-x_0$) in order to avoid the penalty: this corresponds to the flat sections in Figure C.7. When v becomes large enough ($|v| > \sqrt{2K} (\approx 0.63)$), the penalty is recouped in expectation by using the strategy that prevails in the absence of costs: it appears as a sunk cost and the IT selects again the demand $X(v) = v$. This corresponds to the increasing linear sections at the left and right of Figure C.7.

The price function jumps at $d = \pm(1 - x_0)(= \pm 0.9)$ and $d = \pm(1 + x_0)(= \pm 1.1)$. The intuition is as in the linear penalty case. When d exceeds $1 - x_0$, the MM knows that the demand of the IT was larger than $-x_0$ which rules out all values of v at the left of $-\sqrt{2K}$, the left jump of X. Similarly, when d exceeds $1 + x_0$, the MM knows that the demand of the IT was larger than x_0, which rules out all values of v at the left of $\sqrt{2K}$, the right jump of X.

A robustness exercise in the case of Gaussian noise is conducted in Appendix C.2.1 and shows that most of the effects described above qualitatively subsist. We do not prove formally existence of an equilibrium in the case of Gaussian noise; instead we run a fixed-point algorithm on equations (3.6) and (3.7).

3.4 Application: insider trading regulation

3.4.1 The regulator's problem

We now consider a regulator concerned about (i) the post-trade standard deviation of the fundamental, $\sigma(v|d)$, and (ii) the P&L of the uninformed traders:

$$g(u, v) = u(v - P(X(v) + u)). \tag{3.16}$$

(In section 3.4.3, the regulator additionally needs to take care of the expected fine she collects for budget reasons.)

Quantity (i) matters because one would like to have informative prices: when (i) is small, the residual uncertainty about v is also small. Quantity (ii) captures the willingness of the regulator to have liquid markets. In a liquid market, agents who have to trade for non-fundamental reasons do not experience high losses. This corresponds to a situation where g is not too negative. The core issue is that improving upon criterion (i) generally causes criterion (ii) to worsen.

Let

$$S = \mathbb{E}[\sigma(v|d)] \tag{3.17}$$

be the expectation of the post-trade standard deviation of v and

$$G = \mathbb{E}[g(u, v)] \tag{3.18}$$

denote the expected P& L of the NT.

The objective of the regulator can now be stated as the characterisation of the *efficient frontier*, with the following definition:

Definition 12 *(i) A point (G, S) is implementable if it is the outcome of an equilibrium of $\mathcal{K}(C)$ for some admissible penalty C.*

(ii) An implementable point (G, S) is dominated by (G', S') if (G', S') is implementable and $G' \geq G$, $S' \leq S$ with at least one strict inequality.

(iii) The set of implementable non-dominated points is called the efficient frontier.

In section 3.4.3, we will need the following refinement of (ii):

(ii') An implementable point (G, S) belonging to some subset of the plane H is dominated in H by (G', S') if (G', S') is implementable, $G' \geq G$, $S' \leq S$ with at least one strict inequality and $(G', S') \in H$.

Points outside the efficient frontier are irrelevant from the regulator's perspective, as she can improve upon one of her objectives without harming the other one. By contrast, any point belonging to the efficient frontier could be picked by a regulator for a suitable weighting of the objectives. Our goal is to characterise the efficient frontier and the penalties that implement it. We do so in three different settings: without a budget constraint (section 3.4.2), under a budget constraint with non-pecuniary (section 3.4.3) and pecuniary (section 3.4.3) penalties.

First, we introduce some useful notation.

Let

$$\pi^N(v) := \psi_C(X(v), v) \tag{3.19}$$

be the expected net profit of the insider trader in state v,

$$\Pi^N := \mathbb{E}_v[\pi^N(v)] \tag{3.20}$$

be the overall expected net profit (after fine, if any), and

$$F := \mathbb{E}[C(X(v))] \tag{3.21}$$

be the expected penalty that the insider undergoes.

Observe that we can write

$$|G| = \int_0^1 X(v)\left(v - \frac{X(v)}{2}\right)\mathrm{d}v = \underbrace{\int_0^1 \frac{v^2}{2}\,\mathrm{d}v}_{1/6} - \frac{1}{2}\int_0^1 (v - X(v))^2\,\mathrm{d}v. \tag{3.22}$$

This way of seeing the expected losses of the uninformed traders as (an affine transformation of) the L^2 distance between X and the identity will be useful in section 3.4.3.

3.4.2   Efficient frontier without a budget constraint

Theorem 2 *When the regulator does not face a budget constraint, the equation of the efficient frontier is*

$$S = \frac{1}{\sqrt{3}}(1 + 2G), \quad -\frac{1}{6} \le G \le 0.$$

The set of penalties that implements the efficient frontier is exactly the class $\mathcal{O}$ defined as

$$\mathcal{O} = \left\{ C \in \mathcal{C}, \exists K \in \left[0, \frac{1}{2}\right], \quad C(x) \ge x\left(\sqrt{2K} - \frac{x}{2}\right) \text{ for } 0 \le x \le \sqrt{2K}, \right.$$
$$\left. C(x) = K \qquad \qquad \text{ for } \sqrt{2K} < x \le 1 \right\}. \tag{3.23}$$

When $C \in \mathcal{O}$, the demand of the insider writes

$$X_K(v) = \begin{cases} 0 & |v| \le \sqrt{2K}, \\ v & |v| > \sqrt{2K}, \end{cases}$$

for the $K \in [0, \frac{1}{2}]$ associated with C.

Figure C.9 gives a graphical representation of functions in $\mathcal{O}$.

If two penalties in $\mathcal{O}$ are associated with the same K, they implement the same demand schedule X_K. Moreover, it is easy to see that any point in the efficient frontier is implemented by X_K for exactly one value of K.[9] Therefore, K parametrises the efficient frontier. Points associated with a small (resp. large) K are selected by a regulator who puts more weight on information incorporation (resp. on restricting the uninformed traders' losses).

Any regulator that puts nonzero weight on both objectives must at least somewhat reduce insider trading, but not totally. As we shall detail later, the optimal solution is to allow some large trades for large realisations of $|\nu|$, because they incorporate a lot of information; more precisely, the regulator wants to implement $X(\nu) = \nu$ for large values of $|\nu|$. The cutoff point $\sqrt{2K}$ in the schedule X_K then appears as the solution to the equation $\frac{\nu^2}{2} = K$. (Recall that $\frac{\nu^2}{2}$ is the profit of the IT when there is no penalty). This characterises the magnitude of ν above which the penalty appears as a sunk cost to the insider, who then effectively optimises as if there was no penalty and selects the mimicking demand $X(\nu) = \nu$.

Preliminary results on the regulator's objective

Lemma 14 *In equilibrium, the net profits satisfy*

$$\pi^N(\nu) \quad = \quad \int_0^\nu X(s)\,\mathrm{d}s, \tag{3.24}$$

$$\Pi^N \quad = \quad \int_0^1 (1-\nu)X(\nu)\,\mathrm{d}\nu. \tag{3.25}$$

Proof. Consider the parametrised objective function

$$\psi_C : [0,1] \times [0,1] \to \mathbb{R}$$

defined in (3.13). Notice that (i) $\psi_C(x,.)$ is linear in ν and therefore absolutely continuous, (ii) $|\partial_\nu \psi_C(x,\nu)| = |x| \le 1$. (i) and (ii) guarantee that the assumptions of Theorem 2 in Milgrom and Segal (2002) are satisfied. In the present case, this theorem tells us that we can write:

$$\pi^N(\nu) \quad = \quad \pi^N(0) + \int_0^\nu \partial_2 \psi_C(X(s),s)\,\mathrm{d}s$$

$$= \quad \int_0^\nu X(s)\,\mathrm{d}s,$$

---

[9]The penalty $C(x) = K\mathbb{1}_{x \ne 0}$ belongs to $\mathcal{O}$ and implements X_K. A direct calculation shows that the P&L G of the uninformed traders under the demand X_K is $-\frac{1}{6}\left(1 - (2K)^{3/2}\right)$. Hence, the value of K that implements the point (G,S) of the efficient frontier is the solution to $G = -\frac{1}{6}\left(1 - (2K)^{3/2}\right)$.

since the insider does not make any profit when the fundamental v is 0. Finally,

$$
\begin{aligned}
\Pi^N = \frac{1}{2} \int_{-1}^{1} \pi^N(v)\, dv \;\; &= \;\; \frac{1}{2} \int_{-1}^{1} \int_{0}^{v} X(y)\, dy\, dv \\
&= \;\; \int_{0}^{1} \int_{0}^{v} X(y)\, dy\, dv \\
&= \;\; \int_{0}^{1} (1-v) X(v)\, dv.
\end{aligned}
$$

■.

Lemma 15 expresses the expected post-trade standard deviation as a function of the demand profile X.

Lemma 15 *The expected post-trade standard deviation satisfies*

$$
S = \frac{1}{\sqrt{3}} \left(1 - \int_{0}^{1} v X(v)\, dv \right). \tag{3.26}
$$

Proof. By the proof of Lemma 10, $v|d$ is uniform over

$$
I_X(d) \equiv [(X_\ell^{-1}((d-1) \vee (-x_M))\,;\, X_r^{-1}((d+1) \wedge x_M))].
$$

Since the standard deviation of a uniform variable over $[a;b]$ equals $\frac{1}{2\sqrt{3}}(b-a)$, Lemma 15 is an immediate consequence of the following result: *if X is an odd non-decreasing function from $[-1;1]$ to $[-x_M;x_M]$, then the expected length of the interval $I_X(X(v)+u)$ equals* $2\left(1 - \int_0^1 v X(v)\, dv\right)$, which we must now prove.

For $v \in [-1;1]$, define

$$
\begin{aligned}
Y_v &= X_r^{-1}((X(v)+u+1) \wedge x_M) \\
Z_v &= X_l^{-1}((X(v)+u-1) \vee (-x_M)).
\end{aligned}
$$

What we need to prove is that $\mathbb{E}_{v,u}[Y_v - Z_v] = 2\left(1 - \int_0^1 v X(v)\, dv\right)$. By symmetry, $\mathbb{E}_{v,u}[Z_v] = -\mathbb{E}_{v,u}[Y_v]$, thus, it remains to prove that:

$$
\mathbb{E}_{v,u}[Y_v] = 1 - \int_{0}^{1} v X(v)\, dv.
$$

Let us consider v fixed. The random variable Y_v takes values in $[-1,1]$: using Fubini theorem,

$$
\mathbb{E}[Y_v] = \mathbb{E}\left[\int_{-1}^{1} \mathbb{1}_{-1 \le y \le Y_v}\, dy \right] - 1 = \int_{-1}^{1} \mathbb{P}\left(y \le Y_v\right) dy - 1.
$$

By definition of X_r^{-1}, if $X(y) \leq (X(v) + u + 1) \wedge x_M$ then $y \leq Y_v$. Besides, if $y < Y_v$, then using the fact that X is non decreasing, $X(y) \leq (X(v) + u + 1) \wedge x_M$. Thus:

$$\{y \leq Y_v\} \setminus \{X(y) \leq (X(v) + u + 1) \wedge x_M\} \subset \{y = Y_v\}.$$

Let us remark that $Y_v = y$ can hold for two different values of u if and only if X is discontinuous at y or $y = 1$. In particular,

$$\{y \neq 1 | \mathbb{P}(y = Y_v) > 0\} \subset \{y | X(y^-) \neq X(y^+)\}.$$

It follows from this discussion that :

$$\left| \mathbb{E}[Y_v] - \int_{-1}^{1} \mathbb{P}(X(y) \leq X(v) + u + 1) \, \mathrm{d}y + 1 \right| \leq \int_{-1}^{1} \mathbb{P}(Y_v = y) \, \mathrm{d}y \leq \mu\big(\{y | X(y^-) \neq X(y^+)\}\big),$$

where μ is the Lebesgue measure on $[-1, 1]$. Since X is non-decreasing, it has a countable number of discontinuity points. In particular $\mu\big(\{y | X(y^-) \neq X(y^+)\}\big) = 0$ and:

$$\mathbb{E}[Y_v] = \int_{-1}^{1} \mathbb{P}(X(y) \leq X(v) + u + 1) \, \mathrm{d}y - 1.$$

Now,

$$
\begin{aligned}
\mathbb{P}(X(y) \leq X(v) + u + 1) &= \mathbb{P}(u \geq X(y) - X(v) - 1) \\
&= 1 + \left(\frac{1}{2}(X(v) - X(y)) \wedge 0 \right).
\end{aligned}
$$

Going back to the expression of $\mathbb{E}[Y_v]$, we obtain

$$\mathbb{E}[Y_v] = 1 - \frac{1}{2} \int_v^1 X(y) \, \mathrm{d}y + \frac{1}{2}(1 - v)X(v).$$

Integrating over v:

$$
\begin{aligned}
\mathbb{E}_{v,u}[Y_v] &= 1 - \frac{1}{4} \int_{-1}^{1} \int_v^1 X(y) \, \mathrm{d}y \, \mathrm{d}v + \frac{1}{4} \int_{-1}^{1} (1 - v)X(v) \, \mathrm{d}v \\
&= 1 - \frac{1}{4} \int_{-1}^{1} (v + 1)X(v) \, \mathrm{d}v + \frac{1}{4} \int_{-1}^{1} (1 - v)X(v) \, \mathrm{d}v \\
&= 1 - \frac{1}{2} \int_{-1}^{1} v X(v) \, \mathrm{d}v \\
&= 1 - \int_0^1 v X(v) \, \mathrm{d}v,
\end{aligned}
$$

where in line 3, we used the fact that X is odd. This concludes the proof.■

One consequence of Lemma 15 is that large orders associated with large values of the funda-

mental are the ones that contribute the most to incorporating information into prices. Indeed, the values of v such that the product $vX(v)$ is large have the strongest negative impact on S, as can be seen from (3.26).

Shape of the efficient frontier and efficient demand functions

In this section, we give the shape of the efficient frontier and explain what demand schedules are compatible with it. We call these schedules *efficient demand functions.*

Lemma 16 *Let C be a penalty function in $\mathscr{C}$. In the equilibrium of $\mathscr{K}(C)$,*

$$S \geq \frac{1}{\sqrt{3}}(1 + 2G)$$

with equality if and only if there is $v^* \in [0,1]$ such that $X(v) = 0$ for $|v| < v^*$ and $X(v) = v$ for $|v| > v^*$.

Proof. Due to Lemma 15, what we need to show is that

$$-\int_0^1 vX(v)\,dv \geq -2\int_0^1 X(v)\left(v - \frac{X(v)}{2}\right)dv.$$

This is equivalent to

$$\int_0^1 vX(v)\,dv \geq \int_0^1 X(v)^2\,dv,$$

or

$$\int_0^1 X(v)(v - X(v))\,dv \geq 0 \tag{3.27}$$

which holds because $0 \leq X(v) \leq v$ for $v \in [0;1]$.

For the equality to hold, it is necessary and sufficient to have $X(v) = 0$ or $X(v) = v$ almost everywhere. Since X is non-decreasing, it is equivalent to $X(v) = 0$ for $|v| < v^*$ and $X(v) = v$ for $|v| > v^*$, where $v^* = \sup\{v, X(v) = 0\}$. ∎

Equation (3.27) is particularly convenient because it immediately indicates what type of demand function is needed to implement the efficient frontier. Of course, X is an endogenous outcome: what remains to be seen is what regulations implement the efficient demand functions.

Implementation of the efficient demand functions

Lemma 17 *The efficient demand functions derived in Lemma 16 are implemented exactly by the penalties $C \in \mathscr{O}$.*

Proof. See Appendix C.1.4.

By construction, penalties in $\mathcal{O}$ increase quickly as $|v|$ departs from 0 and are flat for large values of $|v|$ (see Figure C.9). Intuitively, this is what is required to implement the efficient demand functions. Indeed, when $|v|$ realises at a small value, the marginal impact of increasing demand on the expected penalty is large, and the IT prefers to refrain from trading. For $|v|$ large, however, the penalty schedule being flat on large demands, a large order allows to cover the expected fine, which appears as a sunk cost. The IT then optimises as in the linear mimicking equilibrium and demands $X(v) = v$.

The proof of Theorem 2 is complete: Lemma 16 characterises the efficient frontier and due to Lemma 17, achieving the efficient frontier can only be done by selecting a cost $C \in \mathcal{O}$, characterised by a $K \in [0, \frac{1}{2}]$.

Illustrations and discussion

Varying K between 0 and $\frac{1}{2}$ allows to cover the entire efficient frontier. As K increases, the losses $(-G)$ of the uninformed traders decrease from $\int_0^1 \frac{v^2}{2} \, dv = \frac{1}{6} \approx 0.167$ to 0, while the expected post-trade standard deviation increases from $\frac{1}{\sqrt{3}}(1 - 2/6) = \frac{2}{3\sqrt{3}} \approx 0.385$ to $\frac{1}{\sqrt{3}} \approx 0.577$.

Each point of Figure C.10 corresponds to a penalty function C; it represents the outcomes $(S, -G)$ in the unique equilibrium of $\mathcal{K}(C)$. For a fixed y-coordinate (a fixed S) the preferred option of the regulator is to select a point with the smallest x-coordinate (that minimises $-G$).

Consistent with Theorem 2, penalties in $\mathcal{O}$ achieve the efficient frontier, which is linear as indicated by Lemma 16.

Outcomes $(S, -G)$ corresponding to quadratic and linear penalties ($C(x) = \alpha x^2$, $C(x) = \alpha|x|$ for varying $\alpha \geq 0$) are also reported in Figure C.10. As one can see, they perform significantly worse than penalties $C \in \mathcal{O}$. This is also the case of penalties with no cost on small trades and big costs on large trades, $C(x) = K^H \mathbb{1}_{|x|>x_0}$. Here K^H is a constant large enough so that the insider never chooses to trade more than x_0. The fact that these particular penalty functions perform poorly compared to penalties in $\mathcal{O}$ is consistent with the intuition given below Lemma 15. Indeed, they imply that $X(v) = v$ for $|v|$ small and $X(v) = 0$ for $|v|$ large (the opposite of the demand functions implied by $C \in \mathcal{O}$), so that the reduction of the expected standard deviation, measured by the term $\int_0^1 v X(v) \, dv$, is low.

Figure C.10 shows that quadratic costs are the most inefficient among the considered costs. In fact, they have the worst performance among *all penalty functions*:

Proposition 14 *Quadratic penalties implement the upper frontier of the locus of outcomes*

(S, G) generated by all penalty functions in $\mathscr{C}$, i.e. they induce the highest possible expected post-trade standard deviation for a given $P \& L$ of the uninformed traders.

Proof. See Appendix C.1.3.

In Appendix C.2.2, we repeat numerically the construction of Figure C.10 in the case of Gaussian noise and obtain similar results.

3.4.3 Efficient frontiers under a budget constraint

So far, our analysis assumes away a real-world constraint on the regulator: investigation costs. Conducting investigations requires time, financial and human resources. How do the regulator's efficient policies change in that case?

In section 3.4.3, we consider the case of non-pecuniary penalties: the regulator cannot use fines to soften her budget constraint. In our model, this caps the maximal expected penalty that can be imposed on insider trades. Hence, in section 3.4.3, the regulator faces the same problem as before–maximising the efficiency of prices for a given level of uninformed traders' losses–yet her set of admissible strategies is narrower.

In section 3.4.3, we study pecuniary fines. We shall assume that the regulator's initial budget is insufficient to cover her investigation expenditures: she needs to levy fines to break even. Hence, in section 3.4.3, the regulator faces a richer problem than before–she now needs to consider a third criterion–yet, this time, there is no restriction *a priori* on her set of admissible strategies.

Non-pecuniary penalties

Let $B > 0$ be the budget allotted to the regulator and $\kappa > 0$ denote the investigation cost. We work under the assumption that the investigation probability α is constant–leaving the relaxation of this hypothesis for future research. The regulator operates under the constraint

$$\alpha \kappa \leq B. \tag{3.28}$$

Recall that the insider trader optimises under an expected penalty schedule $C = \alpha \tilde{C}$, where $\tilde{C}$ is the actual sanction conditional on investigation success.

First, consider the case where there is no constraint on $\tilde{C}$. Then, the regulator could trivially get around its budget constraint (3.28) by reducing α and increasing $\tilde{C}$. In that case, we are

back to the unconstrained problem solved in section 3.4.2.

We now consider the more interesting case where there is an upper bound $\tilde{C}^M$ on $\tilde{C}$.[10] From (3.28), the insider trader faces an expected penalty

$$C = \alpha \tilde{C} \le K := \frac{B}{\kappa} \tilde{C}^M. \tag{3.29}$$

The constraint (3.29) means that we have restricted the regulator's set of admissible penalties:[11]

Definition 13 *In the non-pecuniary case, the set of admissible penalties with a budget constraint is*

$$\mathscr{C}_K = \{C \in \mathscr{C}, C(1) \le K\}.$$

Note that $C(1) \le K$ is equivalent to (3.29) because any penalty in C is symmetrical and non-decreasing over $[0,1]$. Moreover, the budget constraint is an actual constraint for $K \in [0,1/2)$; for $K \ge 1/2$, $\mathscr{C}_K = \mathscr{C}$.

What happens when one restricts the set of admissible penalties? First, some previously efficient points may no longer be feasible. Second, some points that were not previously efficient may no longer be dominated by any point still implementable under the budget constraint. We recast Definition 12 in this new setting:

Definition 14 *In the non-pecuniary case, the efficient frontier under a budget constraint is the set of points (G, S) implementable by a penalty in $\mathscr{C}_K$ that are not dominated by any point implementable by a penalty in $\mathscr{C}_K$.*

As discussed above, the introduction of a constraint on the set of admissible penalties can in general make new efficient points appear. Consider for instance the situation depicted in panel (a) of Figure C.11. The dotted region represents the set of feasible points under the constraint. Points of the previously efficient frontier (oblique straight line) at the right of the dashed line are still feasible and therefore still efficient. Those at the left on the dashed line are not implementable anymore. The lower frontier of the blue area is the new efficient frontier. In particular, new efficient points appear at the left of the dashed line.

---

[10] The existence of $\tilde{C}^M$ can be justified on several grounds. First, as mentioned in Newkirk and Robertson (1998), strong sanctions are difficult to implement (see our discussion in the Introduction). Second, $\tilde{C}^M$ can be seen as the maximal disutility that a non-pecuniary sanction (such as lifetime imprisonment) can impose upon a human being.

[11] Of course, we could obtain the same constraint by ignoring investigation costs, setting $\alpha = 1$ and assuming that the cap $\tilde{C}^M$ on $\tilde{C} = C$ is below $\frac{1}{2}$. The idea here is that if investigation was systematic, the bound $\tilde{C}^M$ would likely be non-binding. It only becomes binding because investigation is costly, which reduces the expected penalty that the IT faces. The extent to which it binds depends on the budget-relevant parameters B and κ: see (3.29).

By contrast, in panel (b), there are no feasible points at the left of the dashed line: the efficient frontier is truncated.

It is a priori unclear in which situation we are. Denote

$$\mathscr{O}_K = \mathscr{O} \cap \mathscr{C}_K.$$

$\mathscr{O}_K$ is the set of efficient penalties derived in section 3.4.2, that are still feasible under the budget constraint. These penalties are still efficient under the budget constraint. Moreover, by direct computation, we obtain that as C varies in $\mathscr{O}_K$, $|G|$ describes the interval

$$\left[|G|_{\min}(K), \frac{1}{6} \right],$$

where

$$|G|_{\min}(K) := \frac{1}{6} \left(1 - (2K)^{3/2} \right). \tag{3.30}$$

The truncation of the previously efficient frontier at the right (in the $(|G|, S)$ plane) of $|G|_{\min}(K)$ is part of the efficient frontier under the budget constraint. In light of the discussion above, the key question is to know what happens at the left of $|G|_{\min}(K)$. Theorem 4 shows that no penalty in $\mathscr{C}_K$ can implement $|G| < |G|_{\min}(K)$ (i.e. we are in the situation of panel (b)). This immediately implies the characterisation of the constrained efficient frontier:

Theorem 3 *The efficient frontier under the constraint $C \leq K$ is the truncation $|G| \geq |G|_{min}(K)$ of the efficient frontier of Theorem 2 and is implemented exactly by penalties in $\mathscr{O}_K$.*

As explained above, Theorem 3 is a consequence of:

Theorem 4 *Let $K \leq 1/2$. Under the constraint $C \leq K$, the expected losses of the uninformed traders are at least*

$$|G| \geq |G|_{min}(K).$$

Proof. See Appendix C.1.5, where we provide the proof as well as several intuitions and graphical interpretations.

One consequence of Theorem 3 is that it is not possible to infer from a regulator's choice of penalty whether she is constrained or not. In the non-pecuniary case, a regulator subject to a binding budget constraint effectively behaves like an unconstrained regulator that would assign less weight to curtailing the losses of the uninformed traders. In the next section, we

study the case of pecuniary penalties and show that, by contrast to the previous result, the introduction of the constraint creates new efficient points.

Pecuniary penalties

We now consider pecuniary penalties, collected by the regulator. We maintain the assumption of a constant α. We suppose that the regulator must have a balanced budget in expectation. The budget constraint (3.28) transforms into

$$\alpha \kappa \leq B + \underbrace{\mathbb{E}[C(X(v))]}_{F}. \tag{3.31}$$

If $B \geq \alpha \kappa$, we are back to the case studied in section 3.4.2. We now consider the more interesting case where $B < \alpha \kappa$ and aim at characterising the new efficient frontier–points where S is minimal for a given G–under the constraint (3.31). This frontier will obtain by projection once we determine the *efficient surface*:

Definition 15 *The efficient surface Σ is the locus of points (G, S, F) generated by any $C \in \mathscr{C}$ such that no $C' \in \mathscr{C}$ can weakly (i) increase G, (ii) decrease S, (iii) increase F with at least one among (i), (ii) or (iii) being strict.*

Hence, our approach is the following:

- consider the three regulator's criteria of interest G, S and F symmetrically and find the efficient surface,

- deduce the efficient frontier in the (G,S) space, under the constraint (3.31), by projection.

• **The efficient surface** Σ.

Define the set of indices $J := \left\{ (x, y), 0 \leq y/(1+y) \leq x \leq y \leq 1 \right\}$.

Theorem 5 *A parametric equation of the efficient surface Σ in the space (G, S, F) is*

$$\left\{ \left(\frac{1}{6}(v_1^2 v_2 - 1); \frac{1}{\sqrt{3}} \left(\frac{2}{3} + \frac{1}{6}(v_1^2 v_2 + v_1 v_2^2) \right); \frac{v_1 v_2}{6}(3 - 2v_1 - v_2) \right) \right\}_{(v_1, v_2) \in J}$$

and it is achieved exactly by the demand schedules $\left(X_{v_1,v_2}\right)_{(v_1,v_2)\in J}$ *where*

$$
X_{v_1,v_2}(v) = \begin{cases} 0 & v \in [0, v_1] \\ \frac{v_2}{v_2-v_1}(v-v_1) & v \in (v_1, v_2] \\ v & v \in (v_2, 1] \\ -X_{v_1,v_2}(-v) & v < 0. \end{cases}
$$

These demand functions can be implemented by the penalties $(C_{v_1,v_2})_{(v_1,v_2)\in J} \in \mathscr{C}$ *where*

$$
C_{v_1,v_2}(x) = \begin{cases} v_1|x| - \frac{v_1}{2v_2}x^2 & |x| \le v_2 \\ \frac{v_1 v_2}{2} & |x| > v_2. \end{cases}
$$

Proof. See Appendix C.1.6.

There is a key difference between proving Theorem 4 and Theorem 5. Here, the most natural candidate optimiser of the weighted objective, i.e. the pointwise minimiser, turns out to be an implementable demand schedule. Reducing the problem to a pointwise minimisation exercise was not possible to prove Theorem 4.

- **Efficient** (G, S) **frontiers for various regulator's budgets**.

The regulator's budget constraint can be written as

$$
F = \mathbb{E}[C(X(v))] \ge F_{\min} := \alpha\kappa - B.
$$

Definition 16 *The F_{min}-efficient frontier is the set of non-dominated points in*

$$
\mathscr{F}(F_{min}) := \left\{ (G(X), S(X)), X \text{ implemented by some } C \in \mathscr{C} \text{ with } \mathbb{E}[C(X(v))] \ge F_{min} \right\}.
$$

We can now construct the $F_{\min}$-efficient frontiers from the efficient surface Σ (denote π_{GS} : $(G, S, F) \mapsto (G, S)$ the projection on the (G, S)-plane):

Proposition 15 *The F_{min}-efficient frontier is the set of points of $\pi_{GS}(\Sigma \cap \{F \ge F_{min}\})$ that are not dominated in $\pi_{GS}(\Sigma \cap \{F \ge F_{min}\})$.*

Proof. See Appendix C.1.7.

This means that to obtain the $F_{\min}$-efficient frontier, one must first project the relevant points (G, S, F) of Σ, and then select those that are efficient in the plane (note that this second step is

necessary, as the projection of a point of the efficient surface will in general not be a point of the efficient frontier). Σ was found by solving an optimisation problem, from which the $F_{\min}$-efficient frontiers are deduced geometrically: we do not need to solve again a minimisation problem.

We are now in a position to provide the $F_{\min}$-efficient frontiers: see Figure C.14.

An important difference emerges with respect to the case of non-pecuniary penalties studied above. Here, the efficient frontiers are not truncatures of the frontier that obtains absent the constraint. Of course, the penalties in $\mathcal{O}$ that implement $F \geq F_{\min}$ are still part of the $F_{\min}$-efficient frontier, but new constrained efficient points emerge (dotted arcs), which are associated with penalty functions and demand schedules that were not previously optimal. The $F_{\min}$ efficient frontier does not even intersect the unconstrained frontier for $F_{\min}$ very large. To see why, note that the maximal expected fine under a penalty in $\mathcal{O}$ is

$$
\begin{aligned}
\max\{F(X), X \text{ implemented by } C \in \mathcal{O}\} &= \max_{0 \leq K \leq 1/2} K\left(1 - \sqrt{2K}\right) \\
&= \frac{2}{27} \approx 0.074, \text{ attained by } K = \frac{2}{9}.
\end{aligned}
$$

This means that if $F_{\min} > \frac{2}{27}$, no penalty in $\mathcal{O}$ allows to balance the regulator's budget.[12]

- **Efficient price and demand functions**.

From Lemma 15, we know that points of the $F_{\min}$-efficient frontier correspond to points in Σ, which means that they are associated with demand schedules of the form X_{v_1, v_2} defined in Theorem 5. To understand how the budget constraint $F \geq F_{\min}$ modifies the nature of the optimal strategies, Figure C.15 plots the (v_1, v_2) used on the $F_{\min}$-efficient frontier for various values of $F_{\min}$.

When $F_{\min} = 0$, we obtain the line $v_2 = v_1$, in which case X_{v_1, v_2} is implemented by a penalty $C \in \mathcal{O}$, consistent with section 3.4.2. We observe that as $F_{\min}$ increases, one needs to widen the gap $v_2 - v_1$. The intuition is that the linear section over $[v_1, v_2]$ of the demand schedule X_{v_1, v_2} best resolves the trade off between large fines and large trade volumes of the insider trader and allows to collect a relatively high amount of fines in expectation. As an example, recall that the demand schedule that implies the highest expected fine (1/12) had $v_2 - v_1 = \frac{1}{2}$.

When the regulator must balance its budget through the collection of pecuniary fines, some previously optimal strategies are no longer feasible as they do not induce the insider trader to pay enough fines in expectation. New constrained efficient points appear, and the class

---

[12] The highest expected fine that the regulator can levy is $F = \frac{1}{12}$. She can do so using the penalty $C_{\frac{1}{2}, 1}$ (defined in Theorem 5). In particular, if $F_{\min} > \frac{1}{12}$, the regulator cannot balance her budget.

of efficient penalties is modified, as well as the equilibrium demand schedules and price functions.

Figure C.16 compares the price functions implied by a demand schedule efficient absent a budget constraint, X_{ν_2, ν_2} and a constrained efficient demand schedule X_{ν_1, ν_2}. Contrary to $P\left(X_{\nu_2, \nu_2}\right)$, $P\left(X_{\nu_1, \nu_2}\right)$ has no flat sections and is everywhere increasing. In particular, in the unconstrained case, the random price is partly discrete: with positive probability, it will be equal to one of the ordinates of the flat sections of $P(X_{\nu_2, \nu_2})$. Conversly, in the case of a strong budget constraint, the random price has a continuous density.

3.5   Conclusion

We have shown how one can obtain an existence and uniqueness result in a Kyle model of insider trading by assuming uniformly–instead of normally–distributed noise. One advantage of our approach is that the proofs bear an intuitive interpretation and the resulting equilibrium is tractable. But the key appeal of our techniques is that they apply indifferently to the cases of legal or illegal insider trading.

We use our existence and uniqueness theorem to contribute to the long-standing debate on insider trading regulation. We solve the problem of a regulator aiming at maximising price efficiency for a given level of noise traders' losses in three cases: unconstrained, constrained with non-pecuniary penalties and constrained with pecuniary penalties.

Our results can be readily used as a building block in a more general model featuring strategic– but illegal–trading of superiorly informed agents. But the scope of our results extends beyond insider trading questions: for instance, one could interpret the penalty functions as size-dependent transaction costs.

We would like to mention two potentially interesting extensions of the chapter. First, one could endogeneise the investigation probability α. Second, one may want to consider a penalty schedule which does not depend solely on the magnitude of the insider's order, but also on other observable quantities–such as the realised profit for example.

A Appendix for Chapter 1

A.1 Proofs

A.1.1 Lemma 1

By definition of $(\tilde{D}_t)$, $\mathbb{E}\left[\tilde{D}_{(t+1)\wedge\zeta_f}|\mathcal{F}_t^I\right] = \mathbb{E}\left[\tilde{D}_{t+1}|\mathcal{F}_t^I\right] = \tilde{D}_t = \tilde{D}_{t\wedge\zeta_f}$ over $\{t < \zeta_f\}$. And $\tilde{D}_{(t+1)\wedge\zeta_f} = \tilde{D}_{t\wedge\zeta_f} = D_{\zeta_f}$ over $\{t \geq \zeta_f\}$. Therefore $\mathbb{E}[\tilde{D}_{(t+1)\wedge\zeta_f}|\mathcal{F}_t^I] = \tilde{D}_{t\wedge\zeta_f}$.

A.1.2 Lemma 2

From Lemma 1, $(\tilde{D}_{t\wedge\zeta_f})$ is a martingale, and it is bounded due to condition (NP). Therefore it is a closed martingale. Now consider a bank at t which is not forced into liquidation. This means that $\zeta_f > t$, and since $(\tilde{D}_{t\wedge\zeta_f})$ is a closed $(\mathcal{F}_t^I)$-martingale, $\mathbb{E}[\tilde{D}_{\zeta_f}|\mathcal{F}_t^I] = D_t$. Now, note that $\zeta_f = \zeta_\ell \wedge \zeta_\phi$ and that

$$\tilde{D}_{\zeta_\ell}\mathbb{1}_{\{\zeta_\ell < \zeta_\phi\}} + \tilde{D}_{\zeta_\phi}\mathbb{1}_{\{\zeta_\ell \geq \zeta_\phi\}} \leq V_{\zeta_\ell}\mathbb{1}_{\{\zeta_\ell < \zeta_\phi\}} + y_{\zeta_\phi}\mathbb{1}_{\{\zeta_\ell \geq \zeta_\phi\}}. \tag{A.1}$$

Taking expectations and noting that the expectation of the right-hand side is the same average of maturity values y_{ζ_ϕ} as $V_t \equiv \mathbb{E}[y_{\zeta_\phi}|\mathcal{F}_t^I]$, we obtain

$$D_t = \mathbb{E}[\tilde{D}_{\zeta_f}|\mathcal{F}_t^I] \leq V_t. \tag{A.2}$$

The bank is solvent.

A.1.3 Lemma 3

Assume voluntary disclosure (the proof in the mandatory disclosure case is included in this one). Consider a time $t < \zeta_f$. If $\delta_t = \emptyset$ and the banker decides to liquidate, she obtains the

value $\alpha V^B < D_0 < D_t$ and her equity is worth zero. Now, if $\delta_t \neq \emptyset$, payoff-relevant information is symmetric:

$$
\begin{aligned}
V_t \equiv \mathbb{E}[y_{\zeta_\phi}|\mathscr{F}_t^I] &= \mathbb{E}[y_{\zeta_\phi}|\mathscr{F}_t^B] \\
\mathbb{E}[\tilde{D}_{\zeta_f}|\mathscr{F}_t^I] &= \mathbb{E}[\tilde{D}_{\zeta_f}|\mathscr{F}_t^B].
\end{aligned}
\tag{A.3}
$$

Since $(\tilde{D}_{t \wedge \zeta_f})$ is a closed martingale,

$$
D_t = \tilde{D}_t = \mathbb{E}\left[\lim_{s \to \infty} \tilde{D}_{s \wedge \zeta_f}|\mathscr{F}_t^I\right] = \mathbb{E}[\tilde{D}_{\zeta_f}|\mathscr{F}_t^I].
\tag{A.4}
$$

Combining (A.3) and (A.4), we obtain

$$
\begin{aligned}
\alpha V_t - D_t &= \mathbb{E}[\alpha y_{\zeta_\phi}\mathbb{1}_{\{\zeta_\phi \leq \zeta_\ell\}} + \alpha y_{\zeta_\ell}\mathbb{1}_{\{\zeta_\phi > \zeta_\ell\}} - \tilde{D}_{\zeta_f}|\mathscr{F}_t^I] \\
&= \mathbb{E}[\alpha y_{\zeta_\phi}\mathbb{1}_{\{\zeta_\phi \leq \zeta_\ell\}} + \alpha y_{\zeta_\ell}\mathbb{1}_{\{\zeta_\phi > \zeta_\ell\}} - \tilde{D}_{\zeta_f}|\mathscr{F}_t^B] \\
&< \mathbb{E}[y_{\zeta_\phi}\mathbb{1}_{\{\zeta_\phi \leq \zeta_\ell\}} + \alpha y_{\zeta_\ell}\mathbb{1}_{\{\zeta_\phi > \zeta_\ell\}} - \tilde{D}_{\zeta_f}|\mathscr{F}_t^B].
\end{aligned}
\tag{A.5}
$$

The first line is the payoff from liquidating today. The last line is the $\mathscr{F}_t^B$-expected payoff of never liquidating strategically.

A.1.4   Lemma 4

First note that for $p \in (0,1)$, in a consistent belief system, $q_t = 0$ after disclosure of the bad state, $q_t = 1$ after disclosure of the good state, and $q_t \in (0,1)$ absent disclosure. Fix such a belief system and show that the sanitisation strategy is optimal. Due to discounting (Blackwell (1965)) we can focus on one-shot deviations. Note that under F and for a given D_0, the event tree is discrete. This is because there are always at most 4 possible states tomorrow, given the state today. Let us consider the choice between playing the sanitisation strategy δ^S at some node or something else, leaving the rest of the strategies unchanged. Let $\mathcal{O}$ be the event tree following playing δ^S at this node and $\mathcal{D}$ the event tree following the other move (the "deviation"). The deviation is either the regulator switching from concealing the bad state to disclosing it, or concealing the good state instead of disclosing it. The former case is equivalent to a strategic default enforced by the regulator, which is never optimal, similarly to Lemma 3. Thus, focus on the latter case and relabel $t = 0$ the deviation time (at which $y_0 = y^G$), and let $y_1, \ldots, y_n, \ldots$ and $J = \zeta_\phi$ be a possible realisation of future asset states and maturity. By condition (M) and induction, the face values $\tilde{F}_0, \ldots, \tilde{F}_n, \ldots$ associated with y_0 undisclosed and the realisations $y_1, \ldots, y_n, \ldots$ disclosed according to the sanitisation strategy satisfy $\tilde{F}_i \geq F_i$, where F_i are the face values in $\mathcal{O}$. Let j be the liquidation time in $\mathcal{D}$. Three cases are possible. (i) $j \leq J - 1$ and there is liquidation at time j in $\mathcal{O}$: then there is liquidation at time j in both $\mathcal{O}$ and $\mathcal{D}$. Since due debt is higher in $\mathcal{D}$ ($\tilde{F}_{j-1} \geq F_{j-1}$), the residual claim is lower in $\mathcal{D}$. (ii)

$j \le J-1$ and there is no liquidation at time j in $\mathscr{O}$: there, debt is lower, and the arguments of the proof of Lemma 3 allow to conclude that the expected residual claim at time j conditional on $\zeta_\phi = J$ is higher in the original tree. (iii) $j \ge J$: the asset matures before liquidation both in $\mathscr{O}$ and $\mathscr{D}$. Since debt is lower in $\mathscr{O}$, the expected residual claim is higher in $\mathscr{O}$. Finally, note that the expected profit at date $t = 0$ is an average of expectations of the residual claim conditional on $y_0 = y^G, y_1, \dots, y_j, \zeta_\phi = J$. Cases (i), (ii) and (iii) above show that these quantities are higher in $\mathscr{O}$ for all J, j, $y_1, \dots, y_j$. Hence, it is optimal for the regulator to play the sanitisation strategy. The belief system consistent with this strategy is then the one given in section 1.3.1.

A.1.5 Lemma 6

Voluntary disclosure case. In equations (1.19) and (1.20), we obtained the recursive relationship

$$q_{\tau+1} = \frac{(1-p)(q_\tau \lambda^{GG} + (1-q_\tau)\lambda^{BG})}{1-p+p(1-q_\tau \lambda^{GG} - (1-q_\tau)\lambda^{BG})}. \tag{A.6}$$

A standard sequence analysis reveals that (q_τ) decreases to the root of G that lies in $[0,1]$, with $G \equiv G_1 - G_2$ and

$$G_1(q) \equiv q(1-p+p(1-q\lambda^{GG}-(1-q)\lambda^{BG})) \tag{A.7}$$

$$G_2(q) \equiv (1-p)\lambda^{BG} + q(1-p)(\lambda^{GG}-\lambda^{BG}). \tag{A.8}$$

This root is the stationary weight in case of voluntary disclosure, q_V^*, and satisfies

$$q_V^* = \frac{1-(1-p)\lambda^{GG}-(2p-1)\lambda^{BG} - \sqrt{(\lambda^{BG})^2 + 2\lambda^{BG}(\lambda^{GG}(p-1)-2p+1) + (1-(1-p)\lambda^{GG})^2}}{2p(\lambda^{GG}-\lambda^{BG})}. \tag{A.9}$$

Mandatory disclosure case. The expression of q_M^* in (1.36) results directly from considering the fixed point of (1.35). We now set out to obtain the inequality $q_V^* < q_M^*$. Since G can only be non-negative for $q \ge q_V^*$, it is sufficient to show that $G(q_M^*; p) \ge 0$ (with obvious notation) for any value of p. Direct calculation shows that

$$\frac{\partial}{\partial p} G(q_M^*; p) \tag{A.10}$$

has the same sign as $1 + \lambda^{BG} - 2\lambda^{GG}$. In particular, it is of constant sign and we obtain

$$G(q_M^*; p) \ge \min\{G(q_M^*; 0), G(q_M^*; 1)\}. \tag{A.11}$$

Since $G(q_M^*; 0) = 0$ and $G(q_M^*; 1) > 0$, we obtain $G(q_M^*; p) \ge 0$.

A.1.6 Proposition 2

We know that in a consistent bank policy, $m_\tau(F(D,\tau)) = D$. The banker picks the lowest F that satisfies this equation, because expected liquidation costs are increasing in F. Hence, in equilibrium,

$$F(D,\tau) = \min\{F \geq 0, m_\tau(F) = D\}. \tag{A.12}$$

In order to find F, we need to make m_τ explicit. If $F \leq y^B$, the promise of F is never defaulted upon: $m_\tau(F) = F$. If $F \leq C(\tau + 1)$, there is one default state (state χ_2, see section 1.3.1) and

$$m_\tau(F) = \phi(\gamma_\tau F + (1 - \gamma_\tau)y^B) + (1 - \phi)F. \tag{A.13}$$

If $F \in (C(\tau + 1), C(0)]$, there are two default states (χ_2 and χ_3) and

$$m_\tau(F) = \phi(\gamma_\tau F + (1 - \gamma_\tau)y^B) + (1 - \phi)(p\gamma_\tau F + \alpha(1 - p\gamma_\tau)V_{\tau+1}). \tag{A.14}$$

If F belongs to $(C(0), y^G]$, there are three default states (χ_2, χ_3 and χ_4) and

$$m_\tau(F) = \phi(\gamma_\tau F + (1 - \gamma_\tau)y^B) + \alpha(1 - \phi)(p\gamma_\tau V_0 + (1 - p\gamma_\tau)V_{\tau+1}). \tag{A.15}$$

A.1.7 Proposition 3

The probability of an announcement tomorrow is $p\gamma_\tau$. The probability of no announcement is $1 - p$. Otherwise, state y^B is disclosed (probability $p(1 - \gamma_\tau)$). First, if $F \leq y^B$, $m_\tau(F) = F$. If $y^B < F \leq C(\tau + 1)$, then

$$m_\tau(F) = \phi(\gamma_\tau F + (1 - \gamma_\tau)y^B) + (1 - \phi)((1 - p(1 - \gamma_\tau))F + p(1 - \gamma_\tau)\alpha V^B). \tag{A.16}$$

If $C(\tau + 1) < F \leq C(0)$, then

$$m_\tau(F) = \phi(\gamma_\tau F + (1 - \gamma_\tau)y^B) + (1 - \phi)(p\gamma_\tau F + (1 - p)\alpha V(\tau + 1) + p(1 - \gamma_\tau)\alpha V^B). \tag{A.17}$$

If $C(0) < F \leq y^G$, then

$$m_\tau(F) = \phi(\gamma_\tau F + (1 - \gamma_\tau)y^B) + \alpha(1 - \phi)(p\gamma_\tau V^G + (1 - p)V(\tau + 1) + p(1 - \gamma_\tau)V^B). \tag{A.18}$$

A.1.8 Proposition 4

Proofs are presented in the voluntary disclosure case, and work identically in the mandatory disclosure case. I first need to introduce the

Lemma 18 *Let τ be a fixed integer and $0 < p^* < 1$. If $\alpha < 1$, there is $K_\tau > 0$ such that for all $p \le p^*$,*

$$C^{[p]}(\tau) \ge \alpha V_\tau^{[p]} + K_\tau, \tag{A.19}$$

where the superscript $[p]$ designates a variable relative to the model solution under the opacity parameter p.

Proof. Promising y^G entails costly liquidation tomorrow unless the asset matures. Hence,

$$m_\tau^{[p]}(y^G) = \phi(1-\alpha)\left(\gamma_\tau^{[p]} y^G + (1 - \gamma_\tau^{[p]})y^B\right) + \alpha V_\tau^{[p]}. \tag{A.20}$$

We obtain the result by setting $K_\tau = \phi(1-\alpha)\gamma_\tau^{[p^*]}$, noting that $C^{[p]}(\tau) \ge m_\tau^{[p]}(y^G)$■.

We now come back to the proof of Proposition 4.

(a) Consider two states (D, τ_{II}) and (D, τ_{IS}) belonging to the II and the IS region, respectively. Because of the monotonicity of the pricing schedule, we only need to verify that $\tau_{II} < \tau_{IS}$. Assume this is not true. Because (D, τ_{II}) is in the II region, there is $F \le C(\tau_{II} + 1) \le C(\tau_{IS} + 1)$ such that

$$D = \phi\left(\gamma_{\tau_{II}} F + (1 - \gamma_{\tau_{II}})y^B\right) + (1-\phi)F. \tag{A.21}$$

Hence,

$$D \le \phi\left(\gamma_{\tau_{IS}} F + (1 - \gamma_{\tau_{IS}})y^B\right) + (1-\phi)F \equiv f(F). \tag{A.22}$$

The function f is continuous and satisfies $f(D) < D$ and $f(F) \ge D$, hence there is $\tilde{F} \in (D, F]$ such that $f(\tilde{F}) = D$. By construction, $\tilde{F}$ provides an II contract in state (D, τ_{IS}), a contradiction with the fact that (D, τ_{IS}) is in the IS region.

(b) We first need to show that for p small, $m_\tau(C(0)) < m_\tau(C(\tau + 1))$. This implies that promising face values between $C(\tau + 1)$ and $C(0)$ does not allow to roll over other debt levels than the ones in the II zone: there is no IS zone. Given the expressions of $m_\tau(C(0))$ and $m_\tau(C(\tau + 1))$, the desired inequality is equivalent to

$$p\gamma_\tau^{[p]}C(0) + \alpha(1 - p\gamma_\tau^{[p]})V_{\tau+1}^{[p]} < C^{[p]}(\tau + 1). \tag{A.23}$$

We conclude by letting $p \to 0$ and using Lemma 18. For the case $p \to 1$, recall that debt capacity is always below the fundamental value from Lemma 2. In the voluntary disclosure case, as p goes to 1, $q_{\tau+1}$ goes to 0, so the fundamental value goes to V^B. Now let $D > V^B$. We have

$$m_\tau^{[p]}(C^{[p]}(\tau + 1)) < C^{[p]}(\tau + 1) \le V_{\tau+1}^{[p]} \to V^B, \tag{A.24}$$

hence D can not be in the II zone for p close enough to 1.

(c) is a consequence of the fact that $m_\tau(C(\tau+1)+\varepsilon) < m_\tau(C(\tau+1))$ for ε close to 0 and $\alpha < 1$. Recall that this is because the face value is only infinitesimally higher, but there will be default in one more state of the world (the non-disclosure state), meaning that the proportional cost $1-\alpha$ now applies to an additional, non-zero probability, state of the world.

(d) (d1) Let $p_1 < p_2$, τ_1, τ_2 such that

$$q_{\tau_1}^{[p_1]} = q_{\tau_2}^{[p_2]} = q. \tag{A.25}$$

The probability to be in state y^G tomorrow is $q' = \lambda^{GG}q + \lambda^{BG}(1-q) = \gamma_{\tau_1}^{[p_1]} = \gamma_{\tau_2}^{[p_2]}$. Then, the probability to be in state y^G tomorrow conditional on no disclosure under parameter p_1 is $\frac{(1-p_1)q'}{1-p_1q'}$. Using the expression of the yield in the IS region, we find

$$
\begin{aligned}
m^{[p_1]}(F) \;=\; & \phi(q'F+(1-q')y^B) \\
+ \;& (1-\phi)\left(q'p_1F+\alpha(1-p_1q')\left[\frac{(1-p_1)q'}{1-p_1q'}V^G+\frac{1-q'}{1-p_1q'}V^B\right]\right). \tag{A.26}
\end{aligned}
$$

From there,

$$m^{[p_1]}(F) - m^{[p_2]}(F) = (1-\phi)(p_2-p_1)q'(\alpha V^G - F), \tag{A.27}$$

which is negative for F close to $C(0)$ by Lemma 18. Given that $D = m^{[p_1]}(F^{[p_1]})$, we have $D < m^{[p_2]}(F^{[p_1]})$, from which we deduce that $F^{[p_1]} > F^{[p_2]}$. Indeed, (D,τ_2) belongs to the IS region under p_2, and $m^{[p_2]}(.)$ is increasing over this region, and must satisfy $D = m^{[p_2]}(F^{[p_2]})$.

(d2) This part of the proposition is clear from the expression of yields. There is equality in the voluntary disclosure case, and strict inequality in the mandatory disclosure, because increasing p increases the probability of having to disclose bad news.

(e) Let $F^V \equiv F^V(D,q)$ and q' be defined as above. We have

$$\phi\left(q'F^V+(1-q')y^B\right)+(1-\phi)F^V = D. \tag{A.28}$$

For $F \le F^V$,

$$
\begin{aligned}
m^M(F) \;\le\; & \phi\left(q'F^V+(1-q')y^B\right)+(1-\phi)\left((1-q')py^B+(1-(1-q')p)F\right) \\
< \;& \phi\left(q'F+(1-q')y^B\right)+(1-\phi)F \\
\le \;& \phi\left(q'F^V+(1-q')y^B\right)+(1-\phi)F^V \\
= \;& D. \tag{A.29}
\end{aligned}
$$

Hence, $m^M(F) < D$ for $F \le F^V$, implying that $F^M > F^V$.

A.1.9 Proposition 5

First, we need to show that for any $\alpha \in \left(\frac{D_0}{V^G}, 1\right)$, $\mathscr{I}_1(\alpha) < \mathscr{I}_0(\alpha)$ (with obvious notation). Under our assumptions, we have

$$t_1(\alpha) = \frac{1}{p_c} \ln \frac{\alpha V^G}{D_0} \tag{A.30}$$

and

$$\mathscr{I}_1(\alpha) = (1 - \alpha) e^{-(p_c + \phi) t_1(\alpha)} V^G. \tag{A.31}$$

Moreover, we know that $t_0(\alpha)$ is the unique solution to $f_\alpha(t) = g_\alpha(t)$, where

$$f_\alpha(t) \equiv \alpha V^G e^{-p_c t} \tag{A.32}$$

$$g_\alpha(t) \equiv D_0 e^{\phi t + \frac{\phi}{p_c}(e^{-p_c t} - 1)}, \tag{A.33}$$

and

$$\mathscr{I}_0(\alpha) = (1 - \alpha) V^G e^{-(p_c + \phi) t_0(\alpha)}. \tag{A.34}$$

Therefore, we need to show that $t_1 > t_0$ over $\left(\frac{D_0}{V^G}, 1\right)$. Since f_α decreases and g_α increases, it is sufficient to show that $f_\alpha(t_1(\alpha)) < g_\alpha(t_1(\alpha))$. This boils down to show

$$D_0 < D_0 \exp\left(\frac{\phi}{p_c} \ln \frac{\alpha V^G}{D_0} + \frac{\phi}{p_c}\left(\frac{D_0}{\alpha V^G} - 1\right)\right), \tag{A.35}$$

or

$$-\ln \frac{D_0}{\alpha V^G} + \frac{D_0}{\alpha V^G} - 1 > 0 \tag{A.36}$$

which holds true because $\ln x < x - 1$ for $x \in (0, 1)$.

Now, $\mathscr{P}_1 < \mathscr{P}_0$ is equivalent to saying that $t_0(\alpha) < t(\alpha) \equiv -\frac{1}{\phi} \ln \mathscr{P}_1(\alpha)$. As before, this is equivalent to $f_\alpha(t(\alpha)) < g_\alpha(t(\alpha))$ for α small, or

$$\frac{\alpha V^G}{D_0} < \exp\left((\phi + p_c) t(\alpha) + \frac{\phi}{p_c}\left(e^{-p_c t(\alpha)} - 1\right)\right). \tag{A.37}$$

Noting that the expressions only depend on the ratio ϕ / p_c and $x = \frac{\alpha V^G}{D_0}$, we can assume w.l.o.g. that $p_c = 1$ and it is sufficient to study when the inequality

$$x < h(j(x)) \tag{A.38}$$

holds, with

$$j(x) \quad = \quad -\frac{1}{\phi}\log\left(\frac{1}{1+\phi} + \frac{\phi}{1+\phi}e^{-(1+\phi)\log x}\right) \tag{A.39}$$

$$h(x) \quad = \quad \exp\left((1+\phi)x + \phi\left(e^{-x} - 1\right)\right). \tag{A.40}$$

j is increasing and

$$j^{-1}(y) = \exp\left(-\frac{1}{1+\phi}\log\left(\frac{\phi+1}{\phi}e^{-\phi y} - \frac{1}{\phi}\right)\right). \tag{A.41}$$

(A.38) is equivalent to $j^{-1}(y) < h(y)$, or, taking logs:

$$r(y) \equiv (1+\phi)y + \phi\left(e^{-y} - 1\right) > s(y) \equiv -\frac{1}{1+\phi}\log\left(\frac{\phi+1}{\phi}e^{-\phi y} - \frac{1}{\phi}\right), \tag{A.42}$$

with $0 \le y \le y_M \equiv \frac{1}{\phi}\log(1+\phi)$. Now note that $r(0) = s(0) = 0$, $r'(0) = s'(0) = 1$, $r''(0) = \phi$, $s''(0) = 1$, $r(y_M^-) < s(y_M^-) = +\infty$. Given these variations, it is now sufficient to show that $(r-s)''$ can only switch sign at most once. But

$$(r-s)'''(y) = -\phi e^{-y} - \phi^3 e^{\phi y}\frac{1+\phi+e^{\phi y}}{(1+\phi - e^{\phi y})^3} < 0 \tag{A.43}$$

for $0 \le y < y_M$. If $\phi < p_c$, $r - s < 0$ over $(0, y_M)$. When $\phi > p_c$, $r - s$ is positive in the neighborhood of 0 and negative close to y_M, so there exists y_0 with $(r-s)(y_0) = 0$ and given the variations of $r - s$ given above, we have $r - s > 0$ over $(0, y_0)$ and $r - s < 0$ over (y_0, y_M). This concludes the proof.

A.1.10 Proposition 6

From Proposition 4, point **(b)**, we know that for p close enough to 1, the II zone disappears in the voluntary disclosure case. This means that disclosure of y^G is necessary to avoid liquidation; the regulator cannot conceal the bad state, which is a default state under both mandatory and voluntary disclosure. But there is a default state that exists only under voluntary disclosure and has positive probability $1 - p$: the event that the regulator actually did not observe the asset ($\omega_t = 0$). It is then immediate from the expressions of the debt capacities and the bond yields in Propositions 1,2 and 3 that the voluntary debt capacity is necessarily attained first, *i.e.* liquidation always occurs weakly after under mandatory disclosure. Since it occurs strictly after with positive probability (the cases where $\omega_t = 0$), we obtain that mandatory disclosure is both strictly more efficient and produces strictly less runs.

A.2 Additional Material

A.2.1 Dynamics in the Continuous-Time Limit

In the limit of vanishing debt maturity, the Markov chain with transition matrix Λ becomes a continuous-time Markov chain with infinitesimal generator

$$A = \begin{pmatrix} -p_c & p_c \\ p_r & -p_r \end{pmatrix}. \tag{A.44}$$

$p_c\,dt$ is the instantaneous probability to move from the good to the bad state. The bank only observes the chain at times $(\zeta_o^k)_{k \geq 1}$, which are exponentially distributed with parameter p_o. If the bank has to disclose the bad state at ζ_o, the game ends; we now consider times before the termination date ζ_f. The corresponding definition of the variable τ in the continuous-time version of the model is

$$\tau = t - \sup\{\zeta_o^k, \ \zeta_o^k \leq t, \ y_{\zeta_o^k} = y^G\}. \tag{A.45}$$

Fundamental values and debt capacities. The transition probabilities of the Markov chain with generator A at horizon t are

$$P_{GG}(t) = \frac{p_r}{p_c + p_r} + \frac{p_c}{p_c + p_r} e^{-(p_c + p_r)t} \tag{A.46}$$

$$P_{BG}(t) = \frac{p_r}{p_r + p_c} - \frac{p_r}{p_r + p_c} e^{-(p_c + p_r)t}, \tag{A.47}$$

and the fundamental values $V^i = \mathbb{E}\left[y_{\zeta_\phi} | y_0 = y^i\right]$ satisfy

$$\begin{pmatrix} V^G \\ V^B \end{pmatrix} = \varphi(\varphi I - A)^{-1}\mathbf{y},$$

or

$$\begin{pmatrix} V^G \\ V^B \end{pmatrix} = \frac{1}{\varphi + p_c + p_r} \begin{pmatrix} \varphi + p_r & p_c \\ p_r & \varphi + p_c \end{pmatrix} \mathbf{y} \tag{A.48}$$

$$= \frac{1}{\varphi + p_c + p_r} \begin{pmatrix} \varphi + p_r \\ p_r \end{pmatrix} y^G \quad \text{if } y^B = 0, \tag{A.49}$$

which we assume from now on. The fundamental value in state τ is then given by

$$V_\tau = q_\tau V^G + (1 - q_\tau) V^B \tag{A.50}$$

where (q_τ) depends on the disclosure regime; and taking the limit in the analytical expression

of debt capacities, we obtain that $C(\tau) = \alpha V_\tau$ in both regimes.

Beliefs dynamics. Under mandatory disclosure,

$$q_\tau = P_{GG}(\tau) \quad \text{for} \quad t < \zeta_f. \tag{A.51}$$

Under voluntary disclosure, the beliefs dynamics is given by the equation

$$dq_\tau = (-p_c q_\tau + p_r(1 - q_\tau) - p_o q_\tau(1 - q_\tau))d\tau :$$

q_τ solves the Ricatti equation

$$q' = p_r - (p_c + p_r + p_0)q + p_o q^2.$$

Let $\varrho_1 = p_r p_0$ and $\varrho_2 = -(p_c + p_r + p_0)$,

$$\kappa_1 = \frac{\varrho_2 + \sqrt{\varrho_2^2 - 4\varrho_1}}{2} < 0$$

$$\kappa_2 = \frac{\varrho_2 - \sqrt{\varrho_2^2 - 4\varrho_1}}{2} < \kappa_1.$$

We have

$$q_\tau = -\frac{1}{p_o} \frac{\kappa_1 k e^{\kappa_1 \tau} + \kappa_2 e^{\kappa_2 \tau}}{k e^{\kappa_1 \tau} + e^{\kappa_2 \tau}}. \tag{A.52}$$

Note that $\pi_\infty = -\frac{\kappa_1}{p_0} = \frac{-\varrho_2 - \sqrt{\varrho_2^2 - 4\varrho_1}}{2p_0}$, which is indeed the unique root of $\pi' = 0$ belonging to $(0, 1)$. k is determined by the condition $q_0 = 1$:

$$k = -\frac{p_o + \kappa_2}{p_o + \kappa_1}.$$

Debt dynamics. *Full transparency.* Away from $C = \alpha V^G$, the only risk is the observation of the bad state, which happens with probability $p_c dt$. Hence

$$D_t = \alpha V^B + (D_0 - \alpha V^B)e^{p_c t}. \tag{A.53}$$

Let

$$t^1(\alpha) = \frac{1}{p_c} \ln \frac{\alpha(V^G - V^B)}{D_0 - \alpha V^B} \tag{A.54}$$

be the maximal time the bank can survive—even in the best scenario. Let ζ_c be the time of jump to the bad state.

Case 1: t^1 realises before ζ_ϕ and ζ_c. This event has probability $e^{-(p_c+\phi)t^1}$. Then, liquidation happens at αV^G, repaying exactly creditors and leaving zero to the banker.

Case 2: ζ_ϕ realises before t^1 and ζ_c. This event has probability $\frac{\phi}{p_c+\phi}\left(1 - e^{-(p_c+\phi)t^1}\right)$. Then, debt is repaid in full and y^G realises.

Case 3: ζ_c realises first. This event has probability $\frac{p_c}{p_c+\phi}\left(1 - e^{-(p_c+\phi)t^1}\right)$. Then, liquidation occurs, at αV^B.

Hence

$$
\mathscr{P} = 1 - \frac{\phi}{p_c+\phi}\left(1 - e^{-(p_c+\phi)t^1(\alpha)}\right) \tag{A.55}
$$

$$
\mathscr{I} = (1-\alpha)\left(e^{-(p_c+\phi)t^1(\alpha)}V^G + \frac{p_c}{p_c+\phi}\left(1 - e^{-(p_c+\phi)t^1(\alpha)}\right)V^B\right). \tag{A.56}
$$

Full opacity. Away from C_t, the only risk is that maturity occurs, in the bad state, so

$$
dD_t = \phi P_{GB}(t)D_t dt, \tag{A.57}
$$

Hence

$$
D_t = D_0 \exp\left(\phi a t + \frac{\phi a}{b}\left(e^{-bt} - 1\right)\right) \tag{A.58}
$$

with $a = \frac{p_c}{p_c+p_r}$ and $b = p_c + p_r$. Let $t^0(\alpha)$ be the unique solution to $\alpha V_t = D_t$.

Case 1: ζ_ϕ realises before t^0. Then with probability $1 - P_{GB}\left(\zeta_\phi\right)$, payoff realises at y^G and debt is fully repaid, and with probability $P_{GB}\left(\zeta_\phi\right)$, the realised payoff is $y^B = 0$.

Case 2: ζ_ϕ realises after t_0. Then there is liquidation at αV_{t^0}.

Hence

$$
\mathscr{P} = e^{-\phi t^0(\alpha)} \tag{A.59}
$$

$$
\mathscr{I} = (1-\alpha)V_{t_0}e^{-\phi t^0(\alpha)}. \tag{A.60}
$$

General case, mandatory disclosure. The debt dynamics under mandatory disclosure is obtained by generalising the dynamics under full opacity, replacing the starting time 0 by

$\zeta_o^{k(t)} = \sup\{\zeta_o^k, \ \zeta_o^k \le t, \ y_{\zeta_o^k} = y^G\}$. We get

$$D_t = \frac{p_o}{p_o + \phi}\alpha V^B + \left(D_{\zeta_o^{k(t)}} - \frac{p_o}{p_o + \phi}\alpha V^B\right)\exp\left((\phi + p_o)a(t - \zeta_o^{k(t)}) + (\phi + p_o)\frac{a}{b}\left(e^{-b(t - \zeta_o^{k(t)})} - 1\right)\right),$$

(A.61)

for $t < \zeta_f$.

General case, voluntary disclosure. The debt dynamics is

$$dD_\tau = \phi(1 - q_\tau)D_\tau \, d\tau,$$

(A.62)

where q_τ is given by (A.52). By integration, we obtain

$$D_t = D_{\zeta_o^{k(t)}}\exp\left(\phi\left(t - \zeta_o^{k(t)}\right) - \phi\left(Q_{t - \zeta_o^{k(t)}} - Q_0\right)\right),$$

(A.63)

for $t < \zeta_f$, where

$$Q_t = -\frac{\kappa_1}{p_o(\kappa_1 - \kappa_2)}\log\left(e^{(\kappa_1 - \kappa_2)t} + k^{-1}\right) - \frac{\kappa_2}{p_o(\kappa_2 - \kappa_1)}\log\left(e^{(\kappa_2 - \kappa_1)t} + k\right).$$

(A.64)

A.2.2 Numerical Approach

The tables in section 1.4.3 are obtained by Monte-Carlo simulations where N is selected in order to have the third significative digit with a 99% confidence level.

Numerical Result 1 is obtained by using the analytical formulas of Appendix A.2.1. For any parameter x, 1000 instances of the parameter set Θ_{-x} are generated randomly and the claims of the Numerical Result are verified along a 30-point grid of admissible values of x.

Numerical Result 2 is obtained by running Monte-Carlo simulations of the continuous-time model for 216 parameter sets within the parameter space. There is no case where voluntary disclosure is more efficient and $\frac{p}{\phi} \ge 20\%$; the values of $\frac{p}{\phi}$ considered being 10%, 20%, 25%, 33%, 50%, 100%, 200% and 400%. For $\frac{p}{\phi} \le 25\%$, because the disclosure regimes are virtually identical, one needs a very large N in order to tell the regimes apart at usual levels of confidence: the differences are economically irrelevant. By taking such a large N, though, one can exhibit examples featuring a very small $\frac{p}{\phi}$ where voluntary disclosure is more efficient, meaning that the bound $p^*\left(\Theta_{-p}\right)$ is not uniformly zero. For larger values of $\frac{p}{\phi}$—when the two regimes meaningfully differ—mandatory disclosure implies a clearly lower inefficiency.

A.3 Figures
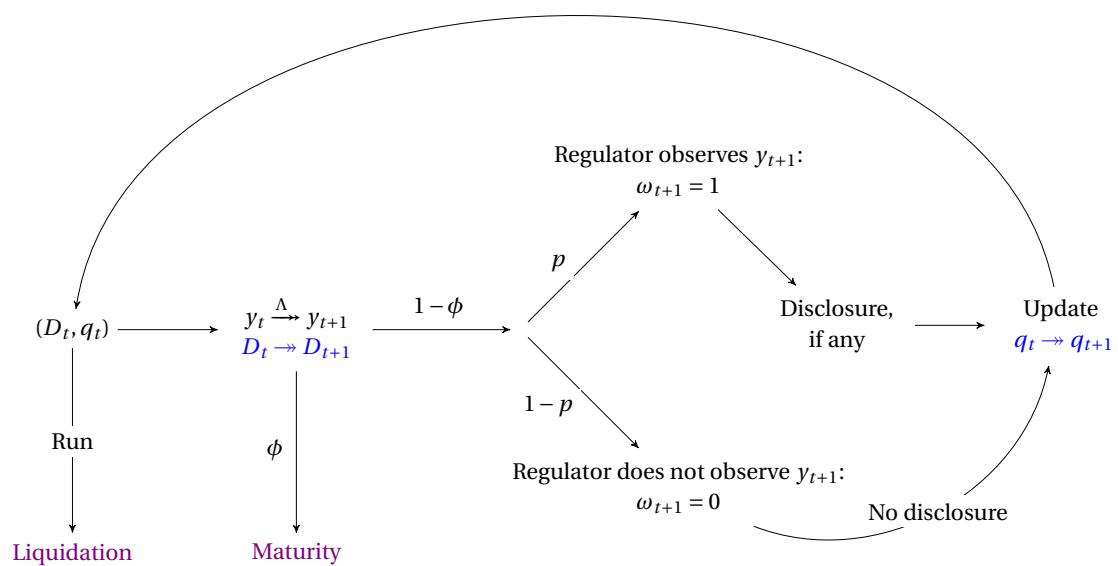
**Figure A.1: Graphical representation of the model.**
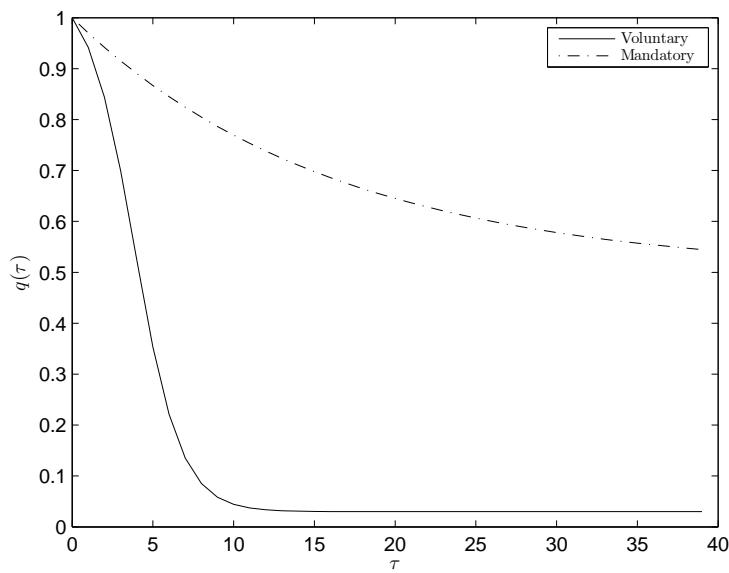
Figure A.2: Probability q_τ to be in state y^G after τ periods of non-disclosure.
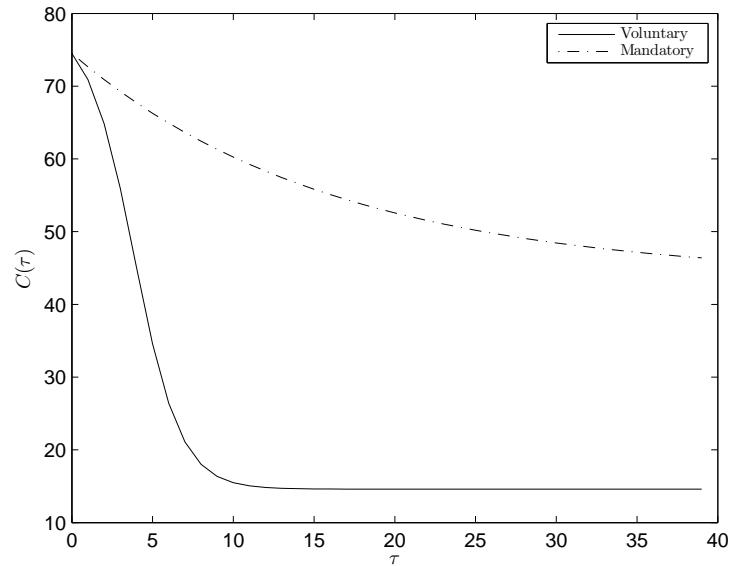
**Figure A.3: Debt capacities under both regimes.**
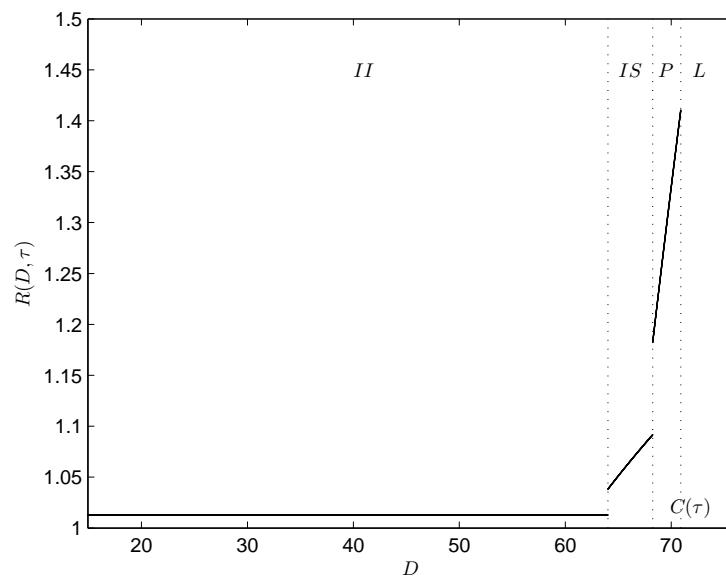
Figure A.4: Bond yields as a function of debt for $\tau = 1$ under voluntary disclosure.
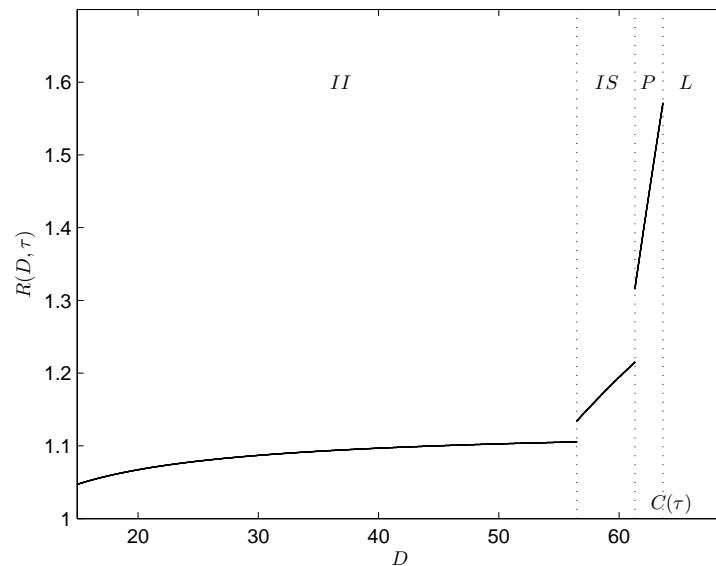
Figure A.5: Bond yields as a function of debt for $\tau = 7$ under mandatory disclosure.
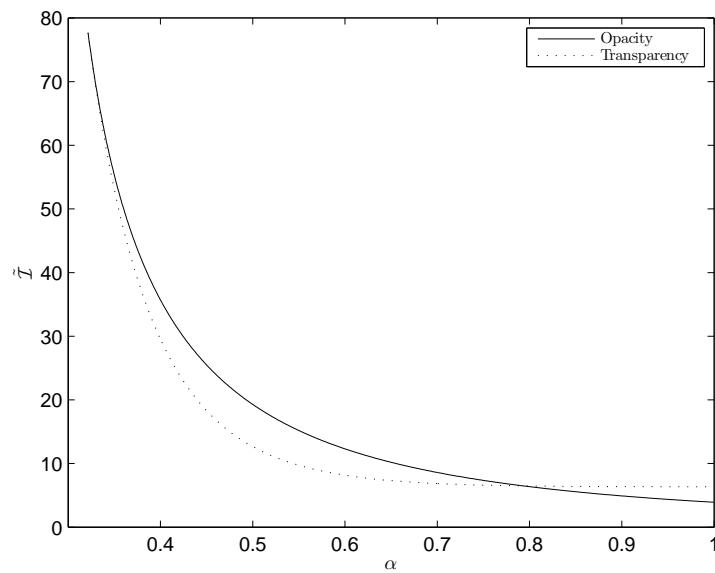

Figure A.6: Inefficiency as a function of the liquidity parameter α.
Continuous-time version with $\phi = 0.1$, $p_c = p_r = 0.04$ and $D_0 = 25$.
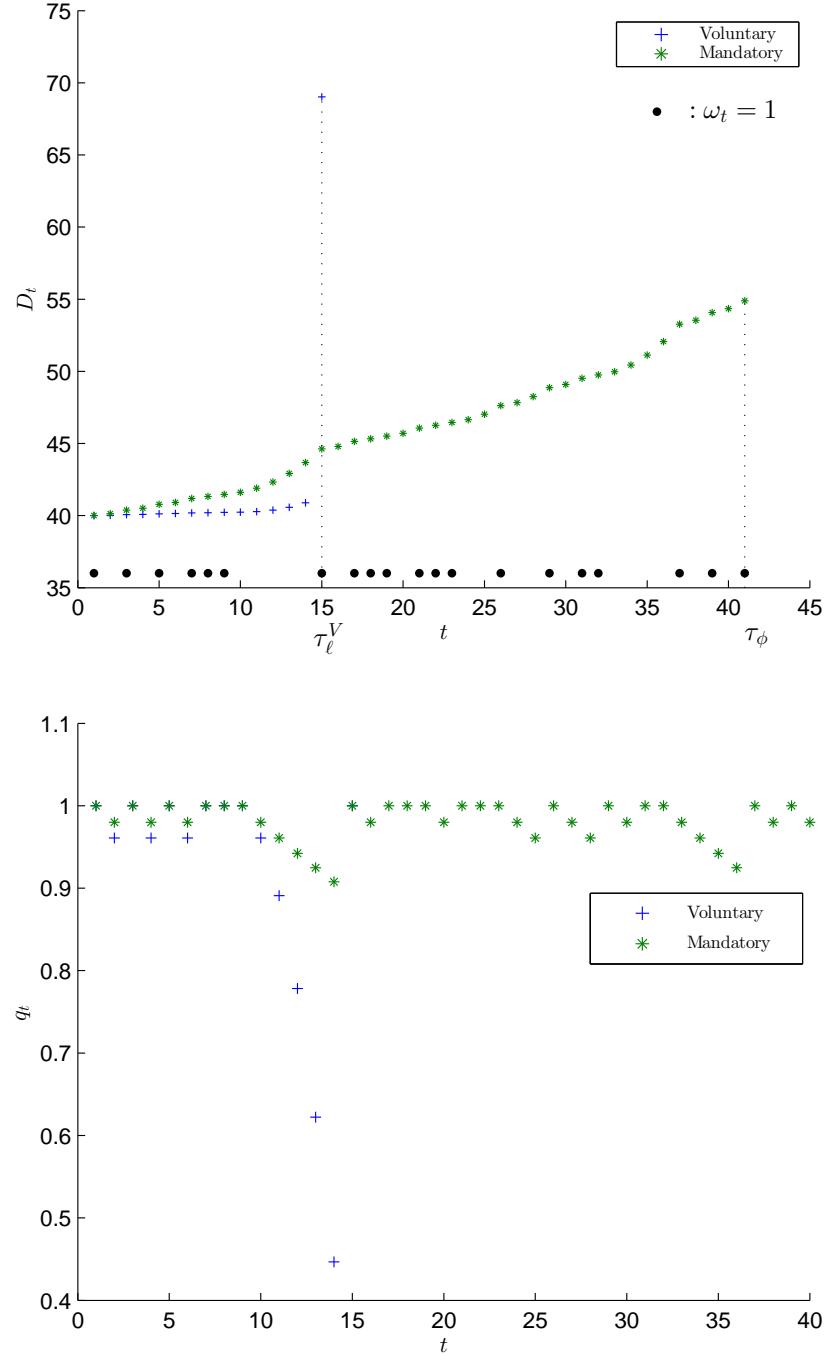
Figure A.7: Debt and beliefs dynamics in a voluntary disclosure-induced run.
$\phi = \lambda^{BG} = 1 - \lambda^{GG} = 2\%$, other parameters are at their baseline value.
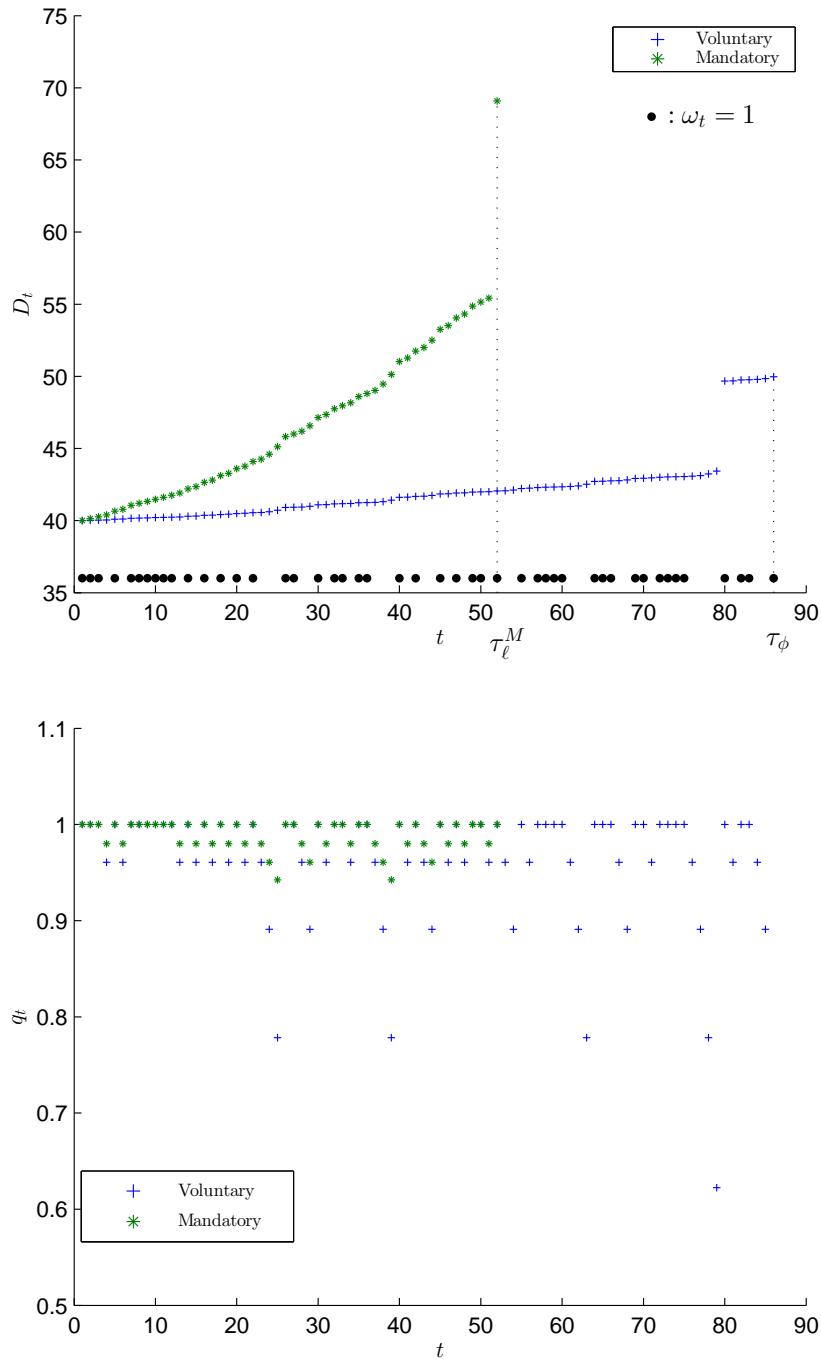
Figure A.8: Debt and beliefs dynamics in a mandatory disclosure-induced run.
$\phi = \lambda^{BG} = 1 - \lambda^{GG} = 2\%$, other parameters are at their baseline value.

B Appendix for Chapter 2

B.1 Proofs

B.1.1 Proposition 7

The proof builds on arguments from Goldstein and Pauzner (2005) and Morris and Shin (2006). Several technical justifications are required in order to adapt them to our particular framework. The thresholds of the dominance regions—described in Appendix B.2.2— are denoted $\underline{\underline{\theta}}$ and $\overline{\overline{\theta}}$.

Existence of a Threshold Equilibrium

Recall that ξ is defined in (2.21). For $\theta < \underline{\underline{\theta}}$, $n \mapsto \xi(\theta, n)$ is of constant, negative sign, and for $\theta > \overline{\overline{\theta}}$, it is of constant, positive sign. Inspection of the expression of ξ shows that $n \mapsto \xi(\theta, n)$ satisfies the single-crossing property when $\theta \in [\underline{\underline{\theta}}, \overline{\overline{\theta}}]$. Hence, the function $\theta \mapsto \xi(\theta, n(\theta, x^*))$ also satisfies the single-crossing property.

The function $x_i \mapsto \Delta(x_i, x_i)$, is negative for x_i low, positive for x_i high, and continuous in x_i, as an immediate consequence of the existence of dominance regions and the expression for Δ in (2.23). Therefore, it crosses 0 at least once: there exists a candidate threshold x^*. We now need to show that x^* is an equilibrium threshold. This is done by showing that the function $x_i \mapsto \Delta(x_i, x^*)$ inherits the single-crossing property of $\theta \mapsto \xi(\theta, n(\theta, x^*))$.

Following Definition 3 in Athey (2002), a function $h : \mathbb{R}^2 \to \mathbb{R}$ is said to be *log-supermodular* if $h \geq 0$ and $h(\underline{u}, \underline{v})h(\overline{u}, \overline{v}) \geq h(u_1, v_1)h(u_2, v_2)$ for all u_1, u_2, v_1 and v_2, where we use the notations $\underline{u} = \min\{u_1, u_2\}$, $\overline{u} = \max\{u_1, u_2\}$, and similarly for $\underline{v}, \overline{v}$.

The family of Gaussian densities $\left(\phi_{m,(\tau_x+\tau_y)^{-1}}\right)_{m\in\mathbb{R}}$ defines an application $h : \mathbb{R}^2 \to \mathbb{R}$ through

$h(m, \theta) = \phi_{m, (\tau_x + \tau_y)^{-1}}(\theta)$, and Lemma 5 in Athey (2002) states that if h is log-supermodular, then

$$m \mapsto \int_{\mathbb{R}} \xi(\theta, n(\theta, x^*)) h(m, \theta) \, d\theta = \mathbb{E}_{m, (\tau_x + \tau_y)^{-1}}[\xi(\theta, n(\theta, x^*))]$$

satisfies the single-crossing property. Since the updated mean $m(x_i, \theta_p)$ is increasing in x_i, then

$$x_i \mapsto \mathbb{E}_{m(x_i, \theta_p), (\tau_x + \tau_y)^{-1}}[\xi(\theta, n(\theta, x^*))] = \Delta(x_i, x^*)$$

also satisfies the single-crossing property. This shows that the candidate threshold x^* is indeed an equilibrium threshold. Verifying the log-supermodularity of h is straightforward, hence existence is established.

Limit Uniqueness

Fix a public signal θ_p. As we are going to let private signal precisions τ_x vary, we recall the dependency in τ_x of $\Delta(x_i, x^*)$ defined by (2.23), by denoting it $\Delta^{\tau_x}(x_i, x^*)$.

From an adaptation of the arguments in Morris and Shin (2006), we know that the convergence

$$\Delta^{\tau_x}(x^*, x^*) \to \frac{1}{1 - \lambda} \int_{\lambda}^{1} \xi(x^*, n) \, dn \equiv \Delta^{\infty}(x^*) \tag{B.1}$$

occurs uniformly on compacts that do not contain $\overline{\overline{\theta}}$ (full details can be found in Carré and Klossner (2018)). Consider a sequence of private signal precisions $\tau_x^k \to \infty$, and let x_k be a sequence of equilibrium thresholds associated with the τ_x^k:

$$\Delta^{\tau_x^k}(x_k, x_k) = 0. \tag{B.2}$$

Assume that x_k converges to $x^* \in \mathbb{R}$. Δ^{∞} is increasing and therefore has at most one zero, denoted z_0. Moreover, Δ^{∞} jumps upwards at $\overline{\overline{\theta}}$, to a value denoted $\Delta^{\infty}(\overline{\overline{\theta}}^+) > 0$.

• *Case 1:* $\Delta^{\infty}(\overline{\overline{\theta}}) > 0$. We first show that if $x^* < \overline{\overline{\theta}}$, then $x^* = z_0$. Let $x^* < \overline{\overline{\theta}}$ and $\underline{x}, \overline{x}$ such that $\underline{x} < x < \overline{x} < \overline{\overline{\theta}}$. Because $x_k \in [\underline{x}, \overline{x}]$ for k large enough and the convergence of $x \mapsto \Delta^{\tau_x^k}(x, x)$ is uniform with respect to $x \in [\underline{x}, \overline{x}]$, the equalities $\Delta^{\tau_x^k}(x_k, x_k) = 0$ can be taken to the limit and we obtain $\Delta^{\infty}(x^*) = 0$. Hence it must be that $x^* = z_0$. We now show that one cannot have $x^* \geq \overline{\overline{\theta}}$. Consider the function $\tilde{\xi}$ obtained without the introduction of an upper dominance region, and let $\tilde{\Delta}, \tilde{\Delta}^{\infty}$ be the expected differential payoff functions associated with $\tilde{\xi}$. Again from the uniform convergence argument, $x \mapsto \tilde{\Delta}^{\tau_x^k}(x, x)$ converges to $\tilde{\Delta}^{\infty}$ uniformly w.r.t. *any*

compact. Moreover, by construction, $\Delta \geq \tilde{\Delta}$. Now since $x_k \to x^*$,

$$
\begin{aligned}
0 = \Delta^{\tau_x^k}(x_k, x_k) &\geq \tilde{\Delta}^{\tau_x^k}(x_k, x_k) \\
&\to \tilde{\Delta}^\infty(x^*) \\
&\geq \tilde{\Delta}^\infty(\overline{\overline{\theta}}) = \Delta^\infty(\overline{\overline{\theta}}) > 0
\end{aligned}
$$

a contradiction. Line 3 is because ξ and $\tilde{\xi}$ agree until $\overline{\overline{\theta}}$ so $\tilde{\Delta}^\infty(\overline{\overline{\theta}}) = \Delta^\infty(\overline{\overline{\theta}})$. Hence, the only possible limit for (x_k) is z_0.

• *Case 2:* $\Delta^\infty(\overline{\overline{\theta}}) \leq 0$. If $x^* < \overline{\overline{\theta}}$, as in Case 1, the uniform convergence result implies that x^* must be a zero of Δ^∞. But $\Delta^\infty(x^*) < \Delta^\infty(\overline{\overline{\theta}}) \leq 0$. Therefore one cannot have $x^* < \overline{\overline{\theta}}$. Similarly, if $x^* > \overline{\overline{\theta}}$, the uniform convergence result implies that $\Delta^\infty(x^*) = 0$, but Δ^∞ is strictly positive over the upper dominance region. Hence, the only possible limit for (x_k) is $\overline{\overline{\theta}}$.

We have obtained that a converging subsequence of (x_k) can only converge to one finite value. The existence of dominance regions rules out the possibility of a subsequence converging to $\pm\infty$. From a standard analysis argument, (x_k) converges, which establishes limit uniqueness.

B.1.2 Proposition 8

Let us first consider the case $\lambda \leq n^{\mathrm{def}}$. Since $Q(n, \delta, b, R) = 1$ for $n \leq n^{\mathrm{def}}$ and $Q(n, \delta, b, R) = 0$ for $n \geq \overline{n}$, we have

$$
\int_\lambda^1 Q(n, \delta, b, R)\, dn = n^{\mathrm{def}} - \lambda + \frac{z}{R} \int_{n^{\mathrm{def}}}^{\overline{n}} \frac{1 - (\delta n - b)/\ell}{\delta(1 - n)}\, dn.
$$

Integrating:

$$
\begin{aligned}
\int_\lambda^1 Q(n, \delta, b, R)\, dn &= n^{\mathrm{def}} - \lambda + \frac{z}{R}\left(\frac{1 + b/\ell}{\delta} \int_{n^{\mathrm{def}}}^{\overline{n}} \frac{1}{1 - n}\, dn - \frac{1}{\ell}\int_{n^{\mathrm{def}}}^{\overline{n}} \frac{n}{1 - n}\, dn \right) \\
&= n^{\mathrm{def}} - \lambda + \frac{z}{\ell}\frac{1}{R}\left(\overline{n} - n^{\mathrm{def}} + \left(1 - \overline{n}\right)\log\left(\frac{1 - \overline{n}}{1 - n^{\mathrm{def}}} \right) \right).
\end{aligned}
$$

We now consider the second case: $\lambda > n^{\mathrm{def}}$.

$$
\begin{aligned}
\int_\lambda^1 Q(n, \delta, b, R)\, dn &= \frac{z}{R}\left(\frac{1 + b/\ell}{\delta} \int_\lambda^{\overline{n}} \frac{1}{1 - n}\, dn - \frac{1}{\ell}\int_\lambda^{\overline{n}} \frac{n}{1 - n}\, dn \right) \\
&= \frac{z}{\ell}\frac{1}{R}\left(\overline{n} - \lambda + \left(1 - \overline{n}\right)\log\left(\frac{1 - \overline{n}}{1 - \lambda} \right) \right).
\end{aligned}
$$

Finally,

$$
\int_\lambda^1 D_1^r(n, \delta, b) = \int_\lambda^{\overline{n}} dn + \int_{\overline{n}}^1 \frac{\ell + b}{\delta n}\, dn = \overline{n} - \lambda - \overline{n}\log\overline{n}.
$$

B.1.3 Proposition 9

From Proposition 8, the probability of success of the risky project at the equilibrium threshold, p^*, can be written as

$$p^* \equiv p(x^*) = \frac{h_0(\delta, b)}{h_1(\delta, b, R)},$$

where

$$
\begin{aligned}
h_0(\delta, b) &= \int_\lambda^1 D_1^r(n, \delta, b)\, \mathrm{d}n \\
h_1(\delta, b, R) &= R \int_\lambda^1 Q(n, \delta, b, R)\, \mathrm{d}n.
\end{aligned}
$$

Since p is strictly increasing, it is equivalent to obtain the comparative statics for p^* or for x^*.

4 is clear, as h_1 is increasing in R due to the absence of bankruptcy costs at $t = 2$, while h_0 is independent of R. However, note that h_1 is not always *strictly* increasing in R. To see why, note that once the probability of being served at $t = 2$ *conditional on the project being successful* drops below one, promising a higher R does not increase the expected payoff of the deposit contract. This is because, for any given depositor, the impact of promising a higher R on the date-2 payoff conditional on being served is exactly offset by the decline in the probability that he will be served:

$$RQ(n, \delta, b, R)\mathbb{I}_{\{Z=z\}} = \frac{1 - (\delta n - b)^+/\ell}{\delta(1 - n)}\mathbb{I}_{\{Z=z\}}.$$

The maximal R which increases the expected date-2 payoff of the deposit contract is equal to the smallest R such that there is default at maturity when the project is successful, for *any* value of n. Denote this deposit rate R^M. The value of $n \geq \lambda$ that maximises the expected payment per residual claimant at maturity is $n_0 = \max\left\{\lambda, \frac{b}{\delta}\right\}$. Therefore, R^M satisfies $(1 - n_0)\delta R^M = 1 - \frac{(n_0\delta - b)^+}{\ell}$, which provides the expression of R^M given in Proposition 9. Combined with our Assumption 5, this yields the following bound on admissible deposit rates:

$$R \leq \overline{R} := \min\{R^M, z/\delta\}.$$

We consider each case, $\lambda < n^{\mathrm{def}}$ and $\lambda \geq n^{\mathrm{def}}$, in turn. Consider first the case $\lambda < n^{\mathrm{def}}$, and let us introduce some notation to derive a more compact expression for h_1. Recalling the definition of n^{def} in (2.7), its expression can be simplified to

$$n^{\mathrm{def}} = \frac{\frac{z}{\ell}\overline{n} - R}{\frac{z}{\ell} - R} = \kappa \overline{n} + (1 - \kappa),$$

by introducing the notation

$$\kappa = \frac{z/\ell}{z/\ell - R}.$$

Note that $\kappa > 1$ since $z/\ell > z/(\ell + b) > z/\delta > R$, where the second inequality follows from Assumption 4 and the third from Assumption 5. Then, using

$$
\begin{aligned}
1 - n^{\text{def}} &= 1 - \kappa\overline{n} - (1 - \kappa) = \kappa(1 - \overline{n}) \\
\overline{n} - n^{\text{def}} &= \overline{n} - \kappa\overline{n} - (1 - \kappa) = -(1 - \kappa)(1 - \overline{n}),
\end{aligned}
$$

we can rewrite h_1 as

$$
\begin{aligned}
h_1(\overline{n}) &= (n^{\text{def}} - \lambda)R + \frac{z}{\ell}\left(\overline{n} - n^{\text{def}} + (1 - \overline{n})\log\left(\frac{1 - \overline{n}}{1 - n^{\text{def}}}\right)\right) \\
&= R\left(1 - \lambda - \frac{\kappa}{\kappa - 1}\log\kappa\right) + \left(R\frac{\kappa}{\kappa - 1}\log\kappa\right)\overline{n} \\
&= A_0 + A_1\overline{n}.
\end{aligned}
$$

Since it is straightforward to show that $\frac{\kappa}{\kappa - 1}\log\kappa > 1$ for all $\kappa > 1$, we have: $A_0 < 0$ and $A_1 > 0$.

From

$$p^* = \frac{h_0}{h_1} = \frac{\overline{n} - \lambda - \overline{n}\log\overline{n}}{A_0 + A_1\overline{n}},$$

we obtain

$$
\begin{aligned}
\text{sgn}\left(\frac{\partial p^*}{\partial\overline{n}}\right) &= \text{sgn}\left(\frac{\partial}{\partial\overline{n}}\frac{h_0}{h_1}\right) \\
&= \text{sgn}\left(-\log\overline{n}(A_0 + A_1\overline{n}) - (\overline{n} - \lambda - \overline{n}\log\overline{n})A_1\right) \\
&= \text{sgn}\left(-A_0\log\overline{n} - A_1(\overline{n} - \lambda)\right) < 0,
\end{aligned}
$$

using $A_0 < 0$, $A_1 > 0$, $\overline{n} < 1$ and $\lambda < n^{\text{def}} \leq \overline{n}$. Since p^* depends on b and δ only through $\overline{n}$, and since $\frac{\partial\overline{n}}{\partial b} > 0$ and $\frac{\partial\overline{n}}{\partial\delta} < 0$, we obtain 2 and 3.

To prove 1, we need to show that

$$\text{sgn}\left(\frac{\partial p^*}{\partial\lambda}\right) = \text{sgn}(-h_1 + Rh_0).$$

is positive. Assume first that $Rp^* \leq 1$. For values of n such that withdrawing early pays off 1 with probability one, the payoff from withdrawing early dominates the payoff from staying, since the latter action results in an expected payoff of $Rp^* \leq 1$. Further, for values of n such that withdrawing early pays off 1 with probability strictly below one, the payoff from staying is null. This implies that, under the Laplacian belief, the payoff of withdrawing early is strictly higher than the payoff of staying, a contradiction with the definition of p^* as the probability

of success at the equilibrium threshold x^*. Hence, $Rp^* > 1$, and

$$-h_1 + Rh_0 = h_1(-1 + Rp^*) > 0.$$

We now turn to the second case $\lambda \geq n^{\text{def}}$. Note that if $\lambda > \overline{n}$, then the bank always defaults at $t = 1$ outside of the upper dominance region, since early withdrawals by impatient investors are so large that the entire risky project is liquidated. It follows that $p^* = p(\overline{\overline{\theta}})$, $\frac{\partial p^*}{\partial n} = 0$, and there is nothing left to prove. Therefore, we only need to consider the case $1 > \overline{n} > \lambda$.

The function h_0 is unchanged, while h_1 and its derivative now take the form

$$h_1(\overline{n}) = \frac{z}{\ell}\left(\overline{n} - \lambda + (1-\overline{n})\log\left(\frac{1-\overline{n}}{1-\lambda}\right)\right)$$

$$\frac{\partial h_1(\overline{n})}{\partial \overline{n}} = \frac{z}{\ell}\left(1 - \log\left(\frac{1-\overline{n}}{1-\lambda}\right) - 1\right) = -\frac{z}{\ell}\log\left(\frac{1-\overline{n}}{1-\lambda}\right),$$

so that

$$\begin{aligned}
\text{sgn}\left(\frac{\partial p^*}{\partial \overline{n}}\right) &= \text{sgn}\left(\frac{\partial}{\partial \overline{n}}\frac{h_0}{h_1}\right) \\
&= \text{sgn}\left(-\log\overline{n}\left(\overline{n} - \lambda + (1-\overline{n})\log\left(\frac{1-\overline{n}}{1-\lambda}\right)\right) + (\overline{n} - \lambda - \overline{n}\log\overline{n})\log\left(\frac{1-\overline{n}}{1-\lambda}\right)\right) \\
&= \text{sgn}\left((\overline{n} - \lambda - \log\overline{n})\log\left(\frac{1-\overline{n}}{1-\lambda}\right) - (\overline{n} - \lambda)\log\overline{n}\right) \\
&= \text{sgn}\left(\iota_\lambda(\overline{n})\right).
\end{aligned}$$

We have $\iota_\lambda(\lambda) = 0$ and

$$\begin{aligned}
\iota'_\lambda(\overline{n}) &= \left(1 - \frac{1}{\overline{n}}\right)\log\left(\frac{1-\overline{n}}{1-\lambda}\right) - \frac{\overline{n} - \lambda - \log\overline{n}}{1-\overline{n}} - \log\overline{n} - \frac{\overline{n}-\lambda}{\overline{n}} \\
&= -\frac{1}{(1-\overline{n})\overline{n}}\left[(1-\overline{n})^2\log\left(\frac{1-\overline{n}}{1-\lambda}\right) - \overline{n}^2\log\overline{n} + \overline{n} - \lambda\right]
\end{aligned}$$

We are left to show that the term inside the square brackets is positive. To that end, fix $\overline{n}$ and let

$$\begin{aligned}
f_{\overline{n}}(\lambda) &= (1-\overline{n})^2\left(\log(1-\overline{n}) - \log(1-\lambda)\right) - \overline{n}^2\log\overline{n} + \overline{n} - \lambda \\
f'_{\overline{n}}(\lambda) &= \frac{(1-\overline{n})^2}{1-\lambda} - 1 \\
f''_{\overline{n}}(\lambda) &= \left(\frac{1-\overline{n}}{1-\lambda}\right)^2 > 0.
\end{aligned}$$

Thus, $f_{\overline{n}}$ is minimised at $\lambda = 1 - (1-\overline{n})^2$, where it takes the value:

$$
\begin{aligned}
f_{\overline{n}}(1-(1-\overline{n})^2) &= -(1-\overline{n})^2 \log(1-\overline{n}) - \overline{n}^2 \log \overline{n} + \overline{n} - 1 + (1-\overline{n})^2 \\
&= -(1-\overline{n})\overline{n}\left(1 + \frac{\overline{n}\log(\overline{n})}{1-\overline{n}} + \frac{(1-\overline{n})\log(1-\overline{n})}{\overline{n}}\right)
\end{aligned}
$$

It is straightforward to show that the function $\tilde{f}$ defined by $\tilde{f} : \overline{n} \in (0;1) \mapsto \frac{\overline{n}\log\overline{n}}{1-\overline{n}}$ and extended by continuity via $\tilde{f}(0) = 0$, $\tilde{f}(1) = -1$ is strictly convex over $(0,1)$. Therefore, its graph lies beneath the chord joining $(0, \tilde{f}(0))$ and $(1, \tilde{f}(1))$, i.e., $\tilde{f}(\overline{n}) < -\overline{n}$ for $n \in (0,1)$. This implies that $\tilde{f}(\overline{n}) + \tilde{f}(1-\overline{n}) < -\overline{n} - (1-\overline{n}) = -1$. Thus, $f_{\overline{n}}(1-(1-\overline{n})^2) = -(1-\overline{n})\overline{n}(1 + \tilde{f}(\overline{n}) + \tilde{f}(1-\overline{n})) > 0$. Consequently, $f_{\overline{n}}$ is strictly positive for all $\overline{n}, \lambda \in (0,1)$. We conclude that $\iota_\lambda < 0$ for all $\overline{n} > \lambda$, i.e. $\frac{\partial p^*}{\partial \overline{n}} < 0$, from which it follows as before that 2 and 3 hold.

Finally,

$$
\begin{aligned}
\operatorname{sgn}\left(\frac{\partial p^*}{\partial \lambda}\right) &= \operatorname{sgn}\left(\frac{\partial}{\partial \lambda}\frac{h_0}{h_1}\right) \\
&= \operatorname{sgn}\left(-\left(\overline{n} - \lambda + (1-\overline{n})\log\left(\frac{1-\overline{n}}{1-\lambda}\right)\right) + (\overline{n} - \lambda - \overline{n}\log\overline{n})\left(1 - \frac{1-\overline{n}}{1-\lambda}\right)\right) \\
&= \operatorname{sgn}\left(-\log\left(\frac{1-\overline{n}}{1-\lambda}\right) - \frac{\overline{n}-\lambda}{1-\lambda} - \frac{\overline{n}-\lambda}{(1-\lambda)(1-\overline{n})}\overline{n}\log\overline{n}\right) \\
&= \operatorname{sgn}\left(J_\lambda(\overline{n})\right).
\end{aligned}
$$

We have $J_\lambda(\lambda) = 0$ and

$$
\begin{aligned}
J'_{\overline{n}}(\overline{n}) &= \frac{1}{1-\overline{n}} - \frac{1}{1-\lambda} - \frac{1}{(1-\overline{n})^2}\overline{n}\log\overline{n} - \frac{\overline{n}-\lambda}{(1-\lambda)(1-\overline{n})}(1+\log\overline{n}) \\
&= \frac{(\lambda + (\overline{n}-2)\overline{n})\log\overline{n}}{(1-\lambda)(1-\overline{n})^2} > 0.
\end{aligned}
$$

where the inequality comes from the facts that $\lambda + (\overline{n}-2)\overline{n} < \overline{n}(\overline{n}-1) < 0$ and $\log\overline{n} < 0$. Therefore, $J_\lambda > 0$ for all $\overline{n} > \lambda$, i.e. $\frac{\partial p^*}{\partial \lambda} > 0$, which concludes the verification of 1.

B.1.4 Proposition 10

We start by providing some intuition about the proof, following the argument of Szkup (2013). For *fixed R* and θ_p, inspection of the limit profit shows that in the limiting case of vanishing noise profit cannot be maximised at any choice $R \neq R^*(\theta; \delta, b)$. However, what is missing in this intuitive approach is the fact that the banker does not observe θ, and thus cannot set $R = R^*(\theta; \delta, b)$. Instead, the banker has to select a R^k solely based on the realisation of the public signal θ_p^k. We must therefore deal with the fact that both signals θ_p^k and deposit rates R^k are not deterministic. This aspect, which Szkup (2013) does not account for, is taken care

of in Step 1, where we obtain a uniform convergence result. Step 2 concludes the proof using the strengthened notion of convergence obtained in Step 1.

Since δ and b are set by the banker once and for all at $t = 0$, we do not report explicitly the dependency of functions on these variables. For instance, $\hat{R}(\theta_p; \delta, b)$ is denoted $\hat{R}(\theta_p)$.

Intuition. Recall that the banker solves the problem

$$\hat{R}(\theta_p) = \underset{R \in [1, \overline{R}_2]}{\arg\max} \quad \mathscr{E}_1(\theta_p, R) \tag{2.32}$$

where $\mathscr{E}_1$ is the expected payoff from holding one unit of equity (which includes the dividend Y_1) *after* realisation of the public signal θ_p. For a given state of the world θ, let θ_p^k be a sequence of public signals about θ, with precisions $\tau_p^k \to \infty$. Consider for now a banker's choice interest rate R which is independent of the signal received, θ_p^k. Then we have the convergence

$$\lim_{k \to \infty} \mathscr{E}_1(\theta_p^k, R) \mathbb{I}_{\{R \neq R^*(\theta)\}} = \left(p(\theta) E_2(n = \lambda, R; z) + Y_1(n = \lambda) \right) \mathbb{I}_{\{R > R^*(\theta)\}}. \tag{B.3}$$

This is because $m(\theta_p^k) \to \theta$ (the mean of the distribution of θ updated after reception of the public signal converges to the true value, θ), and $\tau_y^k \to 0$ (the variance of the distribution of θ updated after reception of the public signal goes to 0). Hence, in the limit the Gaussian distribution $N(m(\theta_p^k), \tau_y^k)$ concentrates all the mass at θ. To obtain (B.3), recall that: (i) for $R > R^*(\theta)$, $n(\theta, x^*(R))$ is arbitrarily close to λ; and (ii) for $R < R^*(\theta)$ $n(\theta, x^*(R))$ is arbitrarily close to 1, in which case $E_2(n(\theta, x^*(R)), R; z) = 0 = Y_1(n(\theta, x^*(R)))$.

Let us now take a closer look at the limiting value of the banker's expected profit, i.e., at the right-hand side of (B.3). The banker wants to avoid the case $R < R^*(\theta)$, because her profit is null in this case. On the other hand, conditional on $R > R^*(\theta)$, the mass of running deposits is arbitrarily close to λ. Thus, the banker has no incentive to increase R further, as this would raise funding costs while having a negligible impact on the probability that her bank suffers a run.

We conclude that the maximiser $\hat{R}$ cannot be strictly to the left of R^* (because the limiting equity value is zero) nor strictly to the right of R^* (because the banker could increase her profit by slightly reducing R).

Step 1. Let $\theta > x^*(\overline{R})$ and ε_1 such that $\theta - \varepsilon_1 > x^*(\overline{R})$. For $\varepsilon_2 > 0$, define

$$
\begin{aligned}
K^1 &= [\theta - \varepsilon_1, \theta + \varepsilon_1], \\
K^2 &= \{ R \in [1, \overline{R}] \mid d(R, R^*(K^1)) \geq \varepsilon_2 \}.
\end{aligned}
$$

To formalise the intuitive limit in (B.3), we first aim at showing that the convergence

$$\lim_{k\to\infty} \mathscr{E}_1(\theta_p, R|\tau_p^k) = \big(p(\theta_p)E_2(n=\lambda, R; z) + Y_1(n=\lambda)\big)\mathbb{1}_{\{R>R^*(\theta_p)\}} \tag{B.4}$$

is uniform with respect to $(\theta_p, R) \in K^1 \times K^2$. The notation $\mathscr{E}_1(\theta_p, R|\tau_p^k)$ designates the expected equity payoff when the public signal realisation is θ_p the deposit rate is R, and the public signal precision is τ_p^k. Notice that the indicator function on the left-hand side of (B.3) no longer appears because, by construction, $R \neq R^*(\theta_p)$ for $(\theta_p, R) \in K^1 \times K^2$.

Here, we consider a fixed public signal, θ_p, and a fixed choice of deposit rate, R. What varies is the public signal precision, τ_p^k. Hence, even though θ_p is fixed, the updated mean $m(\theta_p)$ varies with k. To highlight this dependency, we denote it $m_k(\theta_p)$. Crucially, we require the convergence to be *uniform* in $(\theta_p, R) \in K^1 \times K^2$, because in Step 2 we will account for the randomness of the public signal, which implies a random R. As signals become increasingly precise, these variables (although random) concentrate in small intervals. Due to our uniform convergence result, we will be able to control uniformly the distance between the banker's profit computed off the limit and its limiting value (which is easier to analyze).

Let $\varepsilon, \tilde{\varepsilon} > 0$. By continuity and monotonicity of x^*, for u_3 small enough we have

$$d(R, R^*([\theta - \varepsilon_1 - u_3, \theta + \varepsilon_1 + u_3]) \geq \frac{\varepsilon_2}{2}. \tag{B.5}$$

Let $u_1 > 0$. Then, for k large enough (i.e. τ_p^k large enough) we have

$$m_k(\theta_p) \in [\theta - \varepsilon_1 - u_1, \theta + \varepsilon_1 + u_1],$$

independently of $\theta_p \in K^1$. This is because —whatever the prior mean μ of θ— the update of the mean upon observation of the public signal θ_p becomes arbitrarily close to θ_p as the precision τ_p^k becomes large.

Now, for $u_2 > 0$, the Gaussian distribution $N(m_k(\theta_p), \tau_y^{-1})$ concentrates a mass $1 - \varepsilon$ in

$$[m_k(\theta_p) - u_2, m_k(\theta_p) + u_2]$$

for τ_y large enough. In particular, independently of $\theta_p \in K^1$, it has mass $1 - \varepsilon$ in

$$\tilde{K}^1 = [\theta - \varepsilon_1 - u_1 - u_2, \theta + \varepsilon_1 + u_1 + u_2].$$

Select $u_1, u_2 > 0$ such that $u_1 + u_2 < u_3$. Due to (B.5), we have

$$d(K^2, R^*(\tilde{K}^1)) \geq \frac{\varepsilon_2}{2}. \tag{B.6}$$

The computation of $\mathscr{E}_1$ involves an integration with respect to the posterior distribution of $\theta|\theta_p$. To avoid any notational conflict with θ —the true, unobservable value of the fundamental, which we have fixed here— we denote the possible values of θ given θ_p, from the banker's point of view, by ϑ. The distribution of ϑ conditional on θ_p has mass equal or below ε outside $\tilde{K}^1$ independently of $\theta_p \in K^1$, for large enough signal precisions. For ϑ outside $\tilde{K}^1$, we shall use the naive inequality

$$|E_1(\vartheta, n = \lambda, R)| \le z + b. \tag{B.7}$$

Moreover,

$$\sup_{\vartheta, \vartheta' \in [\theta_p - \tilde{\varepsilon}; \theta_p + \tilde{\varepsilon}]} \left| E_1(\vartheta, n = \lambda, R) - E_1(\vartheta', n = \lambda, R) \right|$$

$$= \sup_{\vartheta, \vartheta' \in [\theta_p - \tilde{\varepsilon}; \theta_p + \tilde{\varepsilon}]} \left| p(\vartheta) \left(\left(1 - \frac{\lambda\delta}{\ell}\right) z - (1 - \lambda)\delta R \right) - p(\vartheta') \left(\left(1 - \frac{\lambda\delta}{\ell}\right) z - (1 - \lambda)\delta R \right) \right|$$

$$\le z \sup_{\vartheta, \vartheta' \in [\theta_p - \tilde{\varepsilon}; \theta_p + \tilde{\varepsilon}]} |p(\vartheta) - p(\vartheta')|$$

$$\le 2\tilde{\varepsilon} ||p'||_\infty z. \tag{B.8}$$

The second line uses the fact that, when $n = \lambda$, the dividend is the same (equal to $(b - \lambda\delta)^+$) regardless of the true fundamental. Therefore, dividends cancel out when we compute the difference of equity payoffs. In fact, the two payoffs only differ through the probabilities of realisation of the good state, which is either $p(\vartheta)$ or $p(\vartheta')$. The last inequality comes from the fact that p' is bounded, so that p is $||p'||_\infty$–Lipschitz and ϑ and ϑ' cannot be distant by more than $2\tilde{\varepsilon}$.

For (θ_p, R) in $K^1 \times K^2$, we have for τ_p^1, τ_p^2 large enough

$$\left| \mathscr{E}_1(\theta_p, R|\tau_p^1) - \mathscr{E}_1(\theta_p, R|\tau_p^2) \right|$$

$$\le 2\varepsilon z + \sup_{\vartheta, \vartheta' \in [\theta_p - \tilde{\varepsilon}; \theta_p + \tilde{\varepsilon}]} \left| E_1(\vartheta, n = \lambda, R) - E_1(\vartheta', n = \lambda, R) \right|$$

$$\le 2\varepsilon z + 2\tilde{\varepsilon} ||p'||_\infty z,$$

where we have used the fact that $\vartheta|\theta_p$ has mass less than ε outside $\tilde{K}^1$, in which case (B.7) applies. And it has mass at least $1 - \varepsilon$ in $\tilde{K}^1$. In that case, by construction of $\tilde{K}^1$ and K^2 and inequality (B.6), n is either λ or 1. When $n = 1$, equity (including dividends) is worthless. So the only contribution to equity value comes from the case $n = \lambda$, and we can use (B.8).

This concludes the proof that the convergence in (B.4) is uniform with respect to $K^1 \times K^2$.

Step 2. Let $\tau_p^k \to \infty$ and a corresponding sequence of public signals θ_p^k. $R^k(\theta_p^k)$ denote the

associated optimal deposit rates. Let $\varepsilon > 0$. We must show that

$$\mathbb{P}(|R^k(\theta_p^k) - R^*(\theta)| \geq \varepsilon) \to 0.$$

Note that given the construction of θ_p^k, we have $\theta_p^k \to \theta$ in probability. First assume that

$$R^k(\theta_p^k) - R^*(\theta) \geq \varepsilon.$$

By continuity of x^*, there is $a > 0$ such that $\theta_p^k \in I_a \equiv [\theta - a, \theta + a]$ implies $R^k(\theta_p^k) \geq R^*(\theta_p^k) + \frac{\varepsilon}{2}$. Now consider $\tilde{R}^k(\theta_p^k) = R^k(\theta_p^k) - \frac{\varepsilon}{4}$. The probability of solvency is bounded away from 0, both under R and $\tilde{R}$. But under $\tilde{R}$, repayment at the final date is always smaller. In the case where the bank is solvent under R, which occurs with positive probability, the repayment under $\tilde{R}$ is smaller by an amount $\frac{\varepsilon}{4}$. Therefore, there exists $\eta > 0$ such that

$$\forall \theta_p^k \in I_a, \ p(\theta_p^k) E_2(n = \lambda, \tilde{R}^k; z) \geq p(\theta_p^k) E_2(n = \lambda, R^k; z) + \eta.$$

But we know by Step 1 that for k large enough, for all $\theta_p^k \in I_a$:

$$\left| \mathcal{E}_1(\theta_p^k, \tilde{R}^k) - p(\theta_p^k) E_2(n = \lambda, \tilde{R}^k; z) - Y_1(n = \lambda) \right| \leq \frac{\eta}{3},$$

$$\left| \mathcal{E}_1(\theta_p^k, R^k) - p(\theta_p^k) E_2(n = \lambda, R^k; z) - Y_1(n = \lambda) \right| \leq \frac{\eta}{3}.$$

In particular, $\mathcal{E}_1(\theta_p^k, \tilde{R}^k) > \mathcal{E}_1(\theta_p^k, R^k)$, a contradiction with the optimality of R^k. This establishes that if $R(\theta_p^k) - R^*(\theta) \geq \varepsilon$, then $\theta_p^k \notin I_a$. Similarly, we can show that $R^*(\theta) - R^k(\theta_p^k) \geq \varepsilon$ implies $\theta_p^k \notin I_a$. In that case, the limit profit under the candidate strategy is 0, and we can obtain a positive expected payoff by switching to a strategy that prevents runs, as above.

Summing up, we can write for k large enough

$$\mathbb{P}(|R^k(\theta_p^k) - R^*(\theta)| \geq \varepsilon) \leq \mathbb{P}(\theta_p^k \notin I_a) \to 0,$$

because $\theta_p^k \to \theta$ in probability.

B.1.5   Corollary 2

First note that $\overline{R}$ is non-decreasing in b and decreasing in δ. The case $\overline{R} = z/\delta$ is obvious. Assume then that $\overline{R} = R^M$. In this case, it must be that $\lambda\delta \geq b$. (If on the contrary $\lambda\delta > b$, then $R^M = 1/(1 - b/\delta) \times z/\delta > z/\delta$; so $\overline{R} = z/\delta$, a contradiction.) Hence, we have: $\frac{\partial R^M}{\partial b} = \frac{1/\ell}{1 - \lambda} \frac{z}{\delta} > 0$ and $\frac{\partial R^M}{\partial \delta} = -\left(\frac{\lambda/\ell}{1 - \lambda} + \frac{1 - (\lambda\delta - b)/\ell}{1 - \lambda} \frac{1}{\delta} \right) \frac{z}{\delta} < 0$.

We prove the comparative statics for the ex-ante *conditional* probability of runs. These results

directly translate into statements about the ex-ante *unconditional* probability of runs, since it is simply a weighted average of the conditional probability. By Proposition 9, $x^*(\delta, b, R; \lambda)$ is decreasing in b, increasing in δ, and decreasing in R. It follows that $x^*(\delta, b, \overline{R}; \lambda)$ is decreasing in b and increasing in δ. The result for μ and $\tau^{-1/2}$ are immediate since these parameters do not impact the equilibrium threshold x^*, but only the distribution of the fundamental θ.

B.1.6 Lemma 8

The investor's utility at dates 1 and 2 is linear in (b^i, e^i, d^i). For ease of exposition, define $\rho^d(\delta, b)$ ($\rho^e(\delta, b)$) as the total expected utility over periods 1 and 2 provided by one unit of deposit (equity),

$$
\begin{aligned}
\rho^d(\delta, b) &= \int_{\underline{\lambda}}^{\overline{\lambda}} \mathrm{d}\lambda\, g(\lambda) \left\{ \lambda \cdot \left(\pi(\delta, b; \lambda) \frac{\ell + b}{\delta} + (1 - \pi(\delta, b; \lambda)) \right) \right. \\
&\qquad \left. + (1 - \lambda) \cdot \beta \left(\pi(\delta, b; \lambda) \frac{\ell + b}{\delta} + (1 - \pi(\delta, b; \lambda)) D_0^s(\delta, b, \lambda) \right) \right\} \quad (\text{B.9})
\end{aligned}
$$

$$
\rho^e(\delta, b) = \int_{\underline{\lambda}}^{\overline{\lambda}} \mathrm{d}\lambda\, g(\lambda)(1 - \lambda) \cdot \beta \, (1 - \pi(\delta, b; \lambda)) \, E_0(\delta, b, \lambda). \quad (\text{B.10})
$$

With this notation, the FOC's of the investor's portfolio choice can be written more compactly as

$$
\begin{aligned}
v'(c_0^i) q_b &\geq \alpha, & = \quad \text{if } b^i > 0, &\qquad (\text{B.11}) \\
v'(c_0^i) q_d &\geq \rho^d(\delta, b), & = \quad \text{if } d^i > 0, &\qquad (\text{B.12}) \\
v'(c_0^i) q_e &\geq \rho^e(\delta, b), & = \quad \text{if } e^i > 0. &\qquad (\text{B.13})
\end{aligned}
$$

We also drop the dependence of the equilibrium threshold, the probability of runs, and the date-0 expected deposit and equity payoff (conditional on no run) on the initial choices of the banker: $x^*(\lambda) \equiv x^*(\delta, b, \overline{R}; \lambda)$, $\pi(\lambda) \equiv \pi(\delta, b; \lambda)$, $E_0(\lambda) \equiv E_0(\delta, b; \lambda)$, $D_0^s(\lambda) \equiv D_0^s(\delta, b; \lambda)$.

Starting from the expression for the date-0 expected utility of banker given in (2.51), we write:

$$
U^B = (1 - \psi) \cdot \mathbb{E}\left[(1 - \pi(\lambda)) E_0(\lambda) \right],
$$

where the expectation is with respect to λ. Then,

$$U^{\mathrm{B}} = (1 - \psi) \times \frac{\mathbb{E}\left[(1-\lambda)(1-\pi(\lambda))E_0(\lambda)\right]}{(1-\mu_\lambda)} \times \underbrace{\frac{(1-\mu_\lambda)\mathbb{E}\left[(1-\pi(\lambda))E_0(\lambda)\right]}{\mathbb{E}\left[(1-\lambda)(1-\pi(\lambda))E_0(\lambda)\right]}}_{:=\Gamma}$$

$$= \Gamma(1-\psi)\frac{\rho_e}{(1-\mu_\lambda)\beta}$$

$$= \Gamma\frac{v'(\omega_0 - I)}{(1-\mu_\lambda)\beta}(q_e - \psi q_e)$$

$$= \Gamma\frac{v'(\omega_0 - I)}{(1-\mu_\lambda)\beta}\left(q_e + q_d\delta - I - (\hat{q}_b + q_b - \hat{q}_b)b\right)$$

$$= \Gamma\frac{\rho_e + \rho_d\delta - \alpha b - v'(\omega_0 - I)(I + (q_b - \hat{q}_b)b)}{(1-\mu_\lambda)\beta},$$

The second line follows from the definition of ρ_e in (B.10). The third line substitutes the equity pricing equation (B.13). The fourth line uses the initial budget constraint (2.2). The fifth line substitutes the pricing equations (B.11)–(B.13).

Using the definitions of ρ_d and ρ_e in (B.9)–(B.10), and the fact that in the limit of vanishing noise: (i) a run precipitates an early default of the bank (Assumption 4); and (ii) conditional on there not being a run, impatient investors are paid with probability one, yields:

$$U^{\mathrm{B}} = \frac{\Gamma}{(1-\mu_\lambda)\beta} \times$$

$$\left\{ \mathbb{E}\left[\lambda\left(\pi(\lambda)\frac{\ell+b}{\delta} + (1-\pi(\lambda))\right) + (1-\lambda)\beta\left(\pi(\lambda)\frac{\ell+b}{\delta} + (1-\pi(\lambda))D_0^s(\lambda)\right)\right]\delta \right.$$

$$\left. + \mathbb{E}\left[(1-\lambda)\beta(1-\pi(\lambda))E_0(\lambda)\right] - (\mu_\lambda + (1-\mu_\lambda)\beta)b - v'(\omega_0 - I)(I + (q_b - \hat{q}_b)b) \right\} \quad \text{(B.14)}$$

For all $\lambda \le \frac{b}{\delta}$, the liquid reserves held by the bank are sufficient to cover withdrawals by impatient agents. Thus, the risky project is kept in whole, and the bank is able to distribute an interim dividend. And since $\frac{b}{\delta} < n^{\mathrm{def}}$, the equity payoff *conditional on there not being a run* is strictly positive when the project succeeds. Thus,

$$E_0(\lambda) = z\mathscr{P}(x^*(\lambda)) - \delta(1-\lambda)D_0^s + (b - \lambda\delta) \quad \text{(B.15)}$$

For all $\lambda > \frac{b}{\delta}$, early withdrawals by impatient agents exceed the liquid reserves of the bank, which is not able to distribute any dividend and is forced to rescale a fraction of the project. Thus, the date-0 expected equity payoff *conditional on there not being a run* is then

$$E_0(\lambda) = \max\left\{\left(1 - \frac{\lambda\delta - b}{\ell}\right)z\mathscr{P}(x^*(\lambda)) - \delta(1-\lambda)D_0^s, 0\right\} \quad \text{(B.16)}$$

where $\mathscr{P}(x^*(\lambda)) := \mathbb{E}[p(\theta)|\theta > x^*(\lambda)]$. Moreover, $\delta(1-\lambda)D_0^s = \left(1 - \frac{\lambda\delta-b}{\ell}\right)z\mathscr{P}(x^*(\lambda))$ when $E_0 = 0$. Using this last fact, substituting (B.15) and (B.16) into (B.14) and rearranging the terms, yields

$$U^B = \frac{\Gamma}{(1-\mu_\lambda)} \cdot$$
$$\mathbb{E}\left[(1-\lambda)\left(\pi(\lambda)\ell + (1-\pi(\lambda))\left(z\mathscr{P}(x^*(\lambda)) - \frac{\lambda\delta-b}{\ell}z\mathscr{P}(x^*(\lambda))\mathbb{I}_{\{b<\lambda\delta\}} + (b-\lambda\delta)\mathbb{I}_{\{b\geq\lambda\delta\}}\right)\right.\right.$$
$$\left. + (1-\pi(\lambda))\lambda D_0^s(\lambda)\delta + \frac{\lambda}{1-\lambda}\frac{\pi(\lambda)\ell + (1-\pi(\lambda))\delta}{\beta} - \frac{v'(\omega_0 - I)(I + (q_b - \hat{q}_b)b)}{(1-\lambda)\beta}\right.$$
$$\left.\left. - (1-\pi(\lambda))\left(\frac{\lambda}{1-\lambda}\frac{1}{\beta} + 1\right)b\right)\right].$$

By defining the expectation operator $\widehat{\mathbb{E}}\left[f(\lambda)\right] = \mathbb{E}\left[\frac{1-\lambda}{1-\mu_\lambda}f(\lambda)\right]$, the expression can be written as

$$U^B = \Gamma \times \widehat{\mathbb{E}}\left[z\mathscr{P}(x^*(\lambda)) - \frac{I}{v'(\omega_0 - I)^{-1}(1-\lambda)\beta} + \frac{\lambda}{1-\lambda}\frac{1-\pi(\lambda)(1-\ell/\delta)}{\beta}\delta\right.$$
$$- \pi(\lambda)\left(z\mathscr{P}(x^*(\lambda)) - \ell\right) - (1-\pi(\lambda))\lambda\delta\left(\frac{z\mathscr{P}(x^*(\lambda))}{\ell} - D_0^s(\lambda)\right)$$
$$+ (1-\pi(\lambda))\left(\frac{z\mathscr{P}(x^*(\lambda))}{\ell}\left(b - (b-\lambda\delta)^+\right) + (b-\lambda\delta)^+\right)$$
$$\left. - \left((1-\pi(\lambda))\left(1 + \frac{\lambda}{1-\lambda}\frac{1}{\beta}\right) + \frac{q_b - \hat{q}_b}{v'(\omega_0 - I)^{-1}(1-\lambda)\beta}\right)b\right] \tag{B.17}$$

where

$$\Gamma(\delta, b) = 1 + \frac{\text{Cov}\left(\lambda, (1-\pi(\lambda))E_0(\delta, b; \lambda)\right)}{\mathbb{E}\left[(1-\lambda)(1-\pi(\lambda))E_0(\delta, b; \lambda)\right]}. \tag{B.18}$$

B.1.7 Proposition 12

Recall the definition of the *liquidity ratio* in (2.6), $\overline{n} = \frac{\ell+b}{\delta}$. ℓ is fixed once and for all. For a given R, Proposition 2.27 shows that the expected payoff of withdrawing under the Laplacian belief, $\int_\lambda^1 D_0^w(n, \delta, b)\, dn$, is a function of $\overline{n}$ only. The same holds true for the expected payoff of staying, $R\int_\lambda^1 Q(n, \delta, b, R)\, dn$, since n^{def} can be expressed as a function of $\overline{n}$, see equation (2.7). Moreover, from Proposition 2.27 and Corollary 2, the run threshold x^* and the ex-ante run probability π are functions of the expected payoffs $\int_\lambda^1 D_0^w(n, \delta, b)\, dn$ and $R\int_\lambda^1 Q(n, \delta, b, R)\, dn$, with R set at $\overline{R}$ (which we have assumed here to be constant). Thus, x^* and π also only depend on $\overline{n}$.

Consider choices of (δ, b) along a *iso-$\overline{n}$ line*, i.e., such that

$$\frac{\ell + b}{\delta} = \overline{n}$$

where $\overline{n}$ is fixed. We first focus on those values of (δ, b) such that there is no dividend at $t = 1$: $\delta \lambda \geq b$. Formally, we are looking at choices $(b, \delta(b))$, with $b \in [0, b^+]$ and

$$
\begin{aligned}
\delta : [0, b^+] &\rightarrow \mathbb{R}_+ \\
b &\mapsto \delta(b) := \frac{\ell + b}{\overline{n}},
\end{aligned}
$$

where $b^+ = \frac{\lambda \ell / \overline{n}}{1 - \lambda / \overline{n}}$ is the point where liquid reserves exactly covers the sure outflows: $b^+ = \lambda \delta(b^+)$. By equations (2.54)–(2.55), given the choice $(\delta(b), b)$ the banker expects the claims issued by her bank to fetch the prices

$$
\begin{aligned}
\hat{q}_d(\delta(b), b) &= v'(\omega_0 - I)^{-1} \rho^d(\delta(b), b), \\
\hat{q}_e(\delta(b), b) &= v'(\omega_0 - I)^{-1} \rho^e(\delta(b), b)
\end{aligned}
$$

where for ease of notation we defined $\rho^d(\delta, b)$ ($\rho^e(\delta, b)$) to be the total expected utility over periods 1 and 2 provided by one unit of deposit (equity) (similarly to (B.9)–(B.10))

$$
\begin{aligned}
\rho^d(\delta(b), b) &= \lambda \cdot \left(\pi(\delta(b), b) \frac{\ell + b}{\delta(b)} + (1 - \pi(\delta(b), b)) \right) \\
&\quad + (1 - \lambda) \cdot \beta \left(\pi(\delta(b), b) \frac{\ell + b}{\delta(b)} + (1 - \pi(\delta(b), b)) D_0^s(\delta(b), b) \right), \\
\rho^e(\delta(b), b) &= (1 - \lambda) \cdot \beta \, (1 - \pi(\delta(b), b)) \, E_0(\delta(b), b).
\end{aligned}
$$

By construction of $\delta(b)$, the liquidity ratio $\overline{n}$, the run/roll strategy of investors x^*, and the ex-ante run probability are constant $\forall b \in [0, b^+]$. This implies that D_0^s is constant $\forall b \in [0, b^+]$. The same holds true for $E_0(\delta(b), b)$. Indeed, for $0 \leq b \leq b^+$ no dividend is paid, hence the equity payoff entirely comes from date-2 payments:

$$E_0(\delta(b), b) = \mathbb{E}_{\mu, \tau^{-1}} [p(\theta) z | \theta > x^*(\delta(b), b, \overline{R})],$$

which is constant since $x^*(\delta(b), b, \overline{R})$ is constant. Therefore, $\rho^d(\delta(b), b)$ and $\rho^e(\delta(b), b)$ are constant, and so are $\hat{q}_d(\delta(b), b)$ and $\hat{q}_e(\delta(b), b)$.

The fraction of equity sold by the banker, ψ, is determined by the fact that $(\delta(b), \psi, b)$ must finance total investment

$$\hat{q}_d(\delta(b), b) \delta(b) + \hat{q}_e(\delta(b), b) \psi = I + q_b b.$$

Since $\hat{q}_d(\delta(b), b)$ and $\hat{q}_e(\delta(b), b)$ are constant, and δ is affine in b by construction, we see that

$$\psi : b \mapsto \frac{I + q_b b - \hat{q}_d(\delta(b), b)\delta(b)}{\hat{q}_e(\delta(b), b)} \tag{B.19}$$

is affine in b. Because $\pi(\delta(b), b)$ and $E_0(\delta(b), b)$ are constant, this implies that the banker's profit

$$b \mapsto (1 - \psi(b))(1 - \pi(\delta(b), b))E_0(\delta(b), b) \tag{B.20}$$

is affine in b. Hence, either $b = 0$ or $b = b^+$ is an optimal choice among the $(\delta(b), b)$ with $b \in [0, b^+]$.

We now focus on values of (δ, b) such that the bank distributes an intermediary dividend: $\delta\lambda < b$. That is, we are looking at choices $(b, \delta(b))$ with $b \in [b^+, \infty)$ and

$$\begin{aligned}
\delta : [b^+, \infty) &\rightarrow \mathbb{R}_+ \\
b &\mapsto \delta(b) := \frac{\ell + b}{\overline{n}},
\end{aligned}$$

The only difference with the previous case is that the time-0 equity value conditional upon no run, E_0, now includes dividend payments:

$$E_0(\delta(b), b) = \mathbb{E}_{\mu, \tau^{-1}}[p(\theta)z|\theta > x^*(\delta(b), b, \overline{R})] + b - \lambda\delta(b).$$

Since δ is affine in b, $E_0(\delta(b), b)$ is affine in b. As in (B.20), we decompose the profit of the banker as

$$(1 - \pi(\delta(b), b))E_0(\delta(b), b) - \psi(b)(1 - \pi(\delta(b), b))E_0(\delta(b), b).$$

The first term is affine in b because $\pi(\delta(b), b)$ is constant. For the second term, notice that E_0 is proportional to $\hat{q}_e$, so by (B.19) $\psi(b)E_0(\delta(b), b)$ is proportional to $I + q_b b - \hat{q}_d(\delta(b), b)\delta(b)$, which is affine in b because $\hat{q}_d$ is constant as before. We conclude that either $b = b^+$ or $b = \infty$ is an optimal choice among the $(\delta(b), b)$ with $b \geq b^+$.

Consider now an arbitrary choice of (δ_0, b_0). This choice yields the liquidity ratio $\overline{n} = \frac{\ell + b_0}{\delta_0}$. But since the banker can increase linearly her utility by moving (up or down) along the iso-$\overline{n}$ line, the choice of (δ_0, b_0) is weakly dominated by the choice of $(\delta(b), b)$, where $b = 0$ (case (i)) or $b = \infty$ (case (iii)) or $b = b^+$. This concludes the proof, because $b^+ = \lambda\delta(b)$, which corresponds to case (ii) in the claim of the proposition.

B.1.8 Lemma 9

The expected consumption of investor i at equilibrium is given by

$$
\begin{aligned}
U^{\mathrm{I}} &= v(c_0^i) + \rho^d(\delta, b) d^i + \rho^e(\delta, b) e^i + \alpha(\omega_1 + T_1 + b^i) \\
&= v(\omega_0 - I) + \alpha\omega_1 + v'(\omega_0 - I)(q_d d^i + q_e e^i) - \alpha(B - b^i) \\
&= v(\omega_0 - I) + \alpha\omega_1 + v'(\omega_0 - I)(I + q_b b) - \alpha b \\
&= v(\omega_0 - I) + \alpha\omega_1 + v'(\omega_0 - I)\left(I + \left(q_b - \hat{q}_b\right) b\right),
\end{aligned}
$$

where the second line uses the first-order conditions with respect to deposit and equity (B.12)–(B.13) and the period-1 government budget constraint $T_1 = -B$, the third line uses (2.53) as well as the market-clearing condition for government bonds, and the last line uses the definition $\hat{q}_b$ in (2.56).

B.1.9 Proposition 13

First, note that if $B = 0$, the result is an immediate consequence of Lemma 9: in this case, the investors' utility is a constant which only depends on the exogenous data of the model; thus, the quantity of deposits that maximises the bankers' utility also maximises social welfare. We now consider the case $B > 0$.

The bankers' budget constraint allows to deduce ψ from δ. Denote by δ^* the privately optimal level of deposit issuance, and by q_b^* the equilibrium bond price. Suppose that banks are constrained to issue another quantity of deposits δ', and let q_b' be the corresponding market-clearing bond price. The bankers' optimal choice of bond holdings are denoted b^* and b', respectively. Lemma 8 gives the bankers' utility $U^{\mathrm{B}}(\delta, b; q_b)$ for a given q_b. We obtain

$$
U^{\mathrm{B}}(\delta', b'; q_b') = U^{\mathrm{B}}(\delta', b'; q_b^*) + \Gamma' b' v'(\omega_0 - I)\beta^{-1}\hat{\mathbb{E}}\left[(1 - \lambda)^{-1}\right](q_b^* - q_b').
$$

If δ' achieves a Pareto improvement, we have $U^{\mathrm{B}}(\delta', b'; q_b') \geq U^{\mathrm{B}}(\delta^*, b^*; q_b^*) \geq U^{\mathrm{B}}(\delta', b'; q_b^*)$; where the second inequality follows by the private optimality of (δ^*, b^*). Since all the terms multiplying $(q_b^* - q_b')$ are strictly positive, it follows that $q_b^* \geq q_b'$, with equality if and only if $U^{\mathrm{B}}(\delta', b'; q_b') = U^{\mathrm{B}}(\delta^*, b^*; q_b^*)$. In words: any mandatory change in the level of deposit issuance only increases the bankers' utility insofar as it reduces the costs of funding the liquid reserves.

Notice that: (i) if $q_b^* > q_b'$, we have $b^* = B$ (banks hold all the bonds) and the investors' utility has varied by $(U_b^{\mathrm{I}})' - (U_b^{\mathrm{I}})^* = v'(\omega_0 - I)(q_b' - q_b^*)B < 0$ (consider successively the cases $q_b' = \hat{q}_b$ and $q_b' > \hat{q}_b$ and use Lemma 9 to observe that we reach this inequality in both cases); (ii) if $q_b^* = q_b'$ then the bankers' utility is unchanged as noted above, and so is the investors' utility

due to Lemma 9.

In both cases, we obtain a contradiction with the fact that δ' achieves a Pareto improvement.

B.2   Additional Material

B.2.1   No Time-0 Dividend

In this section, we derive a simple condition which guarantees that it is not optimal for the banker to raise funds at $t = 0$ for the purpose of distributing an initial dividend, in the case where the optimal funding policy involves a mix of equity and deposit funding.

Start from the no-dividend candidate equilibrium, and consider a deviation of one banker where she raises one additional unit of funds to be paid as an initial dividend. Suppose that she raises this additional unit by issuing equity. For one unit of consumption at time 0, the banker foregoes an expected consumption of $\frac{1}{q_e} \int_{\underline{\lambda}}^{\overline{\lambda}} d\lambda\, g(\lambda)\, (1 - \pi(\delta, b; \lambda))\, E_0(\delta, b; \lambda)$ units. But we have

$$
\frac{1}{q_e} \int_{\underline{\lambda}}^{\overline{\lambda}} d\lambda\, g(\lambda)\, (1 - \pi(\delta, b; \lambda))\, E_0(\delta, b; \lambda) \quad > \quad \frac{1}{q_e} \int_{\underline{\lambda}}^{\overline{\lambda}} d\lambda\, g(\lambda)(1 - \lambda)\beta\, (1 - \pi(\delta, b; \lambda))\, E_0(\delta, b; \lambda)
$$
$$
= \quad \frac{1}{\beta} v'(\omega_0 - I),
$$

where the last equality follows from (2.55). Thus, issuing equity to distribute an initial dividend is suboptimal for the banker as long as the investors' marginal utility of period-0 consumption is not too low: $v'(\omega_0 - I) > \beta$. At the candidate equilibrium, the banker is indifferent between debt and equity financing, so financing the dividend with debt would not be profitable either.

B.2.2   Dominance Regions

Since $p(\theta) \to 0$ as $\theta \to -\infty$, there exists a $\underline{\underline{\theta}} \in \mathbb{R}$ such that for all $\theta < \underline{\underline{\theta}}$ the expected payoff from staying is lower than the payoff from withdrawing early even if no patient investor runs ($n = \lambda$). In this region, the fundamental is so weak that withdrawing is the dominant action for investor i regardless his belief regarding the action chosen by other investors. Therefore, the model features a *lower dominance region.* By contrast, without imposing an additional assumption, the game lacks an *upper dominance region*: realisations of θ where staying is a strictly dominant action. Indeed, every investor running on the bank regardless of the signal realisations, is always an equilibrium (it corresponds to the "bad" equilibrium in Diamond and Dybvig (1983)). Following Goldstein and Pauzner (2005), we deal with this problem by assuming that there is a region of extremely strong fundamentals in which success of the

project is guaranteed, and the bank is able to liquidate the project at no cost. The idea is that a project that is guaranteed to succeed is no longer illiquid.

Assumption 9 *There exists a $\overline{\overline{\theta}}$ such that for $\theta > \overline{\overline{\theta}}$, the project succeeds (i.e. realises z) at $t = 2$ with probability one and can be rescaled at $t = 1$ at no cost (i.e. the liquidation value is $\overline{\ell} = z$).*

Assumption 9 implies that when $\theta > \overline{\overline{\theta}}$, the payoff from staying is always higher than the payoff from withdrawing early, regardless of n. If $\theta > \overline{\overline{\theta}}$ and a mass n of investors run, the fraction of the project remaining in period 2 is $1 - \frac{(n\delta - b)^+}{z}$. Thus, the project pays off $z\left(1 - \frac{(n\delta - b)^+}{z}\right) \geq z - n\delta$ with certainty at $t = 2$, while the remaining debt is $\delta R(1 - n)$. By Assumption 5, $\delta R \leq z$. Therefore, the period 2 payoff is large enough to repay R to the remaining depositors with probability one.

$$z\left(1 - \frac{(n\delta - b)^+}{z}\right) \geq \delta R - n\delta \geq \delta R(1 - n),$$

Because $R \geq 1$, is is preferable to stay rather than to withdraw. Since $\overline{\overline{\theta}}$ can be placed arbitrarily far from the mean, it eliminates the equilibrium where investors ignore their signal and always run, without changing quantitatively the outcome of the game or the payoff of the financial claims issued by banks. In fact, throughout the chapter we assign positive but arbitrarily low probability to the interval $(\overline{\overline{\theta}}, \infty)$, and carry out all payoff computations without explicit mention of the upper dominance region.

B.3 Figures

The probability of being impatient, λ, follows a truncated lognormal distribution, with support $(\underline{\lambda}, \overline{\lambda}] = (0, 0.4]$, expected value $\mu_\lambda = 0.125$, and standard deviation $\sigma_\lambda = 0.05$ For the function $p : \mathbb{R} \to [0,1]$ mapping θ into the success probability of the risky project, we choose the logistic specification $p(\theta) = \frac{1}{1 + e^{-\theta}}$.

Parameter	Description	Value
B	Government bond supply	0.15
I	Investment cost of risky project	1
z	Payoff of risky project if successful	3
ℓ	Liquidation value of risky project	60%
ω_0, ω_1	Endowments of investors	2.5, 0.3
β	Patient agents' discount factor	50%
$\mu, \tau^{-1/2}$	Moments of the prior on θ	0.5, 0.5
$\mu_\lambda, \sigma_\lambda$	Moments of aggregate liquidity needs	0.125, 0.05
$\underline{\lambda}, \overline{\lambda}$	Min. and max. aggregate liquidity needs	0, 0.4

**Figure B.1: Baseline Parameters.**

Appendix B. Appendix for Chapter 2

$t = 0$ $t = 1$ $t = 2$

⋄ bankers choose funding policy   ⋄ investors learn their type   ⋄ project return realises
    and bond holdings   ⋄ public signal realises   ⋄ banks repay their debt
⋄ investors choose portfolio   ⋄ bankers choose deposit rate R ⋄ residual is distributed to
                            ⋄ private signals realise   equity holders
                            ⋄ run decisions
                            ⋄ dividend payment (if any)
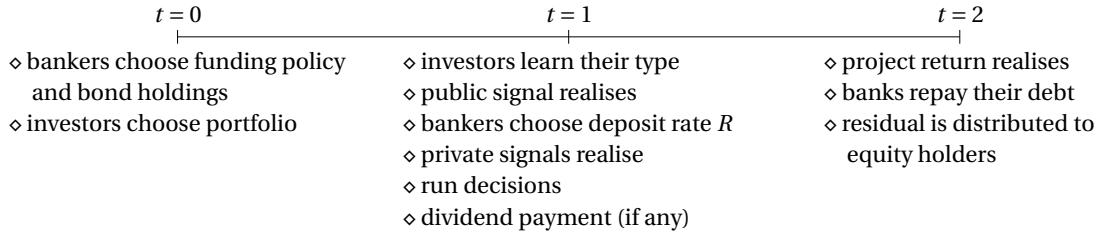
Figure B.2: Timeline of the model.
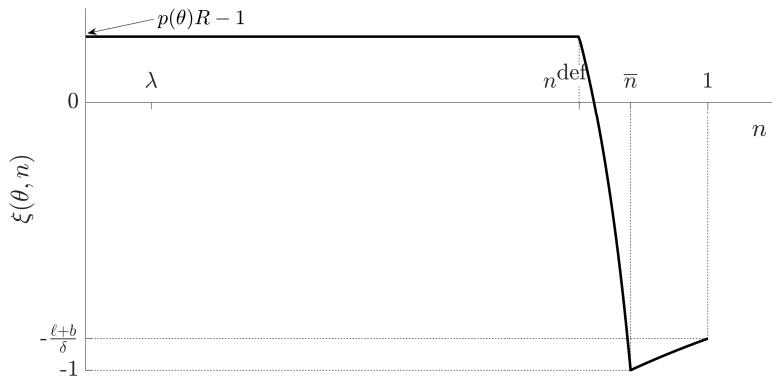
Figure B.3: Payoff gain from staying vs. withdrawing early for a given fundamental.

Figure B.3 depicts the function $n \mapsto \xi(\theta, n)$. ($\theta = 0.5$, $\delta = 0.85$, $b = 0.15$, $\lambda = 0.15$, $R = 2$; other parameters are set at their baseline values, see Figure B.1.)
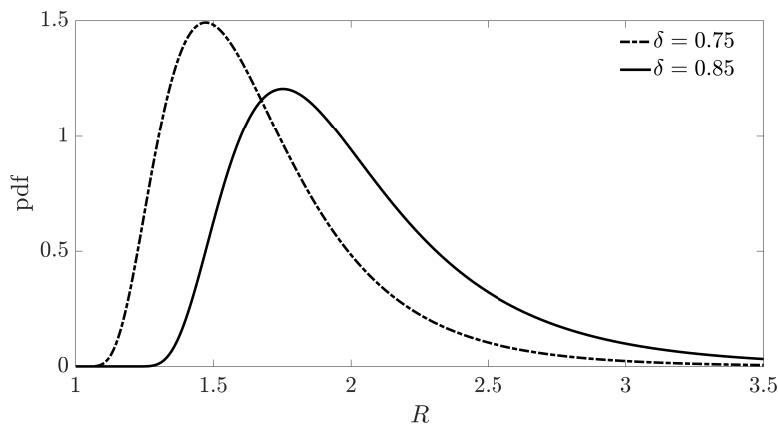
Figure B.4: Probability density of period-2 deposit rate.

Figure B.4 depicts the pdf of the random variable induced by the function $\theta \mapsto R^*(\theta; \delta, b, \lambda)$, i.e. the optimal deposit rate in state θ, for two quantities of deposits. ($\delta = 0.75, 0.85$, $b = 0.15$, $\lambda = 0.15$; other parameters are set at their baseline values, see Figure B.1.)
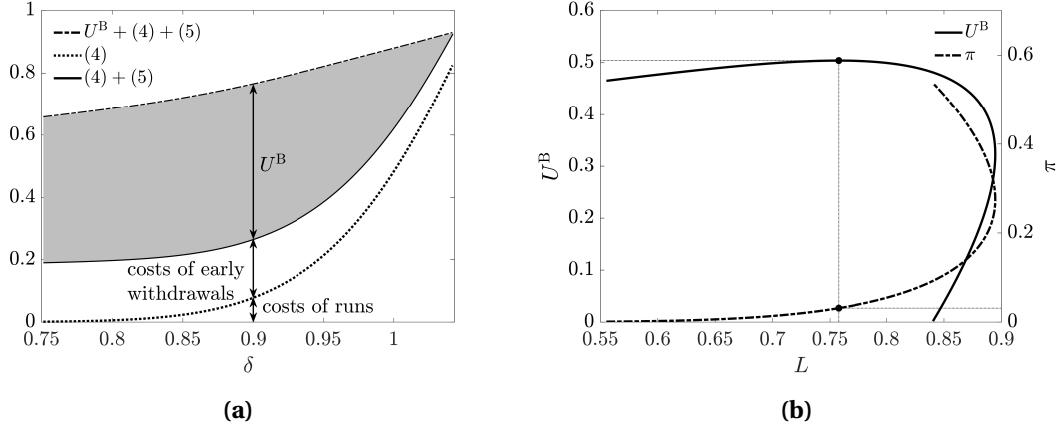
Figure B.5: Banker's choice of funding policy.

Figure B.5a illustrates the decomposition in (2.57). The solid line plots the liquidation costs: the sum of the last two terms in (2.57). The dotted line isolates the part of liquidation costs which is due to runs (fifth term). The banker's utility is given by the height of the shaded area. Figure B.5b shows how the banker's utility, U^B, and the ex-ante run probability, π, vary with the leverage ratio, L. The optimal funding policy consists of a 76/24 debt-equity mix. ($q_b = 0.9585$, $b = 0.15$; all parameters are set at their baseline values, see Figure B.1.)
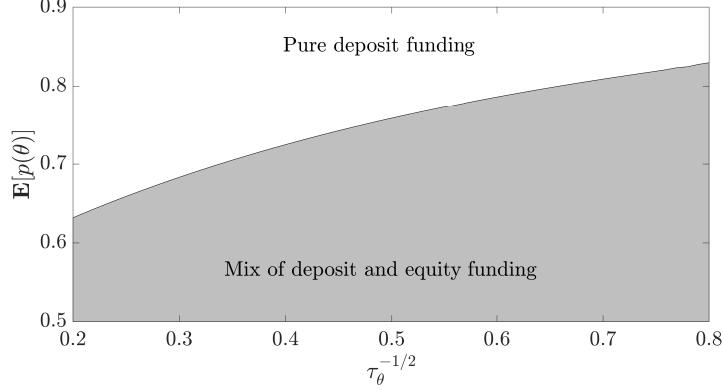
Figure B.6: Boundary between equilibria with pure deposit funding and equilibria with mix of deposit and equity funding.

Figure B.6 shows how the equilibrium type varies with the characteristics of bank assets. The line marks the expected success probability of the project that makes a banker indifferent between a funding policy which involves pure deposit funding and one which involves a mix of deposit and equity funding. It is constructed as follows. For a given a level of volatility of the fundamental, $\tau^{-1/2}$, we solve for the prior mean, μ, such that the first-order condition of the banker's problem (2.51)–(2.52) with respect to δ is equal to zero at $L = 1$. Given a pair $(\mu, \tau^{-1/2})$, the expected success probability is obtained by computing the expectation $\mathbb{E}_{\mu, \tau^{-1}}[p(\theta)]$. For levels of expected success probability above (below) the line, the banker chooses a funding policy with $L = 1$ ($L < 1$). (All parameters are set at their baseline values, see Figure B.1.)
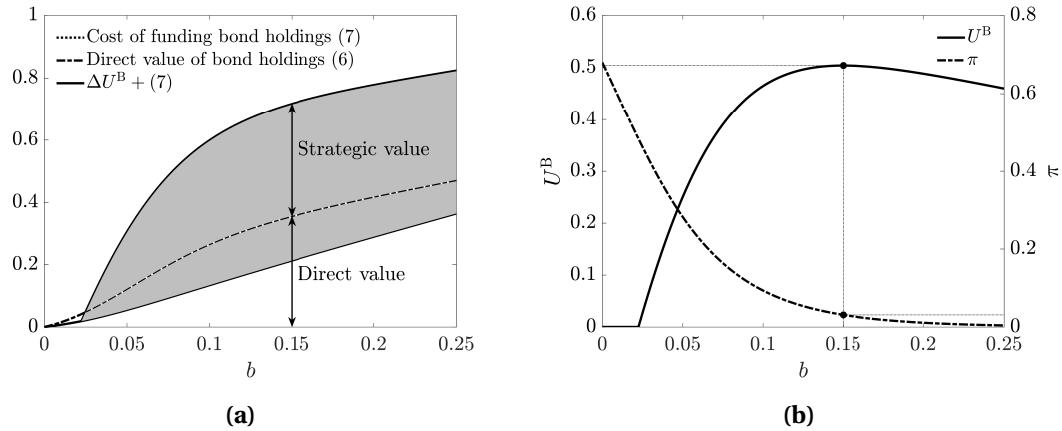
Figure B.7: Banker's choice of bond holdings.

Figure B.7a illustrates the decomposition in (2.57). The dotted line plots the costs of financing the liquid reserves. The dashed-dotted line plots the *direct* value of liquid reserves. The change in the banker's utility, ΔU^{B}, is given by the height of the shaded area. The *strategic* value of liquid reserves is equal to the portion of the utility gain (gross of funding costs) that is not captured by the direct value. Figure B.7b shows how the banker's utility, U^{B}, and the ex-ante probability of runs, π, vary with bond holdings, b. The banker's utility is maximised at $b^* = 0.15$. ($q_b = 0.9585$, $\delta = 0.87$; all parameters set at their baseline values, see Figure B.1.)
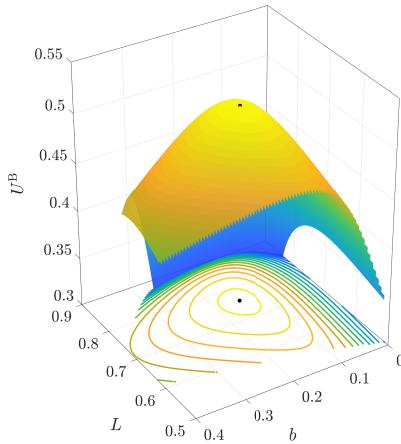
Figure B.8: Banker's utility surface and iso-utility curves.

The surface depicts the bankers' objective function, U^{B}, as a function of leverage L and bond holdings b. The contour diagram drawn on the horizontal plane plots the banker's iso-utility curves: combinations of leverage and bond holdings which yield the same level of expected utility. ($q_b = 0.9585$; other parameters are set at their baseline values, see Figure B.1.)
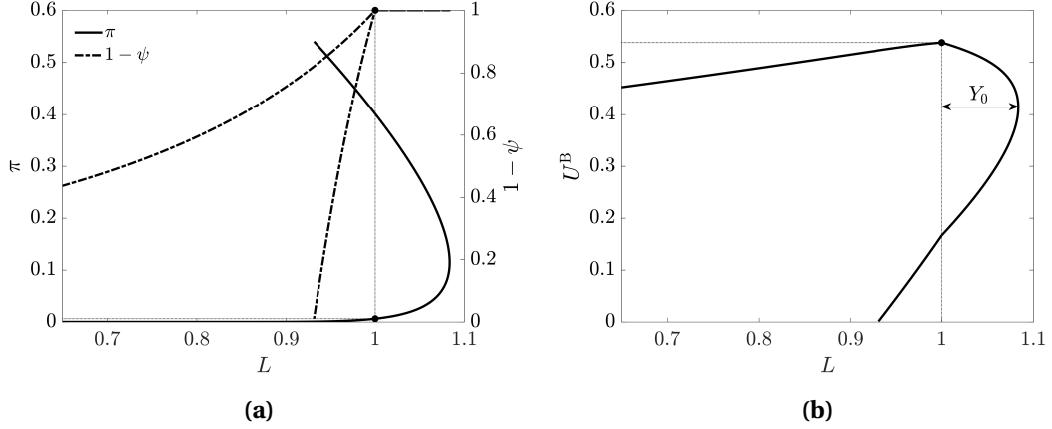
Figure B.9: Equilibrium with pure deposit funding in model without liquid reserves.

Figure B.9a plots the ex-ante run probability, π, and the fraction of outstanding equity retained by the banker, $1 - \psi$, as a function of leverage. Figure B.9b shows how the banker's expected utility, U^{B}, varies with the leverage ratio, L. The banker's expected utility increases approximately linearly with leverage for $L < 1$ and exhibits a kink at $L = 1$, at which point the bank starts paying an initial dividend. The optimal funding policy consists in financing investment entirely with deposits, and to set the initial dividend, Y_0, equal to zero. ($\mu = 0.75$, $\tau = 1/(0.25)^2$; other parameters are set at their baseline values, see Figure B.1.)
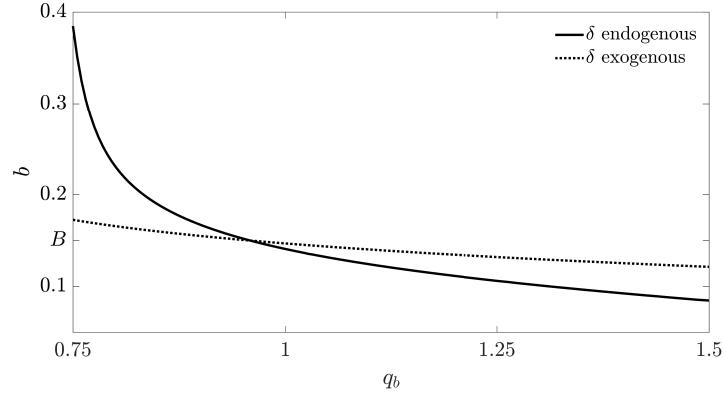
Figure B.10: Bankers' demand for government bonds.

Figure B.10 depicts a banker's *endogenously financed* bond demand $b(q_b)$ (solid) and the *exogenously financed* bond demand $b(q_b; \delta)$ (dotted). The endogenously financed bond demand gives the banker's optimal choice of bond holdings at the *optimal* funding policy. (Parameters are set at their baseline values, see Figure B.1).

(a) **(b)**
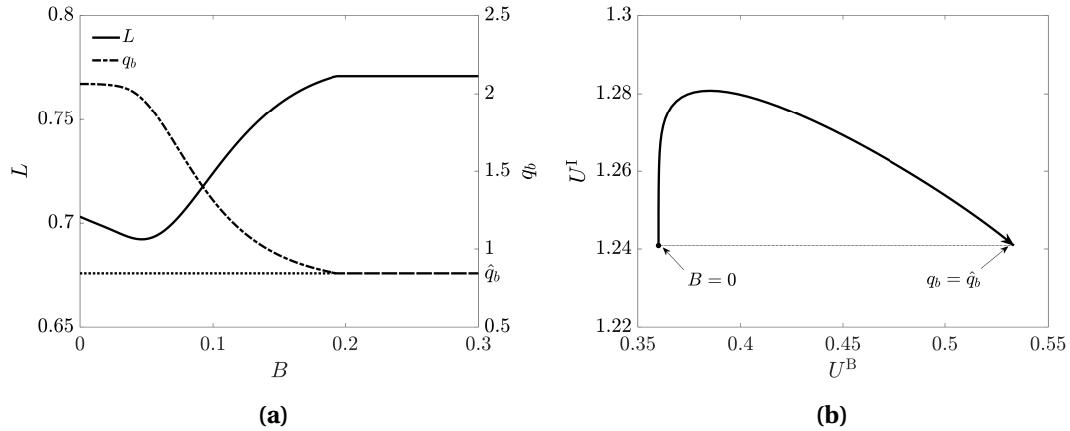
Figure B.11: Impact of government bond supply.

Figure B.11a shows how varying the bond supply of government bonds B impacts the leverage ratio L (solid line), and the bond price q_b (dashed-dotted line). The dotted line plots the investors' bond valuation, $\hat{q}_b$. Figure B.11b traces out the path of the investors' utility U^{I} and bankers' utility U^{B} as the bond supply increases from 0 (arrow start) to 0.3 (arrow head). (All parameters are set at their baseline values, see Figure B.1.)

(a) Volatility of fundamentals **(b) Mean aggregate liquidity needs**
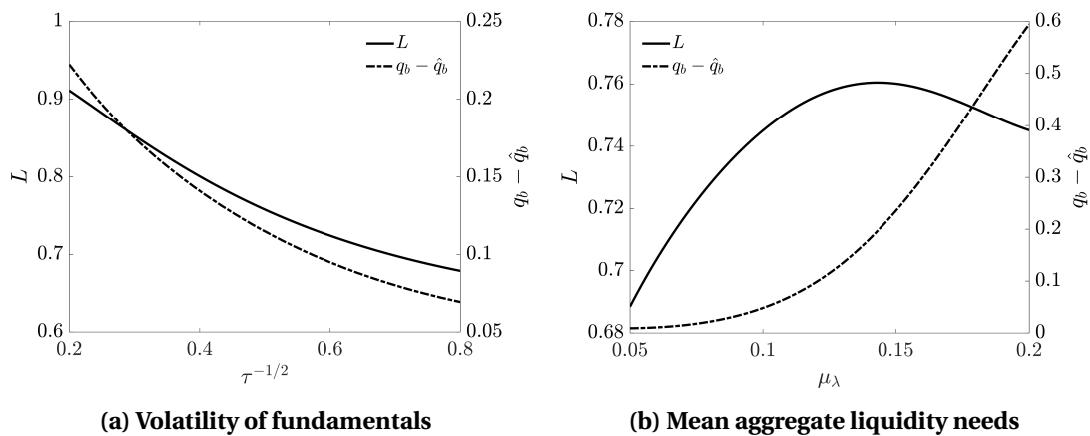
Figure B.12: Comparative statics of bank leverage and the price of public liquidity.

Figure B.12 shows how varying the volatility of the fundamental $\tau^{-1/2}$ (left plot) and the mean aggregate liquidity needs μ_λ (right plot), impacts the leverage ratio L (solid line) and the bond liquidity premium $q_b - \hat{q}_b$ (dashed-dotted line). (All parameters are set at their baseline values, see Figure B.1.)

C Appendix for Chapter 3

C.1 Additional Proofs

C.1.1 Normalisation of supports to $[-1, 1]$

Assume $u \sim U(-a, a)$ and $v \sim U(b, c)$ with $a > 0$ and $b < c$. We want to map an equilibrium with these noise terms and penalty C to an equilibrium with normalised noise. Let $C^0(x^0) = \frac{1}{\sigma} C(ax^0)$ for $-1 \leq x^0 \leq 1$. C^0 defines a penalty in $\mathscr{C}$.

Let (X^0, P^0) be an equilibrium of $\mathscr{K}(C^0)$ under uniform noise distributed over $[-1, 1]$, and admissible demands $I^0 = [-1, 1]$. Let Φ be the linear application mapping $[b, c]$ to $[-1, 1]$:

$$\Phi(v) = \frac{2}{c - b} v - \frac{c + b}{c - b}$$

Similar to Lemma 10, the expected price function must be $\hat{P}(x) = m + \frac{\sigma x}{2a}$ where

$$
\begin{aligned}
m &= \frac{b + c}{2}, \\
\sigma &= \frac{c - b}{2}.
\end{aligned}
$$

For any $v \in [b, c]$, the maximisation program of the IT is

$$\max_{x \in [-a, a]} x\left(v - m - \frac{\sigma}{2a}x\right) - C(x).$$

This can be rewritten as

$$\max_{x^0 \in [-1, 1]} (ax^0)\left(v - m - \frac{\sigma}{2a}(ax^0)\right) - C(ax^0).$$

Or:

$$(a\sigma) \max_{x^0 \in [-1,1]} x^0 \left(v/\sigma - m/\sigma - \frac{x^0}{2} \right) - C^0(x^0),$$

By definition of X^0, the solution of this program is given by $X^0 \left(\frac{v-m}{\sigma} \right) = X^0 \left(\Phi(v) \right)$. Recalling that the actual demand of the IT is $x = ax^0$, we obtain

$$X(v) = aX^0 \left(\frac{v-m}{\sigma} \right) = aX^0 \left(\Phi(v) \right).$$

We can also express the price function using P^0. Since Φ is linear, we can write

$$
\begin{aligned}
P(d) &= \mathbb{E}[v|d] \\
&= \Phi^{-1} \left(\mathbb{E}[\Phi(v)|X(v) + u = d] \right) \\
&= \Phi^{-1} \left(\mathbb{E}\left[\Phi(v) | aX^0(\Phi(v)) + a(u/a) = d \right] \right) \\
&= \Phi^{-1} \left(\mathbb{E}\left[v^0 | X^0(v^0) + u^0 = d/a \right] \right) \\
&= \Phi^{-1} \left(P^0(d/a) \right),
\end{aligned}
$$

because $v^0 = \Phi(v)$ and $u^0 = u/a$ are independent $U(-1,1)$ variables.

So the equilibrium with noise $u \sim U(-a,a)$ and $v \sim U(b,c)$, penalty C and admissible demands $I = [-a,a]$ can be mapped to the equilibrium of $\mathcal{K}(C^0)$ with normalised noise and admissible demands $I^0 = [-1,1]$. By the same procedure, one can do the reverse mapping.

Finally, we note that the model quantities of interest (S, G and F) are mapped one-to-one and ranked identically regardless of the chosen supports, *i.e.* the assertions "$S < S'$","$G < G'$" or "$F < F'$" do not depend on which supports we consider. Therefore, the choice of $[-1,1]$ as the support of the noise is without loss of generality once we assume uniform distributions and a centered uninformed traders' demand.

C.1.2   Lemma 12

First, let us show that $\mathcal{X}_C(v)$ is never empty. Let $v \in [-1,1]$, the function $\psi_C(.,v)$ has a finite upper bound as $C \geq 0$. Let $M = \sup_x \psi_C(x,v) < \infty$ and (x_n) such that $\psi_C(x_n,v) \to M$. There is an extraction of (x_n), still denoted (x_n), such that x_n converges to x and either (i) (x_n) is increasing or (ii) (x_n) is decreasing. By symmetry, we can assume without loss of generality that $x > 0$ or $x = 0$ and the case (ii) holds. Let us first consider case (i). Since C is left-continuous and $x \mapsto x \left(v - \frac{x}{2} \right)$ is continuous, $\psi_C(x_n,v)$ converges to $\psi_C(x,v)$: therefore $\psi_C(x,v) = M$ and $x \in \mathcal{X}_C(v)$. Let us now consider case (ii). Since C is non decreasing, it has a right limit at x denoted by $C(x^+)$ which is greater than $C(x)$. Taking the limit in the definition of $\psi_C(x_n,v)$, the value of $\psi_C(x_n,v)$ converges to $x \left(v - \frac{x}{2} \right) - C(x^+) \leq x \left(v - \frac{x}{2} \right) - C(x)$. Using the fact that

$\psi_C(x_n, v)$ converges to M, we conclude that $C(x^+) = C(x)$ and $\psi_C(x, v) = M$.

Now, let us show that $\mathscr{X}_C$ is a non-decreasing correspondence. Let $v_1 < v_2$ in $[-1; 1]$ and $x_1^* \in \mathscr{X}_C(v_1)$ and $x_2^* \in \mathscr{X}_C(v_2)$. For any $x \in [-1, 1]$:

$$\psi_C(x, v_2) = \psi_C(x, v_1) + (v_2 - v_1)x.$$

Using the fact that $x_1^* \in \mathscr{X}_C(v_1)$ and $v_1 < v_2$, for any $x < x_1^*$,

$$\psi_C(x, v_2) < \psi_C(x_1^*, v_1) + (v_2 - v_1)x_1^* = \psi_C(x_1^*, v_2).$$

By definition, $\psi_C(x_2^*, v_2) \geq \psi_C(x_1^*, v_2)$, thus $x_2^* \geq x_1^*$. Since this inequality holds for any $x_1^* \in \mathscr{X}_C(v_1)$ and $x_2^* \in \mathscr{X}_C(v_2)$, we get that $\sup \mathscr{X}_C(v_1) \leq \inf \mathscr{X}_C(v_2)$: the correspondence $\mathscr{X}_C$ is non-decreasing.

C.1.3 Proposition 14

Using Lemma 15, we can write

$$
\begin{aligned}
-G &= \int_0^1 X(v)\left(v - \frac{X(v)}{2}\right) dv \\
&= 1 - \sqrt{3}S - \frac{1}{2}\int_0^1 X(v)^2 \, dv.
\end{aligned}
\tag{C.1}
$$

By Cauchy-Schwarz inequality

$$
\begin{aligned}
\left(\int_0^1 vX(v)dv\right)^2 &\leq \int_0^1 v^2 dv \int_0^1 X(v)^2 \, dv \\
&\leq \frac{1}{3}\int_0^1 X(v)^2 \, dv
\end{aligned}
\tag{C.2}
$$

$$-\frac{1}{2}\int_0^1 X(v)^2 \, dv \leq -\frac{3}{2}\left(\int_0^1 vX(v)\right)^2 dv = -\frac{3}{2}(1 - \sqrt{3}S)^2.$$

Plugging this into (C.1), we obtain

$$G \geq \sqrt{3}S - 1 + \frac{3}{2}(1 - \sqrt{3}S)^2.\tag{C.3}$$

This inequality determines the highest possible S given G. But there is equality in (C.3) if and only if there is equality in the Cauchy-Schwarz bound (C.2). This is the case if and only if the two functions in the left-hand side are colinear, i.e. if $X(v)$ is proportional to v: $X(v) = \beta v$. Since $0 \leq X(v) \leq 1$ for $0 \leq v \leq 1$, $\beta \in [0; 1]$. We conclude by noting that if $\beta \in [0; 1]$ and $\gamma \in [0; \infty]$ is defined by $\gamma = \frac{1}{2\beta} - \frac{1}{2}$, the quadratic penalty $C(x) = \gamma x^2$ implements $X(v) = \beta v$.

C.1.4 Lemma 17

We first need to introduce some definitions:

Let f be a function defined over $[0,1]$ and $x \in [0,1]$. We define:

$$
\begin{aligned}
\overline{D}^- f(x) &= \limsup_{x' \nearrow x} \frac{f(x') - f(x)}{x' - x}, \\
\underline{D}^- f(x) &= \liminf_{x' \nearrow x} \frac{f(x') - f(x)}{x' - x},
\end{aligned}
$$

One can define similarly $\overline{D}^+ f(x)$ and $\underline{D}^+ f(x)$. Let us recall the first order conditions satisfied by a function at a local maximum.

If x^* is a local maximum of f, then:

$$
\begin{aligned}
\overline{D}^+ f(x^*) &\leq 0, \\
\underline{D}^- f(x^*) &\geq 0
\end{aligned}
$$

We will also use the following real analysis result:

Lemma 19 *Any continuous function f on $]0,1]$ with a null left derivative is contant.*

Let C be a penalty function such that the strategy of the IT satisfies that for any $v \in [0,1]$, $X(v)$ is either 0 or v. Since the strategy of the IT is non-decreasing, there exists v_0 such that $X(v) = 0$ for any $v \in [0, v_0[$ and $X(v) = v$ for any $v \in]v_0, 1]$.

Besides, the penalty function C must be continuous on $]v_0, 1]$. Indeed, if $v' > v \geq v_0$, using the fact that $X(v') = v'$,

$$
v\left(v' - \frac{v}{2}\right) - C(v) \leq v'\left(v' - \frac{v'}{2}\right) - C(v'),
$$

thus, since C is non-decreasing,

$$
0 \leq C(v') - C(v) \leq v'\left(v' - \frac{v'}{2}\right) - v\left(v' - \frac{v}{2}\right).
$$

Taking the limit as v' goes to v, we see that C is right continuous at v. Since by hypothesis it is left continuous on $[0,1]$, the penalty function C is continuous on $]v_0, 1]$.

Let us show that C has a null left derivative on $]v_0, 1]$. If $v \in]v_0, 1]$, we know that v is a profit maximiser at v: $v \in \operatorname{argmax}_x f_v(x)$:. Using the first order condition for the lower left derivative

$\underline{D}^-$ recalled above, at v, $\underline{D}^- f_v(v) \geq 0$. Since $\underline{D}^- f_v(v) = -\overline{D}^- C(v)$, we obtain $\overline{D}^- C(v) \leq 0$. Yet, C is increasing, so the lower and upper left derivatives must be positive : $0 \leq \underline{D}^- C(v) \leq \overline{D}^- C(v)$. Thus:

$$\underline{D}^- C(v) = \overline{D}^- C(v) = 0.$$

This means that the cost function C admits a left derivative at any $v \in]v_0, 1]$, and the value of this left derivative is zero.

Thus C is continuous and has a null left derivative on $]v_0, 1]$. Using Lemma 19, we obtain that C is constant on $]v_0, 1]$. Let us denote by K the value of C on this interval.

The IT does not trade for $v \in [0, v_0)$. In that case, since we know that $0 \leq X(v) \leq v$, we must have

$$\forall x \in [0, v], \quad x\left(v - \frac{x}{2}\right) \leq C(x).$$

By continuity of the left-hand term and the fact that the right-hand term is non-decreasing, we obtain

$$\forall x \in [0, v_0], \quad x\left(v_0 - \frac{x}{2}\right) \leq C(x).$$

There must be equality for $x = v_0$, because otherwise it would not be optimal to select $X(v) = v$ on the right neighborhood of v_0. For the same reason, C can not jump at v_0. This implies that $v_0\left(v_0 - \frac{v_0}{2}\right) = K$, or $v_0 = \sqrt{2K}$ and therefore C must belong to $\mathcal{O}$.

Assume conversely that $C \in \mathcal{O}$. Then for $0 \leq v < v_0$, the insider trader will make negative expected profits if she trades, so that $X(v) = 0$. For $v > v_0$, there are two cases to consider. (i) The IT plays $x \geq v_0$. In that case, the expected penalty K appears as a sunk cost and the best choice is $x = v$, leading to a net profit of $\frac{v^2}{2} - K$. (ii) The IT plays $x \in [0, v)$. The net profit is then

$$
\begin{aligned}
x\left(v - \frac{x}{2}\right) - C(x) &= x\left(v_0 - \frac{x}{2}\right) - C(x) + x(v - v_0) \\
&\leq x(v - v_0) \\
&\leq v_0(v - v_0)
\end{aligned}
$$

where the second line uses the fact that $C \in \mathcal{O}$. Since

$$
\begin{aligned}
\frac{v^2}{2} - K &= \frac{v^2}{2} - \frac{v_0^2}{2} \\
&= \frac{1}{2}(v + v_0)(v - v_0) \\
&> v_0(v - v_0),
\end{aligned}
$$

choice (i) is always preferred. Hence, if $C \in \mathcal{O}$, $X(v) = 0$ for $|v| < v_0$ and $X(v) = v$ for $|v| > v_0$,

which concludes the proof.

C.1.5 Theorem 4

First, define for $0 \le \alpha \le 1 - \sqrt{2K}$:

$$
X_\alpha(v) =
\begin{cases}
v & 0 \le v \le \alpha \\
\alpha & \alpha < v \le \alpha + \sqrt{2K} \\
v & v > \alpha + \sqrt{2K} \\
-X_\alpha(-v) & v < 0.
\end{cases}
$$

We will see that the X_α for $0 \le \alpha \le 1 - \sqrt{2K}$ are exactly the demand schedules that implement the lower bound in Theorem 4. (Note that these demand schedules are implemented by the penalties C_α where $C_\alpha(x) = K\mathbb{1}_{|x|>\alpha}$.)

Step 1: transformation of the problem into a constrained problem of L^2 distance maximisation.

Recall equation (3.22):
$$
|G| = \frac{1}{6} - \frac{1}{2}\int_0^1 (v - X(v))^2 \, dv.
$$

This means that obtaining the bound of the Theorem is equivalent to showing

$$
\max_{C\in\mathscr{C}_K}\int_0^1 (v - X(v))^2 \, dv = \frac{(2K)^{3/2}}{3}, \tag{C.4}
$$

subject to the constraint that $X(v)$ maximises the net profit $\psi_C(.,v)$.

Let $g(v) = v - X(v)$, so that we are looking for an upper bound of $\int_0^1 g^2$. By Lemma 14 and under the constraint $C \le K$, we obtain:

$$
\begin{aligned}
\int_0^1 g &= \int_0^1 v \, dv - \int_0^1 X(v) \, dv \\
&= \frac{1}{2} - \pi^N(1) \\
&\le K.
\end{aligned} \tag{C.5}
$$

This is because, when $v = 1$, the IT can achieve at least a net profit of $\frac{1}{2} - C(1) \ge \frac{1}{2} - K$. Therefore,

the maximum in (C.4) is less or equal to

$$\sup \int_0^1 g^2$$

subject to the constraints (i) $\int_0^1 g \leq K$, and (ii) $g(0) = 0 \leq g(v)$ and $v \mapsto v - g(v)$ is non-decreasing. (i) comes from (C.5), and (ii) is an immediate consequence of the properties of an optimal demand schedule X.

Notice how crucial Lemma 14 is, and therefore how effective the result of Milgrom and Segal (2002) is. Once noted that $C(1) \leq K$ implies a lower bound on the net profit at 1, Lemma 14 allows (i) to incorporate the constraint the X is a maximiser in a parsimonious way, (ii) to reduce the two constraints–$C \leq K$ and X must maximise ψ_C–into a single condition, $\int g \leq K$, which is particularly convenient, as it is a L^1 bound in a L^2 maximisation problem.

Absent the fact that X must be non-decreasing, which translates into the fact that $v \mapsto v - g(v)$ is non-decreasing, the maximisation of $\int g^2$ subject to $\int g = K$ (and $0 \leq g(v) \leq v$) would be standard: to "spread mass as unevenly as possible", one would pick $g(v) = v\mathbb{1}_{v \geq v^*}$ with $\int_{v^*}^1 v \, dv = K$. This is not feasible, however, because it violates the monotonicity constraint. The $g_\alpha : v \mapsto v - X_\alpha(v)$ are then natural candidate maximisers, as they are constructed in a similar spirit of variance maximisation, but respect the monotonicity constraint.

The g_α all have the same L^2 norm, but are away from zero over different intervals. This hints at the fact that for a general function g, when trying to find a bound on $\int g^2$, we will have no way to know where g must be small or large, and therefore little grip on g. The idea is then to consider the repartition function φ of g, because (i) one can reconstruct the moments of g with those of φ (see Step 3) and (ii) it does not matter *where* g is large, only how often it is large. In fact, all the g_α have the same repartition function, which suggests that this is the correct perspective to adopt. See Figure C.17.

For any function f and $x \neq y$, let

$$\tau_{x,y} f = \frac{f(y) - f(x)}{y - x}.$$

Since X is non-decreasing, we have

$$\tau_{x,y} g \leq 1 \tag{C.6}$$

for all $x \neq y$. Now, define

$$\varphi(z) = \mu\left(\{x, g(x) \geq z\}\right).$$

Step 2: (C.6) implies

$$\tau_{x,y}\varphi \leq -1 \qquad\qquad (C.7)$$

for all $x < y$ such that $\varphi(y) > 0$.

g is subject to a monotonicity constraint (namely $v \mapsto v - g(v)$ must be non-decreasing), which we need to transform into a constraint for φ. Clearly, if g increases at speed 1, φ decreases at speed 1. What we show here is that if g increases at speed less than 1 then φ decreases at speed larger than 1.

Since $y > 0$, the set $\{u, g(u) \geq y\}$ is nonempty, so we can consider

$$u^+ = \inf\{u, g(u) \geq y\}.$$

Since $g(0) = 0 \leq x$ we can also define

$$u^- = \sup\{u \leq u^+, g(u) \leq x\}.$$

Because of (C.6), the function g can not jump upwards, hence $g(u^-) = x$ and $g(u^+) = y$. By construction of u^- and u^+, we have:

$$[u^-, u^+) \subset \{u, g(u) \in [x, y)\}. \qquad\qquad (C.8)$$

Since $\tau_{u^-, u^+} g \leq 1$, we have:

$$u^+ - u^- \geq g(u^+) - g(u^-) = y - x, \qquad\qquad (C.9)$$

We can now obtain (C.7):

$$
\begin{aligned}
\tau_{x,y}\varphi &= \frac{\mu\big(\{u, g(u) \geq y\}\big) - \mu\big(\{u, g(u) \geq x\}\big)}{y - x} \\
&= -\frac{\mu\big(\{u, g(u) \in [x, y)\}\big)}{y - x} \\
&\leq -\frac{\mu\big([u^-, u^+)\big)}{y - x} \\
&\leq -1.
\end{aligned}
$$

Line 3 uses (C.8) and Line 4 is a consequence of (C.9).

Step 3: expression of the moments of g as a function of the moments of φ.

Recall that

$$\int_0^1 g = \int_0^1 \varphi$$
$$\int_0^1 g^2 = 2 \int_0^1 y \varphi(y) \, dy. \tag{C.10}$$

Indeed,

$$
\begin{aligned}
\int_0^1 g^2(y) \, dy &= \int_0^1 \int_0^1 \mathbb{1}_{0 \le s \le g^2(y)} \, ds \, dy \\
&= \int_0^1 \mu\left(\{u, g^2(u) \ge s\}\right) ds \\
&= \int_0^1 \mu\left(\{u, g(u) \ge \sqrt{s}\}\right) ds \\
&= 2 \int_0^1 y \varphi(y) \, dy,
\end{aligned}
$$

by using the change of variable $y = \sqrt{s}$. The other equality in (C.10) is proven similarly.

Step 4: translation into a functional maximisation problem with respect to the transform φ.

Using the previous discussion,

$$
\begin{aligned}
\sup_{C \in \mathscr{C}_K} \int_0^1 (v - X(v))^2 \, dv &\le 2 \sup_{\varphi \in \Phi_K^{\le}} \int_0^1 y \varphi(y) \, dy \\
&\le 2 \sup_{\varphi \in \Phi_K} \int_0^1 y \varphi(y) \, dy \tag{C.11}
\end{aligned}
$$

where $\Phi_K^{\le}$ is the set of measurable functions $\left\{ \varphi : [0,1] \to [0,1], \sup_{x,y} \tau_{x,y} \varphi \le -1, \int_0^1 \varphi(y) \, dy \le K \right\}$ and $\Phi_K = \{\varphi \in \Phi_K^{\le}, \int_0^1 \varphi = K\}$. Clearly, in (C.11) the right-hand-side of Line 1 equals the term in Line 2.

Define $\varphi_K(z) = \max\{\sqrt{2K} - z, 0\}$ for $0 \le z \le 1$. Note that $\varphi_K \in \Phi_K$. If $\varphi \in \Phi_K$, $\varphi(0) \ge \varphi_K(0)$. Otherwise, using the fact that $\tau_{0,y} \varphi \le -1$,

$$\varphi(y) \le \varphi(0) - y < \varphi_K(0) - y \le \varphi_K(y).$$

Hence, $\int_0^1 \varphi(y) \, dy$ would be strictly less than $K = \int_0^1 \phi_K(y) \, dy$.

Define $\Delta = \varphi - \varphi_K$: we proved that $\Delta(0) > 0$. Besides, by construction, $\int_0^1 \Delta(y) \, dy = 0$. Define

$$y_0 = \inf\{y, \Delta(y) \le 0\}.$$

Because $\tau_{y_0,y}\varphi \leq -1$, we have $\Delta(y) \leq 0$ for $y > y_0$ and $\Delta(y) \geq 0$ for $y < y_0$. Hence:

$$
\int_0^1 y\varphi(y)\,\mathrm{d}y - \int_0^1 y\varphi_K(y)\,\mathrm{d}y = \int_0^1 y\Delta(y)\,\mathrm{d}y
$$

$$
= \int_0^{y_0} y\Delta(y)\,\mathrm{d}y + \int_{y_0}^1 y\Delta(y)\,\mathrm{d}y
$$

$$
\leq y_0 \int_0^{y_0} \Delta(y)\,\mathrm{d}y + y_0 \int_{y_0}^1 \Delta(y)\,\mathrm{d}y \leq 0.
$$

Figure C.18 provides an illustration.

Thus, the supremum in (C.11) is attained only by the function φ_K and equal to

$$
2\int_0^1 y\varphi_K(y)\,\mathrm{d}y \quad = \quad \int_0^{\sqrt{2K}} y(\sqrt{2K} - y)\,\mathrm{d}y
$$

$$
= \quad \frac{(2K)^{3/2}}{3},
$$

which establishes the bound of the Theorem.

Step 5: The maximum in (C.4) is attained exclusively by the demand schedules $(X_\alpha)_{\alpha \in [0,1-\sqrt{2K}]}$ defined in the Theorem.

First, it is easy to see that these demand schedules achieve the maximum in (C.4). It remains to show that they are the only one to do so. Let X be a demand schedule obtained under a penalty $C \in \mathscr{C}$, $C \leq K$. Let us suppose that it achieves the maximum in (C.4). Consider, as in step 2, the function φ associated with $g(v) = v - X(v)$. The function φ is then a supremum of (C.11) and by step 3, $\varphi = \varphi_K$. Since

$$
\sup_x g(x) \geq \sup\{x, \varphi(x) > 0\} = \sup\{x, \varphi_K(x) > 0\} = \sqrt{2K},
$$

the supremum of $g(v)$ is at least $\sqrt{2K}$. Let us remark that:

$$
\sup_v g(v) = \sup_v \sup_{s \in [0,v]} g(s).
$$

Since $\tau_{.,.}g \leq 1$, the function $\overline{g}(v) = \sup_{s \in [0,v]} g(s)$ is continuous: the supremum of $\overline{g}(v)$ and thus of $g(v)$ is attained at a point v_0. Since $\tau_{.,.}g \leq 1$, $v_0 \geq \sqrt{2K}$ and for $v \in [v_0 - \sqrt{2K}, v_0]$,

$g(v) \geq v - v_0 + \sqrt{2K}$. Since $g \geq 0$, we obtain

$$
\begin{aligned}
\int_0^1 g \quad &\geq \quad \int_{v_0 - \sqrt{2K}}^{v_0} g \\
&\geq \quad \int_{v_0 - \sqrt{2K}}^{v_0} (v - v_0 + \sqrt{2K}) \, dv \\
&\geq \quad K
\end{aligned}
$$

with equality if and only if $g = 0$ outside $[v_0 - \sqrt{2K}, v_0]$ and $g(v) = v - v_0 + \sqrt{2K}$ over $[v_0 - \sqrt{2K}, v_0]$. But there must be equality because $g \in \Phi_K$. Hence g has the above form, and the demand function X, given by $X(v) = v - g(v)$, is equal to X_α as stated in the Theorem, with $\alpha = v_0 - \sqrt{2K}$.

C.1.6 Theorem 5

As a consequence of Lemma 14, in equilibrium the expected fine satisfies

$$
\mathbb{E}[C(X(v))] = \int_0^1 X(v) \left(v - \frac{X(v)}{2} \right) dv - \int_0^1 (1 - v) X(v) \, dv,
$$

and we are working under a constraint $\mathbb{E}[C(X(v))] \geq K_1$.

By Lemma 15, an upper bound constraint on the expected post-trade standard deviation translates into a constraint

$$
\int_0^1 v X(v) dv \geq K_2.
$$

This leads us to consider the following minimisation problem:

$$
\begin{aligned}
\min_X \quad & \int_0^1 X(v) \left(v - \frac{X(v)}{2} \right) dv + \gamma \left(K_1 - \int_0^1 X(v) \left(v - \frac{X(v)}{2} \right) dv + \int_0^1 (1 - v) X(v) \, dv \right) \\
+ \quad & \eta \left(K_2 - \int_0^1 v X(v) \, dv \right),
\end{aligned}
$$

for some weights $\gamma, \eta \geq 0$. Gathering terms, we obtain that this program is equivalent to

$$
\min_X \int_0^1 X(v) \left(\gamma + (1 - 2\gamma - \eta) v + \frac{\gamma - 1}{2} X(v) \right) dv \tag{C.12}
$$

For $0 \leq v \leq 1$, define

$$
\begin{aligned}
P_v : [0, v] \quad &\to \mathbb{R} \\
x \quad &\mapsto x \left(\gamma + (1 - 2\gamma - \eta) v + \frac{\gamma - 1}{2} x \right)
\end{aligned}
$$

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Case 1: $\gamma > 1$. P_v is the restriction to $[0, v]$ of a second-order polynomial with positive leading coefficient. Therefore it reaches its minimum at either 0, v, or when the first order condition is satisfied, say at $x_0(v)$, and $x_0(v)$ achieves the minimum as soon as $0 \leq x_0(v) \leq v$. Given that

$$x_0(v) = \frac{(2\gamma + \eta - 1)v - \gamma}{\gamma - 1},$$

algebra shows that

$$\arg\max P_v = \begin{cases} 0 & v \leq \frac{\gamma}{2\gamma + \eta - 1} \\ x_0(v) & \frac{\gamma}{2\gamma + \eta - 1} \leq v \leq \frac{\gamma}{\gamma + \eta} \\ v & v > \frac{\gamma}{\gamma + \eta}. \end{cases}$$

Let $v_1 = \frac{\gamma}{2\gamma + \eta - 1}$ and $v_2 = \frac{\gamma}{\gamma + \eta}$. We have obtained that with the function X_{v_1, v_2} given in the Theorem, the equality

$$\arg\max P_v = X_{v_1, v_2}(v)$$

holds. Direct calculations show that X_{v_1, v_2} is implemented by C_{v_1, v_2}. This means that we have found an implementable demand schedule that maximises the integral in (C.12) pointwise, which implies that X_{v_1, v_2} is a minimiser of the program (C.12), and it is the only one because the pointwise minimisation of the integral in (C.12) has a unique solution.

Case 2: $\gamma \leq 1$. P_v is now either linear or with a negative leading coefficient, meaning that its minimum is attained either at 0 or v. Algebra shows that $\arg\max P_v = v$ (for $0 \leq v \leq 1$) if and only if

$$\gamma + 2\eta \geq 1 \tag{C.13}$$

and

$$v \geq v^* := \frac{\gamma}{\eta + \frac{3\gamma}{2} - \frac{1}{2}},$$

where, by condition (C.13), $v^* \in [0, 1]$. With $v_1 = v_2 = v^*$ we conclude as before that X_{v_1, v_2} is the unique minimiser of (C.12). Finally, if (C.13) is not satisfied, the minimiser of (C.12) is identically zero, which corresponds to $X_{1,1}$ defined in the Theorem.

Finally, it is easy to see that the (v_1, v_2) constructed above describe the set J as $\gamma, \eta \geq 0$ vary, and J is the family of indices specified in the Theorem. So any index in J corresponds to an efficient demand function. This shows that $\left(X_{v_1, v_2}\right)_{(v_1, v_2) \in J}$ is the family of efficient demand functions.

The proof is complete, because the set of maxima we obtain as $\gamma, \eta \geq 0$ vary is connected, which implies that we have found all the points of the efficient surface.

C.1.7 Proposition 15

(i) We first show that the $F_{\min}$-efficient frontier is included in the set of points of

$$\pi_{GS}(\Sigma \cap \{F \geq F_{\min}\})$$

that are not dominated in $\pi_{GS}(\Sigma \cap \{F \geq F_{\min}\})$.

Let (G, S) be in the $F_{\min}$-efficient frontier. By definition, there is X implemented by $C \in \mathcal{C}$ such that $G = G(X)$, $S = S(X)$ and $F := \mathbb{E}[C(X(v))] \geq F_{\min}$. Only two cases are possible: (a) $(G, S, F) \in \Sigma$ or (b) (G, S, F) is dominated by a point (G', S', F') non-dominated in the closure (in $\mathbb{R}^3$) of all the implementable points, which is exactly Σ. ((G', S', F') is obtained by constructing a sequence (G_n, S_n, F_n) where each point dominates the previous one and define (G', S', F') as its limit, or as (G_N, S_N, F_N) if the procedure stops at N.) In case (a), we see that $(G, S) = \pi_{GS}(G, S, F) \in \pi_{GS}(\Sigma \cap \{F \geq F_{\min}\})$, and since it is in the $F_{\min}$-efficient frontier, it cannot be dominated in that space. In case (b), since (G, S) is in the $F_{\min}$-efficient frontier, we must have $G = G'$ and $S = S'$ and $F' \geq F_{\min}$, so $(G, S) = \pi_{GS}(G', S', F') \in \pi_{GS}(\Sigma \cap \{F \geq F_{\min}\})$ and we conclude as in case (a).

(ii) Let us show the other inclusion. It is enough to prove that if a point is dominated in $\mathscr{F}(F_{\min})$, it is dominated in $\pi_{GS}(\Sigma \cap \{F \geq F_{\min}\})$.

Let $(G, S) \in \pi_{GS}(\Sigma \cap \{F \geq F_{\min}\})$ and F be associated with this point. Assume (G, S) is dominated in $\mathscr{F}(F_{\min})$, say by (G', S'), associated with $F' \geq F_{\min}$. As before, either $(G', S', F') \in \Sigma$ or (G', S', F') is dominated by a point in Σ. In both cases, this means that there exists a point in $\Sigma \cap \{F \geq F_{\min}\}$ whose projection dominates (G, S). This concludes the proof.

C.2 Robustness checks: the case of Gaussian noise

C.2.1 Shape of X and P under Gaussian noise

Which effects of section 3.3.4 are peculiar to uniform noise and which effects are robust when we revert to a normality assumption?

The qualitative behaviour of the demand function X does not depend on the distribution of the noise. Consider for instance a cost $C(x) = K\mathbb{1}_{|x|>x_0}$ with $K, x_0 > 0$. When the magnitude of the optimal demand absent penalties is below x_0, it remains optimal under the penalty C. The IT then blocks its demand at x_0 in order to avoid the expected penalty K, as long as trading does not allow to recoup K on average. For v sufficiently large ($|v| > v_0$ for some $v_0 > 0$), the IT switches back to trading. This creates a jump in the demand function at $\pm v_0$. All these effects

are independent of the assumptions on the noise.

The qualitative behaviour of the price function is robust as far as non-linearity is concerned. Flat sections in the demand schedule X induce steep sections in the price function P. Indeed, when X increases slowly as a function of v, the information that X is likely to have increased a little (obtained through the observation of $d = X(v) + u$) implies that v is likely to have increased a lot. Similarly, steep sections of X induce flat sections of P. Since the introduction of penalties produces steep and flat sections for X, it produces flat and steep sections for P.

What does not hold in general is the fact that P has discontinuities. Those are due to the fact that the uniform distribution has a discontinuous density $\frac{1}{2}\mathbb{I}_{[-1,1]}$. In general, one must have discontinuities in the density of the noise to obtain discontinuities in the price function. With a continuous noise density, jumps are replaced with sections where P increases fast.

To support these arguments, we report the equilibrium (X, P) for the model with Gaussian noise ($u, v \sim N(0, 1)$) and penalty C. We consider the same penalties C as above: quadratic, linear and constant on large trades: see Figures C.4, C.6 and C.8.

C.2.2   Figure C.10 under Gaussian noise

We repeat the construction of Figure C.10 by assuming Gaussian noise: $u, v \sim N(0; 1)$. We obtain Figure C.19. The constant costs upon nonzero trades $C(x) = K\mathbb{I}_{x \neq 0}$ are doing best among the penalty functions considered. This is consistent with the results in the uniform noise case. Other penalties are suboptimal, as before, and the locus of points $(S, -G)$ they generate is very similar in shape.

C.3   Discussion: the mimicking property

We provide two results that complement those of Bagnoli et al. (2001): Result 1 is new and Result 2 provides an alternative proof for a particular case of their work.

Consider a one-period model *à la* Kyle. Let u be the demand of the noise traders, x be the order of the insider trader and v denote the fundamental value of the asset and v_0 its expectation.

The model is said to feature *aggregate orders* when the market maker observes $x + u$ and to feature *individual orders* when she observes the set $\{x; u\}$.

Result 1 *In the individual orders setting, any differentiable and increasing mimicking equilibrium strategy X must be affine in v.*

Proof. Since x and u are indistinguishable, the price function is given by

$$P(\{x, u\}) = \frac{1}{2}(X^{-1}(u) + X^{-1}(x)).$$

Therefore, the maximisation program of the IT is

$$\max_x x\left(v - \mathbb{E}_u\left[\frac{1}{2}X^{-1}(u)\right] - \frac{1}{2}X^{-1}(x)\right).$$

Since $X(v) = u$ in distribution, $v = X^{-1}(u)$ in distribution and the program reduces to

$$\max_x x\left(v - \frac{v_0}{2} - \frac{1}{2}X^{-1}(x)\right).$$

Since X is an equilibrium strategy, the derivative of this expression evaluated at $x = X(v)$ must be zero:

$$
\begin{aligned}
0 &= v - \frac{v_0}{2} - \frac{X^{-1}(x)}{2} - \frac{x}{2}\frac{1}{X'(X^{-1}(x))} \\
&= \frac{v - v_0}{2} - \frac{X(v)}{2X'(v)}.
\end{aligned}
$$

Therefore, X must satisfy the ODE

$$X'(v)(v - v_0) = X(v),$$

i.e. X is linear in $v - v_0$.

Hence, if it is impossible to mimick the noise in an affine manner, we cannot have a mimicking equilibrium. However, if this is possible, we automatically have an equilibrium:

Result 2 *In both the aggregate orders and the individual orders settings, if there exists X increasing and linear in $v - v_0$ such that $X(v) = u$ in distribution, and P is the corresponding pricing function, (X, P) is an equilibrium.*

Proof. In the case of individual orders, this result is immediate from the arguments above: X^{-1} is affine so the first order condition, which is satisfied for $x = X(v)$, characterises a global maximum.

We now consider the case of aggregate orders. Since $X(v)$ and u are indistinguishable, by

symmetry

$$
\begin{aligned}
\mathbb{E}[X(v)|X(v) + u] &= \mathbb{E}[u|X(v) + u] \\
&= \frac{1}{2}\mathbb{E}[X(v) + u|X(v) + u] \\
&= \frac{X(v) + u}{2}.
\end{aligned}
$$

Hence, since X is linear, $P(x+u) = \frac{X^{-1}(x)+X^{-1}(u)}{2}$ as before; and therefore $X(v)$ is still an optimal demand.

C.4 Figures
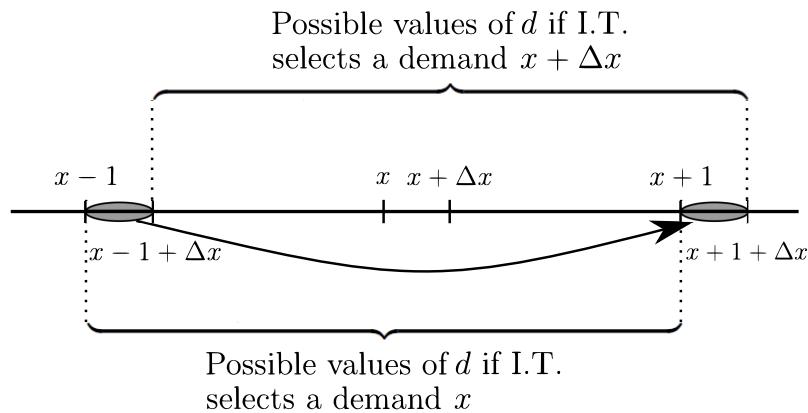
Figure C.1: Marginal expected price impact of an increase in x
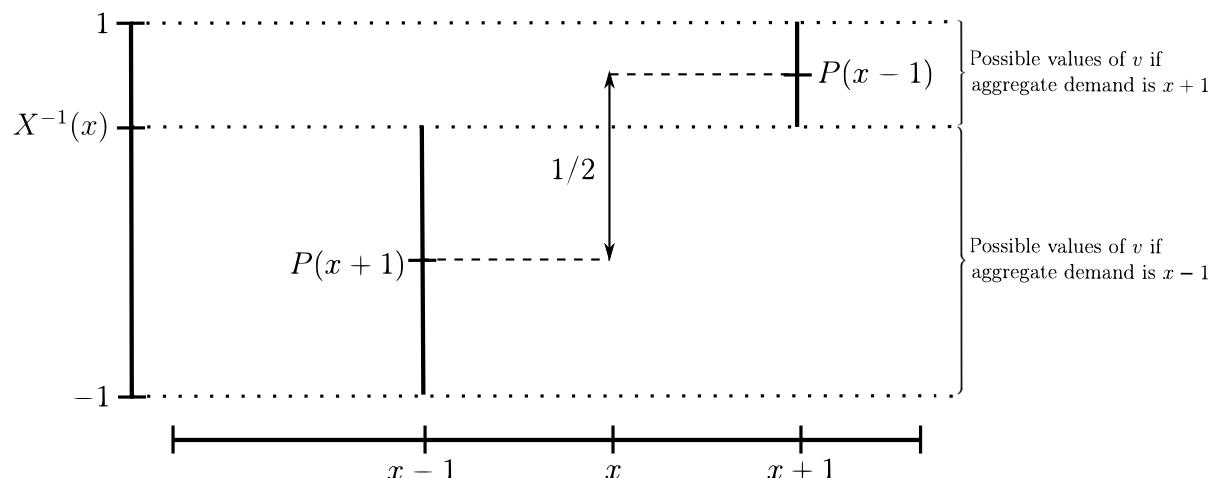
Figure C.2: The marginal expected price impact is constant

(a)   **(b)**
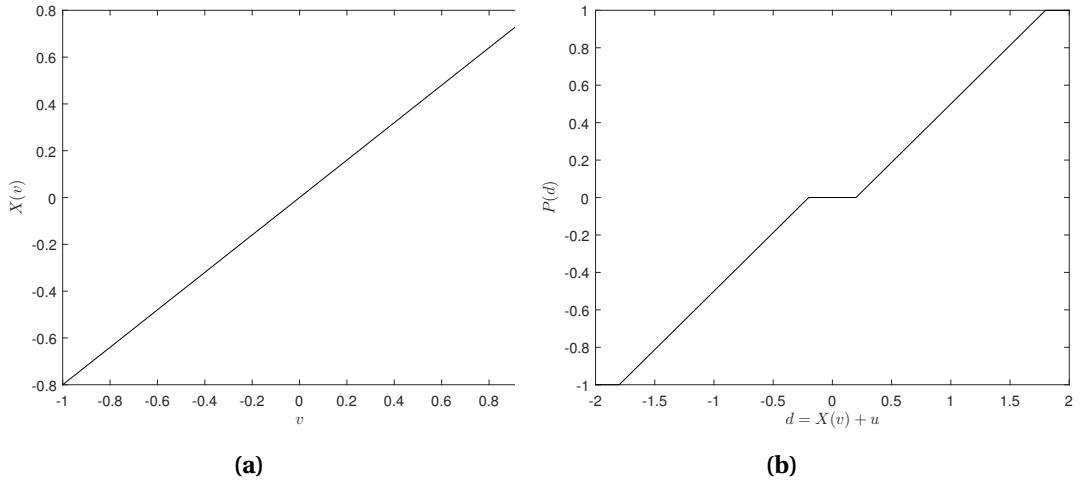
Figure C.3: Insider's demand and pricing under quadratic penalty

$C(x) = \alpha x^2$, $\alpha = 0.125$. Left panel: IT demand X. Right panel: price function P.

Due to the presence of the penalty, the insider trades less than in the linear mimicking equilibrium, so that $X(1) = x_M < 1$ (= 0.8 in this example).

(a)   **(b)**
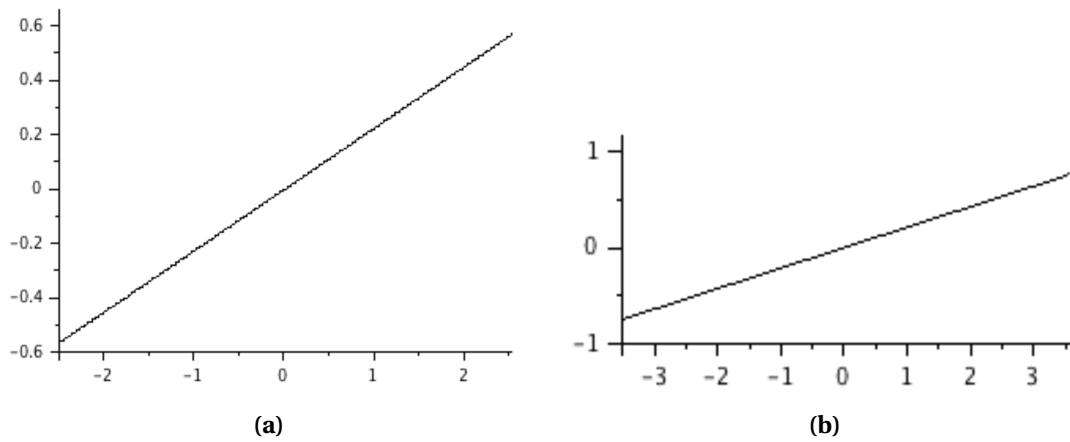
Figure C.4: IT demand and pricing under quadratic penalty, Gaussian case
$C(x) = \alpha x^2$, $\alpha = 2$. Left panel: IT demand X. Right panel: price function P.

(a)

(b)
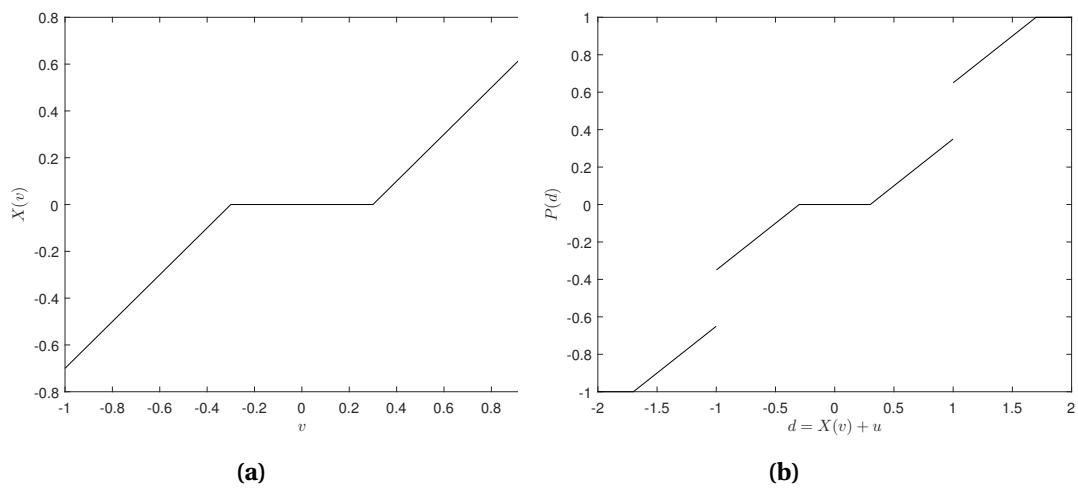
Figure C.5: Insider's demand and pricing under linear penalty

$C(x) = \alpha|x|$, $\alpha = 0.3$. Left panel: IT demand X. Right panel: price function P.

(a)

(b)
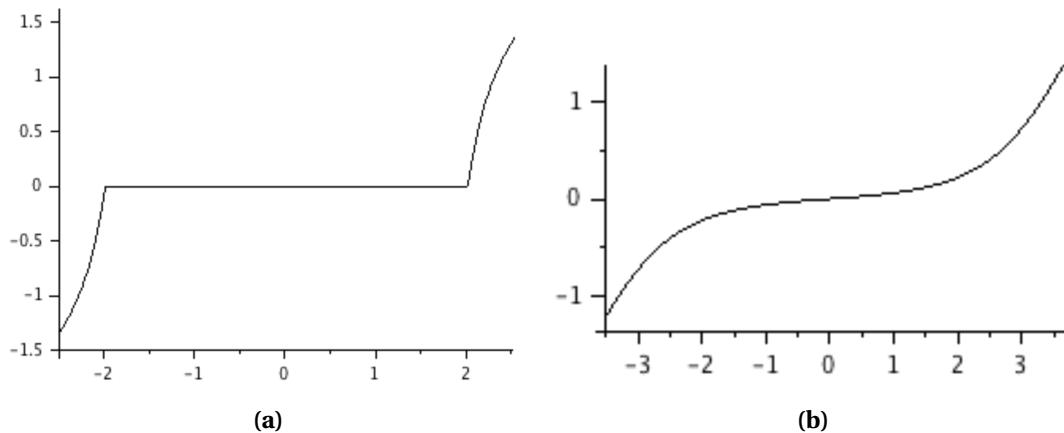
Figure C.6: IT demand and pricing under linear penalty, Gaussian case
$C(x) = \alpha|x|$, $\alpha = 2$. Left panel: IT demand X. Right panel: price function P.

(a)   **(b)**
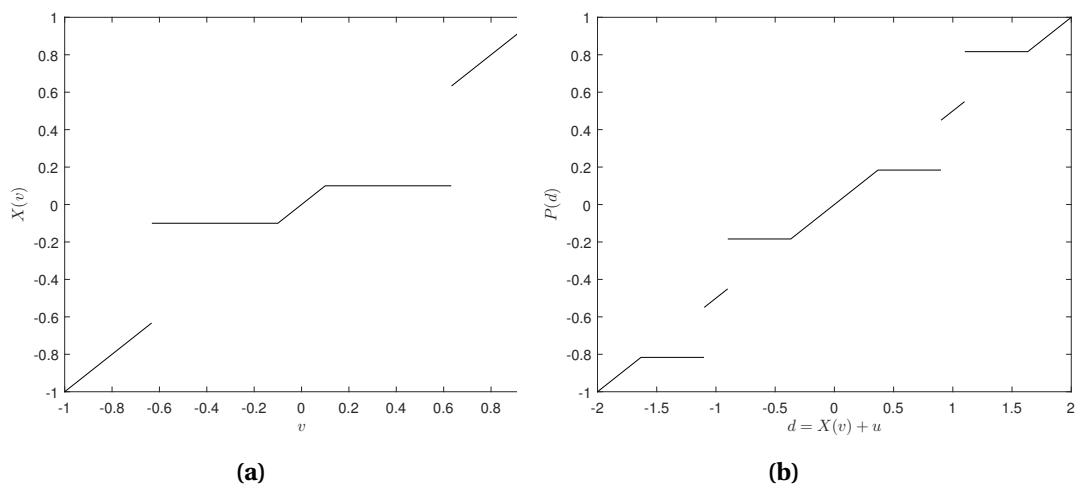
Figure C.7: Insider's demand and pricing under constant penalty on large trades

$C(x) = K\mathbb{1}_{|x|>x_0}$, $K = 0.2$, $x_0 = 0.1$.
Left panel: IT demand X. Right panel: price function P.

(a)   **(b)**
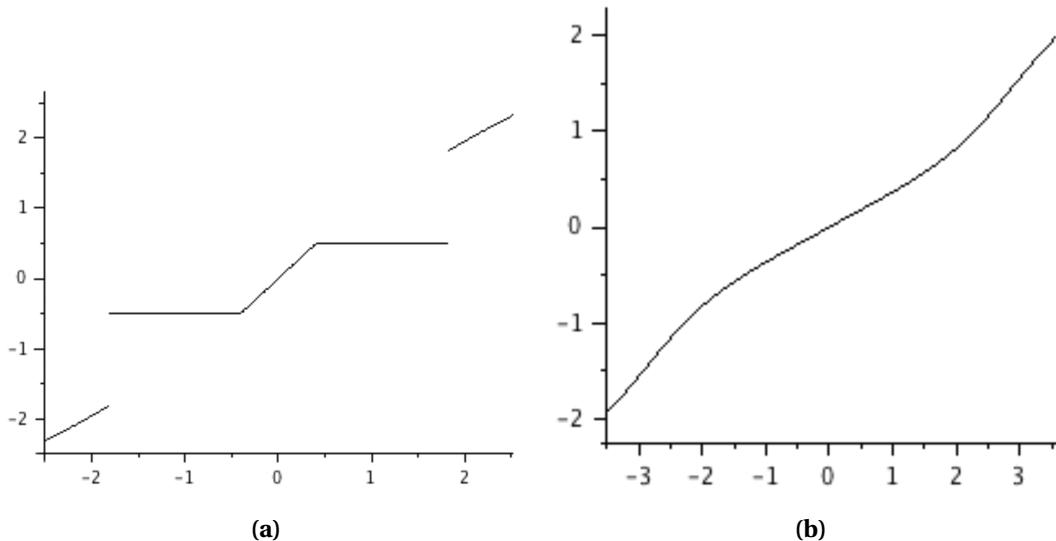
Figure C.8: IT demand and pricing under constant penalty, Gaussian case
$C(x) = K\mathbb{1}_{|x|>x_0}$, $K = 1$, $x_0 = 0.5$.
Left panel: IT demand X. Right panel: price function P.
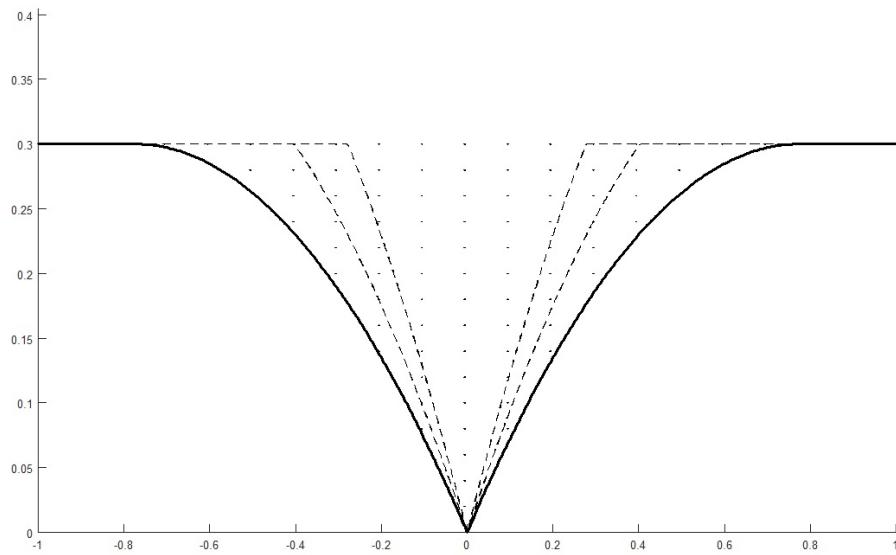
Figure C.9: Some penalty functions in $\mathscr{O}$.

In Figure C.9, the thick line represents the lower bound in the definition of $\mathscr{O}$ when $K = 0.3$ (then, $\sqrt{2K} \approx 0.77$) : any penalty in $\mathscr{O}$ must be above this line. Given that a penalty is symmetrical and non-decreasing over $[0, 1]$, the graph of a function in $\mathscr{O}$ must be included in the dotted area. The two dashed lines represent two such functions.
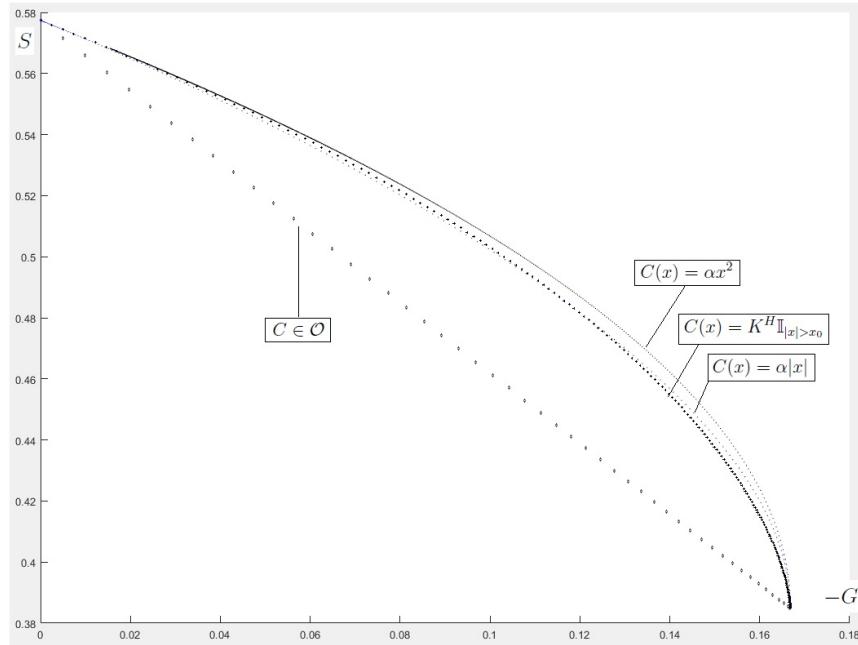
Figure C.10: Locus of $(S, -G)$ for some penalty functions.

(a)   **(b)**
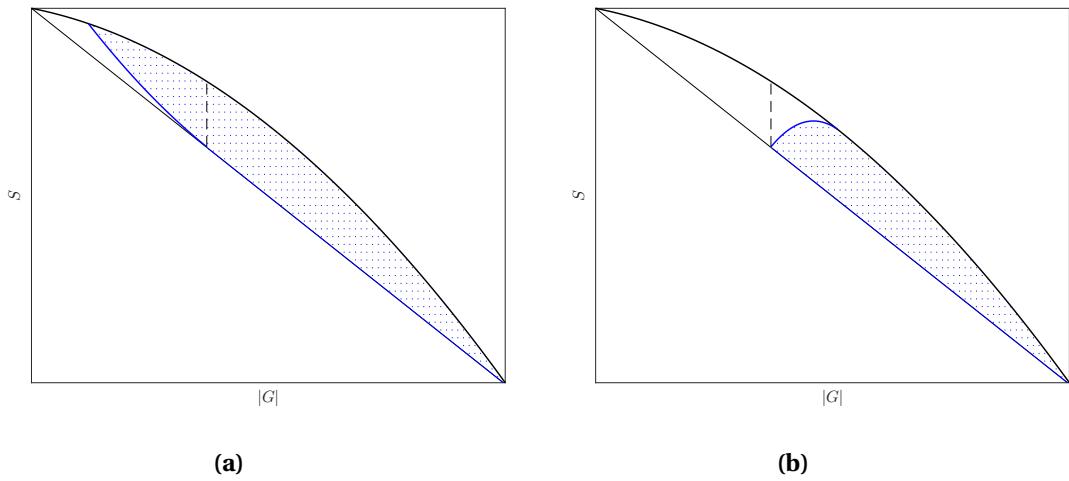
Figure C.11: Introduction of a constraint: two possible scenarios.
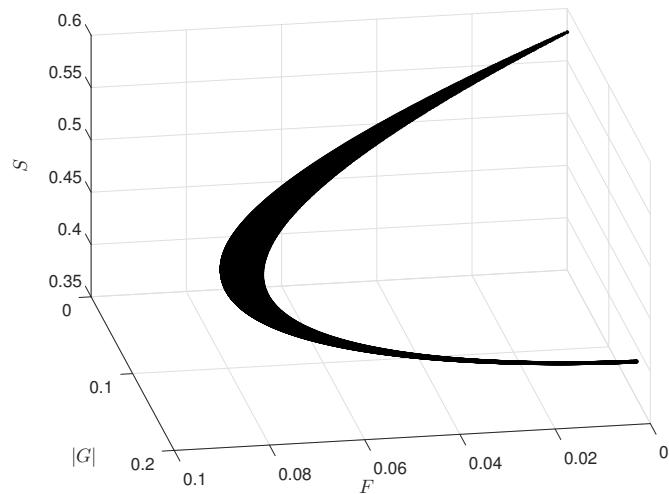
Figure C.12: The efficient surface Σ.
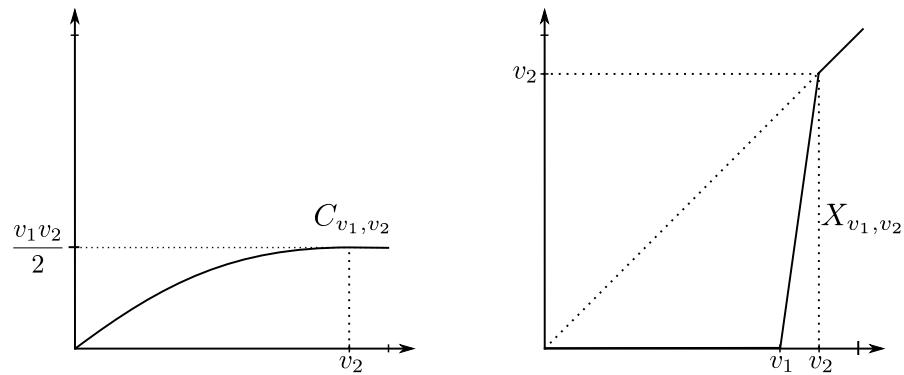
Figure C.13: Efficient demand schedule and penalty function under a budget constraint with pecuniary fines.
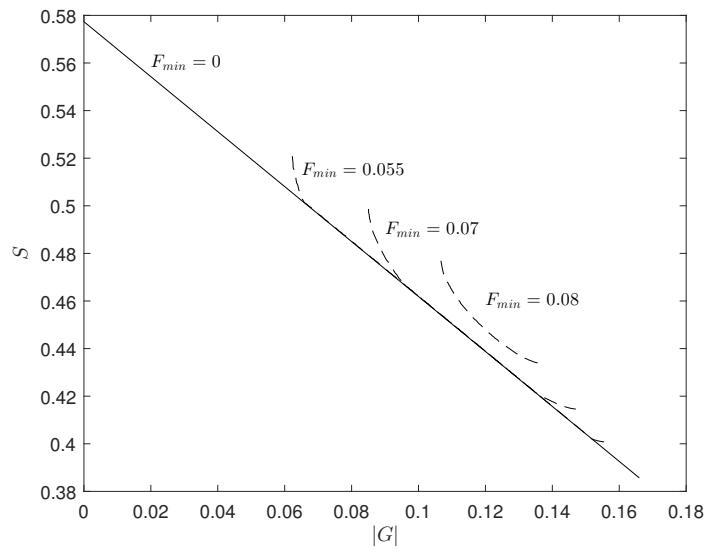

**Figure C.14: Efficient $(|G|, S)$ frontiers under various constraints $F \geq F_{\min}$.**
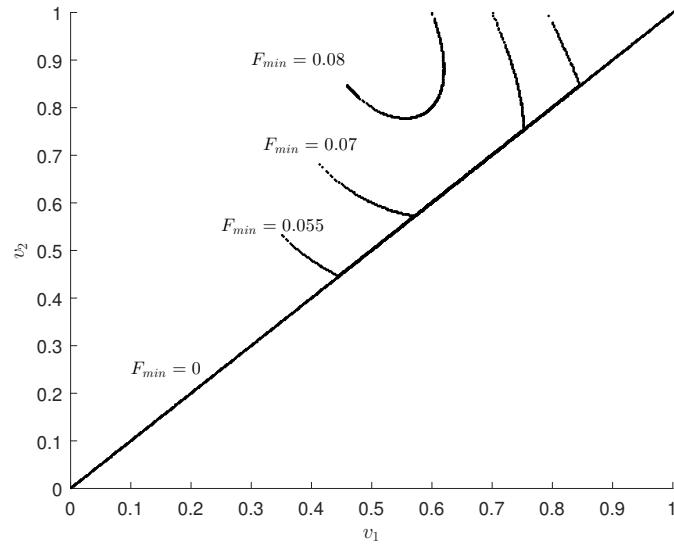
Figure C.15: Indices (v_1, v_2) **of the efficient demand functions** X_{v_1,v_2} **associated with various constraints** $F \geq F_{\min}$.

As an illustration, the red filled dot, which corresponds to $(v_1, v_2) \approx (0.48, 0.61)$ represents the demand schedule $X_{0.48,0.61}$ (where X_{v_1,v_2} is defined in Theorem 5) and indicates that this demand schedule implements one point of the efficient frontier when the budget constraint of the regulator is such that $F_{\min} = 0.07$.
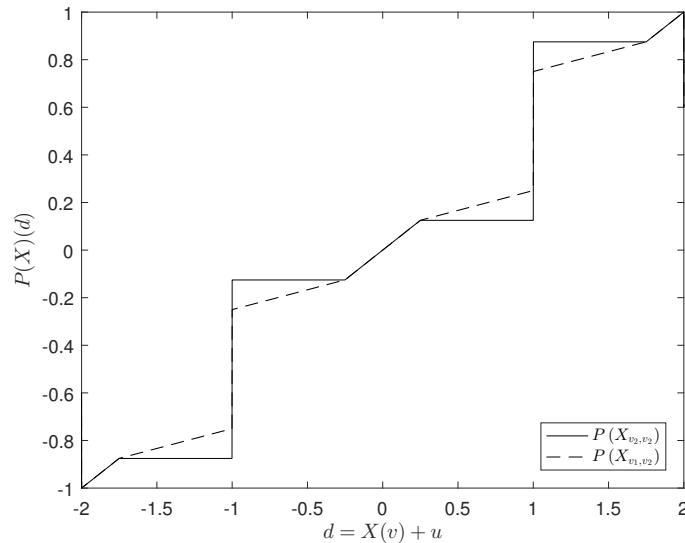

Figure C.16: New patterns of price functions.
$v_1 = 0.5$ and $v_2 = 0.75$.
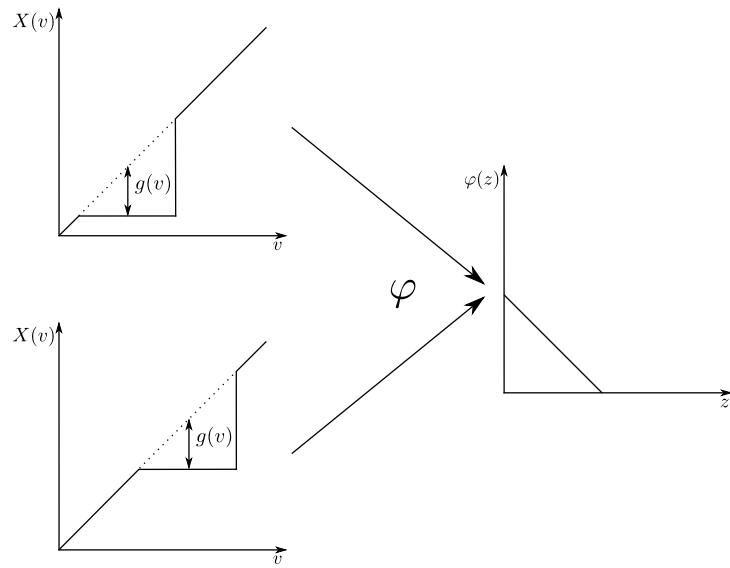
**Figure C.17: Using the repartition function to transform** g
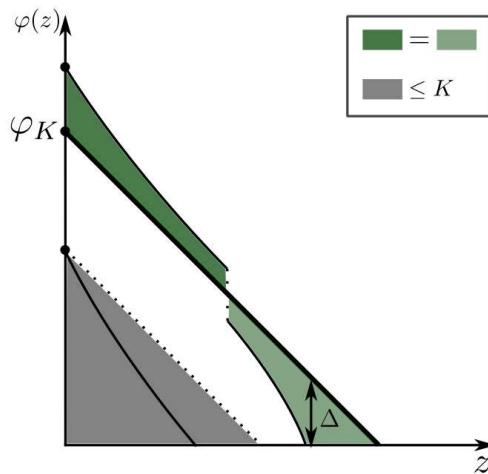
Figure C.18: The transform φ **of a maximiser** g **must be** φ_K**.**

(i) Starting from a point $\varphi(0) < \varphi_K(0)$ (lowest thick dot on the y-axis), φ (solid black curved line) remains below the dotted line and its integral is therefore smaller than the area of the grey region, itself below K. (ii) After crossing φ_K, φ must remain below φ_K. Here, the crossing occurs through a downwards jump of φ.
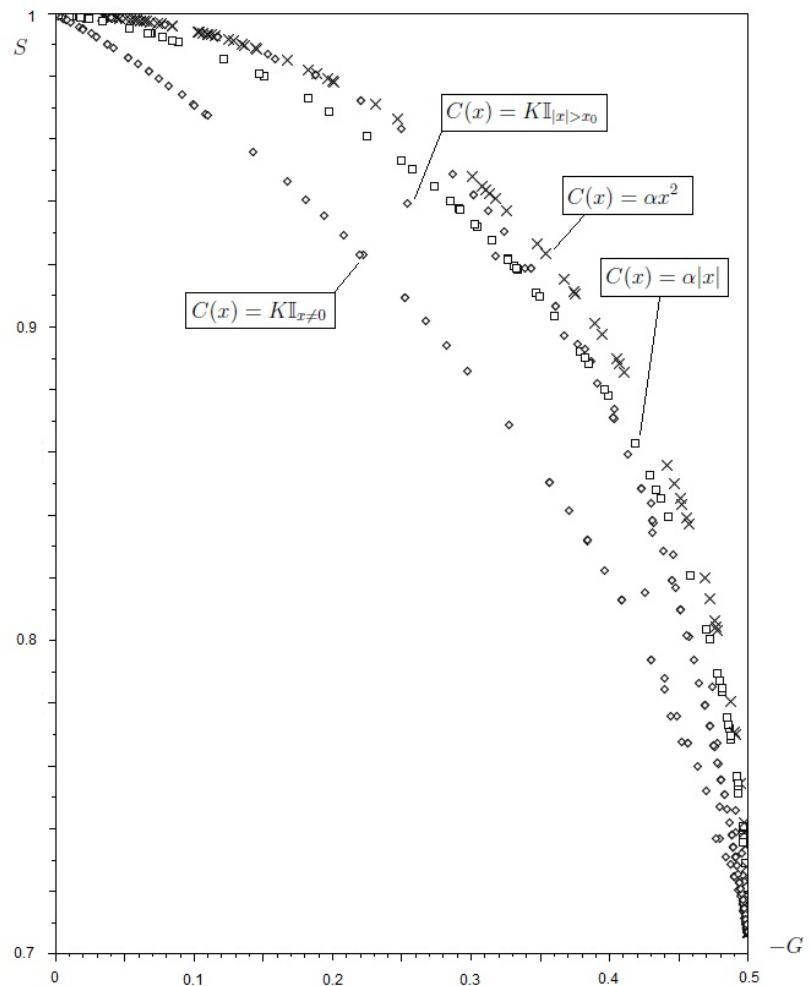
Figure C.19: Locus of $(S, -G)$ **for different penalty functions - Gaussian noise.**

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2007-2012: **École Normale Supérieure** (ENS), Paris.

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2007 - 2009: **MSc in Mathematics** (Probabilités et Modèles Aléatoires), ENS and Paris VI University. Master thesis: Mean field games and applications; advisors: Profs. Jean-Michel Lasry and Olivier Guéant.

Research

Job market paper

"Disclosures, Rollover Risk and Debt Runs", available at https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2702688.

Selected talks (job market paper)

December 2018: Brown bag seminar at USI, Lugano.

September 2018: SFI Academic Job Market workshop, Lausanne.

April 2018: LSE Friday PhD seminar, London.

September 2016: Law and Finance workshop, ETH, Zürich.

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Publication

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Working papers

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2018 - 2019: Financial applications of blockchains and distributed ledgers, MFE (taught by Prof. Alexander Lipton and Dr. Adrien Treccani).

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