Linear and non-linear studies of spurious backward-wave instabilities in a smooth-wall gyrotron beam duct

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Abstract—Recently developed self-consistent models for beam-wave interaction simulations of a gyrotron oscillator are used to study spurious backward-wave instabilities in smooth-wall metallic beam ducts. Starting from a simple case with a basic geometry, the influence of the beam duct geometry and the magnetic field profile on the instabilities threshold and on the quality of the electron beam are studied.

One of the main point hindering high-power gyrotron operation for fusion application is the excitation of spurious instabilities in the beam duct part situated upstream from the resonant cavity. These instabilities, besides being dangerous for the gyrotron components, could possibly degrade the electron beam quality before it enters the cavity. Various solutions have been considered to avoid these instabilities. The solution used for example in Europe is to use a stacked-rings dielectric-loaded beam duct [1], however the complex asymmetric 3D geometry rules out self-consistent simulations and also leads to a challenging and costly manufacturing. Another solution is to use a smooth-wall dielectric loaded beam duct [2], [3]. In this case, the much simpler geometry, azimuthally symmetric, enables to use self-consistent models that are commonly used to simulate beam-wave interaction inside the gyrotron cavity.

Two self-consistent models are used in the present study. A linear code using a spectral approach [4] is used to calculate the self-excitation condition, expressed as a starting current condition. Unlike a time-evolution approach, the spectral approach allows to calculate all stable and unstable modes as shown on figure 1. The eigenvalues calculated for the transverse mode $TE_{25,3}$ in a smooth-wall beam duct geometry (cf. figure 2 with $\theta_1 = 5.1^\circ$) are shown in the complex growth-rate vs frequency plane. The nearly vertically aligned points represent different longitudinal modes for different beam currents. The red circle enlightens the first longitudinal eigenvalue to become unstable, thus giving the starting current for this mode and this operating point. In this example, a time evolution code would compute only the most unstable mode, it would not permit to follow each eigenvalues individually. In case the operating current is above the minimum starting current, a non-linear time-evolution code [5] will be used to evaluate the effect of potential spurious instabilities on the electron beam and its main characteristics, such as the velocity or energy distribution of the electrons at the beam duct output.

The smooth-wall metallic beam duct configurations studied here are shown in figure 2. The reference situation is shown

Fig. 1. Complex frequencies of the eigenvalues computed represented in the complex growth rate vs frequency plane for different values of the beam current. The transverse mode is the $TE_{25,3}$ and the beam-duct geometry is the one shown on figure 2 with $\theta_1 = 5.1$ (Parameters: $U_c = 75$ kV, $\alpha = 1.3$, $R_b = 12$ mm, $B_{max} = 4.7$ T).

with a continuous line. This reference geometry consists of a symmetric beam duct, formed by a constant radius central part and an uptaper at both extremities. The magnetic field profile is flat and a finite conductivity metallic boundary condition is considered for the ohmic losses. Progressively, the parameters related to the beam duct geometry (angle θ_1 and θ_2 on figure 2) and the magnetic field profile are changed towards a real smooth-wall beam duct. Under some assumptions, the losses due to the dielectric coating will be implemented by means of a surface impedance boundary condition.

Fig. 2. Beam duct geometry and magnetic field profile. The full line correspond to the symmetric geometry considered as the reference. The different dashed lines correspond to the different geometries considered in figure 3.

As an illustration of the effect of the geometry, the starting current calculated for the transverse mode $TE_{25,3}$ is shown on figure 3 for four different values of the angle θ_1 , for $\theta_2 = 2.3^\circ$, and for a range of magnetic field between 4.6 and 4.9 T. For this transverse mode, the starting current increases from few mA for the reference geometry up to 40 A for the largest angle ($\theta_1 = 8.5^\circ$). Increasing θ_2 leads also to a significant increase of the starting current of the mode. If one considers the operating beam current of the order of 40 A, this means that this mode would not be excited in this configuration. On the contrary, the starting current depends less strongly on the magnetic field profile. For simplicity, this illustration focused

Fig. 3. Starting current of the $TE_{25,3}$ calculated with the linear code [4] for the four different beam-duct geometries shown in figure 2 (Parameters: $U_c = 75$ kV, $\alpha = 1.3$, $R_b = 12$ mm, $\theta_2 = 2.3^\circ$).

on a single transverse mode. A complete study of spurious instabilities has to take into account all the potentially unstable transverse modes and the minimum starting current among all these modes has to be considered. An example of minimum starting current for the reference case $\theta_1 = 0^\circ$ is shown on figure 4. One notes that in the range of magnetic field between 4.5 and 4.8, the minimum starting current involves three different transverse modes.

Fig. 4. Starting current calculated for the reference situation shown in continuous line in figure 2 (Parameters: $U_c = 75$ kV, $\alpha = 1.3$, $R_b = 12$ mm, $B(z) = 4.7$ T).

For a slightly different situation, closer to a real smooth-wall beam duct, ($\theta_1 = \theta_2 = 8^\circ$, $\sigma_{\text{cond}} = 1.45 \cdot 10^7$ S/m), the electric field profile is shown for the transverse mode $TE_{17,4}$ for a current slightly exceeding its starting current ($I_b = 34.4$ A) (figure 5 a)). The dashed line correspond to the cold-cavity profile, the profile calculated without electron-beam. It is clear that the self-consistent profile is completely

different to the cold-cavity profile, as expected for backward-wave interactions. The electric field profile is mainly peaked at the narrowest part of the beam duct. On figure 5 b), the amplitude of the source term appearing in the wave equation of the model [4], is represented along the longitudinal axis. The location of the peak indicates that the main interaction between the wave and the electron beam takes place in the uptaper, close to the region where the electric field is peaked.

Fig. 5. a) Amplitude of the electric field profile for a beam current slightly exceeding the starting current (in blue, plain) and without electron beam (in green, dot-dashed) for the $TE_{17,4}$ transverse mode. b) Profile of the amplitude of the source term appearing in the wave equation in the linear model [4]. c) Profiles of the beam-duct and magnetic field profile. (Parameters: $U_c = 75$ kV, $\alpha = 1.3$, $R_b = 10$ mm, $\sigma_{\text{cond}} = 1.45 \cdot 10^7$ S/m, $B_{\text{max}} = 4.7$ T).

The preliminary results presented here indicate already that even though a relatively simple model is considered, in the sense that it is a linear model treating a single transverse mode, and that the spectral approach allows for efficient scan of transverse modes or parameters, the study of spurious instabilities in smooth-wall beam duct is intricate and a thorough investigation on all the transverse modes and different configurations has to be considered.

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