On the Modeling of Non-Vertical Risers in the Interaction of Electromagnetic Fields With Overhead Lines

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Abstract—This paper proposes a simple method to take into account non-vertical risers through an equivalent partial inductance. The proposed approach was validated considering several examples and taking as reference full-wave results obtained using a numerical electromagnetics code numerical electromagnetics code (NEC)-4.

Index Terms—Induced current, non-vertical riser, total voltage, transient electromagnetic field, transmission line model.

I. INTRODUCTION

DUE to difficulties in performing experiments related to lightning-induced voltages on power distribution and transmission lines, engineers depend to a great extent on numerical simulations in making assessments concerning the protection of these systems against lightning.

Several models have been used in the literature to estimate the voltages induced on power lines due to lightning [1]–[7]. While some of these models are actually different representations of the solution to the transmission line equations pertinent to the problem under consideration, the others are either incorrect or partially correct solutions [7]. All these models adhere to the approximation that the response of the overhead transmission line to the incident electromagnetic field is quasi-transverse electromagnetic (TEM) [8].

In the analysis to follow, we will adopt the model presented by Agrawal et al. [1] and we will consider a single overhead conductor, but the conclusions to be extracted from the study are also applicable to all the other models and to multiconductor lines.

II. MODEL OF AGRAWAL ET AL.

Let us consider, without loss of generality, a single-wire line located at a height h above a perfectly-conducting ground plane. The line is located along the x-axis and the z-axis is aligned with the vertical. The height of the line is h. The line impinged upon by an exciting (external) electromagnetic field. We represent the vertical and the horizontal components of the exciting electric field, respectively, by $E_x^v$ and $E_z^v$. These exciting electric fields, which are evaluated in the absence of the line conductors, induce currents in the line which, in turn, generate the so-called scattered electric and magnetic fields. Let us define the vertical component of the scattered electric field by $E_z^s$. The two transmission line equations of the model of Agrawal et al. can be written as

$$\frac{\partial v_s^x(x,t)}{\partial x} + R' i(x,t) + L \frac{\partial i(x,t)}{\partial t} = E_x^v(x,0,h,t)$$

$$\frac{\partial i(x,t)}{\partial x} + C' \frac{\partial v_s^x(x,t)}{\partial t} = 0$$

in which $R'$, $L'$, and $C'$ are, respectively, the per-unit-length resistance, inductance, and capacitance of the line, $i(x,t)$ is the current, and $v_s^x(x,t)$ is the scattered voltage defined as

$$v_s^x(x,t) = -\int_0^h E_z^s(x,y,z,t)dy.$$

The total line voltage of the line, $v(x,t)$, can be obtained as

$$v(x,t) = v_s^v(x,t) + v_s^v(x,t) = v_s^v(x,t) - \int_0^h E_z^s(x,0,z,t)dz.$$ 

Note that, unlike the scattered electric field in (4), the coordinate y has been set to zero in the exciting field in the integrand on the right hand side of (3) since the exiting field is, in general, not conservative.

The boundary conditions at the two line ends terminated in impedances, $Z_A$ and $Z_B$, written in terms of the scattered voltages, are given by

$$v_s^v(0,t) = -Z_A i(0,t) + \int_0^h E_x^v(0,0,z,t)dz$$

$$v_s^v(L,t) = Z_B i(L,t) + \int_0^h E_x^v(L,0,z,t)dz.$$ 

The equivalent circuit of the Agrawal et al. model equations is shown in Fig. 1. It should be noted that it is implicitly assumed that the risers at the two line ends are vertical and, therefore, the integrals of the vertical electric field in (4)–(6) are performed along the vertical straight line from the ground plane to the conductor.

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III. TREATMENT OF NON-VERTICAL RISERS

Consider the situation shown in Fig. 2, where the terminal riser is not vertical and is characterized by an arbitrary shape, and not necessarily in the transverse plane.

Unlike the original model of Agrawal et al., since the riser is not vertical, the sources at the terminations are the line integrals of the exiting electric fields along the risers’ non-vertical geometries.

Moreover, the termination impedances, represented by $Z_A$ and $Z_B$ in Fig. 3, are the series combination of the actual termination and an additional inductive impedance stemming from the more general geometry for the risers. In the next section, we present a derivation of these boundary conditions.

A. Boundary Conditions for Non-Vertical Risers

Integrating along the non-vertical riser on the left-hand side of the line shown in Fig. 2, we can write

$$-\int_{\text{Riser}} \vec{E} \cdot d\vec{l} = -I(0)Z_A.$$  \hfill (7)

Separating the total field into the exiting and scattered components, (7) can be written as follows:

$$-\int_{\text{Riser}} \vec{E}^e \cdot d\vec{l} - \int_{\text{Riser}} \vec{E}^s \cdot d\vec{l} = -I(0)Z_A.$$  \hfill (8)

Now, applying the Maxwell–Faraday’s equation to the scattered field around the loop hashed in yellow in the figure, we can write

$$\int_{\text{Riser}} \vec{E}^s \cdot d\vec{l} - \int_0^h \vec{E}_s^z dz = -j\omega \int \vec{B}^s \cdot d\vec{S}.$$  \hfill (9)

Solving for the first term on the left-hand side of (9) and substituting the second term by the scattered voltage [see (3)], we obtain

$$\int_{\text{Riser}} \vec{E}^s \cdot d\vec{l} = -V^s - j\omega \int \vec{B}^s \cdot d\vec{S}.$$  \hfill (10)

Replacing (10) into (8) and solving for the scattered voltage yields

$$V^s = \int_{\text{Riser}} \vec{E}^e \cdot d\vec{l} - I(0)Z_A - j\omega \int \vec{B}^s \cdot d\vec{S}.$$  \hfill (11)

The surface integral in the third term on the right-hand side of (11) is a scattered magnetic flux. Considering the assumption of an electrically small line cross section, this flux is proportional to the termination current, through an inductive term. As a result, we can rewrite (11) as

$$V^s = \int_{\text{Riser}} \vec{E}^e \cdot d\vec{l} - I(0)Z_A - j\omega L_A I(0)$$  \hfill (12)

in which the inductance $L_A$, whose value depends only on the geometry of the riser, is

$$L_A = \frac{\int \vec{B}^s \cdot d\vec{S}}{I(0)}.$$  \hfill (13)

Equation (12) is the result sought. Fig. 3 shows the equivalent circuit for the left-hand side of the line based on (12).

Thus, the integrals of the exciting electric field at the two line ends are evaluated along a path defined by the geometry of the risers.

B. Validation Using Full-Wave Simulations

In this section, we will use full-wave simulations to validate the equivalent inductance (13) that needs to be considered in the case of non-vertical risers. We will consider a single wire of length $L$, height $h$, and radius $r$ above a perfectly conducting ground. The left and right terminations are, respectively, $Z_A$ and $Z_B$. The line is illuminated by a uniform plane wave characterized by an azimuth angle $\theta$, an elevation angle $\psi$, and a polarization angle $\alpha$ [see Fig. 4]. The terminal riser at the right end is assumed to be vertical, while three different configurations will be considered for the left-end riser, as shown in Fig. 5(a)–(c).

For the cases of a non-vertical riser [see Fig. 5(b) and (c)], the equivalent (partial) inductance $L_A$ should be evaluated using (13). The scattered magnetic field is first evaluated using the Biot–Savart law. Then, the total magnetic flux is computed by numerical integration, taking into account the riser image, as shown in Fig. 6.

The field-to-transmission-line coupling equations including the treatment of the non-vertical risers are solved using the Baum–Liu–Tesche (BLT) equations [9]. According to the BLT
Fig. 4. Single-wire transmission line excited by an incident plane wave.

Fig. 5. Cross-section of the three considered geometries for the left-end riser. (a) Vertical. (b) Rectangular. (c) Triangular.

Fig. 6. Illustration of the geometry for the calculation of the total magnetic flux for the left-end riser. (a) Rectangular. (b) Triangular.

The solution for the induced voltages at both ends can be expressed by

\[
V(0, j\omega) = \frac{-\rho_2 (1 + \rho_1) S_1(j\omega) - e^{\gamma L} (1 + \rho_1) S_2(j\omega)}{\rho_1 \rho_2 - e^{2\gamma L}}
\]

\[
V(L, j\omega) = \frac{-e^{\gamma L} (1 + \rho_2) S_1(j\omega) - \rho_1 (1 + \rho_2) S_2(j\omega)}{\rho_1 \rho_2 - e^{2\gamma L}}
\]

where the source terms \( S_1 \) and \( S_2 \) are given by

\[
S_1(j\omega) = \frac{1}{2} \int_0^L e^{\gamma x} E_x(x, j\omega) dx - \frac{1}{2} V_1(j\omega) + \frac{1}{2} V_2(j\omega) e^{\gamma L}
\]

\[
S_2(j\omega) = -\frac{1}{2} \int_0^L e^{\gamma (L-x)} E_x(x, j\omega) dx + \frac{1}{2} V_1(j\omega) e^{\gamma L} - \frac{1}{2} V_2(j\omega)
\]

\[
V_1(j\omega) = -\int_{RiserA} \vec{E}_e \cdot d\vec{l}, \quad V_2(j\omega) = -\int_{RiserB} \vec{E}_e \cdot d\vec{l}
\]

in which \( \rho_1 \) and \( \rho_2 \) are the reflection coefficients at the near-end and far-end of the line, respectively, and \( \gamma \) is the propagation constant along the line. Expressions for the line parameters can be found in [10].

In order to validate the calculation results, the numerical electromagnetics code NEC-4, a full-wave solver based on the method of moments, was used [11]. In what follows, we will consider the three cases for the geometry of the left-end riser shown in Fig. 5.

**Case 1: Vertical Risers:** In this case, we considered a 20-m long wire located at a height of 0.1 m above a perfectly conducting ground. The conductor radius is 1 mm. The line is terminated at both ends in 100 \( \Omega \) resistive loads. The azimuth angle, elevation angle, and polarization angle of the exciting plane wave are 0°, 45°, and 0°, respectively. The frequency range of the wave is 10 kHz–50 MHz, and the amplitude of the \( E \)-field is 1 V/m across the whole frequency spectrum. Fig. 7 presents a comparison between the results obtained by way of the BLT equations and of the NEC-4 code for the case of vertical risers.

As expected, it can be seen that the results based on the transmission line theory (BLT equations) are in excellent agreement with the full-wave results obtained using the NEC-4.

**Case 2: Rectangular Riser at the Left-End:** As in Case 1, we considered a 20-m long wire located at a height of 0.1 m above a perfectly conducting ground and a conductor radius of 1 mm. The line is terminated at both ends in 100 \( \Omega \) resistive loads. The left-end riser corresponds to Fig. 5(b), while the right-end riser is kept vertical. The azimuth angle, elevation angle, and polarization angle of the exciting plane wave are 0°, 45°, and 0°, respectively. The frequency range of the wave is 10 kHz–50 MHz, and the amplitude of the \( E \)-field is 1 V/m across the whole frequency spectrum. We considered two different values for the length \( L_1 \) [see Fig. 5(b)], namely 0.5 and 1 m. For those two lengths, the equivalent inductance \( L_A \) is, respectively, 0.66 and 1.2 \( \mu \)H. The calculated results for the induced currents are
Fig. 8. Induced currents as a function of frequency. (a) Left end. (b) Right end. The left end of the line is terminated in a rectangular shape riser \(L_1 = 0.5 \text{ m}, \text{Fig. 5(b)}, \) and in a vertical riser on its right end. Calculations obtained using the classical transmission line theory, the transmission line theory including the partial inductance, and the NEC-4 are shown, respectively, using a dashed-line, a solid black line, and a solid blue line.

Fig. 9. Induced currents as a function of frequency. (a) Left end. (b) Right end. The left end of the line is terminated in a rectangular shape riser \(L_1 = 1 \text{ m}, \text{Fig. 5(b)}, \) and in a vertical riser on its right end. Calculations obtained using the classical transmission line theory, the transmission line theory including the partial inductance, and the NEC-4 are shown, respectively, using a dashed-line, a solid black line, and a solid blue line.

Fig. 10. Induced currents as a function of frequency. (a) Left end. (b) Right end. The left end of the line is terminated in a rectangular shape riser \(L_1 = 1 \text{ m}, \theta = 45^\circ, \text{Fig. 5(b)}, \) and in a vertical riser on its right end. Calculations obtained using the classical transmission line theory, the transmission line theory including the partial inductance, and NEC-4 are shown, respectively, using a dashed-line, a solid black line, and a solid blue line.

Fig. 11. Induced currents as a function of frequency. (a) Left end. (b) Right end. The left end of the line is terminated in a rectangular shape riser \(L_1 = 1000 \text{ m}, L_2 = 3 \text{ m}, \text{Fig. 5(b)}, \) and in a vertical riser on its right end. Calculations obtained using the classical transmission line theory, the transmission line theory including the partial inductance, and NEC-4 are shown, respectively, using a dashed-line, a solid black line, and a solid blue line.
shown in Figs. 8 and 9. It can be seen that the results calculated using the classical transmission line theory deviate from the full-wave results obtained using NEC-4. Taking into account the non-vertical riser using the partial equivalent inductance yields significantly more accurate results.

Note that, in these figures, as well as in the next ones, we have indicated in the plots the frequency at which the impedance of the equivalent inductance becomes equal to that of the terminal impedance. This frequency gives an indication of the frequencies above which this correction needs to be taken into account.

We now consider another example with the same line configuration but considering an azimuth angle of the exciting field of 45°. The calculated results for the induced current are shown in Fig. 10. Again, one can appreciate the fact that the taking into account of the equivalent inductance results in significant improvement of the results, especially at high frequencies.

We also considered the case of a 1000-m long wire at a height of 5 m above a perfectly conducting ground. The conductor radius is 5 mm. The line is terminated at both ends in 100-Ω resistive loads. The left-end riser corresponds to Fig. 5(b), while the right-end riser is assumed to be vertical. The adopted value for the length $L_1$ is 3 m. In this case, the equivalent inductance $L_A$ is 10.6 μH. The azimuth angle, elevation angle, and polarization angle of the exciting plane wave are 0°, 45°, and 0°, respectively. The amplitude of the $E$-field is 1 V/m across the complete frequency spectrum. The calculated results for the induced current are shown in Fig. 11. Again, it can be seen that the results from the proposed approach agree well with the full-wave results obtained using NEC-4, while the classical transmission line approach fails in reproducing the induced currents in high frequencies.

Case 3: Triangular Riser at the Left-End: In this final case, we considered a 20-m long wire located at a height of 0.1 m above a perfectly conducting ground. The conductor radius is 1 mm. The line is terminated at both ends in 100-Ω resistive loads. The left-end riser corresponds to Fig. 5(c), while the right-end riser is kept vertical. We considered two different values for the length $L_1$, namely 0.5 and 1 m, respectively. In these two cases, the equivalent inductance $L_A$ is respectively 0.49 and 0.96 μH. The calculated results for the induced currents are shown in Figs. 12 and 13. Again, it can be seen that the proposed approach allows to obtain results that are in an excellent agreement with full-wave results obtained using NEC-4, while the classical transmission line approach fails in reproducing the induced currents, especially in high frequencies.

IV. CONCLUSION

In this paper, we showed that the classical transmission line theory is not able to accurately take into account non-vertical risers. We proposed a simple method to take into account non-vertical risers through an equivalent partial inductance. The proposed approach was validated considering several examples and taking as a reference full-wave results obtained using NEC-4.

REFERENCES


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