

Synthesis and Analysis of Linear and Nonlinear Bianisotropic Metasurfaces

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Abstract We present our recent progress pertaining to the synthesis and analysis of bianisotropic linear and nonlinear metasurfaces. These developments are based on the generalized sheet transition conditions (GSTCs), which provide a very complete **and general method for modeling metasurfaces**. Specifically, we present our most recent research, which includes the control of radiation pressure at both the micro- and macroscopic scales, the analysis of second-harmonic scattering from nonlinear metasurfaces, and the surface-wave dispersion retrieval method for bianisotropic metasurfaces in symmetric and asymmetric environments.

I. INTRODUCTION

Metasurfaces are electrically thin arrays of artificial scattering particles engineered to tailor the propagation of electromagnetic waves. By properly designing the shape and material composition of these scattering particles, one may realize very complex electromagnetic properties and thus control the propagation of light in unprecedented ways. As a consequence, metasurfaces have attracted major attention in recent years and have generated an important collection of both theoretical and practical works [1]–[11].

II. SYNTHESIS AND ANALYSIS OF BIANISOTROPIC METASURFACES

In order to facilitate the implementation of metasurfaces, several synthesis techniques have been developed over the past few years [5]–[7], [12]. These techniques may be used to find the optimal material parameters required to achieve a desired electromagnetic transformation, which is typically specified in terms of incident, reflected and transmitted waves. They are of paramount importance not only because they are instrumental in implementing metasurfaces but also because they allow one to unlock and exploit the full potential of metasurfaces. Once the ideal material parameters have been found, the shape and composition of the scattering particles are numerically optimized so that their effective material parameters closely match those determined by the synthesis technique. Over the past few years, we have developed our own synthesis framework for bianisotropic metasurfaces [6], [9], [10]. This framework is based on generalized sheet transition conditions (GSTCs) [13], [14]. To demonstrate the capabilities of our synthesis technique, we have applied it to the implementation of metasurfaces performing many operations bire-

fringent transformations (half- and quarter-wave retardation, beam splitting, orbital angular momentum multiplexing) [15], perfect refraction [16], dispersion engineering [17], remote processing [18] and point emitters light extraction efficiency enhancement [19].

In parallel to the implementation of these synthesis techniques, several metasurface analysis schemes have been developed. They allow one to find the fields scattered by a metasurface with known material parameters for arbitrary illumination. These analysis techniques are particularly important insofar as they allow to characterize the electromagnetic responses of metasurfaces for any non-specified excitations, e.g. plane-wave incidence angle, which differ from that of the initial specification. We have developed several metasurface analysis techniques, which, like their synthesis counterparts, are based on the GSTCs [20]–[23]. These include finite-difference time-domain and frequency-domain schemes as well as finite-element methods, which may be applied for dispersive, time-varying and nonlinear metasurfaces.

III. CONCEPTS AND APPLICATIONS

Countless concepts and applications of metasurfaces have been reported in the literature [24] and it is beyond the scope of this paper to discuss them all. In what follows, we shall rather briefly present our most recently developed metasurface concepts and applications.

A. Radiation Pressure Control

The incredible capabilities of metasurfaces to tailor EM waves may be leveraged to control radiation pressure. This concept may find applications at both macro- and microscopic scales. In the former case, conventional solar sails may be replaced by metasurface sails with flexible/tunable force control allowing the generation of attractive, repulsive, lateral and rotational forces [25]. In the latter case, thousands of nanoparticles may be parallelly steered using surface waves generated with spatially-varying metasurfaces. The direction of propagation of the surface waves, and subsequently of the nanoparticles, may then be controlled by varying the polarization of the light and by patterning the metasurface plane with regions of different spatial variations.

B. Second-Order Nonlinear Metasurfaces

Metasurfaces are expected to play a major role in the future of nonlinear optics thanks to their capabilities of generating giant nonlinear responses. To provide a general synthesis and analysis framework which would apply to nonlinear metasurfaces, we have extended our developments to include the presence of second-order nonlinearities [26], [27]. Based on this extended framework, we have introduced the general second-order nonlinear scattering parameters, which, among others, reveal the asymmetric scattering properties of nonlinear metasurfaces.

C. Surface-Wave Dispersion Retrieval and Synthesis

So far, most surface-wave guiding structures have been realized using impenetrable surfaces. In [28], we extended this concept to penetrable surface-wave guiding metasurfaces. We have now developed a general GSTCs based method to retrieve the dispersion of bianisotropic (penetrable) metasurfaces [29]. The technique consists in solving an eigenvalue problem where the susceptibilities of the metasurface are known and the propagation constant and polarization of the surface-wave modes are to be found. We provide the dispersion relations of birefringent and bianisotropic metasurfaces and show that the latter may be made so as to support surface-wave propagation only on one of their sides, while completely forbidding the existence of surface waves on the other.

REFERENCES

- [1] N. Yu and F. Capasso, "Flat optics with designer metasurfaces," *Nature Mater.*, vol. 13, no. 2, pp. 139–150, 2014.
- [2] N. Yu, P. Genevet, M. A. Kats, F. Aieta, J.-P. Tetienne, F. Capasso, and Z. Gaburro, "Light propagation with phase discontinuities: generalized laws of reflection and refraction," *Science*, vol. 334, no. 6054, pp. 333–337, 2011.
- [3] C. Pfeiffer and A. Grbic, "Metamaterial Huygens' surfaces: Tailoring wave fronts with reflectionless sheets," *Phys. Rev. Lett.*, vol. 110, p. 197401, May 2013.
- [4] A. V. Kildishev, A. Boltasseva, and V. M. Shalaev, "Planar photonics with metasurfaces," *Science*, vol. 339, no. 6125, 2013.
- [5] C. Pfeiffer, C. Zhang, V. Ray, L. J. Guo, and A. Grbic, "High performance bianisotropic metasurfaces: asymmetric transmission of light," *Phys. Rev. Lett.*, vol. 113, p. 023902, Jul 2014.
- [6] K. Achouri, M. A. Salem, and C. Caloz, "General metasurface synthesis based on susceptibility tensors," *IEEE Trans. Antennas Propag.*, vol. 63, no. 7, pp. 2977–2991, Jul. 2015.
- [7] A. Epstein and G. Eleftheriades, "Passive lossless Huygens metasurfaces for conversion of arbitrary source field to directive radiation," *IEEE Trans. Antennas Propag.*, vol. 62, no. 11, pp. 5680–5695, Nov 2014.
- [8] S. Asadchy, V. Y. Ra'di, J. Vehmas, and A. Tretyakov, S. "Functional metamirrors using bianisotropic elements," *Phys. Rev. Lett.*, vol. 114, p. 095503, Mar. 2015.
- [9] K. Achouri, B. A. Khan, S. Gupta, G. Lavigne, M. A. Salem, and C. Caloz, "Synthesis of electromagnetic metasurfaces: principles and illustrations," *EPJ Applied Metamaterials*, vol. 2, p. 12, 2015.
- [10] K. Achouri and C. Caloz, "Design, concepts and applications of electromagnetic metasurfaces," arXiv preprint arXiv:1712.00618, 2017.
- [11] C. Yan, K.-Y. Yang, and O. J. F. Martin, "Fano-resonance-assisted metasurface for color routing," *Light: Science & Applications*, vol. 6, pp. e17017 EP -, Jul 2017.
- [12] T. Niemi, A. Karilainen, and S. Tretyakov, "Synthesis of polarization transformers," *IEEE Trans. Antennas Propag.*, vol. 61, no. 6, pp. 3102–3111, June 2013.
- [13] M. M. Idemen, *Discontinuities in the Electromagnetic Field*. John Wiley & Sons, 2011.
- [14] E. F. Kuester, M. Mohamed, M. Piket-May, and C. Holloway, "Averaged transition conditions for electromagnetic fields at a metafilm," *IEEE Trans. Antennas Propag.*, vol. 51, no. 10, pp. 2641–2651, Oct 2003.
- [15] K. Achouri, G. Lavigne, and C. Caloz, "Comparison of two synthesis methods for birefringent metasurfaces," *J. Appl. Phys.*, vol. 120, no. 23, p. 235305, 2016.
- [16] G. Lavigne, K. Achouri, V. S. Asadchy, S. A. Tretyakov, and C. Caloz, "Susceptibility derivation and experimental demonstration of refracting metasurfaces without spurious diffraction," *IEEE Trans. Antennas Propag.*, vol. 66, no. 3, pp. 1321–1330, March 2018.
- [17] K. Achouri, A. Yahyaoui, S. Gupta, H. Rmili, and C. Caloz, "Dielectric resonator metasurface for dispersion engineering," *IEEE Trans. Antennas Propag.*, vol. 65, no. 2, pp. 673–680, Feb 2017.
- [18] K. Achouri, G. Lavigne, M. A. Salem, and C. Caloz, "Metasurface spatial processor for electromagnetic remote control," *IEEE Trans. Antennas Propag.*, vol. 64, no. 5, pp. 1759–1767, 2016.
- [19] L. Chen, K. Achouri, E. Kallos, and C. Caloz, "Simultaneous enhancement of light extraction and spontaneous emission using a partially reflecting metasurface cavity," *Phys. Rev. A*, vol. 95, p. 053808, May 2017.
- [20] Y. Vahabzadeh, K. Achouri, and C. Caloz, "Simulation of metasurfaces in finite difference techniques," *IEEE Trans. Antennas Propag.*, vol. 64, no. 11, pp. 4753–4759, 2016.
- [21] Y. Vahabzadeh, N. Chamanara, K. Achouri, and C. Caloz, "Computational analysis of metasurfaces," arXiv preprint arXiv:1710.11264, 2017.
- [22] N. Chamanara, Y. Vahabzadeh, K. Achouri, and C. Caloz, "Spacetime processing metasurfaces: Gstc synthesis and prospective applications," in *2016 IEEE International Symposium on Antennas and Propagation (APSURSI)*, June 2016, pp. 365–366.
- [23] S. Sandeep, J. M. Jin, and C. Caloz, "Finite-element modeling of metasurfaces with generalized sheet transition conditions," *IEEE Transactions on Antennas and Propagation*, vol. 65, no. 5, pp. 2413–2420, May 2017.
- [24] S. B. Glybovski, S. A. Tretyakov, P. A. Belov, Y. S. Kivshar, and C. R. Simovski, "Metasurfaces: from microwaves to visible," *Phys. Rep.*, vol. 634, pp. 1 – 72, 2016.
- [25] K. Achouri and C. Caloz, "Metasurface solar sail for flexible radiation pressure control," arXiv preprint arXiv:1710.02837, 2017.
- [26] K. Achouri, Y. Vahabzadeh, and C. Caloz, "Mathematical synthesis and analysis of a second-order magneto-electrically nonlinear metasurface," *Opt. Express*, vol. 25, no. 16, pp. 19013–19022, Aug 2017.
- [27] K. Achouri, G. D. Bernasconi, J. Butet, and O. J. F. Martin, "Homogenization and scattering analysis of second-harmonic generation in nonlinear metasurfaces," arXiv preprint arXiv:1802.10477, 2018.
- [28] K. Achouri and C. Caloz, "Space-wave routing via surface waves using a metasurface system," arXiv preprint arXiv:1612.05576, 2016.
- [29] K. Achouri and O. J. F. Martin, "Surface-wave dispersion retrieval method and synthesis technique for bianisotropic metasurfaces," arXiv preprint arXiv:1805.05116, 2018.