**Abstract**

- We investigate how Geometric Deep Learning can be applied to Fluid flow simulations.
- We propose an end-to-end differentiable architecture that allow object-to-mesh predictions of fluid simulations.
- We provide comparisons with a baseline and visual results on three datasets:
  - Airfoils
  - Backward Facing Step
  - Fixed-wing drones
- Applications range from the initialization of iterative numerical solvers and smart mesh initialization to engineering design and the animation industry.

**Theory on Graph Convolutions**

- In “standard” convolutions (i.e. on euclidean domains) the discrete convolution of a signal by a finite support filter $g$ is given by:
  
  $$ (f * g)[n] = \sum_{m=-M}^{M} f[n-m]g[m] $$

- One possible generalization to non-euclidean domain presented in [1] consists in extracting a local patch of data $D_j(x)$:
  
  $$ D_j(x) = \int f(x') v_j(x,x') dx' , \quad j = 1, ..., J $$
  
  where the $v_j$ are suitably chosen weighting functions such as gaussians

  $$ v_j(u) = \exp \left( -\frac{1}{2} (u - \mu_j)^T \Sigma^{-1} (u - \mu_j) \right) $$

  with learnable mean and covariance matrix. The convolution is then defined as

  $$ (f \ast g)(x) = \sum_j g_j(x) f $$

**Datasets**

- Shapes were given (Airfoils & drone) or automatically generated using gmsh (Backward Facing Step).
- The data has been created by running Computational Fluid Dynamics simulation using OpenFOAM which uses the Finite Volume Method. We solved Reynolds Averaged Navier-Stokes equations with turbulence models $k-\epsilon$ and $k-\omega$.

**Representations**

The architecture we propose combines the positive aspects of 3 representations of 3D objects: Voxelisation, Point Clouds, Meshes.

- **Voxels**
- **Pointcloud**
- **Mesh**

**Backward Facing Step**

Predictions on a backward facing step with different steps and viscosity. The three main fields are shown: $U_x$, $U_y$ and Pressure.

**Airfoils**

Using the same model it is possible to predict any field of interest — here the turbulent viscosity $\nu^t$.

**Drones**

Prediction and groundtruth of fields ($U_x$, $U_y$, $U_z$, Pressure) on the drone dataset, in which we can see the artifacts of a projection error.

**What about the recirculation zone?**

The quality of the recirculation zone can give us a hint about the understanding of the underlying physical phenomena.

The vortex is globally similar but we can see some discrepancies due to the non-conservation of mass. For more "physical" predictions, the conservation of mass could be enforced in the model.

**Acknowledgement & References**

We rely on previous work by Baqué et al. [2] and its continuation by EPFL’s DeepShape project. We build on top of their code base for geometric Deep Learning. We thank Neural Concept for collaboration and Pierre Baqué & Michaël Defferrard for their magnificent support.
