Influence of photovoltaic panel orientation on modern energy systems in residential buildings

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Abstract:
The aim of modern energy systems is to be independent of fossil fuels and to minimize their impact on the environment. Photovoltaic (PV) panels are a common solution for renewable electric energy generation, but the volatility caused by the fluctuation of solar irradiation challenges the electrical power grid.

The aim of this paper is to explore the influence of different orientations of PV panels on the buildings self-sufficiency and self-consumption. This paper is based on a mixed integer-linear programming approach aiming at optimizing the installed size of a range of energy conversion technologies. Compared to conventional building energy system optimization research, the proposed approach offers a more detailed modelling of the energy generated by PV panels that considers the azimuth and the tilt of roof surfaces and PV panels, also including the shading effects between different panels. The proposed methodology is applied to a case study building in the area of Rolle, Switzerland.

The results of this paper suggest that the tilt and azimuth of PV panels and of the roof they are installed on should not be neglected: not considering these aspects generates an error in the estimation of the energy generated by the PV panels, and not including the choice of the roof (and, in case of flat roofs, the choice of PV panel tilt and orientation) where the panels should be installed on as design variables generates a sub-optimization of the system. This is particularly true since the most optimal choice is not necessarily that of filling first flat surfaces and South-oriented ones, a consequence of the better match between generation and demand offered by West-oriented panels, with a higher tilt angle.

Keywords:
Building energy systems, Mixed integer linear programming, Photovoltaic systems, Roof orientation, Renewables integration
1. Introduction

The aim of modern energy systems is to be independent of fossil fuels and to minimize their impact on the environment. The ongoing energy transition therefore requires to increase the share of renewable energy in the existing energy system. Photovoltaics (PV) panels have already become a largely widespread solution for decentralized electric energy generation, and their efficient involvement with residential building energy systems has been the subject of extensive research. However, volatile power generation caused by fluctuation of solar irradiation challenges the capacity of the electrical power grid. Therefore, in addition to maximizing the energy generated from the sun, it is important to reduce the interaction of building energy systems with the electrical power grid by maximizing self-consumption while decreasing the grid’s energy demand.

Most research focusing on designing energy systems, where the installation and sizing of solar panels are used as the optimization parameters, adopts relatively rough approximations when it comes to the modelling of the energy generated by the PV panels. While the effects of tilt and orientation on the energy generation profile of PV panels are commonly included in simulation programs, when dealing with energy system optimization it is common practice to use global irradiation profiles and model the PV panel without orientation or tilt, assuming it is installed flat on the roof, and roofs of buildings are considered to be flat and completely exposed to the sun, without shading. The effect of the temperature on the efficiency of the panel is only included in a few cases [1, 2], while many others neglect this effect [3–5]. However, not only it has been shown that panel/roof orientation and tilt have a substantial influence on the energy generation potential of PV panels, but manipulating these variables (when possible) was also identified as a potential solution for providing a better match between the demand and the energy availability from the PV modules. Litjens et al. studied the influence of electricity demand patterns on the optimal orientation of the PV modules [6], showing that the self consumption rate is higher in general when panels are orientated west. Similar results were obtained by Mondol et al. [7], who simulated the power generated at a location in the United Kingdom at different orientations for time-dependent electricity price profiles and concluded that west oriented panels led to the best economic result, and by Lahmaoui et al. [8], who concluded that a west oriented PV system has a higher share of directly consumed electricity than east oriented systems for a residential demand pattern in Germany.

Researchers have also investigated the interaction between the levelized cost of electricity and the orientation of the panels. Lahmaoui et al. [8] investigated four different orientations in combination with two different tilt angles in a system integrated with a battery for electric energy storage. They found that the storage is cost effective only with South orientation and concluded that the orientation of the PV panels influences the cost optimal storage size [8]. Few researchers focused on including the effects of the tilt and azimuth (both of the roof and of the panels) on the optimal design process. Hartner et al. [9] proposed a nonlinear optimization of the system where the installation angles of the PV panels, in addition to their sizes, were considered as optimization variables. The proposed approach led to the conclusions that few, larger PV systems are more cost optimal than several smaller ones [9]. However, while the
effect of PV tilt and azimuth was included in the optimization, the resulting installation angles were not presented in the results. In addition, the system modelling was particularly focused on the PV panels and excluded the building energy system, e.g. heat pumps and storage. The aim of this paper is to combine the optimization of the building energy system, including energy storage and heat pumps, while considering the PV panels with a more accurate modelling approach which also accounts for the effect of roof tilt and azimuth, and (in the case of flat roofs) includes the tilt and azimuth of the panels as optimization variables. This paper thus aims at bridging a gap between two research topics, thus avoiding the consequences of oversimplifying the modelling accuracy of the contribution of PV panels on one side, and the sub-optimality generated by not looking at the problem from a systems perspective on the other.

2. Methodology

2.1. Building Energy Model Optimization

The objective of this paper is the optimization of building energy systems, including the choices related to the installation of PV panels. As a consequence, unit sizes and installation decisions are the main optimization parameters of interest, with costs as the objective to minimize. The challenge is to propose a computation method providing both the conceptual design and the yearly load scheduling with sufficient precision in a reasonable computing time of a few seconds. For this reason, the optimal integration of the building energy technologies in this paper is formulated as a multi-objective optimization problem based on a mixed integer-linear programming (MILP) formulation.

2.1.1. Problem superstructure

Fig 1 provides a representation of the system to be optimized. Heating requirements can be satisfied by an air-water heat pump, electric auxiliary heaters and a boiler. Energy is stored in either stationary batteries, domestic hot water and buffer tanks or the building envelope. PV panels act as renewable energy sources. The different energy systems are interconnected to the main energy distribution networks: the natural gas, electricity and fresh water grid.

The bulk of the modeling and optimization approaches employed in this paper are derived from [1], to which the reader is referred to for additional details. In the following text, the approach is summarized, while more focus is devoted to the modifications to the original model that were introduced in this work.

2.1.2. Objectives

The optimization of costs involves the combination of two separate contributions: operating (OPEX) and capital (CAPEX) expenses, thus making this a multi-objective optimization (MOO) problem. In this paper, we approached the MOO problem using the \( \epsilon \)-constraint method, thus considering the OPEX as the main problem objective and solving different optimization problems as individual scenarios by constraining the CAPEX at different values. The OPEX comprise both the natural gas, electricity and power grid exchanges. These are defined in Equation (1) where \( (op) \) refers to the grid energy tariffs, \( (E) \) to the electrical power flows, \( (H) \) to the chemical-natural gas-power flows, \( (d) \) to the indexed time step duration, and \( (\Sigma) \) to the set of decision variables reported in [1].
The CAPEX are calculated as the present capital expenses related to the different unit purchases over the project horizon \((N)\). In Equation (2), \((I_{1,u})\) and \((I_{2,u})\) denote the linear cost function parameters, \((y_u)\) the unit existence while \((f_u)\) is the device sizing variable. In addition, \((N_u)\) refers to the unit lifetime, \((r)\) to the project interest rate and \((rep_u)\) to the number of unit replacements over the project horizon.

\[
\sum_{u=1}^{U} (I_{1,u} \cdot y_u + I_{2,u} \cdot f_u) + \sum_{u=1}^{U} \sum_{n=1}^{rep_u\cdot N_u} \frac{1}{(1+r)^n N_u} \cdot (I_{1,u} \cdot y_u + I_{2,u} \cdot f_u) \leq \epsilon_I \tag{2}
\]

### 2.1.3 Problem constraints

#### Energy Balances

The electrical and natural gas energy balances are defined in Equations 3 and 4, where \((E_{b}^-)\) refers to the building uncontrollable load profile.

\[
\begin{align*}
\dot{E}_{grid,p,t}^+ + \sum_{u=1}^{U} \dot{E}_{u,p,t}^+ - \dot{E}_{grid,p,t}^- - \sum_{u=1}^{U} \dot{E}_{u,p,t}^- - \dot{H}_{b,p,t}^- &= 0 \quad \forall p \in P, \ t \in T \\
\dot{H}_{grid,p,t}^- &= \sum_{u=1}^{U} \dot{H}_{u,p,t}^- 
\end{align*}
\tag{3}
\tag{4}
\]

**Figure 1:** Building energy system structure and the respective control variables (blue) [1]
The heat cascade balances the system heat loads while satisfying the second law of thermodynamics. Equations 5 and 6 thus define the thermal energy balance of each temperature interval k where \((Q_{u_h,k}^-)\) represents the released heat of utility \((u_h)\), \((Q_{u_c,k}^+)\) represents the heat demand of utility \((u_c)\), and \((R_k)\) the residual heat cascaded to next interval \((k+1)\).

\[
\dot{R}_{k,p,t} - \dot{R}_{k+1,p,t} = \sum_{u_h=1}^{U} Q_{u_h,k,p,t}^- - \sum_{u_c=1}^{U} Q_{u_c,k,p,t}^+ \tag{5}
\]

\[
\dot{R}_{1,p,t} = \dot{R}_{n_k+1,p,t} \quad \forall p \in P, \ t \in T, \ k \in K \tag{6}
\]

**Cyclic Conditions**

To prevent any energy accumulation between the different independent operating periods \((p)\), the cyclic constraints of Equations 7 to 9 enforce all system states to return to their initial value at the end of each control horizon \((n_t)\). This constraint is enforced on the dwelling temperature \((T_b)\) and on the thermal \((Q)\) and electrical energy \((E)\) stored in the respective storage units.

\[
T_{b,p,1} = T_{b,p,n_t} \tag{7}
\]

\[
Q_{u,p,1} = Q_{u,p,n_t} \tag{8}
\]

\[
E_{u,p,1} = E_{u,p,n_t} \quad \forall p \in P, \ u \in U \tag{9}
\]

**Unit Sizes**

The unit existence \((y_u)\) are expressed in Equation 10 where \((F_{u}^{\min})\) and \((F_{u}^{\max})\) describe the device minimal and maximal sizing values. The output of the units can vary linearly until the installed unit size \(f_u\).

\[
y_u \cdot F_{u}^{\min} \leq f_u \leq y_u \cdot F_{u}^{\max} \quad \forall u \in U \tag{10}
\]

### 2.1.4. Key Performance Indicators

In addition to the problem objective, two additional KPIs are defined to provide additional information concerning the performance of the system. The self-consumption \((SC)\) indicates the share of the total generated electricity that is directly consumed on site. The self-sufficiency \((SS)\) indicates the ratio between self consumed electricity and total electricity demand \([1]\).

\[
SC = \frac{\sum_{p=1}^{P} \sum_{t=1}^{T} (E_{PV,p,t}^+ - E_{grid,p,t}^-) \cdot d_p \cdot d_t}{\sum_{p=1}^{P} \sum_{t=1}^{T} (E_{PV,p,t}^+ - E_{grid,p,t}^+) \cdot d_p \cdot d_t} \tag{11}
\]

\[
SS = \frac{\sum_{p=1}^{P} \sum_{t=1}^{T} (E_{PV}^+ - E_{grid}^- + E_{grid}^+) \cdot d_p \cdot d_t}{\sum_{p=1}^{P} \sum_{t=1}^{T} (E_{PV}^+ - E_{grid}^- + E_{grid}^+) \cdot d_p \cdot d_t} \tag{12}
\]

### 2.2. Photovoltaic Panel Modelling

The main contribution of this study lies in the inclusion of different orientations of the PV Panels. The unit model of the PV panel is stated by Equations 13 to 15. Accordingly, the energy system model additionally consists of the set ”Surface” for describing the building’s envelope and ”Configuration” for describing the different orientation possibilities on this surface.
More specifically, in this paper we consider that the expected energy generated from the installation of PV panels also depends on the orientation and tilt of each roof of the building. Hence, we included in the model a more advanced representation of the energy generated by PV panels during each time step that takes into account the different orientation and tilt of each section of the roof of the building. In the case of non-flat roofs (i.e. roofs with a tilt angle higher than $5^\circ$) the PV modules are assumed to be installed with the roof orientation and only one PV profile is associated with the roof, while in the case of flat roofs (tilt < $5^\circ$), on the other hand, the orientation and the tilt of each panel should be included as optimization variables. In this part of the model we assumed that only eight configurations are possible: PV modules can be oriented to the south with a tilt angle between 0 and $60^\circ$ by step of $10^\circ$ or in a east-west configuration with a tilt angle of $10^\circ$. In this case, the optimization will select one among the eight PV profiles associated with the roof. All the resulting PV profiles used in this paper were generated based on the roof orientation provided by the Federal Office of Topography Swisstopo [10].

The sizing value $f_{PV}$ is the total number of installed PV panels (Eq. 13). The installation of panels is limited by the available surface area (14). In this paper, we assumed the filling rate $\psi_b > A_{b,s}$ to be 70% of the surface area $A_{b,s}$. Furthermore, the footprint $\beta_c$ respects the shading of the panels to each other at different configurations. The final total generated electricity $\dot{E}_{PV,b,p,t}$ results from the sum of the different generated specific electricity $\dot{e}_{PV,b,s,c,p,t}$ from each installed panel of every configuration $c$ on every surface $s$ (Eq. 15). Equation 16 describes the generation of specific PV profiles $\dot{e}_{PV,p,t}$ [W/m$^2$].

$$f_{PV} = \sum_{s \in S} \sum_{c \in C} n_{b,s,c}^{PV}$$  \hspace{1cm} (13)

$$\psi_b \cdot A_{b,s} > = A_{PV} \cdot \sum_{c \in C} \beta_c \cdot n_{b,s,c}^{PV}$$  \hspace{1cm} (14)

$$\dot{E}_{PV,b,p,t} = A_{PV} \cdot \sum_{s \in S} \sum_{c \in C} n_{b,s,c}^{PV} \cdot \dot{e}_{b,s,c,p,t}^{PV} \quad \forall b \in B, \forall p \in P, \forall t \in T$$  \hspace{1cm} (15)

The model of the PV module temperature is expressed as a function of the ambient temperature $T_{amb}$ and the global irradiation on the panel $IRR$.

$$T_{PV,p,t}^{PV} = \frac{U_{PV}^{PV} \cdot T_{amb}^{PV}}{U_{PV}^{PV} - \pi_{PV}^{PV} \cdot IRR_{p,t}^{PV}} + \frac{IRR_{p,t}^{PV} \cdot (\nu_{PV}^{PV} - \eta_{ref}^{PV} - \pi_{PV}^{PV} \cdot T_{ref}^{PV})}{U_{PV}^{PV} - \pi_{PV}^{PV} \cdot IRR_{p,t}^{PV}} \quad \forall p \in P, \forall t \in T$$  \hspace{1cm} (16)

The module heat transfer coefficient $U_{PV}^{PV}$, the absorption coefficient $\nu_{PV}^{PV}$ and the temperature coefficient $\pi_{PV}^{PV}$ are parameters specific of each different PV panel. The performance in reference conditions is given by the reference efficiency $\eta_{ref}^{PV}$ and the reference temperature $T_{ref}^{PV}$.
The proposed approach also takes into account the footprint of each PV module, that is the actual roof area occupied by the module. This is calculated as a function of the required minimum distance between PV modules to prevent shading and of the relative tilt angle to the surface. The distance between two rows of South orientated modules is given by Equation 19.

\[ D = H \cdot \frac{\sin(\alpha + \beta)}{\sin(\beta)} \]  

(19)

where \( H \) is the module height, \( \alpha \) the module tilt and \( \beta \) is the minimum sun elevation to avoid shadowing, which is by default 20° corresponding to the sun elevation at noon during winter in Switzerland. Fig 2 illustrates this geometric correlation.

### 3. Case study

The proposed methodology is applied to a case study located in Switzerland. The case study is a small two floor, residential building with a heated area of 250 m\(^2\) and a large available roof area consisting of four tilted and one flat surfaces (see Table 2).

Electric load profiles were generated using standardized profiles according to Swiss norm [11]. Thermal demand profiles were generated using the methodology in ([12]), based on a combination of physical principles and of a statistical analysis of documented energy demands from

### Table 1: Possible panel configuration for flat roofs

<table>
<thead>
<tr>
<th>Conf.</th>
<th>footprint</th>
<th>tilt</th>
<th>az.</th>
<th>Conf.</th>
<th>footprint</th>
<th>tilt</th>
<th>az.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.00</td>
<td>0</td>
<td>S</td>
<td>5</td>
<td>2.53</td>
<td>40</td>
<td>S</td>
</tr>
<tr>
<td>2</td>
<td>1.46</td>
<td>10</td>
<td>S</td>
<td>6</td>
<td>2.75</td>
<td>50</td>
<td>S</td>
</tr>
<tr>
<td>3</td>
<td>1.88</td>
<td>20</td>
<td>S</td>
<td>7</td>
<td>2.88</td>
<td>60</td>
<td>S</td>
</tr>
<tr>
<td>4</td>
<td>2.24</td>
<td>30</td>
<td>S</td>
<td>8</td>
<td>0.98</td>
<td>10</td>
<td>E/W</td>
</tr>
</tbody>
</table>

### Table 2: Roof sizes, azimuth and tilt for the case study

<table>
<thead>
<tr>
<th>Azimuth</th>
<th>Tilt</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>North-West (352°)</td>
<td>48°</td>
<td>29.5 m(^2)</td>
</tr>
<tr>
<td>South (172°)</td>
<td>27°</td>
<td>29.5 m(^2)</td>
</tr>
<tr>
<td>East (82°)</td>
<td>47°</td>
<td>78.6 m(^2)</td>
</tr>
<tr>
<td>West (262°)</td>
<td>47°</td>
<td>89.7 m(^2)</td>
</tr>
<tr>
<td>Flat roof (-)</td>
<td>0°</td>
<td>41.8 m(^2)</td>
</tr>
</tbody>
</table>
Swiss residential buildings. Temperature and irradiance data are available from [13]. However, running an optimization problem using the standard design reference year for ambient conditions (with 8760 hourly time steps) would involve an unreasonably high number of optimization variables and constraints. For this reason, we used a k-medoids clustering method to decrease the temporal input data of the problem to $8 \times 24$ hours typical operating periods and 2 extreme periods to reflect the most challenging operating conditions. The clustering was based on two independent variables: the daily ambient temperature and the global solar irradiance. Further information on this approach are given in [14] and [1]. Finally, cost parameters were set as to represent the current situation of the energy market in Switzerland: electricity price was assumed to 0.20 CHF/kWh, electricity feed-in revenue 0.08 CHF/kWh, natural gas Price 0.06 CHF/kWh. 20 years lifetime and 0.02 interest rate were used for the annualization of the investment. The installation cost of PV panels is assumed equal to that of standard modules, since what is proposed in the paper only changes the design/optimization approach and does not involve any special or innovative technology.

In order to provide an estimate of the improvement resulting from the more detailed modelling of the PV panels, we also ran the optimization using a "reference" methodology for the optimization of the system. According to the reference methodology, all roofs are assumed to be flat, and all the panels on the roofs are assumed to be installed with no tilt angle. To provide an easier comparison, for each scenario of the MOO we forced the number of installed PV modules to be the same as in the corresponding scenario of the proposed method.

4. Results

Fig 3 shows the Pareto curve of the multi-objective optimization for both approaches the standard without and the proposed with orientation of the PV panels. Scenario 1 corresponds to the first Pareto point with lowest investment cost, while Scenario 13 with lowest operational
expenses. The Pareto front shows, as expected, that increasing the CAPEX allows reducing the OPEX, even to the point of having negative OPEX. This is the consequence of the low electrical load of the building itself compared to the generated electricity.

While not being explicitly objectives of the optimization, the SS and the SC of the system are also quantities of interest. The results show that it is possible to achieve a degree of self-sufficiency up to approximately 80%, with a high surface of PV panels installed. It is however more difficult to keep a high degree of self consumption, especially when the number of installed PV panels increases.

The trends in the installation of PV panels and batteries are shown in Fig 4. PV panels start to be installed already in Scenario 3 and reach the maximum available capacity in Scenario 8, while batteries start to be installed at higher admissible CAPEX. This is due to the fact that the economic advantage of installing batteries is only related to the difference between the price of purchasing and selling electricity to the grid, which has a lower impact on the OPEX than the absolute value of the selling price of electricity. This can also be seen in Fig 4: starting from Scenario 8 both the electricity feed and the electricity import start to decrease, showing the impact of the batteries on the system.

Fig 5 shows both the preferable configuration and the influence of shading. Contrarily to what normally expected, surfaces pointing westwards are installed before the surfaces with south orientation, or the flat roof. This can be explained by the fact that peak electricity demand tend to be in the evening, when surfaces with a south orientation do not receive a large amount of radiation, further reinforces the choice of the optimizer to install PV panels on the west orientated roof first.

Fig 5 also shows the impact of the PV panels configuration and of shading on the flat part of the roof. When the CAPEX constraint of the MOO problem (Equation 2) only allows to invest into a limited number of panels, the chosen solution does not depend on the footprint, as the size of the roof does not constitute a limiting factor. In this case study, high tilt angles are
chosen (see Scenario 5, where configurations 5 and 6 are preferred). Increasing the CAPEX constraint leads to a higher investment into PV-panels, which in turn results into maximizing the number of panels that can be fitted on the roof. This explains the choice of configurations 1 and 8, which are the ones with the lowest footprint (respectively 1 and 0.98, compared to 2.53 and 2.75 for configurations 5 and 6): they allow to install more than twice the amount of panels for the same roof surface. Fig 6 shows the electricity demand and the electricity generation for Scenario 4. In Scenario 4 the west orientated surface is fully covered and none of the south orientated panels are installed (Fig 5). Fig 6 (left) shows that the afternoon peak can be covered to a larger extent if the optimizer is allowed to choose among surfaces with different orientations. This can be seen particularly clearly for typical days 3 and 8. During summer, the installation of batteries can only help to a certain extent: as seasonal storage is not allowed, batteries can provide support in covering the morning and evening peaks, but the low electricity demand makes it necessary to export electricity to the grid (Fig 6 (right)). The comparison of the standard approach for modelling the contribution of PV panels with the one proposed in this paper shows that the two give relatively similar results (in terms of CAPEX and OPEX), for low to medium investment costs (see Fig 3). When the CAPEX increases beyond a certain boundary, however, it can be observed that assuming flat surfaces with perfect irradiation leads to more generated electricity, and hence lower OPEX for the same CAPEX. This can also be seen from the fact that, for the same amount of PV panels installed, the SC level is lower (Fig 3) and the electricity fed to the grid is higher (Fig 4) in the reference case compared to the proposed one, a consequence of both the higher PV generated energy, and of the higher mismatch between the demand and the generation profiles. This is reinforced by the fact that the installed size of the batteries is larger in the reference case compared to the proposed approach (see Fig 4): because of the mismatch between generation and demand, more storage is needed to compensate for it.
5. Discussion and conclusion

The results of this paper suggest that the tilt and azimuth of PV panels and of the roof they are installed on should not be neglected. This consideration is two-fold: on the one hand, the error generated by not considering these aspects generates an overestimation of the energy generated by the PV panels, thereby introducing a bias in the evaluation of the investment. On the other hand, not including the choice of the roof (and, in case of flat roofs, the choice of PV panel tilt and orientation) where the panels should be installed on, generates a sub-optimization of the system. This is particularly true since, as shown in this paper, the most optimal choice is not necessarily that of filling first flat surfaces and South-oriented ones, a consequence of the better match between generation and demand offered by West-oriented panels, with a higher tilt angle. In addition to a better economic performance, this also results in a decrease of the energy that needs to be exchanged with the network, hence leading to higher degrees of self-consumption, or, in alternative, the same degree of self-consumption with a lower installed battery capacity.

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REFERENCES


