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Calculation of High Frequency Electromagnetic Field Coupling to Overhead Transmission Line above a Lossy Ground and Terminated with a Nonlinear Load

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***Abstract*—High frequency electromagnetic fields such as those associated with EMP and IEMI can couple to overhead power lines. Since the height of the overhead power lines can be comparable or even larger than the smallest wavelength of typical EMP and IEMI pulses, the classical TL approximation might not be suitable for evaluating the line response. On the other hand, a traditional full wave solver (*e.g.* MoM) is computationally inefficient, especially when dealing with long lines. To address these problems, we propose an efficient method to handle the high frequency electromagnetic field coupling to overhead lines with nonlinear loads above a lossy ground. In the proposed method, the asymptotic approach is adopted and extended to the case of a lossy ground, which handles the problem in a semi-analytical way and has a much higher computational efficiency in the case of long lines. Although the proposed method applies in the frequency domain, the case of nonlinear loads can be considered through a combination with the time marching method. The proposed method is validated with several numerical examples.**

***Index Terms*—High frequency electromagnetic field; overhead transmission line, transient response; asymptotic method; distributed parameter circuits,**

I. Introduction

OVERHEAD transmission lines are one of the essential components in power systems. High frequency electromagnetic fields such as electromagnetic pulse (EMP) produced by a high-altitude nuclear burst (e.g., [1]- [2]) can induce significant overvoltage and currents, which may cause various effects such as short interruptions, voltage sags and even damage to power components, especially for distribution networks [3]. Furthermore, there is an increased concern about intentional electromagnetic interferences (IEMI) to power systems generated by high power transient electromagnetic sources such as compact radiation systems (e.g., [4]). Therefore, it is important to predict the response of overhead power lines.

The problem of high-frequency electromagnetic field coupling to transmission lines has been thoroughly studied in the past decades (see a review in [5]). One of the commonly-used methods is the classical transmission line (TL) theory [6]-[10]. This method is based on the transmission line approximation, where the line response is assumed to be the quasi transverse electromagnetic (TEM), and the high frequency effects (*e.g.* radiating modes or leaky modes) are neglected. The coupling equations resulting from the classical TL model [11]-[13] can be solved with a relatively low computational cost, and can provide the accurate solutions in the case when the line cross-sectional dimensions are electrically small, namely, smaller than about one tenth of the minimum wavelength of the incoming wave. This requirement might not be satisfied for the case of high frequency electromagnetic fields [14]. For example the IEC standard EMP waveform is characterized by a minimum significant wavelength of about 3 m, which is smaller than the height of typical overhead power lines, which is about 10 m for distribution lines and even higher for the high voltage transmission lines.

To obtain an accurate solution in the case that the cross section of the line is comparable to the wavelength, the antenna theory is commonly used [15]. This method is based on the thin-wire approximation, and can take into account the high frequency effects and provide an accurate solution for the high frequency field illumination. In general, a numerical full-wave technique, *e.g.*, method of moment (MoM) should be applied to solve the mixed potential integral equation (MPIE), thus, leading to a high computational cost especially in the case of overhead power lines whose length extends typically to several kilometers.

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In order to improve the computational efficiency while keeping the high accuracy of the solutions when modeling the high frequency electromagnetic field coupling to overhead lines, some researchers have proposed the so called enhanced or full-wave TL method. Tkachenko *et al*. proposed a TL-like method to handle the high frequency electromagnetic field coupling to a finite length line with open circuit terminals above a perfectly conducting ground that is based on the perturbation theory [16], where the solutions can be obtained iteratively. Tkachenko *et al*. then proposed an asymptotic method to obtain the response of a finite length line terminated by arbitrary linear loads above a PEC ground in a semi-analytical way [17]. The singularity expansion method (SEM) was also adopted to obtain the analytical solution in time domain [18], which can be applied to the case when the finite length line is terminated on open circuit or short circuit terminations. Poljak *et al*. proposed a numerical method to handle the case of a finite line above a lossy ground with open circuit terminals [14].

Since the power lines are typically terminated by nonlinear loads for the sake of protection, it is important to analyze the response behavior and the protection effects by modeling the transmission line with the nonlinear loads. Therefore, to enhance the applicability of the model to real scenarios, this paper aims to propose a method to solve the high frequency electromagnetic field coupling to overhead lines with nonlinear loads above a lossy ground. In the proposed method, the asymptotic approach, characterized by a high computational efficiency, is adopted and extended to the case of a lossy line. The general solution along the asymptotic region in the lossy ground case is developed. First, the scattering coefficients and the reflection coefficients are fitted and calculated in the frequency domain. Then, the response along the entire line can be calculated by these obtained coefficients, allowing therefore a higher computational efficiency than classical full-wave solvers. Since the asymptotic method is based on a frequency domain approach, the time marching method which is a mixed frequency domain and time domain method is adopted to handle nonlinear loads. The proposed method can therefore accurately solve the problem of high frequency uniform plane wave (*e.g.* EMP) coupling to long overhead lines terminated with nonlinear loads above a lossy ground. Additionally, by using the electric field representation method [19]-[21], the proposed method can also be extended to the case nonuniform wave excitations (*e.g.* compact radiation systems in IEMI).

The remainder of this paper is organized as follows. Section II describes the basic concept of the proposed approach. Section III presents the results associated with several case studies to validate the proposed approach. Finally, Section IV presents a summary and general conclusions.

II. Basic Concept of the Proposed Approach

The problem to be investigated is shown in Fig. 1. We consider an overhead lossless conductor[1] of length L, radius r and height of h above a lossy ground, illuminated by an external electromagnetic field. The dielectric constant, electric conductivity and permeability for the air are ε_0, 0 and μ_0, respectively, and the ground is characterized by its electric parameters, namely ε_g, σ_g and μ_0. The incoming wave is a uniform plane wave which has a polarization angle α, azimuth angle θ and an elevation angle ψ. The terminal loads are Z_1 in the left end and Z_2 in the right end. Both of the loads are considered as lumped components connected in the vertical risers at a small height $\Delta/2$ above the ground.

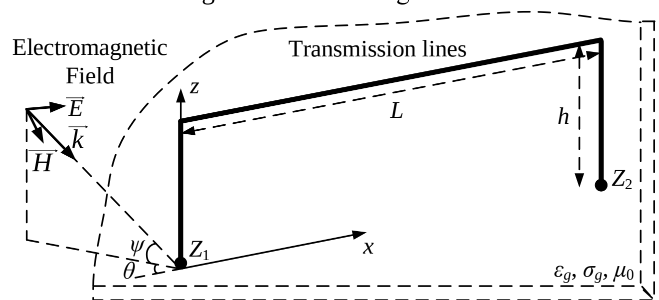

Fig. 1. Overhead transmission line excited by an incident plane wave.

A. Mixed Potential Integral Equation (MPIE) for the Line

Since the conductor is considered as lossless, the x-component of the total electric field along the line is zero.

$$E_x^e(x) + E_x^s(x) = 0 \quad (1)$$

where E_x^e is the x-component of excitation electric field, and E_x^s is the x-component of scattered electric field. The expression for E_x^e is

$$\begin{aligned} E_x^e(x, j\omega) &= E_0(j\omega)[\cos\alpha \sin\psi \cos\theta(e^{jkh\sin\psi} - R_v e^{-jkh\sin\psi}) \\ &\quad + \sin\alpha \sin\theta(e^{jkh\sin\psi} + R_h e^{-jkh\sin\psi})]e^{-jkx\cos\psi\cos\theta} \\ &= E(j\omega)e^{-jk_1 x} \end{aligned} \quad (2)$$

where R_v and R_h are the Fresnel ground reflection coefficients for vertical and horizontal polarizations, respectively. k is the wave number in the free space that is ω/c. The scattered electric field can be expressed as

$$\vec{E}^s = -j\omega\vec{A} - \nabla\varphi \quad (3)$$

where A is the magnetic vector potential and φ is the electric scalar potential. Expression (3) can be written for the x component as

$$E_x^s(x) = -j\omega A_x(x) - \frac{\partial\varphi(x)}{\partial x} \quad (4)$$

where

$$A_x(x) = \frac{\mu_0}{4\pi}\int_0^L I(x')g(x,x')dx' \quad (5)$$

$$\varphi(x) = \frac{1}{4\pi\varepsilon}\int_0^L q(x')g(x,x')dx' \quad (6)$$

where $q(x)$ is the charge distribution along the line, $I(x')$ denotes the induced current along the line, and $g(x, x')$ is the Green's function, whose expression can be found in [14].

The relationship of the charge distribution and the current can be written as

[1] The conductor losses are negligible in typical overhead power lines [F. Rachidi, C.A. Nucci, M. Ianoz, C. Mazzetti, "Influence of a lossy ground on lightning-induced voltages on overhead lines", *IEEE Trans. on Electromagnetic Compatibility*, Vol. 38, No. 3, August 1996.].

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$$q = -\frac{1}{j\omega}\frac{dI}{dx} \tag{7}$$

Inserting (7) into (6), we obtain

$$\varphi(x) + \frac{1}{4\pi\varepsilon j\omega}\int_0^L \frac{\partial I(x')}{\partial x'} g(x,x')dx' = 0 \tag{8}$$

And inserting (1) and (5) into (4), we obtain

$$\frac{\partial\varphi(x)}{\partial x} + j\omega\frac{\mu_0}{4\pi}\int_0^L I(x')g(x,x')dx' = E_x^e(x) \tag{9}$$

Expressions (8) and (9) form the mixed potential integral equations that describe the induced response along the line. It is to be noted that expressions (8) and (9) are derived considering a finite line with open circuit terminals. For the case of arbitrary line terminations, the exact solution of the induced current can be determined by the solution of the complicated Pocklington's equation [10], [17]. Moreover, when the ground has a finite conductivity, the resulting equations would be much more complicated [14], [22]. To obtain the exact solutions using these equations, numerical full-wave methods (*e.g.*, MoM) have to be applied, which are computationally inefficient.

To simplify the problem, the transmission line (TL) theory has been used. For a perfectly-conducting ground, and when the relationship between the line cross section and the incoming wave meets the TL assumptions, namely when kh<<1 and L>>h, the following integral can be approximated as [ref to Tkachenko *et al*]

$$\int_0^h I(x')g(x,x')dx' \approx I(x)\int_0^L g(x,x')dx' \approx 2I(x)\ln\frac{2h}{r} \tag{10}$$

With the approximation of (10) and considering of the definitions of the electric scalar potential φ and the scattered voltage V^s, equations (8) and (9) will reduce to

$$\frac{dI(x')}{dx'} + \frac{4\pi\varepsilon j\omega}{2\ln\frac{2h}{r}}V^s(x) = \frac{dI(x')}{dx'} + j\omega C' V^s(x) = 0 \tag{11}$$

$$\frac{dV^s(x)}{dx} + \frac{j\omega\mu_0 2\ln\frac{2h}{r}}{4\pi}I(x) = \frac{dV^s(x)}{dx} + j\omega L' I(x) = E_x^e(x) \tag{12}$$

Equations (11) and (12) are the field-to-transmission line coupling equation with the TL assumption in the Agrawal *et al*. form [12].

For a lossy ground case, equations (11) and (12) become

$$\frac{dI(x')}{dx'} + Y' V^s(x) = 0 \tag{13}$$

$$\frac{dV^s(x)}{dx} + Z' I(x) = E_x^e(x) \tag{14}$$

where Y' and Z' are the per-unit-length (p.u.l.) transverse admittance and longitudinal impedance of the line [10], respectively.

$$Y' = \frac{j\omega C' Y_g'}{j\omega C' + Y_g'}, \qquad Z' = j\omega L' + Z_g' \tag{15}$$

The Sunde [23] approximation can be used to evaluate the ground impedance Z_g'

$$Z_g' \approx \frac{j\omega\mu_0}{2\pi}\ln\frac{1+\gamma_g h}{\gamma_g h}, \qquad Y_g' \approx \frac{\gamma_g^2}{Z_g'}$$
$$\gamma_g = \sqrt{j\omega\mu_0(\sigma_g + j\omega\varepsilon_g)} \tag{16}$$

B. Asymptotic Method and Its Extension to the Case of a Lossy Ground

When we consider a uniform plane wave coupling to an infinitely-long lossless conductor above a lossy ground, with the thin wire assumption (radius of the wire much smaller than the wavelength and the height of line), the current response along the line can be expressed analytically as [24]

$$I_{\text{inf}}(x, j\omega) = \frac{E_x^e(j\omega)}{Z^e(j\omega)}e^{-jk_1x} = I_0(j\omega)e^{-jk_1x} \tag{17}$$

where Z^e and I_0 are the external impedance of the line and the current coefficient, respectively, which both depend on the line geometry and the frequency. Since I_0 is independent of the location x, the current response has an exponential distribution along the line.

When the line has a finite length, the problem can be solved iteratively and analytically using the perturbation theory [16]. To obtain the analytical formulation in time domain, the singularity expansion method can be applied [18]. This method adopts an iterative approach and is suitable for the more general case of terminals with arbitrary geometries and arbitrary linear loads. Moreover, to pursue a high computational efficiency, this problem can also be solved semi-analytically using the asymptotic method [17]. However, the above-mentioned methods are all based on the perfectly-conducting ground assumption.

In this section, the asymptotic method is extended to the case of a lossy ground. It can be seen from (17) that the current response is expressed as an exponential function of x (along the infinite line). When the line is terminated on a load along a vertical riser, the solution becomes much more complicated, especially in the regions near the terminals. However, in the central part of the line, far enough from both ends, the high-frequency effects generated from the discontinuities would decrease very fast, so that this region can be treated asymptotically as an infinite line [17]. In this way, the entire line can be separated into three regions, as shown in Fig. 2. Region I is the region near the left terminal which contains the left terminal riser and a portion of the horizontal line, $0\le x<l_b-h$, Region III is the region in the right terminal which contains the right terminal riser and a portion of the horizontal line, $L-l_b+h<x\le L$. Region II is the asymptotic region along the line, $l_b-h\le x\le L-l_b+h$. l_b is total length of the terminal region which depends on the height h and the frequency. Typically, a value of l_b of about twice of the height can be adopted [17].

In the asymptotic region, the induced current can be expressed as

$$I(x, j\omega) = I_0(j\omega)e^{-jk_1x} + F_1(x, j\omega) + F_2(x, j\omega) \tag{18}$$

where F_1 and F_2, are unknown functions to be determined.

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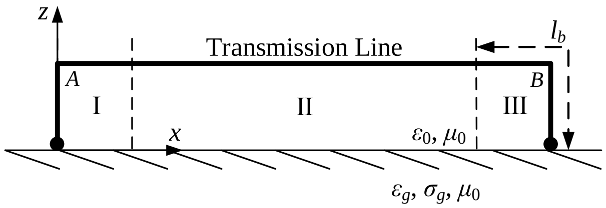

Fig. 2. The three regions along the transmission line (from [17]).

The first term in expression (18) corresponds to the response generated by the exciting field along an infinitely-long line that with the same geometry (height, radius and ground electrical parameters), and which can be obtained by using equation (17). The last two terms F_1 and F_2 in expression (18) represent forward and backward waves propagating from the terminal regions generated as a result of scattering and reflections. These two terms should have the same form as the solution for the same line excited by lumped voltage sources located at line ends. The solutions for the current along a conductor above a lossy ground have been presented in different forms (e.g., [25-29]). These methods involve the solution of the *Sommerfeld* integrals, which are computationally inefficient especially in the case of multiple-frequency simulation. Here we use a simplified approach which is described in what follows.

For the case of a perfectly-conducting ground, the current along the asymptotic region can be expressed as [17]

$$I_P(x, j\omega) = I_{P,0}(j\omega)e^{-jk\cos\psi x} + I_{P,1}(j\omega)e^{-jkx} + I_{P,2}(j\omega)e^{jkx} \quad (19)$$

In order to take into account the presence of a lossy ground, we use the approximate approach in which the functions F_1 and F_2 are still expressed with exponential functions with x as in the case of lossless ground, with the propagation constant evaluated by using the TL theory taking into account the lossy ground. In this way, the current solution along the asymptotic region can be expressed by

$$I(x, j\omega) = I_0(j\omega)e^{-jk_1x} + I_1(j\omega)e^{-\gamma x} + I_2(j\omega)e^{\gamma x} \quad (20)$$

where γ is the line propagation constant is given by

$$\gamma = \sqrt{Z'Y'} \quad (21)$$

in which the p.u.l. parameter Z' and Y' are determined by using (15) and (16). Although these parameters and expressions are to be used within the TL theory, they can still provide accurate solutions for the current along the asymptotic region in a broad frequency range, as will be shown in Section III.

In order now to determine the expressions for I_0, I_1 and I_2, we follow the same approach adopted in [17], considering two semi-infinite lines. First, we consider a right semi-infinite length line, extending from x=0 to infinity, and having the same geometry as that in Fig. 2. When this line is illuminated by the exciting field, the induced current along the line can be expressed as

$$I_+^S(x) = \begin{cases} I_0 F_+^S(x) & 0 \le x \le l_b - h \\ I_0 e^{-jk_1x} + S_+ I_0 e^{-\gamma x} & x >> l_b - h \end{cases} \quad (22)$$

where S_+ is the scattering coefficient associated with the left terminal, F_+^S is the unknown function corresponding to the exact solution in the left terminal region.

Then, we consider that there is no exciting field and a hypothetical current I_2' flowing from right to left (homogeneous solution). The current along the line can be expressed as

$$I_+^R(x) = \begin{cases} I_2' F_+^R(x) & 0 \le x \le l_b - h \\ R_+ I_2' e^{-\gamma x} + I_2' e^{\gamma x} & x >> l_b - h \end{cases} \quad (23)$$

where R_+ is the reflection coefficient associated with the left terminal, F_+^R is an unknown function corresponding to the exact solution in the left terminal region.

The total solution of the right semi-infinite length line can be expressed as the sum of the above-mentioned two parts as

$$I_+(x) = I_+^S(x) + I_+^R(x) \quad (24)$$

Specifically, the total solution along the left terminal and the asymptotic region can be expressed as

$$\begin{aligned} I_{\text{left}+}(x) &= I_0 F_+^S(x) + I_2' F_+^R(x) \\ I_{\text{asy}+}(x) &= I_0 e^{-jk_1x} + (R_+ I_2' + S_+ I_0)e^{-\gamma x} + I_2' e^{\gamma x} \end{aligned} \quad (25)$$

In the same way, if we consider a left semi-infinite line along $-\infty < x \le L$ above a lossy ground, the solution for the total current in the right terminal region and in the asymptotic region can be expressed as

$$\begin{aligned} I_{\text{right}-}(x) &= I_0 e^{-jk_1L} F_-^S(x-L) + I_1' F_-^R(x-L) \\ I_{\text{asy}-}(x) &= I_0 e^{-jk_1x} + I_1' e^{\gamma L} e^{-\gamma x} + (R_- I_1' e^{-\gamma L} + S_- I_0 e^{-jk_1L-\gamma L})e^{\gamma x} \end{aligned} \quad (26)$$

where I_1' is the hypothetical current flowing from left to right. S_- and R_- are respectively the scattering and reflection coefficients associated with the right terminal, F_-^S and F_-^R are unknown functions corresponding to the exact solutions in the right terminal region. It is to be noted that the scattering and reflection coefficients are intrinsic coefficients of the line terminals which are independent on the length of line.

The expressions (25) and (26) are the solutions for the current associated with the two semi-infinite line. By imposing these two solutions along the asymptotic region to be equal, the expression for the unknown parameters I_1 and I_2 for a line length L can be formulated as

$$\begin{aligned} I_1(L) = I_1' e^{\gamma L} &= \frac{S_+ + S_- R_+ e^{-jk_1L-\gamma L}}{1 - R_+ R_- e^{-2\gamma L}} I_0 \\ I_2(L) = I_2' &= \frac{S_- e^{-jk_1L-\gamma L} + S_+ R_- e^{-2\gamma L}}{1 - R_+ R_- e^{-2\gamma L}} I_0 \end{aligned} \quad (27)$$

From (27), it can be seen that the expression for the current along the asymptotic region can be obtained as a function of the scattering and reflection coefficients at the two line ends. To determine these coefficients, we follows the same approach as in [17] where two short auxiliary lines with the same geometry except for the length are simulated by a full-wave solver (*e.g.* MoM). In the simulation, the current response along the entire line is calculated. If we denote L_1 as the length of the auxiliary line 1 and Δx the discretized space step along this auxiliary line, the current response along the asymptotic region can be expressed as

$$I(n\Delta x) = I_0 e^{-jk_1 n\Delta x} + I_1 e^{-\gamma n\Delta x} + I_2 e^{\gamma n\Delta x} \quad (28)$$

where

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$$n = M+1, M+2, ..., N-M$$
$$N = \frac{L_1}{\Delta x}, \qquad M = \frac{l_b}{\Delta x} \tag{29}$$

The coefficients I_0, I_1 and I_2 can be obtained easily from (29) by using a fitting method such as the least squares method [30]. Since the values of I_1 and I_2 depend on the line length, we denote the obtained coefficients as I_0, $I_1(L_1)$ and $I_2(L_2)$. Since there are four unknown coefficients, S_+, S_-, R_+ and R_-, that need to be determined, another auxiliary line with a different length L_2 is also simulated, obtaining I_0, $I_1(L_2)$ and $I_2(L_2)$ using the same fitting way. By using the two groups of the fitted coefficients, the unknown scattering and reflection coefficients can be straightforwardly determined by solving (27). In addition, since the scattering and the reflection coefficients are independent of the line length, when determining these coefficients, the two auxiliary lines can be very short to ensure a high computational efficiency.

With the obtained coefficients, the expressions for the current at both terminal regions can also be formulated. In the case of the auxiliary line of length L_1, according to (25), the solution in the left terminal region is

$$I_{\text{left}}^{L_1}(x) = I_0 F_+^S(x) + I_2(L_1) F_+^R(x) \tag{30}$$

In the case of the auxiliary line of length L_2, the solution in the left terminal region is

$$I_{\text{left}}^{L_2}(x) = I_0 F_+^S(x) + I_2(L_2) F_+^R(x) \tag{31}$$

Since the current solution along the left terminal at these two cases are already obtained by using a full-wave approach, the unknown functions F_+^S and F_+^R can be easily determined using (30) and (31). Thus, the current response along the left terminal region of a line with any length L can be calculated analytically as

$$I_{\text{left}}(x) = \frac{I_{\text{left}}^{L_2}(x) - I_{\text{left}}^{L_1}(x)}{I_2(L_2) - I_2(L_1)}(I_2(L) - I_2(L_2)) + I_{\text{left}}^{L_2}(x) \tag{32}$$

The expression for the current along the right terminal region can also be obtained in the same way.

$$I_{\text{right}}(x) = \frac{I_{\text{right}}^{L_2}(x+L_2-L)e^{jk_1(L_2-L)} - I_{\text{right}}^{L_1}(x+L_1-L)e^{jk_1(L_1-L)}}{I_1(L_2)e^{jk_1L_2-\gamma L_2} - I_1(L_1)e^{jk_1L_1-\gamma L_1}}$$
$$\times (I_1(L)e^{jk_1L-\gamma L} - I_1(L_2)e^{jk_1L_2-\gamma L_2}) + I_{\text{right}}^{L_2}(x+L_2-L)e^{jk_1(L_2-L)} \tag{33}$$

Hence, the expressions of the current response along the entire line is obtained. By using these expressions, the current response of any line that has the same geometry but with any different length can be calculated analytically.

It is worth mentioning that the incoming wave was assumed to be a uniform plane wave where the exciting horizontal electric field along the line is an exponential function with x. It can be seen from (20) that the first term of the general solution along the asymptotic region corresponds to the solution of an infinitely-long line excited by the exciting field, which has the same form of exponential function as that of the horizontal exciting electric field along the line (2). Based on this characteristic, the proposed method can also be extended into the case that the incoming wave is a nonuniform wave, by fitting the horizontal electric field along the line with exponential functions using the electric field representation technique [19]-[21]. In doing so, the proposed method can be applied to the problem of IEMI excitation of overhead power lines. Additionally, this method can also be conveniently extended to the case of a multi-conductor transmission line [31]-[33].

C. Asymptotic Method for the Case of a Lumped Voltage Source Excitation

The problem of a line excited by a lumped voltage source located at an arbitrary location has been thoroughly discussed [34]-[36]. In this paper, we present a solution to solve this problem based on the asymptotic method.

Assume there are two lumped voltage sources V_1 and V_2 which are located at the two line ends. Since there is no exciting field, the general solution along the asymptotic region becomes

$$I(x, j\omega) = I_1(j\omega)e^{-\gamma x} + I_2(j\omega)e^{\gamma x} \tag{34}$$

Similar to the case when the exciting source is an incoming wave, two semi-infinite lines are considered. The solution along the right semi-infinite line excited by a lumped source located at the left end can be expressed as

$$I_+^S(x) = \begin{cases} F_+^P(x) & 0 \le x \le l_b - h \\ P_+ e^{-\gamma x} & x >> l_b - h \end{cases} \tag{35}$$

where P_+ is a coefficient that describes the current solution along the asymptotic region as a result of a lumped source excitation at the left terminal. F_+^P is the unknown function corresponding to the exact solution in the left terminal region excited by a lumped voltage source.

Assuming there is no lumped voltage source at the left end and a hypothetical current I_2' propagating from right to left (homogeneous solution), the current can be expressed as

$$I_+^R(x) = \begin{cases} I_2' F_+^R(x) & 0 \le x \le l_b - h \\ R_+ I_2' e^{-\gamma x} + I_2' e^{\gamma x} & x >> l_b - h \end{cases} \tag{36}$$

The total solution for the right semi-infinite line can be obtained as the sum of the above-mentioned two parts as

$$I_{\text{left}+}(x) = F_+^P(x) + I_2' F_+^R(x)$$
$$I_{\text{asy}+}(x) = (R_+ I_2' + P_+)e^{-\gamma x} + I_2' e^{\gamma x} \tag{37}$$

In the same way, when a left semi-infinite line is considered, the total current solution along the line can be expressed as

$$I_{\text{right}-}(x) = F_-^P(x-L) + I_1' F_-^R(x-L)$$
$$I_{\text{asy}-}(x) = I_1' e^{\gamma L} e^{-\gamma x} + (R_- I_1' e^{-\gamma L} + P_- e^{-\gamma L})e^{\gamma x} \tag{38}$$

where P_- is the coefficient that describes the current solution along the asymptotic region as a result of a lumped source excitation at the right terminal, and F_-^P is the unknown function corresponding to the exact solution in the right terminal region, excited by a lumped voltage source at the right terminal.

By imposing these two solutions along the asymptotic region to be equal, the expressions for the unknown parameters I_1 and I_2 for line of length L can be formulated as

$$I_1(L) = \frac{P_+ + P_- R_+ e^{-\gamma L}}{1 - R_+ R_- e^{-2\gamma L}}, \qquad I_2(L) = \frac{P_- e^{-\gamma L} + P_+ R_- e^{-2\gamma L}}{1 - R_+ R_- e^{-2\gamma L}} \tag{39}$$

A similar procedure as in the case of a field excitation can be adopted. Two short auxiliary lines of length L_1 and L_2 excited by the two lumped sources are simulated by a full-wave solver. The coefficients $I_1(L_1)$, $I_1(L_2)$, $I_2(L_1)$ and $I_2(L_2)$ can be obtained by the least squares method. And then the coefficients P_+, P_-, R_+ and R_- can be calculated by using (39). The expressions for the current in the two terminal regions can be formulated as

$$\begin{aligned} I_{\text{left}}(x) &= \frac{I_{\text{left}}^{L_2}(x) - I_{\text{left}}^{L_1}(x)}{I_2(L_2) - I_2(L_1)}(I_2(L) - I_2(L_2)) + I_{\text{left}}^{L_2}(x) \\ I_{\text{right}}(x) &= \frac{I_{\text{right}}^{L_2}(x+L_2-L) - I_{\text{right}}^{L_1}(x+L_1-L)}{I_1(L_2)e^{-\gamma L_2} - I_1(L_1)e^{-\gamma L_1}} \\ &\times (I_1(L)e^{-\gamma L} - I_1(L_2)e^{-\gamma L_2}) + I_{\text{right}}^{L_2}(x+L_2-L) \end{aligned} \quad (40)$$

Specifically, if there is only one lumped voltage source, the current solution can also be determined in a similar way. Assuming that there is a lumped voltage source V located at the right end, the coefficients F_+^P and P_+ would both reduce to zero. Since the other coefficients R_+, R_- and P_- are all independent of the lumped source at the left end, they can be calculated assuming that the line is excited by two identical lumped voltage sources V located at the both ends of the line. The expressions of the coefficients I_1 and I_2 will be in this case

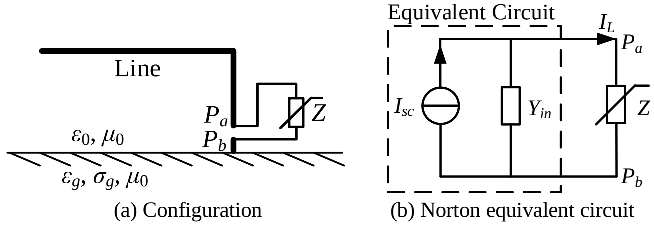
$$I_1(L) = \frac{P_- R_+ e^{-\gamma L}}{1 - R_+ R_- e^{-2\gamma L}}, \quad I_2(L) = \frac{P_- e^{-\gamma L}}{1 - R_+ R_- e^{-2\gamma L}} \quad (41)$$

And the current in the terminal regions can be expressed as

$$\begin{aligned} I_{\text{left}}(x) &= \frac{I_{\text{left}}^{L_2}(x) - I_{\text{left}}^{L_1}(x)}{I_2(L_2) - I_2(L_1)} I_2(L) \\ I_{\text{right}}(x) &= \frac{I_{\text{right}}^{L_2}(x+L_2-L) - I_{\text{right}}^{L_1}(x+L_1-L)}{I_1(L_2)e^{-\gamma L_2} - I_1(L_1)e^{-\gamma L_1}} \\ &\times (I_1(L)e^{-\gamma L} - I_1(L_2)e^{-\gamma L_2}) + I_{\text{right}}^{L_2}(x+L_2-L) \end{aligned} \quad (42)$$

D. Nonlinear Load

The above-mentioned procedure is based on a frequency-domain approach and, therefore, cannot be directly applied to the case involving nonlinear loads. Methods allowing to take into account nonlinear loads within a frequency domain analysis have been discussed in the literature (e.g., [37]-[38]). This paper adopts a mixed frequency-time domain method that can be integrated into the proposed asymptotic method to solve the problem of a long overhead line terminated with a nonlinear load in an efficient manner. Assume that there is a lumped nonlinear load between two points P_a and P_b along the vertical riser of a terminal, as illustrated in Fig. 3(a). In this case, the Norton equivalent circuit between the two points P_a and P_b can be determined (see Fig. 3(b)), in which I_{sc} is the equivalent Norton source which is equal to the current at the same point for a short-circuit termination (Z = 0), and Y_{in} is the equivalent admittance of the circuit.



(a) Configuration (b) Norton equivalent circuit

Fig. 3. The (a) configuration and the (b) Norton equivalent circuit of the nonlinear load terminated.

The relationship of the current in Fig. 3(b) can be simply expressed as

$$I_{sc}(j\omega) = I_L(j\omega) + Y_{in}(j\omega)U_L(j\omega) \quad (43)$$

The relationship between the voltage and current on the load in time domain reads

$$u_L(t) = F[i_L(t)] \quad (44)$$

where F is the U-I characteristic of the nonlinear load.

The time domain formulation of (43) can be expressed as

$$\begin{aligned} i_{sc}(t) &= i_L(t) + y_{in}(t) * F[i_L(t)] \\ &= i_L(t) + \int_{-\infty}^{t} y_{in}(t-t')F[i_L(t')]dt' \end{aligned} \quad (45)$$

Equation (45) can be re-arranged into the following form, in which Δt is chosen to be sufficiently small

$$\begin{aligned} i_{sc}(t) &= i_L(t) + \int_{-\infty}^{t-\Delta t} y_{in}(t-t')F[i_L(t')]dt' + \int_{t-\Delta t}^{t} y_{in}(t-t')F[i_L(t')]dt' \\ &\approx i_L(t) + \int_{-\infty}^{t-\Delta t} y_{in}(t-t')F[i_L(t')]dt' + y_{in}(0)F[i_L(t)] \end{aligned} \quad (46)$$

Since the starting time of the exciting field is t=0, (46) can be written as

$$i_L(t) + y_{in}(0)F[i_L(t)] = i_{sc}(t) - \int_{0}^{t-\Delta t} y_{in}(t-t')F[i_L(t')]dt' \quad (47)$$

It can be seen from (47) that if the U-I characteristic of the nonlinear load is determined, the current solution i_L in time domain can be solved step by step using the time marching method.

To solve the current response of the line which is terminated by the nonlinear load in time domain, the key step is to determine the Norton equivalent circuit parameters, namely i_{sc} and y_{in}. In this paper, these two parameters are solved by the proposed asymptotic method in the frequency domain, and then transformed into time domain by using the IFFT method.

III. Numerical Validation of the Proposed Algorithm

In this section, four numerical examples are considered to assess the proposed algorithm.

A. Example 1: General Solution for the Current along the Asymptotic Region for the Case of a Lossy Ground

In the first example, the assumption that the general solution of the current response along the asymptotic region in the lossy ground case could be approximately expressed by the exponential functions, and the propagation constant γ could be evaluated by the TL theory is assessed. To this aim, we consider three cases which include both lumped and external field excitations. Since this example only aims at validating the general solution, the current along the horizontal part of the line (x=0~L) is entirely calculated by the general solution.

In the first case, a 100-m long, 10-m high overhead conductor of 1 mm diameter above a lossy ground is considered. The conductivity and the relative dielectric constant of the ground are assumed to be 0.01 S/m and 10, respectively. The terminal loads at both ends are 50 Ω. The line is excited by two 1-V lumped voltage sources located at both terminals along the

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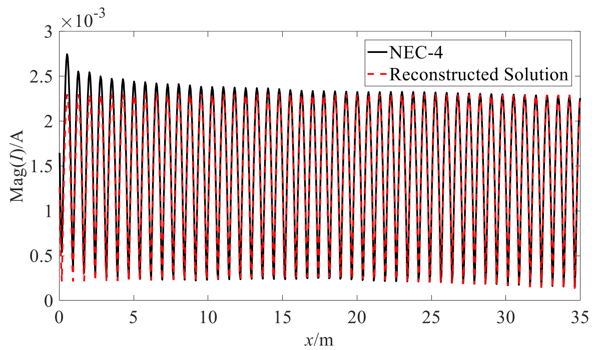
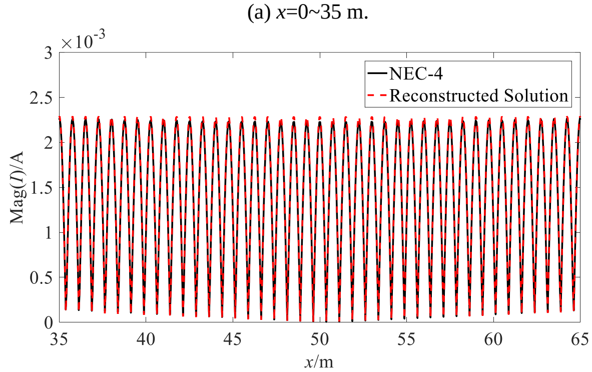
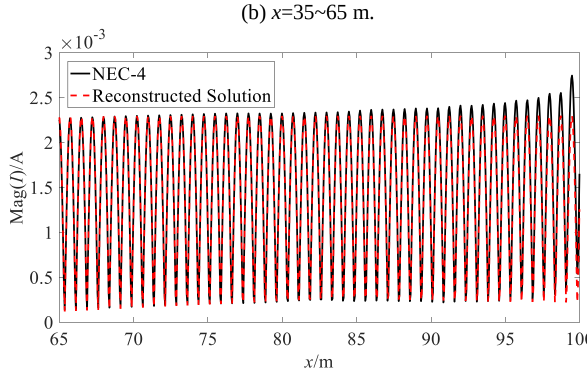
horizontal line (points A and B in Fig. 2). The frequency of the lumped voltage sources is 200 MHz. It should be noted that in this case the wavelength is 1.5 m which is much smaller than the height of the line, thus the classical TL approximation is not valid for this case. The current response along the line is simulated by the Numerical Electromagnetics Code (NEC-4) [39] which is a full-wave solver based on the MoM. Since there is no exciting field, the general solution along the asymptotic region is assumed given by (34). The current solutions along the whole asymptotic region is evaluated and then the unknown coefficients I_1 and I_2 are fitted by using the least squares method. The bound of the asymptotic region l_b is set to 20 m. When the unknown coefficients I_1 and I_2 are obtained, the current solution along the line are reconstructed using these fitted coefficients. The comparison between the original solutions obtained using NEC-4 and the reconstructed solutions using the fitted coefficients are shown in Fig. 4. The fitted coefficients are shown in Table I. It can be seen that in the asymptotic region the reconstructed waveforms agrees well with the original NEC-4 solutions.



(a) x=0~35 m.



(b) x=35~65 m.



(c) x=65~100 m.

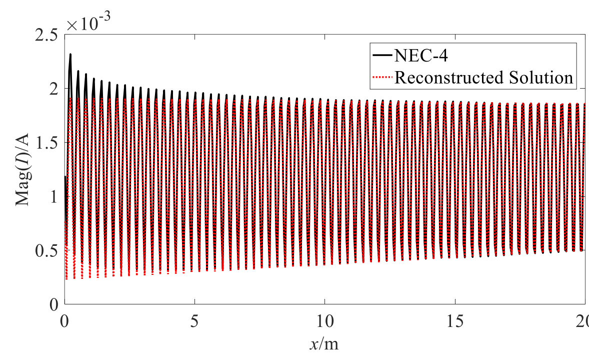
Fig. 4. Comparison between the original solutions obtained using NEC-4 and the reconstructed solution. The line is formed by a 100-m long, 10-m high, 1-mm diameter conductor above a ground of conductivity 0.01 S/m. Lumped source excitation with a frequency f = 200 MHz.

In the second case, the length of the line is 40 m, the conductivity of the ground is 10^{-5} S/m and the frequency of the lumped voltage sources is 500 MHz, while other parameters are the same as those in the first case. This case aims to validate the general solution in a more 'difficult' case characterized by a very poor ground conductivity and for a higher frequency. Note that such a low conductivity ground is not typical and is only used for the numerical validation of the proposed method.

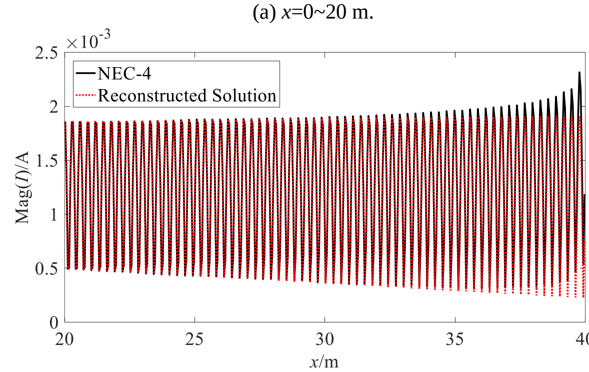
In the same way, the current solution was calculated first using NEC-4, and the unknown coefficients were fitted by the least squares method, and then the current solution along the line was reconstructed by the obtained fitted coefficients. The comparison between the original NEC-4 solution and the reconstructed one are shown in Fig. 5. It can be seen that the reconstructed waveforms agree well with the NEC-4 solution in the asymptotic region.

TABLE I
THE FITTED COEFFICIENTS

I_1	I_2
11.65-4.21i	1.47+10.52i



(a) x=0~20 m.



(b) x=20~40 m.

Fig. 5. Comparison between the original solutions obtained using NEC-4 and the reconstructed solution using the fitted coefficients. The line is formed by a 40-m long, 10-m high, 1-mm diameter conductor above a ground of conductivity 10^{-5} S/m. Lumped source excitation with a frequency f = 500 MHz.

In the third case, the line is excited by an impinging field. The parameters of the line are the same as those in the second case, while the excitation is a plane wave with the amplitude of 1 V/m; the polarization angle, the azimuth angle and the elevation angle are 0°, 0° and 45°, respectively. The current response is calculated using both NEC-4 and the classical TL method. Then the reconstructed solution is also calculated by the fitted

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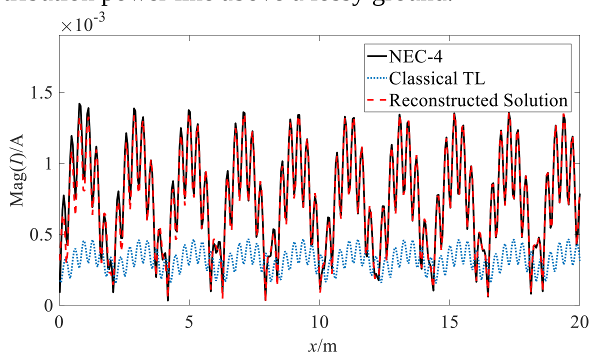
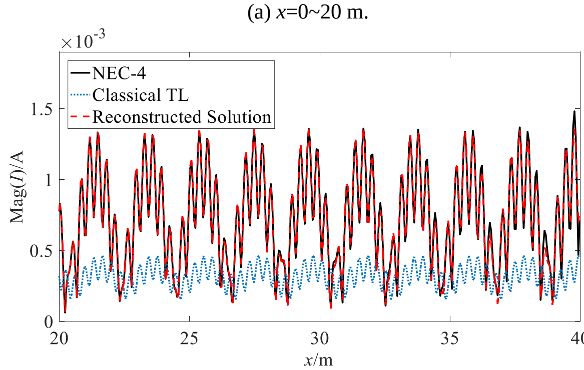
coefficients obtained from the solutions of NEC-4 using (29). The comparison between the induced currents calculated by NEC-4, classical TL method, and the reconstructed solution are shown in Fig. 6. It can be seen that the reconstructed solution agrees well with the solution obtained from the NEC-4 in the asymptotic region, whereas the solution obtained from the classical TL method is inaccurate.

These cases prove that the general solution in the asymptotic region expressed by (20) is an excellent approximation in the case of lumped and external field excitations. Moreover, the propagation constant estimated using the TL theory could provide the solutions with a high accuracy along the asymptotic region. The proposed approach is even reasonable for a 'difficult' case characterized by an unusually poor ground conductivity and for a broad frequency range covering a spectrum which extends to frequencies much beyond those of a typical EMP or even some of the IEMI sources. Above all, this example shows that the proposed approach is sufficiently reasonable for the typical case of EMP coupling to a distribution power line above a lossy ground.



(a) x=0~20 m.



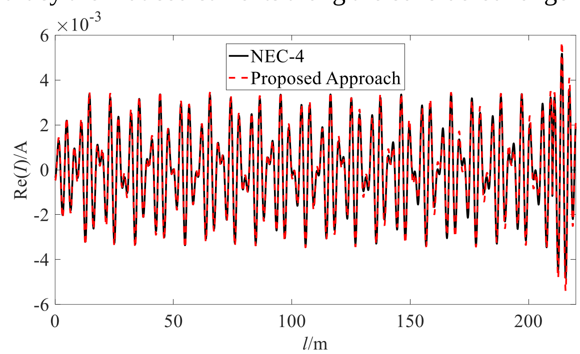
(b) x=20~40 m.

Fig. 6. Comparison between the original solutions obtained using NEC-4, the solution obtained using the classical TL theory and the reconstructed solution using the fitted coefficients. The line is formed by a 40-m long, 10-m high, 1-mm diameter conductor above a ground of conductivity 10^{-5} S/m. The line is excited by a 1-V/m, 500-MHz plane wave. The polarization angle, the azimuth angle and the elevation angle are 0°, 0° and 45°, respectively.

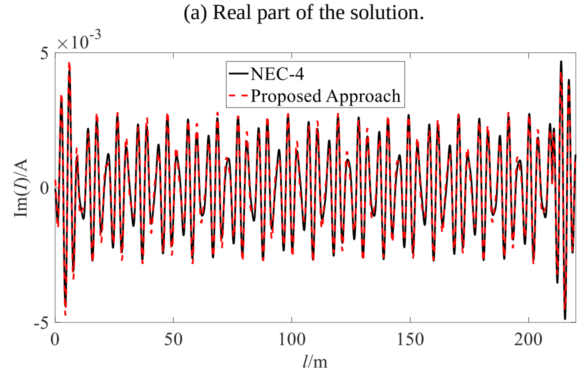
B. *Example 2: Long Overhead Line above a Lossy Ground Illuminated by an Exciting Field*

In the second example, the proposed method is entirely validated for several cases. Therefore, the current in the terminal regions are also taken into account.

In the first case, we consider a 200-m long, 10-m high, 1-mm diameter conductor above a lossy ground. The ground conductivity and relative permittivity are 0.01 S/m and 10, respectively. The terminal loads at both ends are 50 Ω. The line is excited by a plane wave with an amplitude of 1 V; the polarization angle, the azimuth angle and the elevation angle are 0°, 0° and 45°, respectively. Since both vertical risers are taken into account, we use coordinate l that describes the location along the line including both risers. In this way, the coordinates of the load Z_1 which is at x=0, y=Δ/2 corresponds to l=Δ/2, and that of the load Z_2 which is at x=L, y=Δ/2 corresponds to l=L+2h-Δ/2. The line response was evaluated using NEC-4 and the proposed method. The current responses along the entire line at the frequency of 100 MHz are shown in Fig. 7. It can be seen that the solution along the entire line calculated using the proposed approach agrees remarkably well with that the results obtained using NEC-4. The lengths of the two auxiliary lines used in the example were 60 m and 61.5 m, respectively. Figures 8-11 present similar comparisons but considering longer lines (400 m and 1000 m). It can be seen that the proposed method allows to reproduce with a reasonable accuracy the induced currents along the considered longer lines.



(a) Real part of the solution.



(b) Imaginary part of the solution.

Fig. 7. Comparison between the proposed approach and results obtained using NEC-4. Induced current on a 200-m long, 10-m high, 1-mm diameter conductor above a ground of conductivity 10^{-2} S/m and relative permittivity 10. The line is excited by a 1-V/m, 100-MHz plane wave. The polarization angle, the azimuth angle and the elevation angle are 0°, 0° and 45°, respectively.

To quantify further the validity range of the proposed method as a function of the frequency, the relative error for the induced current at the line end with respect to the NEC results are evaluated using the following expression

$$\varepsilon = \frac{\left\| I_n \right| - \left| I_a \right\|}{\max(\left| I \right|_n, \left| I_a \right|)} \tag{48}$$

where I_n is the magnitude of the current result obtained from NEC-4, and I_a is the magnitude of the current result obtained from the proposed method.

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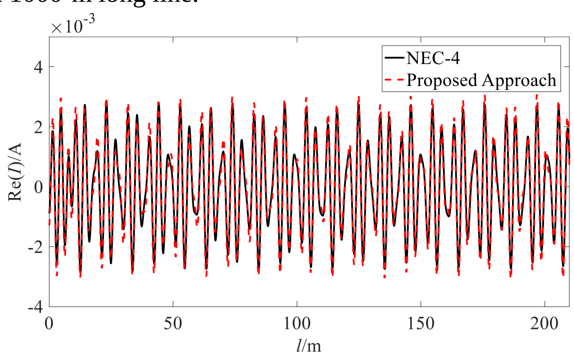
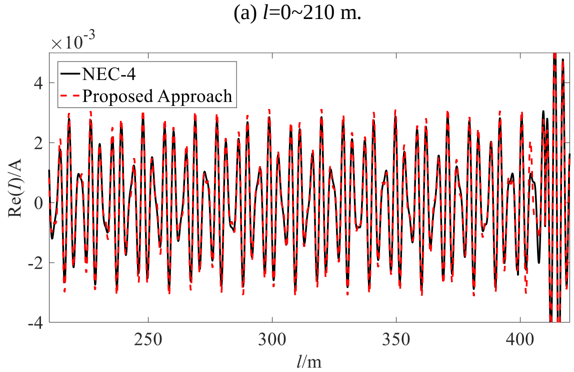
The relationship between the relative error and the frequency is shown in Fig. 12 for the case of the 1000-m long line. It can be seen from the figure that the relative error increases with the frequency. However, the results show that the relative error remains relatively low up to a frequency of about 250 MHz, which means that the proposed method is suitable for the simulation of HEMP coupling to overhead lines since significant frequency components of HEMP are within 100 MHz or so. To increase the accuracy for higher frequencies (*e.g.* >300 MHz), other more accurate formulas for the ground impedance (*e.g.*, [40]) can be used. This issue requires more in-depth investigations and will be the subject of future work.

The computation time of the proposed approach for the three considered line lengths are reported in Table II. In the same table, the same figures related to NEC-4 are also reported for comparison. In the calculation, the discretization step along the line was 0.1 m. The adopted software were NEC-4 and Matlab, which run on a PC with 3.2 GHz CPU and 16 GB RAM. It can be seen that the CPU cost of NEC-4 increases geometrically with the line length. When the length is 1000 m, about 1650 s is needed to calculate the results for a single frequency, which can become unaffordable when it is applied to solve a time-domain solution. At the same time, since the proposed method adopts a semi-analytical scheme in which numerical processes are only applied to two very short auxiliary lines, a much higher computational efficiency is obtained. It can be seen that only 3.2 s is needed when applying the proposed method in the case of a 1000-m long line.

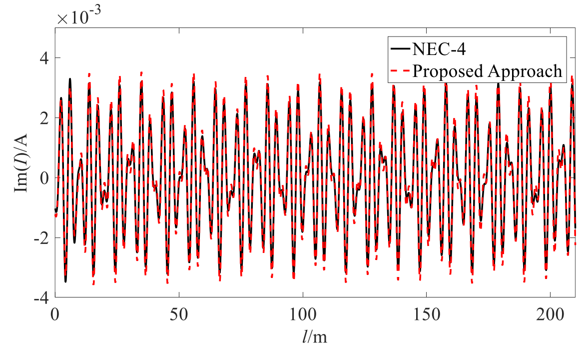


(a) *l*=0~210 m.

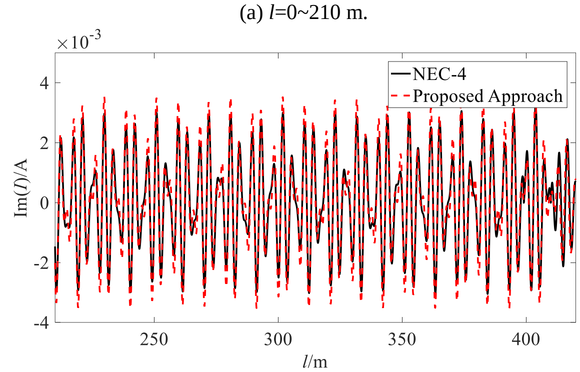


(b) *l*=210~420 m.

Fig. 8. Comparison between the real part solutions obtained using proposed approach and that using NEC-4. Same as Fig. 7 but with a line length of 400 m.

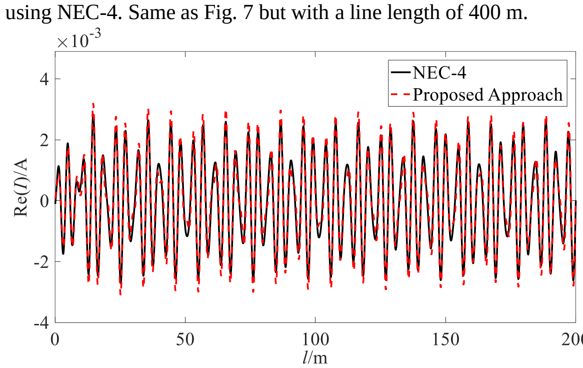


(a) *l*=0~210 m.

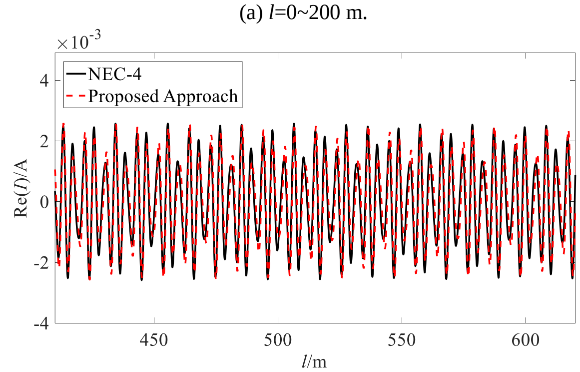


(b) *l*=210~420 m.

Fig. 9. Comparison between the imaginary part using proposed approach and that using NEC-4. Same as Fig. 7 but with a line length of 400 m.



(a) *l*=0~200 m.



(b) *l*=410~620 m.

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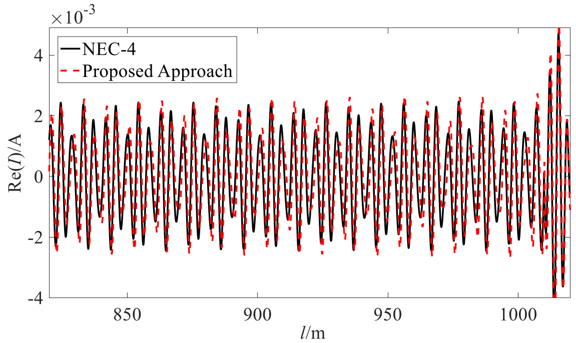
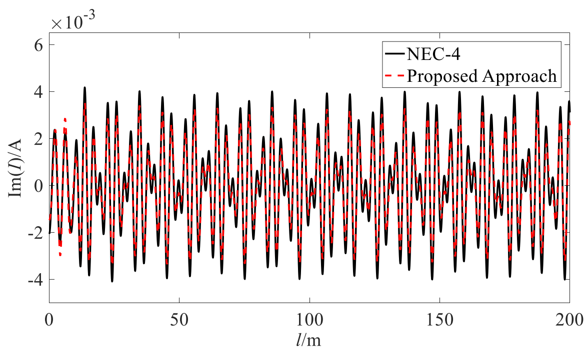
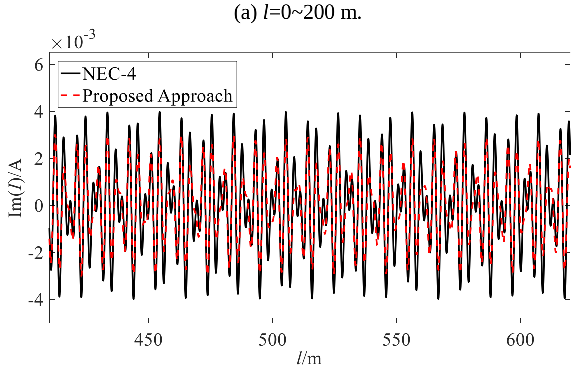


(c) l=820~1020 m.

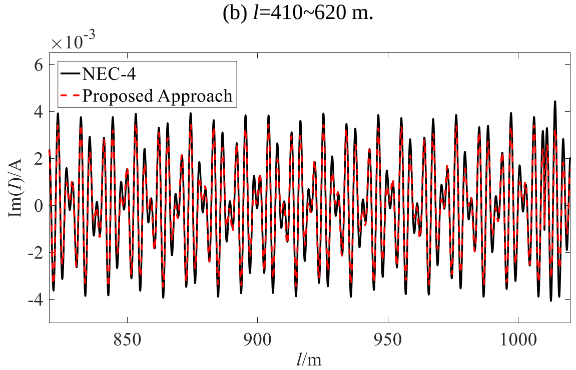
Fig. 10. Comparison between the real part solutions obtained using proposed approach and that using NEC-4. Same as Fig. 7 but with a line length of 1000 m.



(a) l=0~200 m.



(b) l=410~620 m.



(c) l=820~1020 m.

Fig. 11. Comparison between the imaginary part solutions obtained using proposed approach and that using NEC-4. Same as Fig. 7 but with a line length of 1000 m.

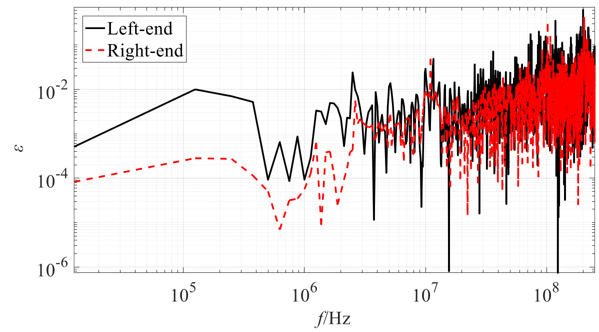


Fig. 12. Relationship between the relative error and the frequency with a line length of 1000 m.

TABLE II

THE CPU COST OF THE TWO METHODS (PROPOSED APPROACH VERSUS NEC-4) AS A FUNCTION OF LINE LENGTH WITH FREQUENCY OF 100 MHZ (UNIT: S)

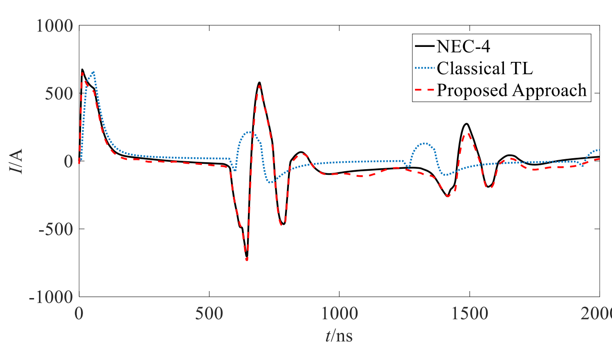
NEC-4			Proposed Approach		
200 m	400 m	1000 m	200 m	400 m	1000 m
19.96	123.29	1650.43	3.17	3.18	3.20

In order to investigate the solution in the time domain, the time-domain current solutions at the two terminal loads Z_1 and Z_2 are calculated. The IEC standard waveform is adopted as the waveform of the electric field of the incoming wave, which is defined in the IEC 61000-2-9 as

$$E(t) = E_0 k_0 (e^{-\alpha t} - e^{-\beta t}) \tag{49}$$

where E_0=50 kV/m, k_0=1.05, α=4×10^7 s^{-1}, β=6×10^8 s^{-1}.

The full-wave approach (NEC-4), the classical TL method and the proposed method are adopted to calculate the current response at the two loads Z_1 and Z_2 in the frequency domain, and then the time domain solutions are obtained by using the IFFT method. The comparison among the time domain solutions for the induced current at the loads Z_1 and Z_2 is shown in Fig. 13. It can be seen that the results obtained using the proposed approach agrees very well with those obtained from NEC-4. On the other hand, the results calculated using the classical TL deviate significantly from the full-wave results. Moreover, it can be seen clearly that the classical TL method results in an underestimation of both of the gradient of the rising edge and the amplitude of the current response, in agreement with the findings of [14].



(a)

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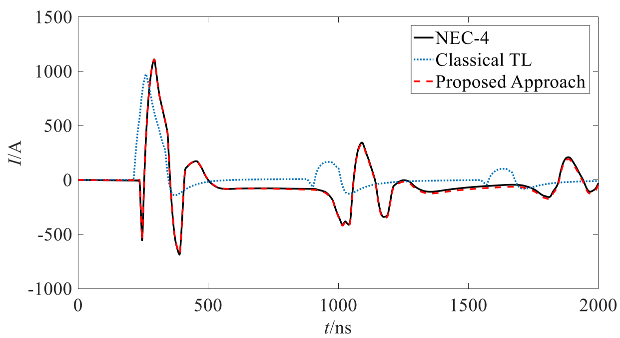
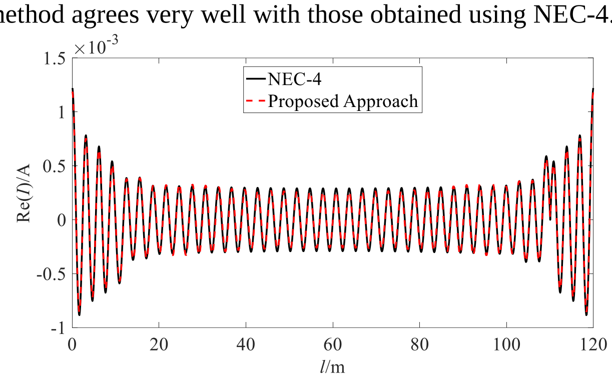


(b)

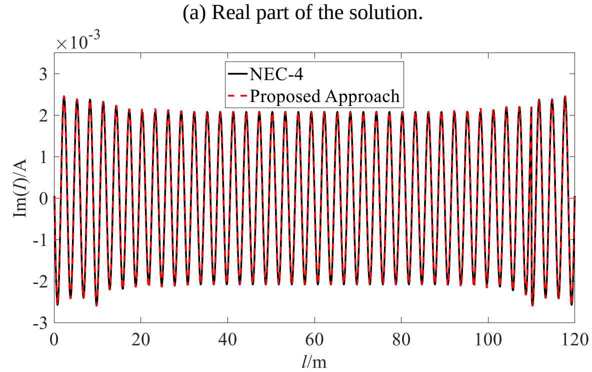
Fig. 13. Comparison between the proposed approach and results obtained using the classical TL approach and NEC-4. Induced current on a 1000-m long, 10-m high, 1-mm diameter conductor above a ground of conductivity 10^{-2} S/m and relative permittivity 10. The line is excited by an IEC 61000-2-9 plane wave plane wave. The polarization angle, the azimuth angle and the elevation angle are 0^o, 0^o and 45^o, respectively. (a) Induced current in the left-end impedance (b) Induced current in the right-end impedance.

C. *Example 3: Long Overhead Line above a Lossy Ground Excited by a Lumped Voltage Source*

We consider the case of a 100-m long, 10-m high, 1-mm diameter conductor above a lossy ground. The conductivity and the relative dielectric constant of the ground are assumed to be 0.01 S/m and 10, respectively. The terminal loads at both ends are 50 Ω. The line is excited by two 1-V, 100 MHz lumped voltage sources located at both terminal ends. The comparisons between the current solutions calculated using the proposed approach and those obtained using NEC-4 are shown in Fig. 14. It can be seen that the results calculated using the proposed method agrees very well with those obtained using NEC-4.



(a) Real part of the solution.



(b) Imaginary part of the solution.

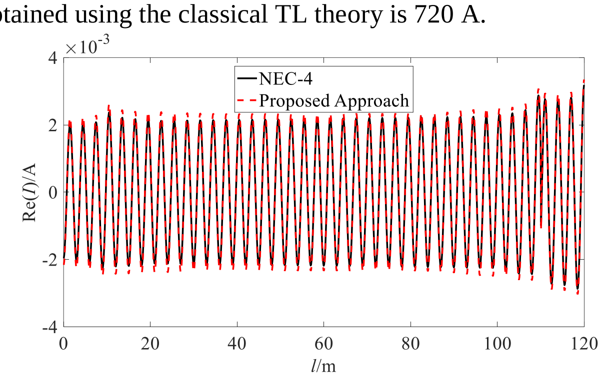
Fig. 14. Comparison between the proposed approach and results obtained using NEC-4. Induced current on a 100-m long, 10-m high, 1-mm diameter conductor above a ground of conductivity 10^{-2} S/m and relative permittivity 10. The line is excited by a two 1-V, 100-MHz lumped voltage sources at each end of the line.

In the second case, the geometry of the line is the same like that in the first case, while only one lumped voltage source which equal to 1V located at the right end of line. The comparison between the current solutions which calculated from the NEC-4 and proposed approach is shown in Figs. 15. It can be seen that the results calculated from the proposed method agrees well with that from the NEC-4.

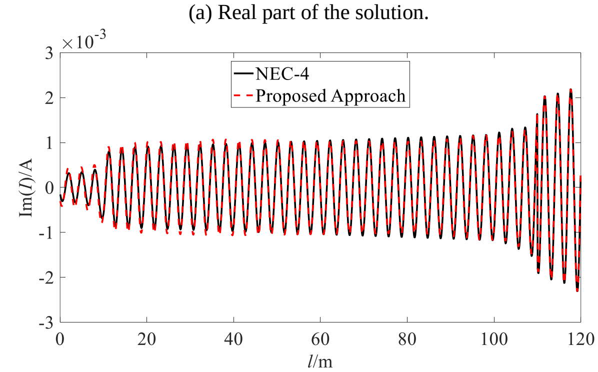
D. *Example 4: Long Overhead Line above a Lossy Ground Terminated with a Nonlinear Load*

We consider the same line configuration of Section III.B. The terminal load at the left end is 50 Ω, while that at the right end is formed by the parallel connection of a 50 Ω load with a nonlinear load. The *U-I* characteristic of the nonlinear load is shown in Fig. 16 and corresponds to the behavior of n typical voltage limiting equipment. The line is excited by the same IEC plane wave. According to the proposed method in Section II, the equivalent circuit source i_{sc} is calculated by using the asymptotic method when setting the nonlinear load as zero. The equivalent admittance y_{in} is calculated when the line is excited by a lumped voltage source located at the right terminal. The current response is calculated using the proposed method, as well as using the classical TL method. The calculated current across the nonlinear load and the 50 Ω load at the right terminal are shown in Fig. 17.

It can be seen from the Fig. 17 that the classical TL approach results in an inaccurate and underestimated induced current. In particular, the amplitude of the current through the nonlinear load calculated using the proposed method is 890 A while that obtained using the classical TL theory is 720 A.



(a) Real part of the solution.



(b) Imaginary part of the solution.

Fig. 15. Same as in Fig. 14 but with only one voltage source located at the line right-end.

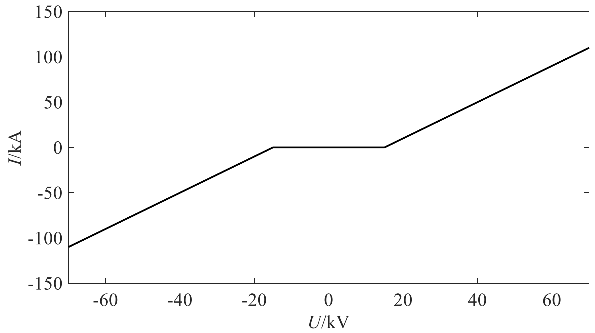
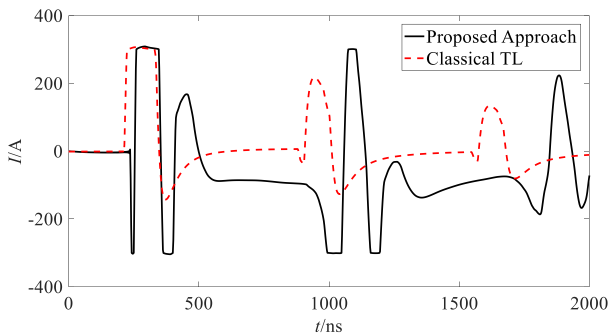
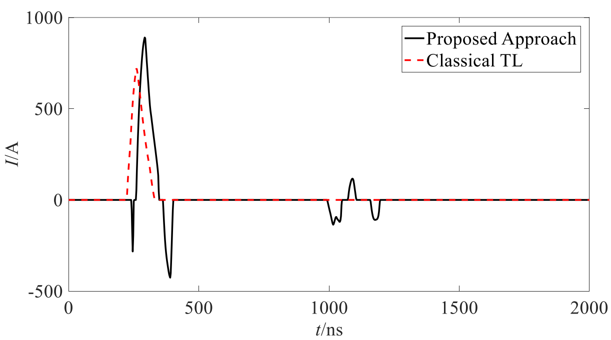


Fig. 16. *U-I* characteristic of the nonlinear load.



(a) Current through the 50 Ω load.



(b) Current through nonlinear load.

Fig. 17. Induced current in the right terminal (a) across the 50 Ω load and (b) the nonlinear load. 100-m long, 10-m high, 1-mm diameter conductor above a ground of conductivity 10^{-2} S/m and relative permittivity 10. The line is excited by an IEC 61000-2-9 plane wave. The polarization angle, the azimuth angle and the elevation angle are 0 º, 0 º and 45º, respectively

IV. Conclusion

This paper proposed an efficient method to model the high frequency electromagnetic field coupling to long overhead line terminated with nonlinear loads above a lossy ground. The cross-section of the line can be comparable to the wavelength of the incoming wave. In the proposed method, the asymptotic approach which is a semi-analytical method is adopted and extended to the case of a lossy ground. General solutions along the asymptotic region for the case of a lossy ground case are developed. The scattering and reflection coefficients of the line are fitted from the numerical results obtained by using a full wave solver to auxiliary lines with short line lengths. Once the coefficients are determined, the response of the entire line can be calculated analytically, resulting in a high computational efficiency compared to full wave solvers, especially for long lines. Since the asymptotic method is a frequency-domain approach, the time marching method which is a mixed frequency and time domain method is adopted to handle nonlinear loads.

The proposed method has been validated considering different numerical examples, and taking as reference full-wave simulations obtained by a full-wave solver based on the Method of Moments (NEC-4). The results showed that the proposed method can accurately and efficiently predict the response of a long lines to either lumped sources or external field excitation.

Future work will be devoted to investigate the numerical performance of the proposed method. The method will also be extended to the case of a multi-conductor transmission lines, inhomogeneous (multilayered) soil, and nonuniform excitation sources. Moreover, the use of more accurate expressions for the ground impedance instead of the Sunde's formula will also be investigated.

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