Essays on adaptation to rare disasters

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Abstract

This thesis aims to further investigate rare natural disasters and studies adaptation decisions under uncertainty by solving several computational economic models. The modeling of rare natural disasters depends on the treatment of catastrophic outcomes with a low probability. The impact of rare disasters on economic activities has been explored in the literature. This thesis is dedicated to answering a question: How do we optimally adapt or alleviate the risk of uncertain rare natural disasters?

Uncertainty is central to this dissertation. I overview the sources of uncertainty and present how these uncertainties are addressed in the literature of environmental economics. I demonstrate why rare natural disasters are worth studying and present the advantages that can be gained by investigating them. Subsequently, I present my choice of adaptation measures, spatial adaptation and adaptive capital stock, to better situate my study in the existing literature of adaptation. I investigate adaptation decisions by using an approach based on numerical methods.

I employ the existing computational general equilibrium model GENESwIS in Chapter 1. I propose a new simulation approach, the hazard myopia, for a better consideration of spatial adaptation for future high impact floods in Switzerland. The hazard myopic agent solves an intertemporal optimization problem by adopting his subjective belief of the risk of flooding. Capital stocks are recursively updated based on the actual damage. However, my simulation results contradict the real Swiss situation. To correctly handle rare but catastrophic events, one promising approach is to convert the model from deterministic to fully stochastic so that the precautionary saving is endogenized.

Why does an economic agent save for a less productive but risk-free capital or a non-productive but adaptive capital stock? Chapter 2 aims to address this question by solving dynamic stochastic equilibrium models. I present the optimal policy functions for spatial adaptation and adaptive capital stock in a stylized but reproducible setting. I quantitatively show that the certainty-equivalent deterministic model underestimates the risk of rare natural disasters.

Chapter 3 discusses the qualitative argument established by Schelling (1992, Section IV). I quantitatively demonstrate that the developing economy's best strategy to adapt to future disasters is to advance its economic development. A developed economy bolsters non-productive yet adaptive capital stock to prepare for future uncertainties.

Throughout Chapters 2 and 3, my implementations are massively parallelized on a high-end computation cluster to speed up the solving processes. The efficient usage of parallel computing is an emerging field in economics. This dissertation demonstrates the applicability of exploiting the massive power of modern computing in economics.

KEYWORDS: Environmental economics, uncertainty, adaptation, natural disasters and their management, computable general equilibrium models, dynamic stochastic general equilibrium models, adaptive sparse grids, parallel computing.

Résumé

Cette thèse vise à analyser les conséquences économiques des catastrophes naturelles rares et à étudier l'efficacité de mesures d'adaptations en cas d'incertitudes grâce à un modèle économique numérique. La modélisation des catastrophes naturelles dépend de comment sont traités les évènements à faible probabilité. Bien que les conséquences économiques des catastrophes naturelles aient été étudiées dans la littérature, une question centrale est toujours en suspens : Quelle est la manière optimale d'atténuer le risque et de s'adapter à ces évènements catastrophiques, rares et incertains ? Cette thèse est dédiée à répondre à cette question.

La modélisation des incertitudes est au coeur de cette dissertation. Je passe en revue les principales sources d'incertitude en économie de l'environnement et j'explique comment ces incertitudes sont traitées dans la littérature. Je montre pourquoi l'étude des catastrophes naturelles rares est pertinente et je détaille les avantages que l'on peut tirer d'une telle analyse.

De plus, je présente les mesures d'adaptations incluses dans cette dissertation : l'adaptation spatiale et l'investissement dans du capital d'adaptation, qui protège le capital vulnérable. Je discute comment les mesures d'adaptation ont été étudiées dans la littérature, notamment à l'aide de méthodes numériques, afin de souligner ma contribution.

Dans le Chapitre 1, j'essaie d'implémenter dans le modèle d'équilibre général GENESwIS les catastrophes naturelles ainsi que les mesures d'adaptation. Je propose une nouvelle approche de simulation, la myopie aux risques (*hazard myopia*), pour mieux prendre en compte l'adaptation spatiale aux futures grandes inondations en Suisse. Un agent myope aux risques résout un problème d'optimisation intertemporelle en évaluant subjectivement le risque d'inondation. En revanche, les stocks de capital sont mis à jour de manière récursive en fonction des dommages objectifs, c'est-à-dire des dégâts réels. En effet, le modèle d'équilibre général, déterministique, ne prend pas en considération le phénomène d'épargne préventif, qui est un comportement déterminant en présence d'incertitude. Pour correctement représenter les évènements rares mais catastrophiques, il est ainsi nécessaire d'utiliser un modèle stochastique.

En situation d'incertitude, un agent économique va investir davantage dans un capital moins productif mais sans risque ou dans un capital non productif qui protège le capital productif vulnérable. Comment expliquer ce phénomène d'épargne préventive ? Je traite cette question dans le Chapitre 2 en résolvant un modèle d'équilibre général dynamique stochastique. Dans ce cadre stylisé mais reproductible, j'obtiens les politiques d'investissement optimales pour deux mesures d'adaptation, l'adaptation spatiale et l'investissement dans du capital d'adap-

tation. Grâce à des simulations numériques, je montre que le modèle déterministique, qui utilise l'équivalent certain, sous-estime le risque de catastrophes naturelles.
Dans le Chapitre 3, je discute et je confirme quantitativement l'argument défendu par Schelling (1992, Section IV). En effet, je démontre que la meilleure stratégie d'adaptation à un désastre naturel future varie en fonction du développement économique des pays. Un pays en voie de développement devrait prioriser son développement économique en investissement dans du capital productif au détriment des mesures d'adaptation. A l'inverse, un pays développé devrait renforcer son capital d'adaptation, qui est non-productif mais qui permet d'atténuer les conséquences d'une catastrophe.
A travers les Chapitres 2 et 3, j'ai eu recours à des techniques de parallélisation sur un cluster de calcul haut de gamme dans le but d'accélérer les processus de résolution. L'utilisation du calcul parallèle est un développement récent dans le domaine de l'économie. Cette thèse illustre l'utilité des techniques informatiques modernes et vise également à encourager leur utilisation en économie.

MOTS CLÉS : Economie de l'environnement, incertitude, adaptation, catastrophes naturelles et leur gestion, modèle d'équilibre général, modèles d'équilibre général dynamique stochastique, grilles adaptives éparses, calcul parallèle.

Contents

List of Figures

List of Tables

General introduction

The concept of rare disasters, such as a great recession or world war, is introduced and studied in economics by Rietz (1988), Barro (2006) and Gourio (2012), among others. Their motivation is to solve the classic equity-risk premium puzzle in a financial market, proposed by Mehra and Prescott (1985). When a rare disaster occurs with low probability, outputs jump down sharply (Barro, 2006). Barro (2006) demonstrates that the properties of rare disasters are analogous to fat-tailed distributed shocks in climate change. Judd, Maliar, and Maliar (2011), Posch and Trimborn (2013) and Fernández-Villaverde and Levintal (2018) mainly propose numerical methods to analyze the negative impacts of rare disasters on production activities.

The financial crisis is not the only example of rare disasters. *Rare natural disasters* such as massive floods, great earthquakes or long droughts can cause extreme damages to economic activities. The probability of high impact natural disasters is small but possible to occur (Weitzman, 2011). These natural disasters cause damage not only to material assets but also to intangible values including human life. Weitzman (2009) proposes the first theoretical treatment of low probability but extreme impact events. However, it still includes many aspects that are yet to be developed.

Tsur and Zemel (1998), Ikefuji and Horii (2012) and Bretschger and Vinogradova (2017) examine the impact of rare disasters from an environmental economics perspective. However, except for a few contributions, such as Millner and Dietz (2015) and Grames et al. (2016), one critical point is still missing: *how do we adapt or alleviate the risk of uncertain rare natural disasters?* This question would be consistently pursued throughout this dissertation.

I briefly summarize the five key concepts in this thesis: *environmental economics and uncertainty, mitigation and adaptation to climate change, rare natural disasters, numerical methods in economics* and a *certainty-equivalent deterministic model.* Following this, I present the organization of this thesis.

Environmental economics and uncertainty

Knight (1921) formalized a distinction between risk and uncertainty in 1921. According to Knight (1921), *risk* refers to situations where we can correctly and objectively measure the probability of each event. *Uncertainty*, on the other hand, refers to situations where we cannot know the probability of each event. In reality, the case of risk is rarely true. Every decision maker postulates the probabilities of uncertain events *subjectively* in the situation of uncertainty. According to Savage (1954), the choice behavior of every decision maker is one as if he subjectively attaches the probability on each event and takes expected utilities.

Uncertainty is central to environmental economics. Pindyck (2007), Pindyck (2013a) and Weitzman (2013) summarize the sources of uncertainty in modern environmental economics. Stern (2016a) criticizes that the current economic models in this field fail to sufficiently model these uncertainties. Farmer et al. (2015) overview the shortcomings in the current modeling approaches and propose how economic models should be elaborated to address these uncertainties correctly. In this chapter, based on the above arguments, I present the sources of uncertainty in environmental economics and how these uncertainties have been tackled mainly in the literature of numerical modeling below. Then I present which uncertainty this dissertation concerns.

Literature review

The *discount rate* is one of the most notorious and controversial uncertainties. The impact of climate change and related environmental policies would persist for over decades or centuries. Long time horizons make it difficult to discount the distant future. Suppose that we rely on an exponential discounting scheme, the model tends to trivialize events in the distant future and results critically depend on the choice of a discount rate, especially in the context of cost-benefit analyses. One well-known example about uncertainty in the discount rate is in the evaluation of the social cost of carbon (SCC), which is the discounted value of the marginal damage to economic activities caused by carbon emissions. Stern (2007) and Nordhaus (2008), among others, calculate SCC and achieved strikingly different estimations as well as optimal abatement policy. Nordhaus (2008) uses his integrated assessment model, namely the DICE model. According to Nordhaus (2008), the optimal abatement should start gradually, consistent with SCC at around 30 USD per ton of carbon in 2005 or more, depending on the abatement target. On the other hand, Stern (2007) suggests a drastic and immediate CO_2 emission is necessary, which requires around 350 USD per ton of carbon in 2005. This considerable difference stems from many assumptions behind the economic and climate models, and the role of the discount rate is crucial in their estimations.[1] Nordhaus (2008) assumes that the pure rate of social time preference equals 1.5 percent per year. Stern (2007) assumes almost zero for the pure rate of time preference.[2] There is still no consensus between economists about the discount rate, as Weitzman (2011) summarizes, and the choice of the discount rate remains a subjective issue.

Environmental economists widely recognize *parametric uncertainty*. A typical example of a

[1] Indeed, in Nordhaus (2008), Nordhaus adopts Stern's discount rate and calculates substantially different SCC compared to his original estimations.

[2] Besides this underlying assumption on the discount rate, many challenge that Stern's assumptions about economical formulations and abatement cost, among others, are too pessimistic, but Stern (2008) defends these assumptions.

parametric uncertainty is in the choice of the relative risk-aversion parameter η. The role of η can be understood in two ways: A large relative risk-aversion means a small marginal utility of consumption in the future. In other words, avoiding climate change today offers a smaller benefit in the future. If there is uncertainty in the future, a large η means a more risk-averse and precautious economic agent. He expects a more considerable loss in the future welfare and the benefits from today's abatement become meaningful. Pindyck (2013b) claims the former effect dominates the latter; however, there is no definite conclusion about an appropriate value of η in the literature. In addition to the relative risk-aversion parameter, many parametric uncertainties are reported, and these are typically addressed using Monte-Carlo simulations. Löschel and Otto (2009), for instance, test the sensitivity of modeling results on technological uncertainties concerning the performance of backstop technologies. Babonneau et al. (2012) execute Monte-Carlo simulations for climate sensitivity, technological progress, economic growth, and oil and gas price.

The *damage function* is "the most speculative element of the analysis" of climate change (Pindyck, 2013a). It translates the temperature change to economic activities, such as factor productivity. Economic models are susceptible to the functional form of the damage function; however, there is little agreement on it. Functional form is still very arbitrary, and no theoretical or empirical foundations have been justified to date, for instance, see Tol (2002a) and Tol (2002b) for the FUND model, Hope (2006) for the PAGE model and Nordhaus (2008) for the DICE model. Damage functions are typically calibrated to analyze the temperature range from 2 to 3°C. Therefore if the temperature increase exceeds 5°C, the reliability of the function largely deteriorates.

The *climate sensitivity* parameter defines the increase in temperature relative to preindustrial levels, which corresponds to double the atmospheric CO_2 concentration. Figure 1 illustrates the estimated probability distribution functions for a climate sensitivity parameter from the published studies. Climate scientists use climate models to estimate the parameter based on the variety of assumptions involving parameters, physical processes and observational data among others. As Figure 1 shows, there is enormous uncertainty. Allen and Frame (2007) conclude that climate sensitivity is "unknowable." Also one should note that the probability distribution of climate sensitivity exhibited is fat-tailed, which is potentially another source of uncertainty: *catastrophic events*.

Catastrophic events are another source of uncertainty that environmental economists have realized but failed to address in a model correctly. IPCC (2007) estimates that the global average surface temperature is likely to be in the range from 2 to 4.5°C with the best estimate of 3°C, and it is very improbable to be less than 1.5°C. However, the climate outcomes tend to follow a fat-tailed distribution; thus, catastrophic events such as 7°C or 8°C temperature increase are still possible to happen. These events have a very low probability of occurring but are realized with a catastrophic consequence. Given the specific nature of the catastrophic events, the expected damage may be inappropriate to represent all catastrophic events, and this fact makes modeling implausible (Weitzman, 2009, 2011). The literature on this topic is

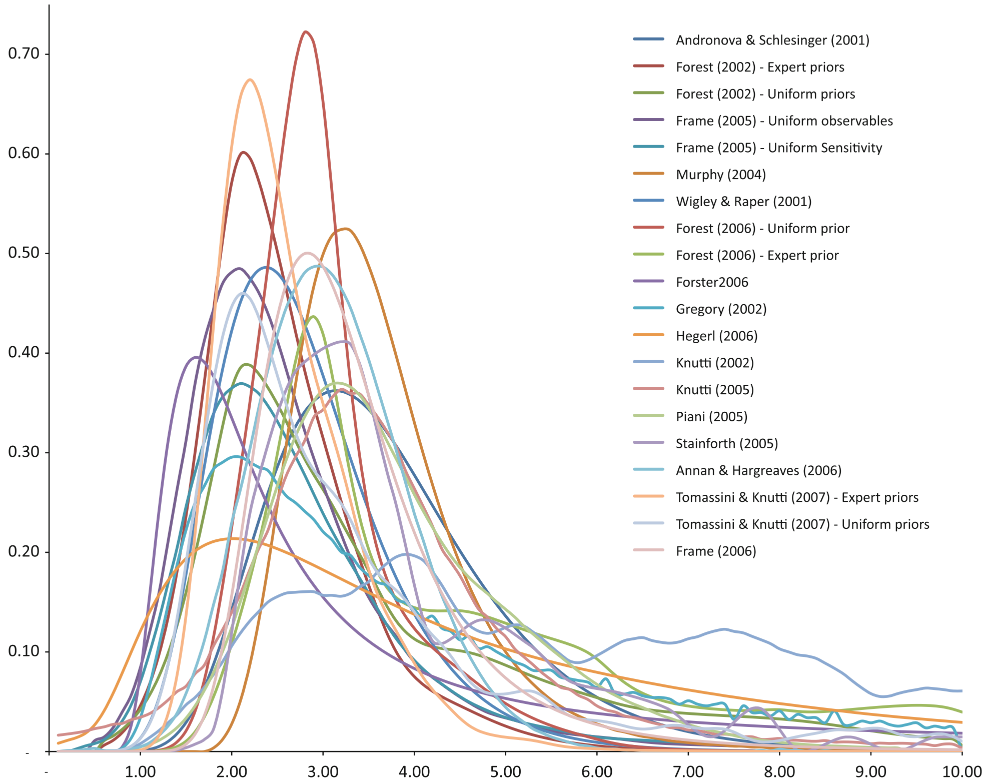


Figure 1 – Estimated probability density functions for a climate sensitivity parameter, adopted from Millner, Dietz, and Heal (2013)

limited apart from a few mentions from Dietz and Stern (2015), for instance.

Tipping points are coming to be considered as a further source of uncertainty. Lenton et al. (2008) define "tipping point" as a critical threshold. Once a state passes a tipping point, the state of a system is completely altered and may not return to the original state. They shortlist nine tipping elements and conclude that the melt of the Arctic Sea ice and the Greenland ice sheet are the greatest threats to our planet. Tipping elements have been grossly underestimated by conventional modeling, however, some recent literature discuss this issue for instance Lontzek, Cai, and Judd (2012), Lemoine and Traeger (2014), Lontzek et al. (2015) and Cai, Judd, and Lontzek (2015).

Uncertainty in this dissertation

Environmental economics suffer from a wide range of uncertainties. To make matters worse, climate scientists claim that some of the listed uncertainties are "unknowable". Given these potential difficulties, this dissertation focuses on two sources of uncertainty. One is a *rare natural disaster with a small probability of occurring but causes catastrophic damage to the environment* such as great floods or massive earthquakes. The probability of significant natural disasters is estimated to be very small, but it has happened in the past and causing extensive damage to the national or global GDP, as has been recorded. Rare natural disasters share common natures with catastrophic events.

The second source of uncertainty is *a small but frequent productivity shock*. The productivity shock is standard in the economics literature. I aim to include this uncertainty to maintain consistency with other fields of economics.

Mitigation and adaptation to climate change

Climate change is primarily regarded as one of this century's most pertinent global common concerns. Stabilization of greenhouse gas concentrations is necessary to prevent irreversible damage to the environment, which is attributable to anthropogenic interference. After a seminal contribution by Nordhaus (1991), there is a growing demand from the economic side to curb CO_2 emissions to sustain economic growth. Mitigation measures are a priority to avoid irreversible damage to the environment. However, additional adaptation efforts are required to reduce the adverse impacts of the projected climate change. In this chapter, I overview these approaches. First I briefly review mitigation measures and subsequently I discuss the adaptation to climate change, which is a central focus of this dissertation. Finally, I present the contributions of this dissertation to the existing literature.

Mitigation of climate change

Mitigating climate change is primal research interests. Carbon tax and carbon emission trading are the best examples of mitigating ongoing climate change. Price (tax) and quantity (permit) based regulatory instruments provide different incentive motives after a seminal contribution by Weitzman (1974). This price and quantity regulation is widely discussed in the theoretical assessment of environmental policy, for instance see Montero (2002), Kelly (2005), Krysiak (2008), Krysiak and Oberauner (2010) and Storrøsten (2014) among others.

William D. Nordhaus, a Nobel laureate in 2018, is one of the founders of the economic analysis of anthropogenic impacts on the environment. His seminal contribution is to propose a stylized and reproducible mathematical model, namely an integrated assessment model (IAM), where an economic and a climate module are synthesized. Today IAMs have become one of the main tools to estimate the efficiency of a variety of mitigation measures. Depending on the scope of the modeling exercises, existing IAMs are roughly classified into two approaches: *top-down* and *bottom-up*.

Top-down IAMs focus on the macroeconomic structure of target regions. A representative agent solves a static or an intertemporal optimization problem subjected to several constraints, deciding how much output to consume and how much to save for the future. IPCC (2001) categorizes top-down IAMs into two subcategories: a *policy optimization model* (POM) and a *policy evaluation model* (PEM).

POMs focus on the cost-benefit aspect of mitigation measures. For instance, the DICE model

(Nordhaus, 2008), the RICE model (Nordhaus and Boyer, 2000), the FUND model (Tol, 2002a,b) and the PAGE model (Hope, 2006) have been developed and deployed to evaluate mitigation strategies. POMs are often cast as a Ramsey growth model with a simple climate model and a damage function.

PEMs have detailed sectoral input-output information to analyze a general or partial equilibrium, while POMs highly aggregate sectoral details. The computational general equilibrium (CGE) model is regarded as an example of PEMs. A representative agent in a CGE model is either complete myopic, such as the EPPA model (Paltsev et al., 2005) or the GEMINI-E3 model (Bernard and Vielle, 2008), or has perfect foresight, such as the EPPA-FL (Babiker et al., 2009) model or the GENESwIS model (Vöhringer, 2012). POMs are largely implemented as an optimization problem, while PEMs are often categorized as a simulation model.

Bottom-up IAMs represent the technological side of target regions in greater detail. Bottom-up IAMs are often cast as a non-linear optimization model and analyze a partial equilibrium at the sectoral, regional or global level. MERGE (: Model for Evaluating the Regional and Global Effects of climate change policies) is a well-known example of (Manne, Mendelsohn, and Richels, 1995). Barreto and Kypreos (2004) study the impact of R&D investments, which is an essential factor to push technological progress within a given technology, on the diffusion of renewable technologies, using the bottom-up energy system model ERIS. Usui, Furubayashi, and Nakata (2017) endogenize the energy-related R&D investment for a renewable technology, which requires technological innovation to be deployed in the mature electricity market in Japan, and optimize the timing of R&D investment within the framework of a partial equilibrium model. The carbon capture and storage (CCS) system is one option to reduce a significant amount of CO_2 emission from carbon-intensive power plants. Technological change and the cost of carbon mitigation are widely discussed (Grübler, 2003; Kypreos, 2008). Endogenous technological development and induced technological change have been widely studied especially with a technology-rich energy system model such as van der Zwaan et al. (2002), Buonanno, Carraro, and Galeotti (2003) and Castelnuovo et al. (2005).

Adaptation to climate change

Due to the inertia of the climate system and the demands from public administration, there is a growing need for adaptation strategies. Adaptation aims to reduce possible impacts of climate change on the economy and the environment. In this sense, adaption to climate change could become a substitute or a complementary action of mitigation.

Tol (2005) argues three differences between mitigation and adaptation instruments. First, as summarized in Table 1, mitigation is more likely to be a public good and requires a concerted international effort. On the other hand, adaptation could be implemented on a national or a regional scale, providing sufficient implementation speed and geographical scope. Second, adaptation can be carried out with a more specific geographical scope and shorter temporal scale than mitigation. Third, mitigation and adaptation are substitute goods. Mitigation takes resources away from adaptation and vice versa.

Table 1 – Mismatches between mitigation and adaptation adapted from (Tol, 2005)

	Scale of implementation	Decision makers	Time scale
Mitigation	National scale in the context of international negotiations	Ministries or Governments	Long-term
Adaptation	Local scale of individual households or companies in the context of a regional economy	Local managers	Short- or medium-term

Fankhauser, Smith, and Tol (1999) developed some basic rules to design efficient adaptation strategies that enrich the flexibility and the resilience of systems to climate shocks, reflecting the long time scale and the prevailing uncertainties about climate change. They classify adaptation measures into four types: reactive, anticipatory, autonomous and planned adaptation.

The objectives of adaptation measures can be diverse. One approach is to try to cancel all climate-related impacts, maintaining the status quo. Another is to cancel adverse impacts and to collect positive opportunities. In the presence of limited resources, adaptation strategies involve some trade-offs with multiple policy alternatives such as economic development and climate mitigation. Therefore, there is a growing demand for the understanding of the economic nature of adaptation in the context of climate change; however, most aspects of adaptation measures are yet to be addressed (Ciscar and Dowling, 2014). Most adaptation efforts create local public good as opposed to mitigation, which contributes to the global public good, creating a concern that such actions would not receive enough attention from private parties.

Private of public actors motivate many forms of adaptation measures. These adaptations are undertaken in the context of local or international negotiations (Chambwera et al., 2014). General forms of adaptation and the primary stakeholders are summarized in Table 2. One notable and successful example is about adaptation to great floods in Switzerland. Swiss national and local authorities have realized the risk of great floods after the catastrophic flood in 2005 and have installed early warning systems, weather regulation services. Those facilities worked correctly in 2007, even the magnitude and the catchment of the flood was very similar to the flood in 2005 (NZZ, 2007a,b). Japan is known for an earthquake-prone country. Public facilities in Japan are supposed to install earthquake resisting function. Risk communication, as well as evacuation training, are regularly conducted by every local municipality. Some of the actions are public, i.e., they are funded by public actors such as governments, NGOs, or international organizations.

Figure 2 illustrates the link between the cost of climate change and the adaptation cost, for the case that, either, optimal adaptation is possible, or not. A fraction of the climate cost can be reduced without any payment as shown in both panels. It is called adaptation without any payment. Most adaptation efforts, like mitigation, require additional investment to reduce

Table 2 – Adaptation measures and their main stakeholders

Adaptation measures	Stakeholder(s)
Altered patterns of enterprise management, facility investment, enterprise choice or resource use	Mainly private
Direct capital investments in public infrastructure	Mainly public
Technology development through research	Private and public
Creation and dissemination of adaptation information	Mainly public
Human capital enhancement	Private and public
Redesign or development of adaptation institutions	Private and public
Changes in norms and regulations to facilitate autonomous actions	Mainly public
Changes in individual behavior	Private with possibly public
Emergency response procedures and crisis management	Mainly public

the negative impact of future climate change. The cost of climate change can be reduced monotonically with increasing adaptation costs. The left panel shows the case when optimal adaptation is possible. The optimal adaptation level can be found by equating the marginal adaptation cost with the marginal benefit of adaptation. On the other hand, as shown in the right panel, optimal adaptation is in general not possible. Any adaptation instrument can not overcome some of the climate costs since some instruments are too cost-inefficient or too ambitious to be implemented. There is also a technology limit. Fankhauser and Soare (2013) claim the role of government intervention to overcome barriers that prevent smooth or autonomous adaptation. Barriers to adaptation include shortcomings in the institutional and regulatory environment, market failures such as asymmetric information and moral hazard, as well as the lack of behavioral and information.[3] Because of these barriers and technological constraints, the marginal cost of adaptation cannot be equal to the marginal benefit of adaptation in this setting.

Adaptation is not a static project. Rather, it evolves, taking into account changing climate conditions, technology availability and its maturity (Chambwera et al., 2014). Uncertainty, the lack of reliable data and the patterns of future climate change are also an important aspect since the longer analytic periods are, the more uncertainty there is. However, in most cases, especially with a low-probability but high-impact catastrophic disaster, it may be impossible to identify reasonable probabilities for alternative outcomes and forecast the set of possible future outcomes (Weitzman, 2009, 2011). Data quality and quantity are also significant sources of uncertainty (Chambwera et al., 2014). Poor or sparse data limits the accuracy of numerical estimations. It is especially the case for developing countries for which adaptation to climate change is a more urgent matter (Millner and Dietz, 2015).

[3]Moser and Ekstrom (2010) define barriers as obstacles that could be stemmed from all phases and sub-processes throughout the adaptation process. Common barriers are listed up by Moser and Ekstrom (2010, and references therein).

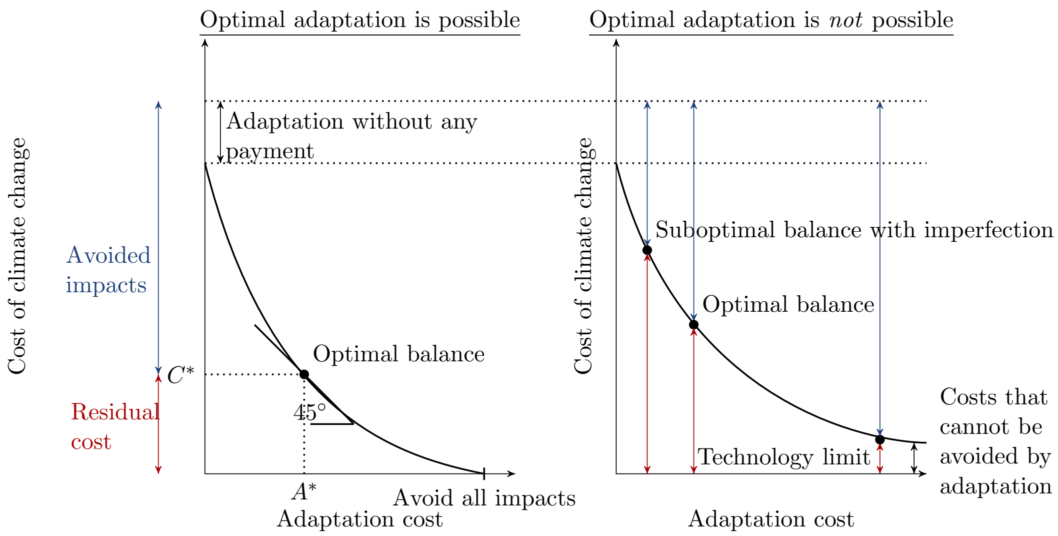


Figure 2 – Cost of adaptation and the residual cost of climate change, the left panel shows the case where optimal adaptation is possible, while the right panel is the case where optimal adaptation is not possible. The figures are cited from Chambwera et al. (2014), but has been slightly modified by the author.

Adaptation is classified into two forms; flow and stock adaptation. Flow adaptation occurs in a single period, and stock adaptation involves intertemporal investment decision-making. Some existing economic models consider flow or stock adaptation. de Bruin, Dellink, and Tol (2009) are some of the first to model adaptation as an endogenous choice variable in IAM, AD-DICE. Bosello, Carraro, and De Cian (2010) discuss the optimal balance between mitigation and adaptation in another IAM, AD-WITCH. In analytical literature, stochastic dynamic optimization over a continuous time is a standard tool to study optimal adaptation decisions, for instance, see Tsur and Zemel (1998). Bréchet, Hritonenko, and Yatsenko (2013) solve the optimal balance between mitigation and adaptation by using the deterministic Solow growth model. Millner and Dietz (2015) introduce adaptive capital stock together with productive capital stock, to the deterministic Ramsey-Cass-Koopmans growth model and solve the intertemporal optimization model numerically for the consumption, as well as for the investment to adaptive and productive capital stock. Grames et al. (2016) present a continuous dynamic stochastic growth model with adaptive capital stock to address uncertain floods.

Adaptation in this dissertation

In this dissertation, I focus only on adaptation and mainly address two adaptation measures: *spatial adaptation* and the introduction of *adaptive capital stock*.[4] In the spatial adaptation, I classify the existing productive capital stock into two types, depending on its vulnerability to

[4] In terms of terminology, spatial adaptation is well accepted in the literature on flood risk, for instance, see Koks et al. (2014). The concept of adaptive capital is relatively new, but my definition is similar to Millner and Dietz (2015) and Grames et al. (2016).

rare natural disasters. Spatial adaptation can be cast as intertemporal investment decision-making between vulnerable and non-vulnerable capital stock. Spatial adaptation is one form of “stock” adaptation. Spatial adaptation is “planned” adaptation in the classifications of Fankhauser, Smith, and Tol (1999) and is an example of “direct capital investments in public infrastructure” in Table 2.

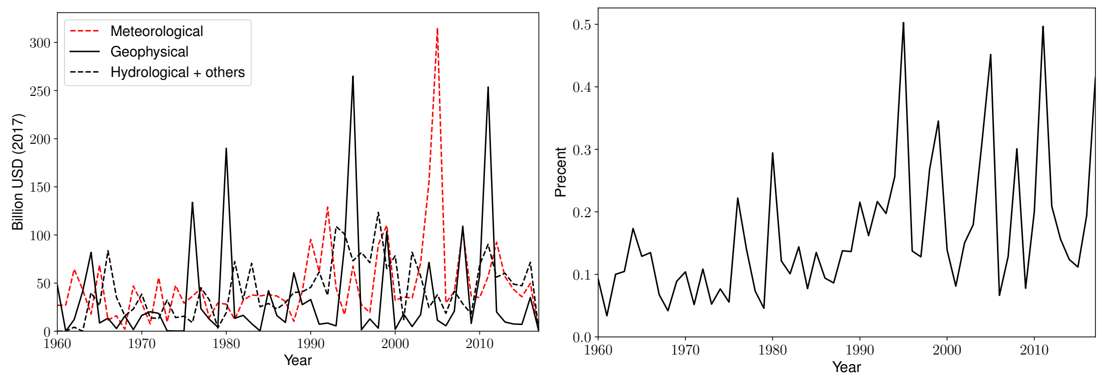
The adaptive capital stock is assumed not to contribute to production activities and becomes only effective when a disaster occurs. Therefore, an investment in adaptive capital stock has a precautionary sense. Adaptive capital stock requires dynamic investment decision-making between productive and adaptive capital stock. Therefore, the adaptive capital stock is another form of “stock” adaptation. The introduction of adaptive capital stock is an “anticipatory” action in the classifications of Fankhauser, Smith, and Tol (1999) and is again an example of “direct capital investments in public infrastructure” in Table 2.

The dissertation relates or contributes to the existing literature in the following ways: First, both spatial adaptation and adaptive capital stock are stock adaptations, as in Agrawala et al. (2011), and have either a planned or an anticipatory sense, as in Fankhauser, Smith, and Tol (1999). In Chapter 1, I aim to evaluate the efficiency of several spatial adaptation policies, using an existing CGE model. The dynamic stochastic equilibrium analyses in Chapters 2 and 3 are analogous to analytical studies such as in Tsur and Zemel (1998), Bréchet, Hritonenko, and Yatsenko (2013) and Millner and Dietz (2015). In general, closed-form solutions require stylized assumptions especially under uncertainty, though I use a state-of-the-art numerical method to approximate an Euler equation with keeping reasonable assumptions directly. Finally, Grames et al. (2016) discuss the optimal adaptation decision for uncertain hazards over continuous time, while my models are formulated in discrete time. The consideration of spatial adaptation is an additional asset.

Rare natural disasters

Rare natural disasters have caused notable damage to human welfare. Economic damage from natural disasters on the global GDP from 1960 are summarized in Figure 3 (CRED and Guha-Sapir, 2018). Figure 3a displays the long term economic damage from natural disasters. Natural disasters have caused an extreme impact on economic activities; however, the trend exhibited is considerably random.

The ratio of damage to the global GDP is shown in Figure 3b. Again the trend presented is substantially random, but I weakly conjecture that the ratio continues on a steady and upward trend. The continuous economic growth supports this observation. The economic growth is expected to continue for the next decades and centuries. If I can assume some degree of relationship between economic growth, climate change and the probability of having a rare natural disaster, the economic development itself threatens sustainable development. An adaptation to rare natural disasters is also demanded within this context.



(a) Economic damage from natural disasters

(b) Ratio of damage from natural disasters to global GDP

Figure 3 – Economic damage from natural disasters on a global scale. The label "others" includes biological, climatological and extra-terrestrial disasters. Data source is CRED and Guha-Sapir (2018) and The World Bank (2018).

Numerical methods in economics

Numerical methods have become the third pillar in most areas of science.[5] In the past, science relied on observational and experimental facts. Theories were developed to offer mathematical reasoning. All of the theories are mathematically tractable and offer implications in some closed-form expression. Models are highly simplified and require stylized assumptions. The theories are useful to illustrate general rules, but a computational approach is necessary when a model is analytically intractable by adding additional elements.

Nowadays, computational approaches are widely accepted in most (hard) sciences such a physics, chemistry, astronomy, meteorology, pharmacy among others. Many economists emphasize on the analytical *assumption-theorem-proving* style, but some realize the importance of computing power. We need to specify all the assumptions such as functional forms and parameters. However, if we want to address some complexities that have been omitted to illustrate a closed-form solution, such as uncertainty or heterogeneity, numerical methods could become complements or substitutes to analytical solutions.

Computational general equilibrium (CGE) is the most mature area of computational economics (Judd, 1997). A CGE model is based on the general equilibrium theory, which was first pioneered by Léon Walras and mathematically formalized mainly by Kenneth J. Arrow and Gerard Debreu. The basic idea is that all prices are adjusted until the demand and supply are equalized in all considered markets. The Scarf's algorithm is the first solution to solve CGE models. Shoven and Whalley (1984) survey the artistic contributions in this field.

After seminal contributions by Thomas F. Rutherford and James R. Markusen, among others, CGE models have now become one of the standard policy evaluation tools in economics.

[5] Judd (1997) is a comprehensive essay about computational economics.

Most CGE models are calibrated and solved using the General Algebraic Modeling System (GAMS), a high-level modeling language for mathematical programming (Rosenthal, 2013). Mathematical Programming System for General Equilibrium (MPSGE) has been developed as a subsystem of GAMS (Rutherford, 1999). MPSGE greatly facilitates the programming cost of CGE modeling. Advances in a mixed complementarity approach are noteworthy to support the development of CGE modeling, for instance, the PATH solver (Ferris and Munson, 2000).

Numerical methods for solving non-linear dynamic problems have been continuously developed. One of the most active research fields is to propose a solution algorithm to solve rational expectation models. Judd (1992) demonstrates how rational expectation models can be solved by using a *projection method*. Projection (or time iteration collocation) method has been widely employed to solve a variety of economic models, including dynamic games as (Judd, 1996) and asymmetric information models (Ausubel, 1990). Classical projection methods depend on the orthogonal global polynomials such as Chebyshev or Legendre polynomial. Approximation based on the orthogonal polynomials becomes more expensive when one aims to solve a high dimensional rational expectation model. It is known as the *curse of dimensionality*. The Smolyak algorithm is a well-known example to alleviate the curse of dimensionality (Krueger and Kubler, 2004).

High-performance computing (HPC) would become more critical for serious computational works in economics. The clock speed of a single CPU is reaching its transition limit, and it is expensive to make a single processor faster. Serial computing would also reach its limitation soon and is already incapable of solving a numerically expensive economic model. HPC enables us to access the modern and massive power of the high-end computing system. However, compared to other (hard) sciences, applications of HPC in economics are limited, for instance, see Aldrich et al. (2011), Cai et al. (2015) and Brumm and Scheidegger (2017) among others.

There are many more numerical methods and algorithms capable of solving interesting economic models than I can include here. There is no golden algorithm to solve all economic models. We need to select the correct numerical methods depending on the characteristics of the models that we want to solve numerically. Computing time and approximation quality are worth considering. Aruoba, Fernández-Villaverde, and Rubio-Ramírez (2006) and Juillard and Villemot (2011) compared some solution algorithms and resulted in approximation qualities for some examples. Some are interested in selecting the best programming language for economic modeling and comparing them (Aruoba and Fernández-Villaverde, 2015).

This dissertation employs two numerical models. In Chapter 1, I use a computational general equilibrium model. In Chapters 2 and 3, I solve dynamic stochastic equilibrium models by using an approach based on the time iteration collocation method. The presented models in Chapters 2 and 3 are too expensive for a serial desktop computer. Therefore, I implement and massively parallelize them on the university computing cluster via the message passing interface (MPI).

Certainty-equivalent deterministic model

Stochastic models have no steady state as deterministic models do. A certainty-equivalent deterministic model offers a useful benchmark solution for the subsequent analyses with a stochastic model (Miranda and Fackler, 2002). The most convenient way to transform stochastic models to the certainty-equivalent deterministic models are to take an expected value of uncertain parameters.

Suppose that I want to solve the following stochastic programming:

$$\min_{x} \mathbb{E}\left[f(x;\xi)\right] \tag{1}$$

$$\text{s.t.} \quad x \in \mathscr{X}(\xi) \tag{2}$$

where x is the vector of decision variables and ξ is the vector of (uncertain) parameters. $\mathscr{X}(\cdot)$ stands for the feasible set of the decision variables. I assume ξ follows *ex-ante* probability distributions. I can convert the stochastic model to the certainty-equivalent deterministic model by taking:

$$\bar{\xi} = \mathbb{E}[\xi] \tag{3}$$

However, the modeling results in Eq. (1) is different from the certainty-equivalent deterministic one:

$$\mathbb{E}\left[f\left(x^*;\xi\right)\right] \neq f\left(x^*;\mathbb{E}[\xi]\right) \tag{4}$$

where x^* is the vector of the optimal control of the associated problem.

In some applications, especially if the model is linear and an uncertain parameter distributes uniformly, the certainty-equivalent deterministic model offers an acceptable approximation quality for the corresponding stochastic model. However, the certainty- equivalent approximation is known to pose poor approximation, if the model contains a non-linear equation or a fat-tailed distributed parameter. The presented thesis addresses highly non-linear stochastic models with rare but catastrophic disasters. In this context, the certainty-equivalent approximation is expected to be the wrong choice for this thesis.

Outline of the thesis

The dissertation studies adaptations to rare natural disasters under uncertainty using an approach based on numerical economic models. *How do we (optimally) adapt or alleviate the risk of future rare natural disasters?* The dissertation is dedicated to answering this question and is organized as follows:

Chapter 1 presents the first numerical simulations for uncertain high impact floods in Switzerland. I employ the existing CGE model GENESwIS (Vöhringer, 2012), and consider spatial adaptation. I start this chapter by classifying the existing Swiss productive capital stock amongst flood vulnerable and non-vulnerable capital stock. Flood hazards involve a high degree of uncertainty; therefore, the conventional perfect-foresight agent is inappropriate. The spatial adaptation is one form of the stock adaptations; thus, investment decision making should be based on an intertemporal optimization that a conventional myopic economic agent misses. Regarding this aspect, I propose a new simulation approach, namely the *hazard-myopia*. The hazard myopic economic agent has perfect-foresight of all macroeconomic conditions, but he is still myopic when it comes to uncertain floods. Instead of objectively observing information on flooding, he subjectively forms an expectation on an upcoming flood. The agent solves an intertemporal optimization problem based on his subjective belief. The capital stocks in the next period are recursively updated considering the realized damage scale. I address uncertainty around the timing of high impact floods with a large number of Monte-Carlo simulations. In each Monte-Carlo simulation run, the timing of hazards are exogenously defined and substituted into the model.

However, the simulated results are counterintuitive. The hazard myopic economic agent invests more in vulnerable capital stock, even though he realizes the risk of significant flooding in the future. The model is calibrated based on the Swiss empirical data; however, the modeling insights contradict the real Swiss situation.

To model and to exploit stochasticity in the improved representation, I study optimal adaptation decisions on rare natural disasters by using an approach based on a dynamic stochastic equilibrium model in Chapter Chapter 2.

As an example of adaptation measures, I consider spatial adaptation and the introduction of adaptive capital stock. The scope of Chapter 2 is to carefully study the optimal adaptation decisions on uncertain rare natural disasters with either spatial adaptation or adaptive capital stock. The models are formulated as a social planner problem. First-order autoregressive productivity shock is considered. I present a modeling way that includes uncertain rare natural disasters in discrete time and solves the models by applying the time iteration collocation with adaptive sparse grid (Brumm and Scheidegger, 2017). To speed up the solving processes, my implementations are massively parallelized using high-performance computing architecture. I numerically approximate the optimal stochastic policy functions for the economy either with spatial adaptation or adaptive capital stock. Furthermore, I demonstrate the applicability of the stochastic adaptation decisions over the deterministic ones, focussing on small probability but catastrophic natural disasters.

Chapter 2 contributes not only to adaptation decisions but also to the literature of computational economics. The models have several capital stocks. Therefore, occasionally binding irreversible investment constraints are required. The occasionally binding constraints impose non-smooth regions in the interpolant and that largely deteriorates the quality of approximation (Judd, Kubler, and Schmedders, 2000). Adaptive sparse grid algorithm correctly detects non-smoothness regions. My approximation quality is good enough compared to the existing literature (Brumm and Grill, 2014).

Chapter 3 extends the models and the discussions in Chapter 2. Adaptation is likely to be installed in the regional and specific environment (Tol, 2005). Optimal adaptation decisions depend on the development state of the target economy. Spatial planning and adaptive capital stock are complementarities. A tradeoff between the two adaptation measures should be considered in the same manner. I develop a social planner's problem with two adaptation measures in Chapter 3. The model is solved using the time iteration collocation with an adaptive sparse grid algorithm.

Computed stochastic policy functions are applied to the initially well developed and developing economy. I claim that if the economy is still developing, it is socially optimal to invest more in productive capital stock at first, to secure the resources available for adaptation. If the initial state of the economy is sufficiently developed, the initial growth rate of the adaptive capital stock exceeds that of the productive capital stock in preparation for future uncertainty. Schelling (1992, Secion IV) qualitatively claims that the developing countries' "best defense against climate change may be their continued development". My results quantitatively support his statement.

1 Spatial adaptation in the CGE model

1.1 Introduction

Switzerland has been identified, as have many other European countries, vulnerable to flooding. Floods cause significant damage not only to material assets but also to intangible valuables. Floods can be triggered by diverse weather conditions such as thunderstorms, long-lasting rainfall or snow-melt. Schmocker-Fackel and Naef (2010) analyzed up to 105 years of streamflow data in Switzerland to determine whether the flood frequency has increased as a result of the changes in the atmospheric circulation. Flood risk analysis takes into account practical information about floods, such as frequency and magnitude, as well as the exposure and vulnerability in flood-prone areas. Fuchs et al. (2017) and Röthlisberger, Zischg, and Keiler (2017) carry out a case study in Switzerland. It is a pervasive and active research field. Recent scientific evidence demonstrates that the impact of climate change is visible in the hydrological data such as heavy precipitation and peak flows, implying that flood risk has already increased in Switzerland (CH2018, 2018). Swiss authorities have realized the risk of floods and have installed some early warning systems and physical instruments to alert these associated uncertainties.

How do we adapt to the risk of uncertain, rare but catastrophic floods? I would tackle this question using an approach based on the existing simulation model GENESwIS (Vöhringer, 2012). GENESwIS is a dynamic computational general equilibrium (CGE) model and represents the Swiss economy as a small open economy. I aim to deliver the ex-ante estimation of adaptation to great floods within the context of Switzerland. First, I implement spatial adaptation on GENESwIS as an example of an adaptation instrument.[1] In the spatial adaptation, I categorize the existing Swiss productive capital stock into two types, depending on its vulnerability to floods: vulnerable and non-vulnerable capital stock.

An increasing number of papers have applied CGE models to study the economic impact of natural disasters and the distribution of damage within the economy. Earthquakes disrupt life

[1] Spatial adaptation is a well-considered adaptation measure to flooding, for instance see Koks et al. (2014).

and transportation infrastructures and cause a significant impact on the economy. Rose and Guha (2004) estimate the indirect impacts of large earthquakes on business activities. Rose and Liao (2005) study the sectoral and regional impacts of a water disruption caused by an earthquake in the Portland Metropolitan. Tsuchiya, Tatano, and Okada (2007) and Tatano and Tsuchiya (2008) employ a spatial CGE model and investigate the impacts of transportation infrastructure disruptions caused by an empirical or a projected massive earthquake in Japan. High impact floods and their distributional effects have been widely studied. Pauw et al. (2011) conduct a case study for Malawi to estimate the losses caused by possible extreme weather events, such as droughts and floods. Haddad and Teixeira (2015) study the economic impact of floods in São Paulo, Brazil, by integrating a spatial CGE model and GIS information. Carrera et al. (2015) estimate the damage to the physical stock and apply the model to an empirical flood in Italy. The potential economic losses attributable to general natural hazards have been estimated and reported. Berrittella et al. (2007) investigate the impact of water scarcity in the framework of international trade. Pycroft, Abrell, and Ciscar (2016) evaluate the impacts of sea-level rise at a global scale. Finally, Gertz and Davies (2015) is the first to use a dynamic CGE model to examine recovering processes after significant flooding in Metro Vancouver, where an economic agent has perfect foresight.

Natural hazards involve a high degree of uncertainty. The timing and the magnitude of natural hazards are unpredictable; therefore, the conventional perfect-foresight economic agent is not a reasonable assumption within the context of uncertainty. Spatial adaptation is one form of stock adaptation. This fact makes the conventional myopic economic agent inappropriate since he does not consider an intertemporal tradeoff.

The first contribution of this chapter is to propose a *hazard myopic* simulation algorithm. There are three critical assumptions behind the hazard myopia. First, the representative agent has perfect-foresight of all macroeconomic conditions except for uncertain natural disasters. Second, the agent subjectively forms an expectation of the scale of damage of future floods. This subjectively formed expectation is directly considered when the agent solves an intertemporal optimization problem. This subjective belief reflects how much the representative agent is concerned with the risk in the future. When the agent finds that there is a high future risk, he prepares for future uncertainty by assuming higher additional damage on vulnerable capital stock. When he underestimates the risk of floods, his belief in the additional damage naturally decreases.

The third assumption behind the hazard myopic simulation is in the timing of capital updating. The representative agent correctly observes whether a flood has occurred or not in each period. Capital stocks need to be recursively updated based on the realized observed damage scale, in line with the complete myopic economic agent. The representative agent again solves an intertemporal optimization problem in the next period. In short, there are two laws of motion of capital stock. One is based on a subjective belief, and it is used when the agent solves an optimization problem. The other is based on an observed damage scale, depending on whether a flood occurs or not, and capital stocks are recursively updated based on this law of motion every period.

I extend the GENESwIS model so as to include a hazard myopic economic agent. I assume he is risk-averse. The risk-averse economic agent subjectively evaluates the risk of future flooding. The more he is concerned about the risk of future great floods, the more he invests into less productive but risk-free capital stock, such as non-vulnerable capital stock. After checking the credibility and the stability of the new simulation approach, I evaluate the spatial adaptation strategies for the selected major five floods in Switzerland. I address uncertainty in the timing of floods by a large number of Monte-Carlo simulations.

However, my modeling offers counterintuitive results and contradicts the real Swiss situation. That is, the representative agent realizes the risk of flooding, but he invests his saving mainly in vulnerable capital stock, even though he is assumed to be risk-averse. The more he is concerned about the risk of future floods, the more he invests in more productive but risky capital stock, such as vulnerable capital stock, instead of investing into non-vulnerable capital stock.

Given this result, I decided to contribute to the modeling community by providing reasons why my attempts had failed to model adaptation decisions that could be consistent with risk-averse behavior. I systematically compare some advanced economic models with my study. I conclude that the treatment of uncertainty in my approach is inadequate. Investment in non-vulnerable capital stock is a precautionary saving with which an economic agent with income shocks is known to save excessively rather than determinate (Ljungqvist and Sargent, 2012, Section 18.14). The economic agent invests in a less productive but risk-free capital based on a precautionary motive, but this effect can be strengthened under uncertainty.

The remainder of this chapter is organized as follows. Section 1.2 briefly overviews the flood history in Switzerland and selects the major five floods for simulation analyses. Section 1.3 presents the structure of the existing computational general equilibrium model GENESwIS and proposes a new modeling approach: the hazard myopia. Section 1.4 summarizes the simulation results. However, the simulation results contradict what we observe in reality. Therefore, I discuss how the existing CGE model can be elaborated to include correct adaptation behavior to rare but catastrophic disasters in Section 1.5, and I conclude in Section 1.6.

1.2 Switzerland and floods

The Swiss Federal Research Institute for Forest, Snow and Landscape Research (WSL) has systematically accumulated data on damage caused by naturally triggered floods, debris flows and landslides since 1972 (Hilker, Badoux, and Hegg, 2009). Figure 1.1 illustrates the cumulative flood damage costs and their spatial distribution in Switzerland since 1972. The regions Espace Midland and Central Switzerland each account for more than one-quarter of the absolute total loss. When the recent history of flood damage is reviewed, as shown in Figure 1.2, we can observe several prominent floods in terms of their damage to the Swiss

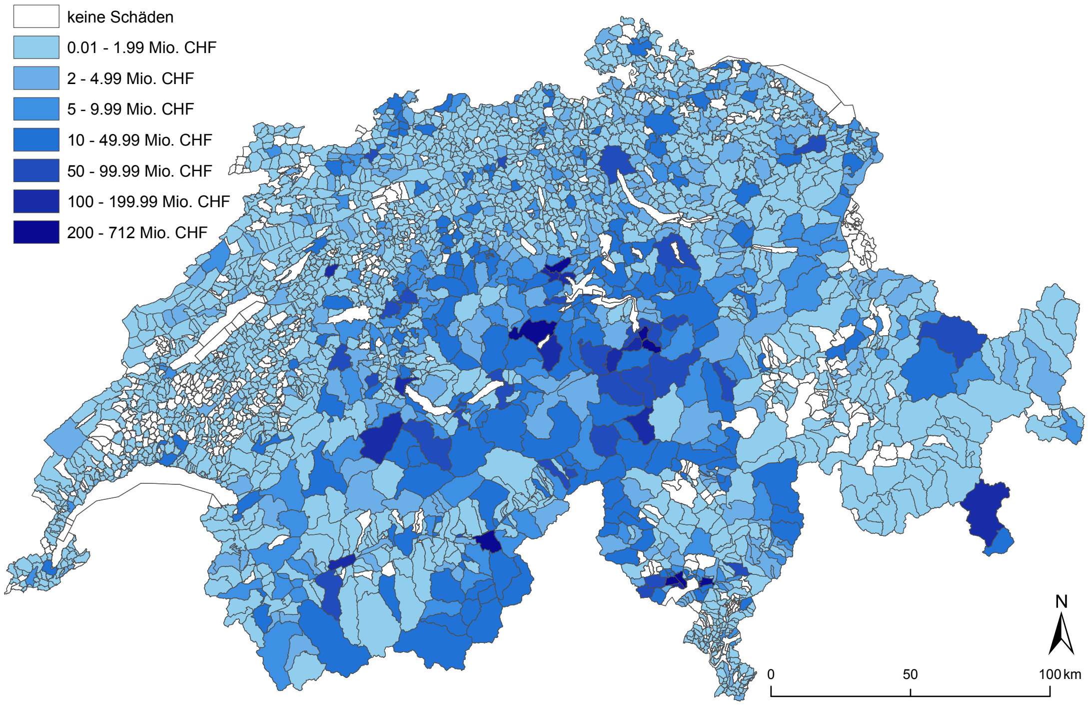


Figure 1.1 – Spatial distribution of cumulative costs of flood damage in Switzerland from 1972 to 2013. The figure is initially provided by the WSL Storm Damage Database of Switzerland (Hilker, Badoux, and Hegg, 2009).

economy. National and local authorities have successfully applied the WSL storm damage database in Switzerland in a decision-making process.

Together with our project partners at the University of Bern (Ole Rössler, Luise Keller and Alexandra Gavilano), we have selected the major floods in Switzerland. I briefly present reasons why the selected five floods are interesting for the subsequent simulation analyses with the existing computational general equilibrium model: GENESwIS.

Flood in August 2005

From the 19th to the 24th of August 2005, torrential rain fell mainly in Canton Bern and Central Switzerland.[2] In some observatory stations, the amount of precipitation surpassed the existing record. Especially the rivers Aare and Reuss generated extraordinary levels of discharge. Flooding, erosion, landslides and debris flows killed seven people. The flood caused the largest monetary damage on the Swiss economy in history and approximately 900 communes were affected.

[2]Flood information is taken from Sturmarchiv Schweiz (http://www.sturmarchiv.ch/) and National Platform for Natural Hazard (http://www.planat.ch/en/).

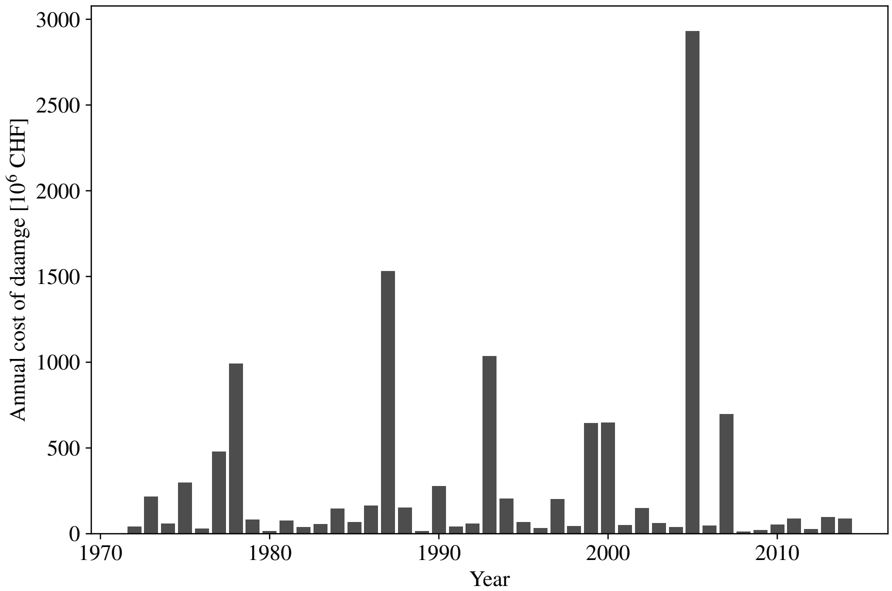


Figure 1.2 – The annual cost of damage caused by flooding in Switzerland. Cost data is converted into Swiss francs (CHF) in 2014.

Flood in May 1999

The northern part of Switzerland experienced extremely heavy precipitation from the 4th to 22nd May 1999. Persistent high temperatures in this time and heavy precipitations accelerated the melting of the snow in the mountains and the melt-water significantly increased the level of the discharge. Due to the long-lasting rainfall and melt-water, the rivers Thur, Aare, Linth and Lake Bodensee were flooded. Canton Zürich and Bern were affected. The event is recognized as a centennial flood. The catchments and the flooding process are different from those in 2005.

Flood in October 2000

A severe storm hit Canton Valais on October 14th 2000. A disaster happened in the village Gondo. A mixture of water, soil mud and parts of the protection wall avalanched down and destroyed one-third of the village within seconds. The event happened so quickly that the installed warning system gave insufficient notice to take action. Lake Maggiore was flooded during the period. This flood is also recognized as an example of a lake case, which differs from other floods.

Flood in October 2011

In 2011, there was heavy precipitation between October 6th and 10th. Due to the amount of warm air flowing from subtropical latitudes, a considerable amount of fresh snowfall melted.

Table 1.1 – Empirical floods in Switzerland

Flood	Date	Return periods [year] [a]	Nominal damage [10^6 CHF] [b]	Real damage [10^6 CHF in 2008] [c]	Damage scale Δ_f^O [-] [d]
1	August 2005	150	2977.60	3105.89	7.54×10^{-2}
2	May 1999	150	577.25	644.07	1.56×10^{-2}
3	October 2000	100	668.55	737.18	1.79×10^{-2}
4	October 2011	100	84.99	87.13	2.12×10^{-3}
5	August 2007	150	379.18	403.39	9.80×10^{-3}

[a] Annual probability of flood (π_f) is the inverse of the return periods.
[b] WSL Storm Damage Database of Switzerland.
[c] We convert the nominal damage by using the Consumer Price index reported by Federal Statistical Office (2018). Details are in Eq. (1.1).
[d] Benchmark share of vulnerable capital stock is from Röthlisberger, Zischg, and Keiler (2017).

The rainfall and the large volume of snow caused floods in the region Bernese Oberland in Canton Bern, Canton Valais and Canton Glarus. The frequency of heavy precipitation is increased due to climate change (Milly et al., 2002; CH2018, 2018).

Flood in August 2007

The intense rainfall between the 8th and 9th of August 2007 caused a rapid increase in the discharge in the river Birs and Aare. Not only was the flooding process, but also the catchments, were very similar to those of the 2005 flood. Early warning systems as well as weather regulation services, which had been installed after the flood in 2005, worked correctly (NZZ, 2007a,b).

I collect the nominal economic damage recorded in the flooded date and convert them to real terms in the benchmark year by using the Consumer Price index (Federal Statistical Office, 2018). I assume that the economic damage can be understood as direct depreciation on vulnerable capital stock. In 2008, there are 4.12×10^4 CHF million of capital stock in total. Röthlisberger, Zischg, and Keiler (2017) overlay empirical information about buildings and inhabitants on a map about flood-prone areas and report that 20.4% of capital stock is installed in vulnerable regions in Switzerland. I compute the damage scale of each flood listed in Table 1.1 by using the following simple formula:

$$\Delta_t^O = \frac{\text{Nominal damage}_t \times \frac{CPI_{2008}}{CPI_t}}{\underbrace{4.12 \times 10^4}_{\text{Capital in 2008}} \times \underbrace{0.204}_{\text{Share of vulnerable capital stock}}} \quad \forall i \in \{2005, 1999, 2000, 2011, 2007\} \tag{1.1}$$

Table 1.2 – Highly aggregated domestic production sectors

Sector	Abbreviation in GENESwIS
Agriculture	AGR
Industry	IND
Energy	ENE
Service	SRV
Transport	TRS
Housing and real estate services	HOU

1.3 Methodology

1.3.1 The GENESwIS model

GENESwIS, a dynamic computational general equilibrium model for the Swiss economy, was first introduced in Vöhringer (2012), and since then it has been widely employed in climate policy simulations.[3] GENESwIS assumes a perfect foresight and risk-averse representative agent who solves an intertemporal optimization problem subject to the budget constraint. Similar to other perfect foresight CGE models, such as FL-EPPA model (for details, see Babiker et al. (2009)), GENESwIS is part of the family of the standard Ramsey growth model. GENESwIS is calibrated based on the Swiss Input-Output table in 2008 and has a flexible sectoral aggregation. I highly aggregate domestic production sectors compared to the original version in Vöhringer (2012) mainly to stabilize a solving process and to finalize Monte-Carlo simulations within reasonable computational expense. Table 1.2 presents the adopted sectoral aggregation in this study. Besides the representative household, the model has a government whose role is to equate tax revenue and expenses for public goods provision. The time horizon is from 2010 to 2100 and I assume 5 year time steps.

In numerical CGE simulations, the model is often represented with a market equilibrium where the representative household aims to maximize his utility, with several cost-minimizing firms and the government interact with each other (Rutherford, 1999). Rutherford (1995) characterizes the equilibrium conditions with the following three classes of non-linear equations: the zero-profit conditions, the market-clearance conditions and the income balance. These three classes of equations typically take the form of a mixed complementarity program (MCP). GENESwIS is developed on the General Algebraic Modeling System (GAMS) modeling language and solved numerically by employing the PATH solver (Ferris and Munson, 2000; Rosenthal, 2013). Furthermore, the coding is greatly facilitated by the support of MPSGE, which is a subsystem of GAMS (Rutherford, 1999).

In the following sections, I briefly discuss the zero profit conditions, the market clearance and the income balance. Instead of showing full algebra, I present the so-called nested constant elasticity of substitution (CES) trees. The nested CES tree can graphically illustrate

[3] GENESwIS is classified into a *policy evaluation model* according to the discussion by IPCC (2001).

the structure of the model. Full algebra is deferred to Section A.1.

1.3.2 Vulnerable and non-vulnerable capital stock

I classify the existing Swiss production capital stock into two categories, depending on the vulnerability to uncertain but large flood shocks: vulnerable and non-vulnerable capital stock. On the one hand, the vulnerable capital stock is installed in the flood-vulnerable region and faces the risk of massive depreciation when a flood occurs. On the other hand, the non-vulnerable capital stock is accumulated in the flood non-vulnerable region; thus, it is a risk-free capital. Both capital stocks are productive and substitute goods.

Stern (2016b, Chapter 4) claims that the economic damages are generally introduced as a negative impact on the production function or a direct depreciation on the capital stock. In this section, I adopt the latter approach. Stochastic element $\tilde{\Delta}_f^O$ is introduced in the law of motion of the vulnerable capital stock:

$$KV_{t+1} = \left(1 - \delta - \tilde{\Delta}_f^O\right) KV_t + I_t^{KV} \quad \text{where } \tilde{\Delta}_f^O = \begin{cases} \Delta_f^O & \text{with probability } \pi_f \\ 0 & \text{with probability } \left(1 - \pi_f\right) \end{cases} \tag{1.2}$$

Note that δ stands for the annual depreciation rate. Δ_f^O represents an *observed* damage scale of flood f, which the representative agent in the model cannot correctly observe. Table 1.1 summarizes the estimated empirical damage rate and annual probability for the selected floods.

Non-vulnerable capital stock is risk-free capital, and therefore, it follows the standard law of motion:

$$KNV_{t+1} = (1 - \delta)\, KNV_t + I_t^{KNV} \tag{1.3}$$

1.3.3 Spatial adaptation

Spatial adaptation is one of the most popular and has been widely considered adaptation measures to flooding in the literature of hydrology; for instance, see Koks et al. (2014). In this study I define the spatial adaptation by classifying the existing Swiss production capital stock into two types, depending on their vulnerability to uncertain significant floods. More details about the vulnerable and non-vulnerable capital stock can be found in Section 1.3.2. When an economic agent realizes the risk of great floods in the future, he would invest more in non-vulnerable capital stock to prepare for future uncertainty. If the agent underestimates the risk of future flooding, he allocates more investment to vulnerable capital stock that exhibits higher marginal productivity than non-vulnerable capital stock. Spatial adaptation is one form of stock adaptation, and intertemporal investment decision-making between vulnerable and non-vulnerable capital stock is central for spatial adaptation.

1.3.4 Uncertainty treatment

The stochastic element in this study is in the uncertain flood shocks, and it is introduced in the law of motion of vulnerable capital stock as presented in Eq. (1.2). To make the model tractable, instead of developing a stochastic model, I replace the random element $\tilde{\Delta}_f^O$ by a deterministic parameter Δ_f^S. Δ_f^S is an agent's *subjectively* formed expectation for the uncertain flood f.[4] For instance, if an economic agent correctly expects uncertain flood f, his subjective expectation is:

$$\Delta_f^S = \mathbb{E}\left[\tilde{\Delta}_f^O\right] = \Delta_f^O \times \pi_f \tag{1.4}$$

It is equivalent to having a *certainty-equivalent deterministic* model. The choice of Δ_f^S is arbitrary for the agent. However, the level of Δ_f^S represents the agent's expectation for the risk of uncertain floods. I assume the agent is risk-averse. Here I aim to define his attitude toward uncertain floods by measuring how much his subjective expectation deviates from the certainty-equivalent level. The discussions around the risk attitude lie outside the scope of this paper. Subjective belief can be interrupted in the following manner:

$$\begin{cases} \Delta_f^S > \mathbb{E}\left[\tilde{\Delta}_f^O\right] & \text{(Overestimate the risk of floods)} \\ \Delta_f^S = \mathbb{E}\left[\tilde{\Delta}_f^O\right] & \text{(Certainty-equivalent)} \\ \Delta_f^S < \mathbb{E}\left[\tilde{\Delta}_f^O\right] & \text{(Underestimate the risk of floods)} \end{cases} \tag{1.5}$$

Introducing the subjective expectation Δ_f^S has two advantages. First, the model is now deterministic, and this makes modeling much easier and more tractable. Second, the agent's belief of uncertain floods is represented in the choice of Δ_f^S. Natural disasters involve a high degree of uncertainty, and it is, in general, impossible to form a correct estimation. Economic agents have their own belief for future uncertainty, and it can be embedded in the choice of Δ_f^S. When the agent is more concerned about the risk of future floods, he saves more for the vulnerable capital stock by increasing the level of Δ_f^S.

The representative agent solves an intertemporal optimization problem based on his subjective expectation Δ_f^S. Specifically, the law of motion of vulnerable capital stock that the representative agent is concerned about concerns is given in Eq. (1.6).

$$KV_{t+1} = \left(1 - \delta - \Delta_f^S\right) KV_t + I_t^{KV} \tag{1.6}$$

The representative agent assumes that the vulnerable capital stock is more depreciated than the non-vulnerable capital stock. Why does a representative agent decide to build new capital stock in a place assumed to be vulnerable to natural hazards? Two effects motivate this

[4] There is no link with risk aversion. The representative economic agent determines subjective belief based on his available knowledge. When he is correctly informed, his subjective belief would be equal to the expected value. Subjective belief measures how much his belief deviates from the expected value.

observation. First, vulnerable and non-vulnerable capital stock are assumed to be substitutes in production functions. Second, in equilibrium, the marginal productivity of the vulnerable capital stock is higher than that of the non-vulnerable capital stock by Δ_f^S.[5] Vulnerable areas are in general more productive than non-vulnerable regions.

1.3.5 The nested CES structure

The nested CES structure represents the input-output structure of the target economy assuming a constant degree of substitutability. In the following sections, I present the so-called nested CES tree. More detailed algebra is deferred in Appendix A.

Zero-profit condition

For each domestic production sector s in Table 1.2, each firm produces a product by using capital, labor and intermediate inputs. The nested CES structure for the production sector s is provided in Figure 1.3. A nested constant elasticity of substitution (CES) function models the substitution effect between the factor and the intermediate inputs. σ_s^Y, σ_s^{KL} and σ_s^A define the elasticity of substitution for the corresponding nest in the sector s. Furthermore, in the top nest, the produced domestic good in sector s is transformed into a domestic usage with the price index $p_{s,t}^Y$ and export with the price index p_t^{FX} based on the constant elasticity of transformation (CET) function. The elasticity of transformation in sector s is given as η_s^Y.

Given the unit cost function $C_{s,t}^Y\left(r_t^{KV}, r_t^{KNV}, w_t, p_{i,t}^A\right)$ and the unit revenue function $R_{s,t}^Y\left(p_{s,t}^Y, p_t^{FX}\right)$, the following zero-profit condition for the domestic production sector $Y_{s,t}$ can be formulated as:

$$-\pi_{s,t}^Y = C_{s,t}^Y\left(r_t^{KV}, r_t^{KNV}, w_t, p_{i,t}^A\right) - R_{s,t}^Y\left(p_{s,t}^Y, p_t^{FX}\right) \geq 0 \quad \perp \quad Y_{s,t} \geq 0 \quad \forall s, t \tag{1.7}$$

where the notation $\pi_{s,t}^Y$ donates the unit profit function and the symbol $\perp$, throughout this chapter, indicates the complementarity slackness condition.

Adopting the Armington (1969) assumption, I treat domestic production and imports as imperfect substitutes. These two types of goods are combined based on the CES function, as Figure 1.4 illustrates, where the constant elasticity of substitution for good i is given as σ_i^A. Given the unit cost function $C_{i,t}^A$, the zero-profit condition for this production block is in Eq. (1.8), where $A_{i,t}$ is the Armington good production from sector i in time t:

$$-\pi_{i,t}^A = C_{i,t}^A\left(p_{i,t}^Y, p_t^{FX}\right) - p_{i,t}^A \geq 0 \quad \perp \quad A_{i,t} \geq 0 \quad \forall i, t \tag{1.8}$$

[5] It is a well-known result from a neoclassical growth model with physical and human capital, for instance, see Acemoglu (2009, Secion 10.4).

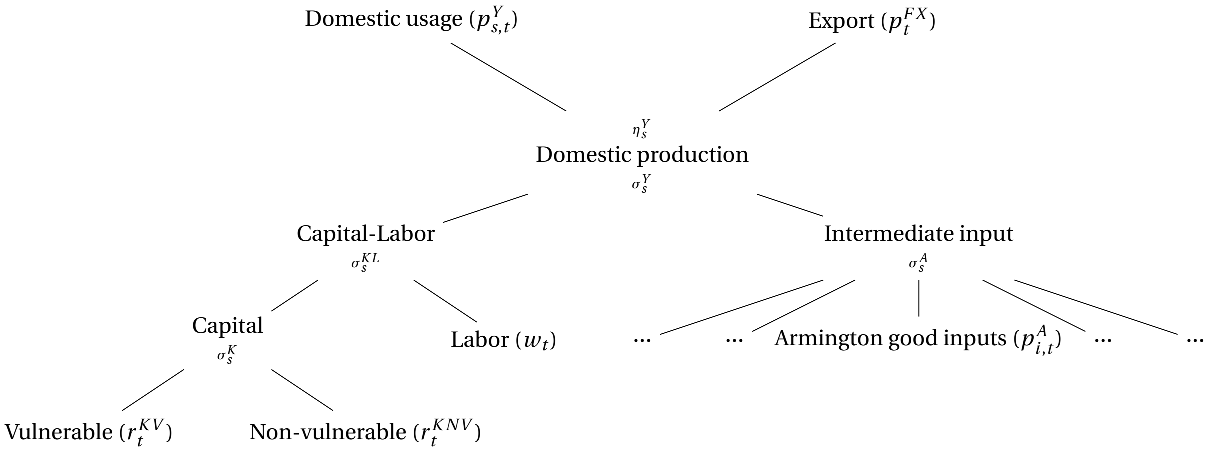


Figure 1.3 – Nested CES structure of the domestic production sector s

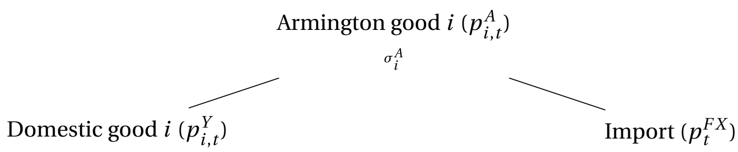


Figure 1.4 – Nested CES structure of the Armington good production i

Figure 1.5 presents the CES structure of consumption in which the representative household consumes leisure, goods and services based on the given constant elasticity of substitution σ^C. I assume $\sigma^C = \sigma^{CA} = 1$, which represents a Cobb-Douglas form of substitution. Defining the unit cost function $C^C\left(w_t, p^Y_{i,t}\right)$, the zero-profit condition for this nesting can be defined in Eq. (1.9):

$$-\pi^C_t = C^C_t\left(w_t, p^Y_{i,t}\right) - p^C_t \geq 0 \quad \perp \quad C_t \geq 0 \quad \forall t \tag{1.9}$$

The representative household solves an intertemporal optimization problem. I suppose a constant intertemporal elasticity of substitution (CIES) utility function for the welfare over

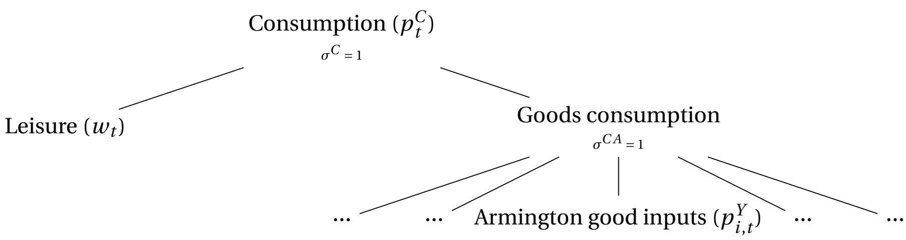


Figure 1.5 – Nested CES structure of the consumption

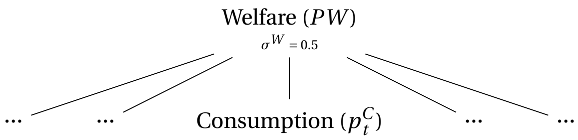


Figure 1.6 – Nested CES structure of the welfare

the whole computational time horizon.[6] The CES structure is displayed in Figure 1.6. I set the intertemporal elasticity of substitution σ^W equal to 0.5 (Babiker et al., 2009). The zero-profit condition of the block is provided in Eq. (1.10) where $C^W\left(p_t^C\right)$ is the unit cost function:

$$-\pi^W = C^W\left(p_t^C\right) - p^W \geq 0 \quad \perp \quad W \geq 0 \tag{1.10}$$

The zero-profit condition for the investment activities can be formulated taking into account intertemporal substitution.[7] The vulnerable capital stock faces the risk of significant depreciation when an uncertain flood occurs. The non-vulnerable capital stock is risk-free. The zero-profit condition for investment in vulnerable capital stock is presented in Eq. (1.11). The condition claims that the gross price of the vulnerable capital stock in the next period $t+1$ is equal or less than the unit cost of an aggregated investment $C_t^{IKV}\left(p_{i,t}^A\right)$:

$$-\pi_t^{IKV} = C_t^{IKV}\left(p_{i,t}^A\right) - p_{t+1}^{KV} \geq 0 \quad \perp \quad I_t^{KV} \geq 0 \tag{1.11}$$

The zero-profit condition for investment in non-vulnerable capital stock is in Eq. (1.12).

$$-\pi_t^{IKNV} = C_t^{IKNV}\left(p_{i,t}^A\right) - p_{t+1}^{KNV} \geq 0 \quad \perp \quad I_t^{KNV} \geq 0 \tag{1.12}$$

Eq. (1.13) presents the zero-profit condition for the accumulation of the vulnerable capital stock. The unit returns of the vulnerable capital stock plus the gross price of capital stock in period $t+1$ consider the annual depreciation δ and the subjective damage Δ_f^S. It is greater or equal to the price of the vulnerable capital stock in period t.

$$-\pi_t^{KV} = r_t^{KV} + \left(1 - \delta - \Delta_f^S\right) p_{t+1}^{KV} - p_t^{KV} \geq 0 \quad \perp \quad KV_t \geq 0 \tag{1.13}$$

Finally, the zero-profit condition for the accumulation of the non-vulnerable capital stock is

[6] CIES utility function is a monotonic transformation of a better known CRRA utility function. Both utility functions show the same intertemporal characteristics. Therefore, the Walrasian or Hicksian demand functions for both utility functions are the same. MPSGE directly handles the CIES utility function. I would discuss this point more closely in Section 1.5.1.

[7] Paltsev (2004) is pedagogic but provides thoughtful discussions about the zero-profit conditions for investment activities and capital accumulations.

found in Eq. (1.14).

$$\pi_t^{KNV} = r_t^{KNV} + (1-\delta)\, p_{t+1}^{KNV} - p_t^{KNV} \geq 0 \quad \perp \quad KNV_{s,t} \geq 0 \tag{1.14}$$

Market-clearance condition

The market-clearance condition claims that there is no excess demand in the market of product i, and the corresponding price $p_{i,t}$ is positive. If a market for good i exhibits an excess supply, the corresponding price is zero. Mathematically this is complementarity and takes the following form:

$$D_{i,t} - S_{i,t} \geq 0 \quad \perp \quad p_{i,t} \geq 0 \tag{1.15}$$

Compensated demands of each good $D_{i,t}$ are obtained from the corresponding expenditure function by applying Shephard's Lemma.

Income balance

The income balance condition must hold for each period. In every period t, the total expenditure is equal to total factor income and some transfers from the government.

1.3.6 Dynamic steady state with vulnerable and non-vulnerable capital stock

All dynamic CGE models need to be calibrated to replicate the dynamic steady state. In the dynamic steady state, all economic activities are on the balanced growth path.[8] In this section, I present the calibration strategy with vulnerable and non-vulnerable capital stock. My task here is to reasonably manipulate the empirical social accounting matrix (SAM) to support the dynamic steady state.

In the balanced growth path, vulnerable and non-vulnerable capital stocks have an exogenous growth rate g:

$$KV_{t+1} = \left(1+g\right) KV_t \tag{1.16}$$

$$KNV_{t+1} = \left(1+g\right) KNV_t \tag{1.17}$$

All the prices in period $t+1$ can be discounted to the present value with a given interest rate r:

$$p_{t+1} = \frac{p_t}{1+r} \tag{1.18}$$

I am revisiting the zero-profit conditions for two investments provided in Eqs. (1.13) and (1.14).

[8] For more details, see Paltsev (2004) who provides pedagogic and thoughtful treatments on this issue.

Assumed that the zero-profit conditions are strictly binding, the investment activities are positive ($I_t^{KV}, I_t^{KNV} \geq 0$):[9]

$$p_t^{KV} - \left(1 - \delta - \Delta_f^S\right) p_{t+1}^{KV} - r_t^{KV} = 0 \tag{1.19}$$

$$p_t^{KNV} - (1 - \delta)\, p_{t+1}^{KNV} - r_t^{KNV} = 0 \tag{1.20}$$

By applying Eq. (1.18) to Eqs. (1.19) and (1.20), the rental price of each capital stock is given in Eqs. (1.21) and (1.22):

$$r_t^{KV} = \left(\delta + \Delta_f^S + r\right) \frac{p_t^{KV}}{1+r} = \left(\delta + \Delta_f^S + r\right) p_t \tag{1.21}$$

$$r_t^{KNV} = (\delta + r)\, \frac{p_t^{KNV}}{1+r} = (\delta + r)\, p_t \tag{1.22}$$

If the market is competitive and all production technologies exhibit constant returns to scale, the marginal productivity of production function F of each capital stock is equal to its rental price:

$$F_{KV,t} = \left(\delta + \Delta_f^S + r\right) p_t \tag{1.23}$$

$$F_{KNV,t} = (\delta + r)\, p_t \tag{1.24}$$

Eqs. (1.23) and (1.24) argue that the vulnerable capital stock is more productive than the non-vulnerable capital stock by Δ_f^S.

Finally, from Eqs. (1.6) and (1.16) for the vulnerable capital stock and Eqs. (1.3) and (1.17) for the non-vulnerable capital stock, in the dynamic steady state, I confirm that each investment and each capital stock satisfy the following conditions:

$$I_t^{KV} = \left(\delta + \Delta_f^S + g\right) KV_t \tag{1.25}$$

$$I_t^{KNV} = \left(\delta + g\right) KNV_t \tag{1.26}$$

As I have mentioned above, the GENESwIS model needs to be calibrated based on the existing 2008 Swiss Input-Output table that represents the benchmark static economy. In general, the raw Input-Output table does not support a dynamic steady state under the assumed rates for growth, interest and capital depreciation. It is necessary to manipulate the Swiss Input-Output table to have the dynamic steady state with vulnerable and non-vulnerable capital stock.

The Swiss Input-Output table in 2008 reports the total *value* of the capital stock VK_{2008}. Without loss of generality, I can suppose that all prices in the benchmark year are equal to one $p_{2008} = 1$. Therefore, I can regard that all of the numbers in the Input-Output table are

[9] In CGE analyses, we usually analyze the target economy around the dynamic steady state, which excludes the probability to have a zero investment (a corner solution).

a unit-free *amount*. Röthlisberger, Zischg, and Keiler (2017) overlay the building footprints and the map of flood-prone areas within a GIS and estimate 20.4% buildings are located in flood-vulnerable regions in Switzerland. I use this value as a benchmark share of vulnerable capital stock. Supposing this share, the benchmark investment demands of each capital stock is:

$$I_{2008}^{KV} = \sum_s I_{s,2008}^{KV} = \sum_s \frac{\delta + \Delta_f^S + g}{\delta + \Delta^S + r} KV_{s,2008} \quad (1.27)$$

$$I_{2008}^{KNV} = \sum_s I_{s,2008}^{KNV} = \sum_s \frac{\delta + g}{\delta + r} KNV_{s,2008} \quad (1.28)$$

Due to Eq. (1.27), there is extra investment demand. I subtract this extra investment demand from the consumption to balance the Input-Output table again.[10]

1.3.7 Hazard myopic approach

Natural hazards involve a high degree of uncertainty. In this context, an economic agent with perfect-foresight is a strong assumption. Recursive dynamic modeling with a complete myopic economic agent is an alternative. However, in this latter approach, the saving preference is exogenously given. Spatial adaptation is stock adaptation; therefore, investment decisions should be intertemporal and endogenized. Given two considerations, I propose a new modeling approach: namely *hazard myopia*.

A hazard myopic economic agent has perfect foresight of all macroeconomic conditions, as in a perfect-foresight model. However, he is entirely myopic when it comes to uncertain natural shocks that are an exogenous process.

Numerical economic models, in general, does not deal with an infinite-time horizon; thus, a terminal condition is introduced to replicate an infinite time horizon following Lau, Pahlke, and Rutherford (2002). Furthermore, I extend a terminal period from T to T' to mitigate the possible effect of the terminal condition. I retrieve and report simulation results from period 0 to T in the following section as illustrated in Figure 1.7.

Law of motion of vulnerable capital stock is central in the hazard myopic simulation. Instead of observing correct information about floods, he subjectively forms expectations of uncertain floods, Δ_f^S. As a perfect-foresight economic agent does, the hazard myopic representative agent solves an intertemporal optimization model from period t to the terminal period T, having a law of motion of a vulnerable capital stock based on Δ_f^S as in Eq. (1.6).[11] Between

[10] This treatment is reasonable especially for the Swiss economy, which is highly capital dependent.

[11] This is a potential drawback of the hazard myopia. In a perfect CGE modeling, the capital stock in the first year is exogenously given by the benchmark data but the capital stock in the subsequent periods is endogenously defined by solving an intertemporal optimization problem. Another source of the first year effect stems from the fact that policies are applied from the second periods. An agent with perfect foresight consumes more in the first period due to the anticipation of future floods. I treat the first year different from the other periods in these two

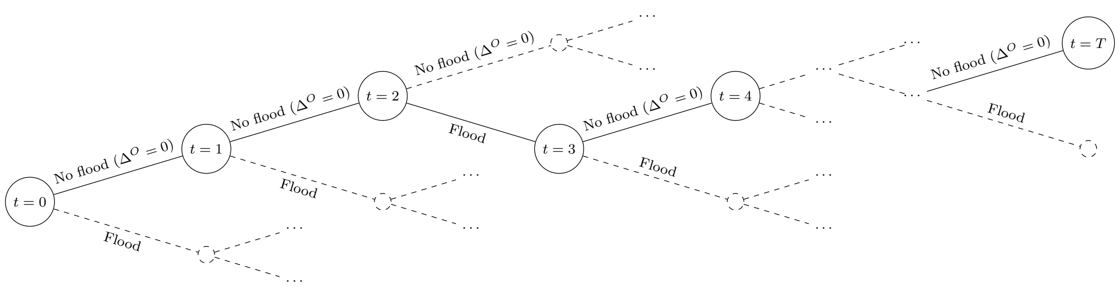


Figure 1.7 – Schematic image of a hazard myopic simulation

period t and $t+1$, both capital stocks are updated like a recursive approach. Vulnerable capital stock follows Eq. (1.2) and non-vulnerable capital stock follows Eq. (1.3). When a flood f occurs with a probability π_f, the observed damage scale Δ_f^O is realized. When there is no flood, there is no damage to the vulnerable capital stock.

Figure 1.7 presents a schematic overview of the hazard myopic simulation. In a hazard myopic simulation, I need to solve in total T intertemporal optimization problems. I retrieve the optimized decisions from each sub-model to derive the whole simulation path. For instance, in Figure 1.7, I solve at first an intertemporal optimization model from period $t = 0$ to $t = T$. Both capital stocks are updated recursively. The vulnerable capital stock is updated with an observed damage scale zero (no flood). Similar to the period $t = 2$ and $t = 3$, the hazard myopic economic agent solves an intertemporal model from period $t = 2$ to the terminal period T. Both capital stocks are updated, but the vulnerable capital stock is largely damaged by Δ_f^O.

The timing of flood f is assumed to follow a Poisson process. A Poisson arrival rate is an inverse of return periods of a target flood. The Poisson process has been employed in the literature of economics to model a random event, for instance see Barro (2006), Posch and Trimborn (2013) and Bretschger and Vinogradova (2017), among others.

I address uncertainty around flood hazards by a large number of Monte-Carlo simulations. In each simulation run, the timing of floods is exogenously drawn from the corresponding Poisson process and exogenously given to the deterministic CGE model. During flooding periods, the vulnerable capital stock is further depreciated by the observed damage scale Δ_f^O. I solve the set of intertemporal optimization models and retrieve the price and activity indexes to draw simulation paths for the whole computational periods. The solution algorithm is provided in Algorithm 1.

1.4 Result

In this section, I present the simulation results with the GENESwIS model. I first check the credibility of the hazard myopic approach with the stylized flood in Section 1.4.1. In

points.

Algorithm 1: Monte-Carlo simulation of a hazard myopic agent with vulnerable and non-vulnerable capital stocks

Initialization:

Set the maximum number of Monte-Carlo iterations, $\overline{n}$.
Set the computational time horizon from t_0 to T.
Define the observed damage scale Δ_f^O and the probability π_f.
The representative household subjectively forms an expectation of a damage scale Δ_f^S on the vulnerable capital stock.
Adjust the Input-Output table based on Δ_f^S and calibrate the model for the dynamic steady state.
Define the flooding time periods t^F from the corresponding Poisson process.
for $n \leq \overline{n}$ **do**
  Solve the model from t_0 to T considering Δ_f^S.
  for $t \in \{t_0, t_1, \cdots T-1\}$ **do**
    Chop the time horizon from $t+1$ to T.
    Update the time dependent parameters.
    Non-vulnerable capital stock in the next period is updated by
      $KNV_{t+1} = (1-\delta)\, KNV_t + I_t^{KNV}$
    if $t = t^F$, **then**
      Vulnerable capital stock in the next period is updated by
        $KV_{t+1} = \left(1-\delta-\Delta_f^O\right) KV_t + I_t^{KV}$
    else
      Vulnerable capital stock in the next period is updated by
        $KV_{t+1} = (1-\delta)\, KV_t + I_t^{KV}$
    Solve the model with updated stock variables from $t+1$ to T.
  Integrate all simulation paths from t_0 to T.
  if $n \leq \overline{n}$, **then**
    Initialize the model and go to the next Monte-Carlo iteration $n+1$.
  else
    break
Retrieve all the simulation paths.
Each price and activity index have $T \times \overline{n}$ dimensions.

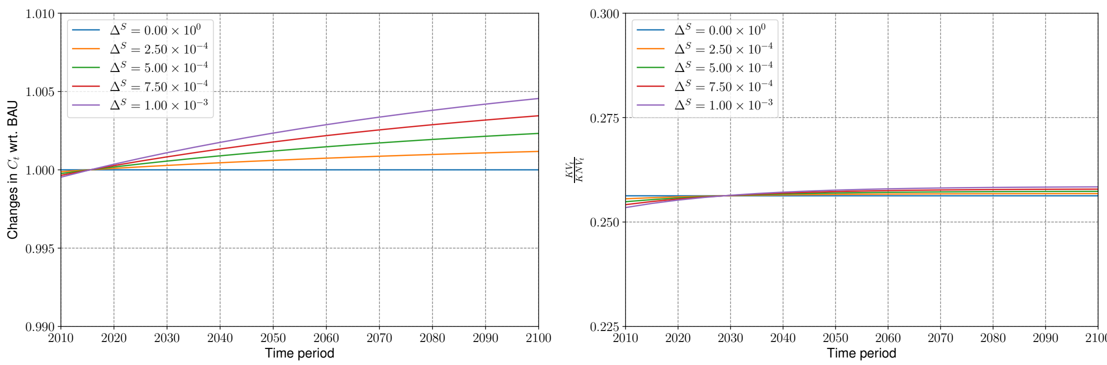
Section 1.4.2, I empirically analyze five floods in Switzerland. Detailed information about the floods is summarized in Table 1.1.

1.4.1 Test simulation with a stylized flood

This test simulation aims to check the credibility of my advanced modeling algorithm, hazard myopia. I assume a stylized flood, which information is in Table 1.3, for testing purposes. Following the standard CGE simulations, I measure a deviation from the business as usual (BAU) case. The BAU refers to the case where (1) no flood occurs throughout computational

Table 1.3 – Stylized flood to test the hazard myopic approach

Return periods	Poisson rate λ	Actual damage Δ^O	Expected value $\mathbb{E}\left[\tilde{\Delta}^O\right]$
100	0.01	0.05	5×10^{-4}



(a) Consumption

(b) Capital ratio

Figure 1.8 – Consumption and capital ratio path without exogenous floods

periods and (2) the representative agents ignore the risk of floods ($\Delta^S = 0$).[12]

The primary goal of this test is to check general modeling behavior when the subjectively formed expectation Δ^S changes. I focus on consumption and the capital ratio of vulnerable to non-vulnerable capital stock.

No flood scenario

The first simulation starts from the simplest case where there are no floods throughout the computational periods. The vulnerable capital stock is updated by Eq. (1.2) in which Δ_f^O is equal to zero between every period. I change the subjectively formed damage expectation Δ^S from zero to $\Delta^S = 1.0 \times 10^{-3}$. Note that the case where $\Delta^S = 5 \times 10^{-4}$ represents the certainty-equivalent deterministic model. The representative agent has the right expectation for the stylized flood.

Figure 1.8a plots changes in consumption with respect to the BAU case. The higher the subjective expectations the representative agent forms, the smaller the consumption demand in the first period becomes. It is due to the benchmark calibration. When the representative agent is concerned more about the risk of future floods, he subjectively forms a higher expectation for the uncertainty. It becomes an additional motive to save more for the vulnerable capital stock. The consumption demand decreases concerning the strictly binding resource constraints.

[12] In general, the BAU scenario includes more detailed assumptions such as the growth rate of population and GDP. However, I prefer to have a simple BAU scenario for my testing purposes.

Moreover, the higher the subjective expectations the representative agent forms, the higher the consumption profile becomes in the long run. It is due to two reasons. Firstly, recall that in equilibrium the difference between the marginal productivities of the vulnerable and non-vulnerable capital stock are equal to the subjectively formed expectation Δ^S. Having a higher subjective expectation is identical to assuming higher marginal productivity for the vulnerable capital stock. Secondly, the agent's intertemporal optimization problem is based on $\Delta^S \geq 0$. I update both capital stocks based on the law of motion with the observed damage scale Eq. (1.2) for the vulnerable capital and Eq. (1.3) for the non-vulnerable. When there is no flood, there is an excess investment in the vulnerable capital stock, since the representative agent assumes that the vulnerable capital stock is depreciated more by Δ^S.

Figure 1.8 presents the capital ratio of the vulnerable to the non-vulnerable capital stock. To aid the reader to interpolate the capital ratio:

$$\frac{d}{dt}\left(\frac{KV_t}{KNV_t}\right) = \frac{KV_t}{KNV_t}\left(\frac{\dot{KV}_t}{KV_t} - \frac{\dot{KNV}_t}{KNV_t}\right) \tag{1.29}$$

where $\dot{KV}_t$ and $\dot{KNV}_t$ represent the time derivative of each variable, and $\frac{\dot{KV}_t}{KV_t}$ and $\frac{\dot{KNV}_t}{KNV_t}$ measure the growth rate of each capital stock. As $KV_t, KNV_t > 0,\ \forall t$, if the slope of $\frac{KV_t}{KNV_t}$ shows an upward trend, the growth rate of the vulnerable capital stock exceeds that of non-vulnerable capital stock, and vice versa.

There are two revelations of Figure 1.8b. First, the higher the subjective expectation the agent forms, the lower the benchmark capital ratio of the vulnerable to the non-vulnerable capital stock is. The representative agent relies more on the risky capital stock when he is subjectively concerned about future risk. Second, the growth rate of the vulnerable capital stock always exceeds that of the non-vulnerable capital stock. The reason is the same as why the consumption path is elevated with a higher subjective expectation.

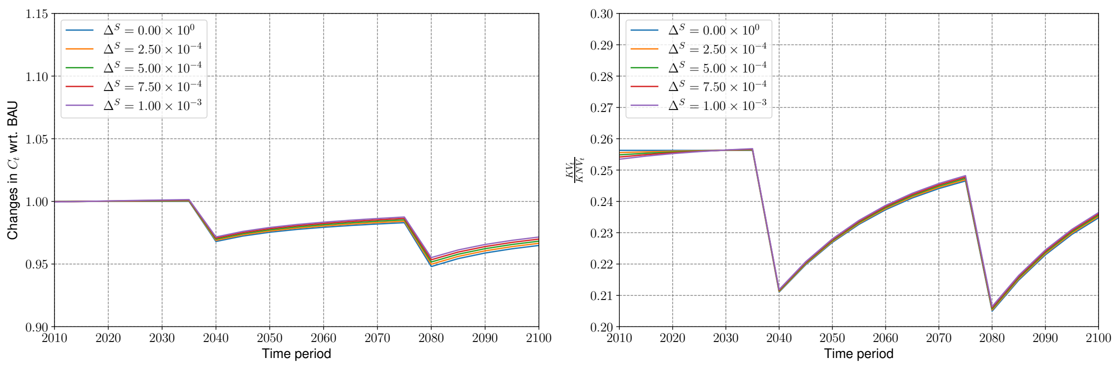
Two deterministic floods in 2040 and 2080

I assume that floods occur in 2040 and 2080.[13] As the frequency of the stylized flood is every 100 years, it is unlikely that they would be 40 years apart but worth testing to check the robustness of our method.

When there is a flood, the vulnerable capital stock is updated by Eq. (1.2) with $\Delta^O = 0.05$. I check the sensitivity of the Δ^S parameter by changing Δ^S from 0 to 1.0×10^{-3}.

Figure 1.9a shows the consumption profile. If a flood occurs, there is a great reduction in consumption. The reason is twofold. On the one hand, the economic outcomes from all production sectors are largely impaired, since the vulnerable capital stock has significant damage. On the other hand, a large amount of investment is allocated to vulnerable capital

[13] A flood occurs between 2035 and 2040, and 2075 and 2080 respectively since I adopted a five years time step.



(a) Consumption

(b) Capital ratio

Figure 1.9 – Consumption and capital ratio path during two exogenous floods

stock. This saving behavior largely constrains the available resource for consumption.

Figure 1.9b shows the capital ratio of the vulnerable capital stock to the non-vulnerable capital stock. When a flood occurs, the ratio drops. However, after flooding, it shows an upward trend. The growth rate of the vulnerable capital stock exceeds that of non-vulnerable capital stock. The representative agent allocates more investment in the vulnerable capital stock to recover from the damage of flooding.

It is worth noting that having a higher subjective risk expectation leads to an increase in welfare as shown in Figures 1.8a and 1.9a. Furthermore, the ratio of the vulnerable capital to the non-vulnerable capital stock is elevated. The representative agent realizes the risk of flooding but invests his saving mainly in vulnerable capital stock, and as a result, the economy becomes more flood vulnerable.

Monte-Carlo analyses

I perform 1,000 Monte-Carlo simulations with a stylized flood to check the stability of the hazard myopic simulation. I change the subjectively formed expectation Δ^S from 0 to 7.5×10^{-4} where the case $\Delta^S = 5.0 \times 10^{-4}$ replicates the certainty-equivalent case. The Monte-Carlo simulation algorithm is presented in Algorithm 1. Hereunder, I present the distribution with a focus on consumption.

Figure 1.10 summarizes the Monte-Carlo simulation results with a stylized flood. The mean of the Monte-Carlo simulated consumption paths, as well as the two worst consumption paths, are presented. The latter is provided mainly to offer insights behind Monte-Carlo simulations. Figure 1.10 is similar to what I have observed in Figures 1.8a and 1.9a. The higher subjective expectation the representative agent forms, the higher consumption profile the representative agent has.

In Figure 1.11 I present the 10%, 25%, 50%, 75% and 90% quantiles as well as a range of possible

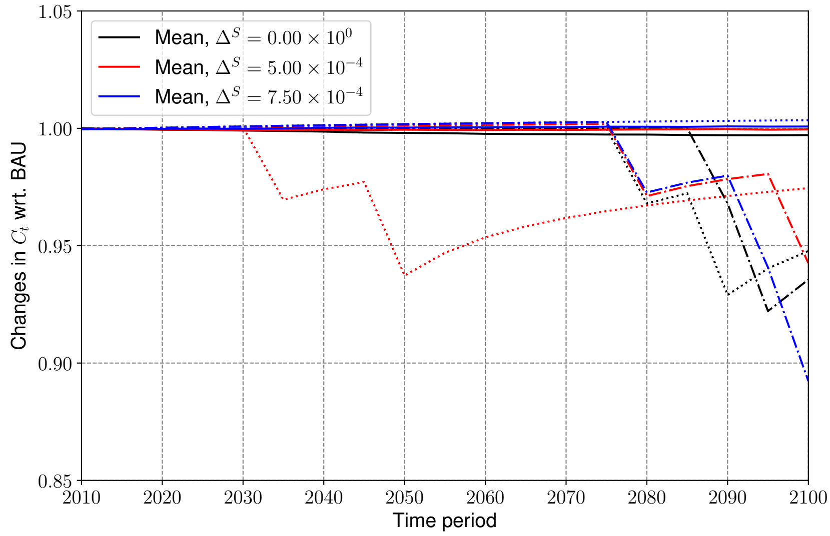


Figure 1.10 – The mean of consumption for the stylized flood. Dashed trajectories show the possible two worst paths.

sample paths (1% to 99%) in the gray cloud. I change the subjectively formed expectation Δ^S. Figure 1.11 shows why it is difficult to estimate the damage caused by rare disasters by using a (certainty-equivalent) deterministic model. The upper four quantiles (25%, 50%, 75% and 90%) are overlapped. It illustrates that, in most simulated paths, there is no flood. Recalling that the probability of flood is 1% per year. In other words, simulated paths are possibly distributed as in the gray cloud. These simulated paths are equivalent to the upper range of the distribution with at least a 75% probability.

My revelation from these observations is that the risk of a stylized flood in the future is not big enough for the representative agent to invest his saving in non-vulnerable capital stock. He invests more in vulnerable capital stock that exhibits higher marginal productivity than a non-vulnerable capital stock in the equilibrium. He subjectively forms higher expectations of the vulnerable capital stock, not as a precautionary concern about the risk of floods, but to increase the marginal productivity of risky capital - vulnerable capital stock.

1.4.2 Simulations with empirical floods in Switzerland

In this section, I apply the proposed approach, the hazard myopia, to the empirical Swiss floods in Table 1.1. Unfortunately, the main findings are similar to the results in Section 1.4.1. This is why I only show the simulated results with the worst flood in 2005 and the flood in 2007. The other floods in 1999, 2000 and 2011 are deferred in Appendix B.

I present the deviation from the BAU case. In the BAU case, there is no flood, and the representative agent completely ignores the risk of a flood f, $\Delta^S_f = 0$. In general, the BAU case in the CGE analyses includes more detailed assumptions, such as the exogenous growth rate of the

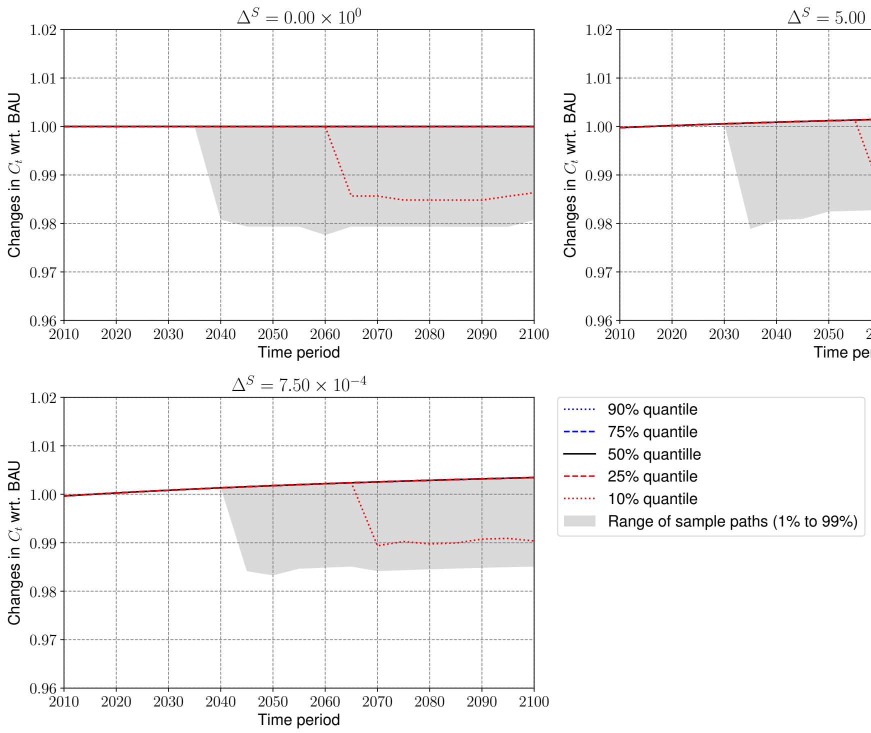


Figure 1.11 – Monte-Carlo simulated results for the stylized flood when the subjectively formed depreciation rate Δ^S is changed.

population and GDP. In the sections below, I set the BAU case as the dynamic steady state to keep the model simple.

Flood in August 2005

The flood in August 2005 was the most costly in term of insured economic damages and is considered a 150 years great flood. Nominal damage was around 3 billion CHF, and seven people were killed. The damage expectation that the representative agent subjectively forms of this event is equal to the expected value of the flood in 2005. It is a certainty-equivalent deterministic model:

$$\Delta^S_{2005} = \mathbb{E}\left[\tilde{\Delta}^O_{2005}\right] = 7.54 \times 10^{-2} \times \frac{1}{150} \approx 5.03 \times 10^{-4} \tag{1.30}$$

I present three scenarios. In the first scenario, the representative agent completely neglects the risk of flooding $\Delta^S_{2005} = 0$. In the second scenario, the agent correctly observes the risk of flooding and his subjective expectation is equal to the expected value $\Delta^S_{2005} = \mathbb{E}\left[\tilde{\Delta}^O_{2005}\right] = 5.03 \times 10^{-4}$. In the third scenario, I assume that the representative agent is more concerned about the risk of future floods and he sets $\Delta^S_{2005} = \mathbb{E}\left[\tilde{\Delta}^O_{2005}\right] \times 1.5 = 7.54 \times 10^{-4}$.

Figure 1.12 shows the mean of 1,000 Monte-Carlo simulated consumption paths. The dashed

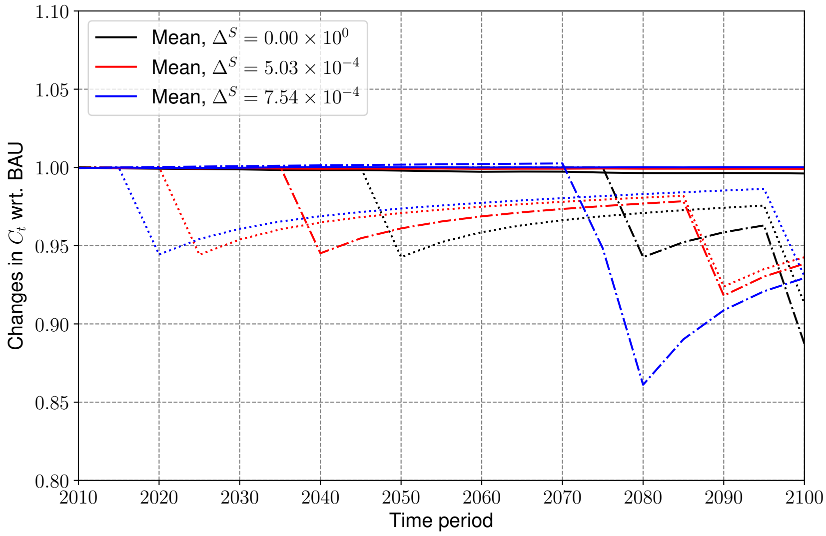


Figure 1.12 – The mean consumption for the flood occurred in 2005. Dashed trajectories show the possible two worst paths.

trajectories show the possible worst cases. The main trend in the mean is the same as in the case with the stylized flood. Forming a higher subjective expectation improves welfare. When I focus on the possible worst case, the number of floods is at most two, given that the flood in 2005 has a return period of 150 years. Note that once a flood occurs, the consumption path cannot be returned to the original. Floods completely alter a system; however, the risk of having a post-flooded economy is too small for the economic agent to allocate more investment in non-vulnerable capital stock, since the probability of flooding is small enough.

Figure 1.13 shows the distribution of Monte-Carlo simulated consumption paths. Upper quantiles (90%, 75%, 50% and 25%) are overlapped, given that the probability of the flood in 2005 is $\frac{1}{150}$. 1% to 99% of the consumption paths are distributed within the gray cloud as illustrated; however, the simulated paths converge on the upper bound with a probability of more than 75% (no flood).

1.5 Discussion

The main simulation exercises presented in Section 1.4 offer a peculiar policy recommendation. Having a higher subjective depreciation rate for the vulnerable capital stock is intended to replicate a situation where the risk of future floods is a more significant concern of the representative agent. I suppose that the higher depreciation rate the representative agent subjectively forms, the more investment he would allocate to the capital stock and the less is his consumption. However, the simulation results contradict my assumption. The higher depreciation rate the representative agent subjectively forms, the larger consumption profile the representative agent achieves. Furthermore, the vulnerable capital stock is more rapidly

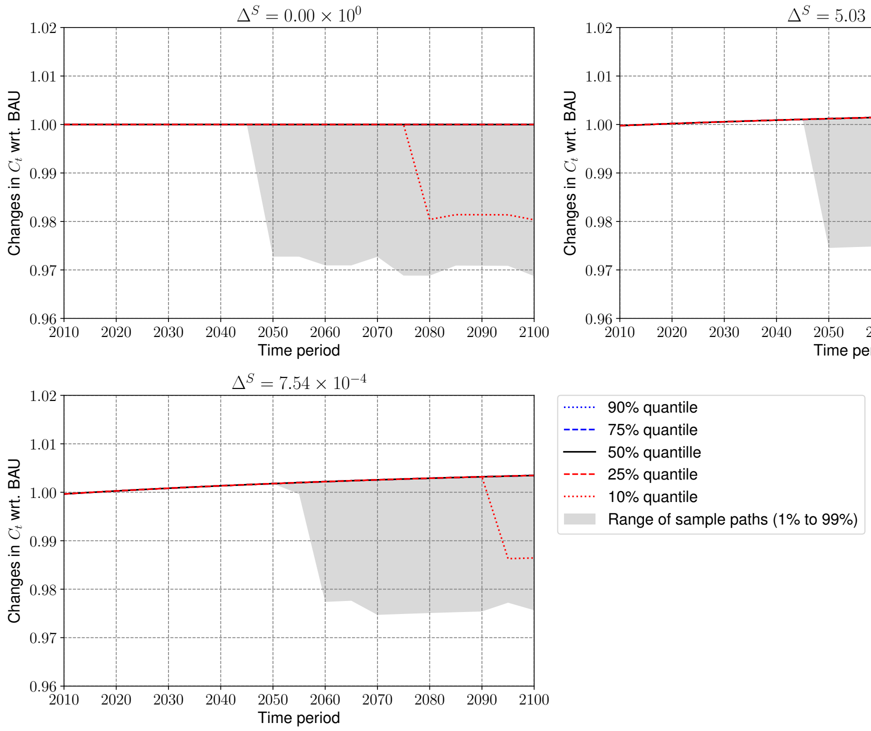


Figure 1.13 – Monte-Carlo simulated results for the flood occurred in 2005 when the subjectively formed depreciation rate Δ^S is changed.

accumulated than the non-vulnerable capital stock. These two findings suggest that, in the simulation model, the risk of floods is not significant enough for the representative agent to invest his saving in risk-free but less productive capital stock, non-vulnerable capital stock, even though I assume a risk-averse agent. The simulation results contradict the real Swiss situation.[14] It is also a problematic issue, especially for a policy evaluation model.

I aim to investigate this issue closely; however, as far as I have noticed, there is no literature from the CGE community that discusses rare but catastrophic events under uncertainty. When I turn my attention to the literature of IAMs, some advanced IAMs correctly internalize tipping elements in their framework. IAMs and dynamic CGE models start from the Ramsey model and rely on many common economic assumptions, though their structure and the modeling approaches differ. The IAM literature offers a good starting point if there is no reliable literature directly employed in CGE models.[15]

[14] Swiss Cantons learned from the flood in 2005 and a protective structure, as well as an alarm system, had been installed (NZZ, 2007a). Those systems forewarned the inhabitants in 2007 moreover, contributed to minimizing flood damage (NZZ, 2007b).

[15] Some early literature such as Mastrandrea and Schneider (2001), Yohe, Andronova, and Schlesinger (2004), Keller, Bolker, and Bradford (2004) and Nordhaus (2010) include the damage from tipping elements such as the collapse of the North Atlantic thermohaline circulation (THC) and the sea level rise. Their major approach is to increase the curvature of the damage function, and all of them are deterministic. Since CGE models do not include a damage module; these studies are not suitable to support my arguments here.

Cai, Judd, and Lontzek (2012) is the first to introduce a dynamic stochastic integrated model of climate and economy (DSICE). They introduce a stochastic productivity shock and tipping elements to the deterministic DICE model (Nordhaus, 2008). Lemoine and Traeger (2014) is based on the DICE model but introduces stochastic tipping points. One of their contributions is to differentiate the pre-tipping and post-tipping value function and show that the tipping points increase the level of the optimal carbon tax. Cai, Judd, and Lontzek (2015) extends Cai, Judd, and Lontzek (2012) to discuss a recursive utility function and allows for a more detailed climate module.

Table 1.4 summarizes the significant difference between the existing IAMs with tipping points and my study. In the following sections, I first compare the existing models with my study. Then I aim to propose how existing CGE models could be extended to handle rare but catastrophic events correctly.

Table 1.4 – Comparison between IAMs with tipping points and my study

Reference	Type	Deterministic or stochastic	Utility function[b]	Production function	Discount factor	Time horizon	Numerical methods
Cai, Judd, and Lontzek (2012)[a]	IAM	Stochastic	CRRA ($\eta = 2$)	Cobb-Douglas	≈ 0.985	Infinite	VFI[d] and Chebyshev polynomial
Lemoine and Traeger (2014)	IAM	Stochastic	CRRA ($\eta = 2$)	Cobb-Douglas	Time dependent effective discount factor[c]	Infinite	VFI and Chebyshev polynomial
Cai, Judd, and Lontzek (2015)	IAM	Stochastic	Epstein-Zin ($\eta = 10$ and $\sigma = 1.5$)	Cobb-Douglas	0.985	Infinite	VFI and Chebyshev polynomial
My study	CGE	Deterministic	CIES ($\eta = \frac{1}{\sigma} = 2$)	CES	≈ 0.957	Finite[e]	MPSGE/MCP

[a] Cai, Judd, and Lontzek (2012) presents the DCICE model, which is a stochastic version of the DICE model (Nordhaus, 2008). The DCICE model has been used by the authors for instance in Lontzek, Cai, and Judd (2012) and Lontzek et al. (2015).

[b] η is the degree of risk-aversion and σ is the intertemporal elasticity of substitution.

[c] Detail discussions is in Lemoine and Traeger (2014, p. 163)

[d] VFI: Value function iteration

[e] Terminal constraints are introduced to replicate the infinite time horizon.

1.5.1 Utility function

Perfect-foresight CGE models, especially based on MPSGE, employ a constant intertemporal elasticity of substitution (CIES) function, for instance see Babiker et al. (2009), Bretschger, Ramer, and Schwark (2011) and Vöhringer (2012).

$$U = \left[\sum_{t=0}^{T} \left(\frac{1}{1+\rho} \right)^{t} C_t^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{1.31}$$

Eq. (1.31) claims that the representative agent utility U depends on consumption C where ρ denotes the annual rate of the time preference. θ is the inverse of the elasticity of intertemporal substitution and T is the terminal period.[16] The utility function in Eq. (1.31) has the same intertemporal characteristics as the better known time-separable additive utility function, namely the constant relative risk aversion (CRRA) utility function in Eq. (1.32).

$$\tilde{U} = \sum_{t=0}^{T} \left(\frac{1}{1+\rho} \right)^{t} \frac{C_t^{1-\theta} - 1}{1-\theta} \tag{1.32}$$

Rutherford (2004) confirms that the CRRA utility function in Eq. (1.32) is a monotonic transformation of the CIES utility function in Eq. (1.31).

$$\tilde{U} = V\left(U\right) = \left[\left(1-\theta\right) U + \sum_{t=0}^{T} \left(\frac{1}{1+\rho} \right)^{t} \right]^{\frac{1}{1-\theta}} \tag{1.33}$$

Moreover, the marginal rate of substitution of the two utility functions is the same:

$$\frac{\frac{\partial U}{\partial C_{t+1}}}{\frac{\partial U}{\partial C_t}} = \frac{\frac{\partial \tilde{U}}{\partial C_{t+1}}}{\frac{\partial \tilde{U}}{\partial C_t}} = \frac{1}{1+\rho} \left(\frac{C_t}{C_{t+1}} \right)^{1-\theta} \tag{1.34}$$

Economically, the CRRA and CIES utility functions in Eq. (1.31) and Eq. (1.32) are equivalent. The optimization problem based on both utility functions yields the same demand functions (Mas-Colell, Whinston, and Green, 1995). The MPSGE can directly handle the CIES utility function in Eq. (1.31), therefore, the CIES utility function becomes a preferred option in the literature of dynamic CGE models, especially based on MPSGE.

As summarized in Table 1.4, Cai, Judd, and Lontzek (2012) and Lemoine and Traeger (2014) adopt the CRRA utility function with the same degree of risk-aversion as myself. Cai, Judd,

[16] CGE models cannot deal with an infinite time horizon, thus, a terminal period must be specified and a terminal condition needs to be introduced to replicate an infinite time horizon, for instance see Lau, Pahlke, and Rutherford (2002).

and Lontzek (2015) employ the Epstein-Zin recursive social welfare function (Epstein and Zin, 1989) as in Eq. (1.35), where the annual utility function is in Eq. (1.36):[17]

$$U_t = \left[\left(1-\beta\right) u\left(C_t, L_t\right) + \beta \left\{ \mathbb{E}_t \left[U_{t+1}^{1-\eta} \right] \right\}^{\frac{1-\frac{1}{\sigma}}{1-\eta}} \right]^{\frac{1}{1-\frac{1}{\sigma}}} \tag{1.35}$$

$$u\left(C_t, L_t\right) = \frac{\left(\frac{C_t}{L_t}\right)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} L_t \tag{1.36}$$

β is the discount factor, σ is the intertemporal elasticity of substitution, and η is the risk aversion parameter. Epstein-Zin preference allows separating the effect of risk aversion and intertemporal substitution. Crost and Traeger (2011) suggest that the Epstein-Zin preference disentangles risk aversion from intertemporal substitutability and this resolves the equity-premium and risk-free rate puzzles. Moreover, it offers better calibration results primarily in an integrated assessment model.

I am personally skeptical about embedding the Epstein-Zin preference in the framework of CGE models. It would be an advantage to discuss the intertemporal substitution and the risk-aversion in a separated manner. However, I am not so confident with how much modeling insights from CGE models can be enriched by taking into account this utility function.

1.5.2 Production function

CGE models have a detailed sectoral input-output structure. To produce one product, a firm requires not only factor inputs but also other goods as an intermediate input good. The substitution effect between factors or intermediate inputs is implemented by using a constant elasticity of substitution production function.

On the other hand, Cai, Judd, and Lontzek (2012), Lemoine and Traeger (2014) and Cai, Judd, and Lontzek (2015) develop a one sector growth model. All of them employ the standard Cobb-Douglas production function where the substitution effect between capital and labor input is considered.

Both CES and Cobb-Douglas production function exhibit constant returns to scale. Furthermore, the Cobb-Douglas function is the special case of the CES function.[18] I conclude that the difference in the production function cannot explain the reasons to have counter-intuitive results in my simulations.

[17] Epstein-Zin preference is gaining attention as an alternative to the CRRA utility function, for instance see Traeger (2014) and Fernández-Villaverde and Levintal (2018).

[18] When the elasticity of substitution is equal to one, the CES function is identical to the Cobb-Douglas function.

1.5.3 Discount factor

All of the models employ almost the same time preference to discount values in the distant future. Cai, Judd, and Lontzek (2012), Lemoine and Traeger (2014) and Cai, Judd, and Lontzek (2015) are recursively formulated as an infinite time horizon model, while in my attempt I specify the terminal period and introduce the terminal constraint to approximate an infinite time horizon. For details see Lau, Pahlke, and Rutherford (2002).

1.5.4 Uncertainty and precautionary saving

All of my simulations with a CGE model are deterministic but with exogenous random shocks. I start this subsection by reviewing my simulation approach, called hazard myopia. More details on this approach can be found in Section 1.3.7.

Assuming that a spatial adaptation decision for flood f is developed, where the observed damage scale is known to be Δ_f^O and the annual hazard probability is p_f. Instead of developing a dynamic stochastic model, I suppose that the representative agent in the model subjectively forms an expectation Δ_O^S for the flood f with his available information. If he correctly observes the flood f, his subjective damage expectation is:

$$\Delta_f^S = \mathbb{E}\left[\tilde{\Delta}_f^O\right] = \Delta_f^O \times p_f \tag{1.37}$$

This is equivalent to the certainty-equivalent deterministic model.

If the representative agent is more afraid of the risk of future floods, his subjective expectation is larger than its expected value. When he is less conscious of the risk of floods, he would form a lower subjective expectation. In other words, the representative agent saves more for the vulnerable capital stock when his subjective expectation is larger than the certainty equivalent level. He saves less when he forms a little subjective expectation. The simulated results presented in Section 1.4 consider the above discussions, however, illustrate the contradicted behavior.

This modification dramatically simplifies modeling but corrupts the stochasticity around flood events. I fail to include the *precautionary saving*, even though a risk-averse economic agent is assumed.

Precautionary saving acts as self-insurance.[19] Extra saving is demanded by the uncertainty in future income rather than determinate. It leads to a steeper consumption profile for a consumer. Precautionary saving would lead to excessive accumulation of capital than the economy would have if there were no uncertainty. Precautionary saving is in response to the consumer's risk behavior. Necessary conditions on a utility function are $u(\cdot) \in C^3$, $u'(\cdot) \geq 0$, $u''(\cdot) \leq 0$ and $u'''(\cdot) \geq 0$. The CRRA utility function is one of the best examples that induces

[19] General discussions about precautionary saving are found vastly throughout literature. Ljungqvist and Sargent (2012, see Section 18.14 and reference therein) offers a comprehensive discussion on this topic.

precautionary saving if there is uncertainty in the future.

Precautionary saving that is induced by risk-averse behavior and uncertainty in the future becomes an additional motive to invest more in capital stock. Weitzman (2009, 2011) argues that the probability of catastrophic events is very small but non-negligible, and therefore, the calculation of "expected damage" may be inappropriate to have a certainty-equivalent deterministic model. Indeed, such a model underestimates the risk of catastrophic events. Cai, Judd, and Lontzek (2015) achieve the same conclusion by using their stochastic IAM.

I conjecture that the reason why my modeling results are counter-intuitive is due to precautionary saving. Stochasticity is not correctly handled in my CGE attempts which corrupts a precautionary effect. On the other hand, the IAMs in Table 1.4 are developed in a dynamic stochastic setting which correctly supports the optimal consumption profile with a risk-averse economic agent.

Rutherford and Meeraus (2005, see, p.17) suggest in his presentation that "without heading possibilities [all possible future states] stochasticity cannot be exploited". My attempt failed to respect this argument. I explore the uncertainty around rare but catastrophic disasters by running Monte-Carlo simulations with a certainty-equivalent deterministic model and ex-ante probability distributions. Pindyck (2013a) suggests that this combination still prevails to evaluate uncertainty. However, we should recall that an investment in a non-vulnerable capital stock, which is less productive than a vulnerable capital stock in equilibrium, is motivated by precautionary effects.

Developing a fully stochastic CGE model (maybe with MPSGE) is not an easy task. As far as I have realized, there is no literature about stochastic CGE models so far. Chang and Rutherford (2016) solve a dynamic stochastic growth model by using a mixed complementarity approach. They rely on the standard numerical recipe, the value function iteration, as all of the advanced numerical economic models presented in Table 1.4.

1.6 Conclusion

I aimed to address spatial adaptation decisions for the major floods in Switzerland by solving an existing CGE model, GENESwIS. In addition to the conventional approaches in the literature of CGE, which are recursive and perfect-foresight dynamics, I developed a new approach called hazard myopia. A hazard myopic agent has perfect-foresight of all macroeconomic conditions except uncertain disasters. Intertemporal decisions in period t are the same as those of the perfect foresight agent. Between period t and $t+1$, the capital stocks are recursively updated, as a recursive dynamic CGE model does, depending on the realized observed damage scale.

I conclude that the reason why I end up with problematic simulation results is in the treatment of the stochasticity. Instead of developing a stochastic dynamic model, I converted the model to a certainty-equivalent deterministic one by taking an expected value of uncertain flood parameters. By doing this, the modeling has become tractable, but I corrupted a precautionary saving motive with which a representative agent is known to save more than he would do under deterministic. The excessive accumulation of capital stock, especially of non-vulnerable capital stock which is less-productive but risk-free, can be thought to be closely related to the precautionary saving. I surmise that full integration of stochasticity is promising, especially when one aims to address small probability events, but I remain this as a future task.

The proposed modeling approach in this chapter is closely related to Cai, Judd, and Steinbuks (2017) who demonstrate that a non-linear certainty equivalent approximation method (NLCEQ) is suitable to solve a dynamic stochastic general equilibrium model. NLCEQ transforms a stochastic model to deterministic one by taking the mean of uncertain parameters or replace a stochastic law of motion by a deterministic but applicable law of motion. As I do in this chapter, NLCEQ solves a finite time horizon model with a terminal constraint instead of dealing with an infinite time horizon model. Then NLCEQ approximates the results of the sequence of optimization problems and computes for the value or policy function. NLCEQ seems to be a promising modeling approach to solve a dynamic stochastic general equilibrium model; however, I would note that Cai, Judd, and Steinbuks (2017) deal with only a small but frequent shock. The implementation of rare disasters on NLCEQ remains a future work, and a comparative study between the two approaches is demanding.

Parametric uncertainty should be carefully addressed. Climate economic models, especially CGE models, heavily rely on parameters that have almost no scientific or statistical support. Babonneau et al. (2012) address parametric uncertainty through Monte Carlo simulations, and recently (Harenberg et al., 2018) propose a systematic uncertain quantification (: UQ) approach. Systematic UQ is desirable, but I leave parametric uncertainty as one of the most appealing future researches.

Finally, the proposed hazard myopic modeling is rather a descriptive approach. The choice of subjective expectation of the risk of future flooding is arbitrary, and I do not count any connection with the degree of risk-aversion behind. Furthermore, it is known that people tend to be loss-averse, especially for a catastrophic shock. Prospect theory might be one of the most well-known examples to combine these behavioral observations.[20] I find, especially in numerical simulations, it might be an interesting extension to deviate from the conventional expected utility maximization theorem, however, I leave this for future research.

These conclusions encourage me to develop a rather stylized but stochastic dynamic model with adaptation measures. Adaptation decisions, as well as rare but catastrophic events, are relatively new topics in modern environmental economics. Thus, it is beneficial for us to know the essential characteristics of adaptation to rare but catastrophic events under uncertainty. In the next chapters, I demonstrate optimal adaptation decisions to rare natural disasters by using an approach based on a dynamic stochastic equilibrium model.

[20] For instance, see Wakker (2010).

2 Dynamic stochastic equilibrium analysis with adaptation

2.1 Introduction

Rare natural disasters, such as a massive flood or a great earthquake, are studied in Tsur and Zemel (1998), Ikefuji and Horii (2012) and Bretschger and Vinogradova (2017) in the context of climate change or in Judd, Maliar, and Maliar (2011), Posch and Trimborn (2013) and Fernández-Villaverde and Levintal (2018) in the literature of numerical methods in economics. These papers contribute to present how rare disasters are modeled in the framework of a dynamic (stochastic) economic model or how we can solve the models numerically. This literature mainly focuses on the macroeconomic impact of rare disasters on economic activities, however, one critical point is still missing: except for a few contributions such as Millner and Dietz (2015) and Grames et al. (2016), *how do we optimally adapt or alleviate the risk of uncertain rare natural disasters?*

To tackle this question requires a dynamic stochastic equilibrium model with adaptation measures. As an example of adaptation measures, I consider two adaptation schemes: spatial adaptation and the introduction of adaptive capital stock.[1] In the spatial adaptation, I classify the existing productive capital stock into vulnerable and non-vulnerable capital stock, depending on the vulnerability to rare natural disasters. Vulnerable capital stock faces the risk of massive depreciation caused by a disaster; however it exhibits higher marginal productivity than non-vulnerable capital stock in equilibrium.[2] Adaptive capital stock does not contribute to the production function; however, we can alleviate the damage on productive capital stock by accumulating the adaptive capital stock relative to the productive capital stock.[3] Given that the adaptive capital stock becomes effective only when a natural disaster occurs, investment

[1] Spatial adaptation is widely studied in literature within the context of adaptation of significant flooding, for instance, see Koks et al. (2014). The concept of adaptive capital stock is similar to Millner and Dietz (2015) and Grames et al. (2016).

[2] If an economic agent knows a priori that vulnerable capital stock faces the risk of natural disasters, the only reason why he invests in vulnerable capital stock is that the capital stock is more productive than in a non-vulnerable region.

[3] Typical adaptive capital stock I suppose is a dike or a dam for flooding or an antiseismic device for an earthquake.

in adaptive capital stock is regarded as a sort of precautionary saving.

My contributions to this chapter are mainly threefold. First, I present a model that processes uncertain rare disasters, in addition to the conventional autoregressive productivity shock, with adaptation measures in the framework of a dynamic stochastic equilibrium model in discrete time. Natural disasters present a small probability with a catastrophic consequence. Our modeling lessons, therefore, are substantially different from conventional dynamic stochastic (general) equilibrium analyses with real business cycle shocks that are small but frequent. As another modeling highlight, I introduce occasionally binding irreversible constraints on investment decisions. It is a well known numerical issue, for instance, see Judd, Kubler, and Schmedders (2000), that an occasionally binding constraint poses substantial difficulties for derivative-based optimization solvers. Consumption and saving decisions might substantially differ between pre and post disasters. Therefore I solve the models globally, not locally.[4] I overcome these numerical difficulties by implementing the time iteration collocation with adaptive sparse grid algorithm (Brumm and Scheidegger, 2017). To speed up our solution processes, the models are massively parallelized via the Message Passive Interface (MPI) and implemented on a high-end computing cluster.

Second, I carefully study the optimal adaptation behavior for spatial adaptation and the introduction of adaptive capital stock. I numerically approximate stochastic policy functions for the model with either spatial adaptation or adaptive capital stock and graphically present policy functions in the state space. Policy functions return an optimal decision given an exogenous state. Non-smooth regions in a policy function are correctly detected by the solution algorithm and reasonably approximated with a tolerated approximation error. It is worthwhile to note that the derived stochastic policy functions internalize uncertainty around rare natural disasters and productivity shocks in the *ex-ante* manner.

The final remark in this chapter is to quantitatively demonstrate the applicability of a stochastic model over a certainty-equivalent deterministic model by comparing the Monte-Carlo simulated paths. A certainty-equivalent deterministic model treats uncertainty in the *ex-ante* manner. Since the probability of rare natural disasters is small enough, there is no striking difference between the means of Monte-Carlo simulated paths from either a stochastic or certainty-equivalent deterministic model. Monte-Carlo simulated paths with a certainty-equivalent deterministic model present a substantially random trend with much higher variance than those with a stochastic model, as Cai, Judd, and Lontzek (2015) achieve the same conclusion in the estimation of the social cost of carbon. I conclude that adaptation decisions should be based on a stochastic model because it supports low variance economic paths and correctly accounts for risk-averse and precautionary behavior for the risk of uncertain disasters.

[4] Local method, such as the perturbation method, approximates equilibrium solutions with tolerated approximation error if the solutions are close enough to its steady state. However, it is a known issue that the approximated quality rapidly deteriorates when the solution deviates from its steady state. Judd (1998) provides comprehensive discussions about the local and global method. Lontzek and Narita (2011) solve a stochastic optimization model of a climate-economic system by both methods and compare the approximation quality of both methods.

The remainder of this chapter is organized as follows. Section 2.2 presents two models implemented with either spatial adaptation or adaptive capital stock and numerical methods that I adopt. Section 2.3 graphically presents stochastic policy functions for the two models and shows Monte-Carlo simulated paths. Section 2.4 discusses the applicability of the stochastic approach over the deterministic approach. Section 2.5 concludes this chapter.

2.2 Models and numerical methods

I develop a stylized neoclassical growth model implemented with either spatial adaptation or adaptive capital stock. Dynamic stochastic economic problems do not have a closed-form solution in general.[5] The analytical solutions from a further stylized model pose subtle implications; therefore in modern economics, there is a growing demand to employ numerical methods.[6] The two models have formulated as an infinite-time horizon dynamic stochastic social planner problem. We solve each social planner problem for the Euler equations analytically, and numerically solve the system of non-linear equilibrium conditions. In Section 2.2.1, I present the basic framework of a social planner problem. In Section 2.2.2, I study the optimal adaptation decisions for spatial adaptation. In Section 2.2.3, the adaptation capital stock is introduced. Finally in Section 2.2.4, I briefly overview the time iteration collocation with adaptive sparse grid, integration and a parallelization scheme.

2.2.1 Social planner problem

The objective function of the social planner is to maximize the sum of discounted and time separative utility, as in Eq. (2.1), subject to some occasionally binding investment constraints and a resource constraint:

$$\max \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(C_t) \right] \tag{2.1}$$

β stands for a discount factor and $\mathbb{E}_0$ is an expectation operator from the initial time period 0. We assume the neoclassical utility function $u(\cdot)$, mathematically $u(\cdot) \in \mathbb{C}^3$, $u_C \geq 0$, $u_{CC} \leq 0$, $u_{CCC} \geq 0$ and $\lim_{c \to 0} u_C = \infty$. We assume the time separative constant relative risk aversion(: CRRA) for the utility function:

$$u(C_t) = \frac{C_t^{1-\eta}}{1-\eta} \tag{2.2}$$

Since the choice of the functional form of a utility function controls the intertemporal saving

[5] One of the most famous exceptions might be the logarithmic preferences and Cobb-Douglas production (Ljungqvist and Sargent, 2012, see, Section 3.1.2.).

[6] Judd (1998) and Miranda and Fackler (2002) are a classical but standard reference for the numerical methods in economics.

and consumption decisions, it is worthwhile to discuss the other forms of a utility function.[7] The logarithmic utility function is still rather standard and ensures good analytical tractability, for instance, see Golosov et al. (2014). However, the CRRA utility function exhibits a more general form of the risk-averse preference. A recursive utility function, especially the Epstein-Zin preference (Epstein and Zin, 1989), becomes an alternative to the CRRA utility function in the very advanced numerical studies, see Traeger (2014), Cai, Judd, and Lontzek (2015) and Fernández-Villaverde and Levintal (2018) among others. Epstein-Zin preference can capture the risk-aversion and the effect of the intertemporal elasticity of substitution within a recursive formula. We aim to study the optimal adaptation decision to rare disasters under uncertainty, and in our point of view, the CRRA utility function is the most appealing to achieve our objectives with the reasonable computational expense.

2.2.2 Spatial adaptation

I introduce spatial adaptation by classifying the existing productive capital stock into two classes based on their vulnerability to uncertain shocks, namely vulnerable (KV_t) and non-vulnerable capital stock (KNV_t). Both capital stocks are productive and only vulnerable capital faces the risk of large depreciation caused by uncertain shocks.

I formulate the social planner's problem whose objective function is in Eq. (2.3), subject to irreversible investment constraints of vulnerable capital stock Eq. (2.4) and of non-vulnerable capital stock Eq. (2.5) and a resource constraint Eq. (2.6).

$$\max_{\{C_t, KV_{t+1}, KNV_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(C_t) \right] \tag{2.3}$$

$$\text{s.t.} \quad I_t^{KV} = KV_{t+1} - \left(1 - \delta - \tilde{\Delta}^z\right) KV_t \geq 0 \quad \perp \quad \mu_t^{KV} \geq 0$$

$$\text{where} \begin{cases} \tilde{\Delta}^{z=0} = 0, & \Pr[z=0] = \pi(0) \\ \tilde{\Delta}^{z=1} = \Delta^1, & \Pr[z=1] = \pi(1) \\ \tilde{\Delta}^{z=2} = \Delta^2, & \Pr[z=2] = \pi(2) \end{cases} \tag{2.4}$$

$$I_t^{KNV} = KNV_{t+1} - (1-\delta) KNV_t \geq 0 \quad \perp \quad \mu_t^{KNV} \geq 0 \tag{2.5}$$

$$a_t f(KV_t, KNV_t) - I_t^{KV} - \frac{q^{KV}}{2} KV_t \left(\frac{I_t^{KV}}{KV_t} - \delta - \tilde{\Delta}^z \right)^2 - I_t^{KNV} - \frac{q^{KNV}}{2} KNV_t \left(\frac{I_t^{KNV}}{KNV_t} - \delta \right)^2$$

$$- C_t \geq 0 \quad \perp \quad \lambda_t \geq 0 \tag{2.6}$$

$$\ln a_{t+1} = \rho \ln a_t + \epsilon_{t+1}, \quad \epsilon_{t+1} \sim N\left(0, s^2\right) \tag{2.7}$$

Utility function remains the same as in Eq. (2.2). Uncertainty around rare disasters is introduced through $\tilde{\Delta}^z$ in Eq. (2.4). In time t, the social planner observes in which state he is standing. When state 0 occurs with probability $\pi(0)$, there is no shock, thus the realized

[7] Among others, Barro (2006), Lontzek and Narita (2011) and Bretschger and Vinogradova (2017) adopt the CRRA utility function.

damage scale Δ is zero. State 1 is realized with probability $\pi(1)$ where a moderate and frequent shock occurs. The shock imposes additional depreciation on vulnerable capital stock by Δ^1. Rare natural disasters occur with probability $\pi(2)$ and its damage scale is given by Δ^2. Clearly $\pi(0)+\pi(1)+\pi(2)=1$. Eqs. (2.4) and (2.5) are an occasionally binding irreversible constraint on investment to each capital stock.[8] Eq. (2.6) is the strictly binding resource constraint considering a convex quadratic investment adjustment cost for both capital stocks, for instance, see Juillard and Villemot (2011). Parameters q^{KV} and q^{KNV} govern the intensity of the quadratic part. If $q^{KV}, q^{KNV} = 0$, there is no adjustment cost. If $q^{KV}, q^{KNV} > 0$, to have a different level of capital stock between subsequent periods is costly due to the quadratic cost. It becomes an incentive to allocate more investment to damaged capital stock in state 1 and 2 to recover from disasters.

Eq. (2.7) defines a law of motion of the first-order autoregressive productivity shock, for instance, see Juillard and Villemot (2011). AR(1) productivity shock is a well-studied example of a small but frequent shock such as a real business cycle shock (Juillard and Villemot, 2011; Judd et al., 2014; Brumm and Scheidegger, 2017, among others).[9] Dynamics, as well as an integration method of rare disasters, are comparatively different from those of small but frequent shocks (Judd, Maliar, and Maliar, 2011). This study, mainly following (Judd, Maliar, and Maliar, 2011), examines the modeling way of two different types of exogenous shocks.

Log productivity follows independent and identically distributed (i.i.d.) normal distribution with the standard deviation s. ρ characterizes an autocorrelation process.

The production function $f(\cdot)$ in Eq. (2.6) satisfies the following neoclassical assumptions: $f(\cdot) \in \mathbb{C}^2$, $f_{KV} > 0$, $f_{KNV} > 0$, $f_{KVKV} < 0$, $f_{KNVKNV} < 0$, $\lim_{KV\to 0} f_{KV} = +\infty$, $\lim_{KV\to\infty} f_{KV} = 0$, $\lim_{KNV\to 0} f_{KNV} = +\infty$ and $\lim_{KNV\to\infty} f_{KNV} = 0$. Since both vulnerable and non-vulnerable capital stock are productive, I employ the nested constant elasticity of substitution (CES) production function as in Eq. (2.8):

$$f(KV_t, KNV_t) = \left[\theta KV_t^{\frac{\sigma-1}{\sigma}} + (1-\theta)\, KNV_t^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}\alpha} \tag{2.8}$$

The CES production function satisfies the neoclassical assumptions. In the bottom nest, we model the substitution between vulnerable and non-vulnerable capital stock based on the constant elasticity σ. θ measures the benchmark share of vulnerable capital stock over the sum of two capital stocks. In the top nest, the aggregated capital stock and labor input are combined based on the Cobb-Douglas function. I normalize, for simplicity, the labor input to one over the time horizon. α represents the initial input share of capital stock.

Since the irreversible investment constraints are occasionally binding, the Karush-Kuhn-

[8] Irreversible investment constraint prevents negative investment, meaning that capital stock cannot be converted back to a re-investment good once it is invested: *putty-clay* capital stock.

[9] Another common way to model a real business cycle shock is to adopt the finite state Markov chain, for instance, see Stachurski (2009). Hamilton (2005) estimates the finite Markov transition matrix for the U.S. business cycle.

Tucker multiplier (KKT) multipliers μ_t^{KV} for Eq. (2.4) and μ_t^{KNV} for Eq. (2.5) have to be considered. λ_t is an associated Lagrange multiplier for the strictly binding resource constraint. Finally note that in Eqs. (2.4) to (2.6), the symbol $\perp$ indicates a complementarity slackness condition. We summarize the model parameters in Table 2.1. These values might be well accepted in the neoclassical literature, and the parametric uncertainty regarding these values lie outside the scope of the paper.

By formulating the Lagrangian and solving for the first-order equilibrium conditions, I can derive the following two Euler conditions where the symbol $\mathbb{E}_t$ is an expectation operator from time t. I define the growth rate $g_t^{KV} := \frac{KV_{t+1}}{KV_t} - 1$ for vulnerable capital stock and $g_t^{KNV} := \frac{KNV_{t+1}}{KNV_t} - 1$ for non-vulnerable capital stock. More details on the Lagrangian is deferred to Section C.2.

$$\lambda_t \left[1 + q^{KV} g_t^{KV}\right] - \mu_t^{KV} = \beta \mathbb{E}_t \left[\lambda_{t+1} \left\{ a_{t+1} f_{KV}\left(KV_{t+1}, KNV_{t+1}\right) + \left(1 - \delta - \tilde{\Delta}^z\right) + \frac{q^{KV}}{2} g_{t+1}^{KV} \left(g_{t+1}^{KV} + 2\right) \right\} - \mu_{t+1}^{KV} \left(1 - \delta - \tilde{\Delta}^z\right) \right] \tag{2.9}$$

$$\lambda_t \left[1 + q^{KNV} g_t^{KNV}\right] - \mu_t^{KNV} = \beta \mathbb{E}_t \left[\lambda_{t+1} \left\{ a_{t+1} f_{KNV}\left(KV_{t+1}, KNV_{t+1}\right) + (1 - \delta) + \frac{q^{KNV}}{2} g_{t+1}^{KNV} \left(g_{t+1}^{KNV} + 2\right) \right\} - \mu_{t+1}^{KV} (1 - \delta) \right] \tag{2.10}$$

The two Euler equations characterize the intertemporal equilibrium conditions for choice variables between time t and $t+1$, hedging the three discrete states in time $t+1$ and the random and continuous productivity shock a_{t+1}. The state and the level of productivity shock in period $t+1$ are uncertain for the social planner in period t. Therefore, it is necessary to introduce an expectation operator $\mathbb{E}_t$.[10]

Our system of non-linear equilibrium conditions includes the two Euler conditions Eqs. (2.9) and (2.10), the two occasionally binding irreversible investment conditions Eqs. (2.4) and (2.5), and the resource constraint Eq. (2.6).

2.2.3 Adaptive capital stock

In the second class of adaptation, I introduce the adaptive capital stock, which is non-productive but alleviates the damage on both productive and adaptive capital stocks. The role of the adaptive capital stock is similar to the existing literature such as de Bruin, Dellink, and Tol (2009), Bosello, Carraro, and De Cian (2010), Millner and Dietz (2015) and Grames et al. (2016). However, modeling adaptive capital stock depends on how we treat exogenous damages. As Stern (2016b, Chapter 4) discusses, the economic damages are normally modeled

[10]Discussions about how to discretize the continuous productivity shock and to evaluate the expectation operator in Eqs. (2.9) and (2.10) are deferred in Section 2.2.5.

as a negative impact on the production function, such as de Bruin, Dellink, and Tol (2009), Bosello, Carraro, and De Cian (2010) and Millner and Dietz (2015), or as direct damage on capital stock, such as Grames et al. (2016). In this chapter, I focus on the optimal investment balance between productive and adaptive capital stock. Moreover, the adaptive capital stock is assumed to directly introduced on the productive capital stock.[11] Considering my research objective, I consider direct damage on capital stock and discuss how the damage can be alleviated by accumulating adaptive capital stock relative to productive capital stock. I introduce the adaptive function directly in the law of motion of productive and adaptive capital stock, which is in line with Grames et al. (2016).

The social planner solves the following infinite time horizon dynamic stochastic optimization problem. The objective function Eq. (2.11) is to be maximized subject to the irreversible investment conditions, Eqs. (2.12) and (2.13), and the resource constraint, Eq. (2.14). Eq. (2.15) defines a law of motion of the first-order autoregressive productivity shock.

$$\max_{\{C_t,K_{t+1},A_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u(C_t)\right] \tag{2.11}$$

$$\text{s.t.} \quad I_t^K = K_{t+1} - \left(1-\delta-\tilde{\Delta}^z h(K_t,A_t)\right)K_t \geq 0 \quad \perp \quad \mu_t^K \geq 0 \tag{2.12}$$

$$I_t^A = A_{t+1} - \left(1-\delta-\tilde{\Delta}^z h(K_t,A_t)\right)A_t \geq 0 \quad \perp \quad \mu_t^A \geq 0 \tag{2.13}$$

$$a_t f(K_t) - I_t^K - \frac{q^K}{2}K_t\left(\frac{I_t^K}{K_t}-\delta-\tilde{\Delta}^z h(K_t,A_t)\right)^2 - I_t^A - \frac{q^A}{2}A_t\left(\frac{I_t^A}{A_t}-\delta-\tilde{\Delta}^z h(K_t,A_t)\right)^2$$

$$-C_t \geq 0 \quad \perp \quad \lambda_t \geq 0 \quad \text{where}\begin{cases}\tilde{\Delta}^{z=0}=0, & \Pr[z=0]=\pi(0)\\ \tilde{\Delta}^{z=1}=\Delta^1, & \Pr[z=1]=\pi(1)\\ \tilde{\Delta}^{z=2}=\Delta^2, & \Pr[z=1]=\pi(2)\end{cases} \tag{2.14}$$

$$\ln a_{t+1} = \rho \ln a_t + \epsilon_{t+1}, \quad \epsilon_{t+1} \sim N\left(0,s^2\right) \tag{2.15}$$

I employ the CRRA utility function, provided in Eq. (2.2), and the neoclassical production function $f(\cdot)$, mathematically, $f(\cdot)\in\mathbb{C}^2$, $f_K>0$, $f_{KK}<0$, $\lim_{K\to 0} f_K=+\infty$ and $\lim_{K\to\infty} f_K=0$. I assume the Cobb-Douglas production function and keep the exogenous labor input equal to one:

$$f(K_t) = K_t^{\alpha} \tag{2.16}$$

In Eqs. (2.12) and (2.13), μ_t^K and μ_t^A stand for a KKT multiplier and λ_t in the resource constraint Eq. (2.14) is a Lagrange multiplier. The symbol $\perp$ represents complementarity slackness. The model parameters are summarized in Table 2.1.

As provided in Eqs. (2.12) and (2.13), the productive capital stock K_t and the adaptive capital

[11] Typical examples of an adaptation capital are a dike or a dam to mitigate flooding, and an antiseismic structure installed on a building to counteract earthquakes, among others.

stock A_t face a risk of catastrophic uncertain shocks that cause damage to the capital stock by $\tilde{\Delta}^z$. The role of the adaptation function $h(\cdot)$ is to define how much uncertain damage $\tilde{\Delta}^z$ can be mitigated through the accumulation of the adaptive capital stock relative to the productive capital stock. $h(\cdot)$ in Eqs. (2.12) to (2.14), which defines the adaptive function with the following mathematical properties.[12]

$$h : \mathbb{R}_+ \times \mathbb{R}_+ \mapsto (0,1] \tag{2.17}$$

$$\forall K_t, h(K_t, 0) = 1 \tag{2.18}$$

$$h_K(\cdot) \geq 0, h_{KK}(\cdot) \leq 0 \tag{2.19}$$

$$h_A(\cdot) \leq 0, h_{AA}(\cdot) \geq 0 \tag{2.20}$$

$$h_{AK}(\cdot) < 0 \tag{2.21}$$

Eq. (2.17) claims that it is impossible to fully adapt to the damage ($h(\cdot) > 0$). Eq. (2.18) shows that, if there is no adaption effort, i.e., $A_t = 0$, the economy will accept the full damage ($h(\cdot) = 1$). The main assumption behind Eq. (2.19) is that the marginal unit of productive capital stock always requires the additional adaptive capital stock to achieve the same level of adaptation, but the necessary amount of adaptive capital stock exhibits decreasing returns to scale. Eq. (2.20) claims that the marginal unit of adaptation always contributes to alleviating the damage, but it exhibits decreasing returns to scale. In the end, Eq. (2.21) implies the marginal unit of adaptive capital stock reduces the damage on both capital stocks more effectively when the level of productive capital stock is small, rather than when it is large.

By solving the Lagrangian for the first-order conditions, I can derive the following Euler equations. More details on the Lagrangian is deferred in Section C.2.

$$\begin{aligned}
&\lambda_t \left[1 + q^K g_t^K\right] - \mu_t^K = \\
&\quad \beta \mathbb{E}_t \Bigg[\lambda_{t+1} \Big\{ a_{t+1} f_K(K_{t+1}) + \big(1 - \delta - \tilde{\Delta}^z h(K_{t+1}, A_{t+1})\big) - \tilde{\Delta}^z h_K(K_{t+1}, A_{t+1})(K_{t+1} + A_{t+1}) \\
&\quad + \frac{q^K}{2} g_{t+1}^K \left(g_{t+1}^K + 2\right) \Big\} - \mu_{t+1}^K \left\{ \big(1 - \delta - \tilde{\Delta}^z h(K_{t+1}, A_{t+1})\big) - \tilde{\Delta}^z h_K(K_{t+1}, A_{t+1}) K_{t+1} \right\} \\
&\quad + \mu_{t+1}^A \tilde{\Delta}^z h_K(K_{t+1}, A_{t+1}) A_{t+1} \Bigg]
\end{aligned} \tag{2.22}$$

$$\begin{aligned}
&\lambda_t \left[1 + q^A g_t^A\right] - \mu_t^A = \\
&\quad \beta \mathbb{E}_t \Bigg[\lambda_{t+1} \Big\{ \big(1 - \delta - \tilde{\Delta}^z h(K_{t+1}, A_{t+1})\big) - \tilde{\Delta}^z h_A(K_{t+1}, A_{t+1})(K_{t+1} + A_{t+1}) \\
&\quad + \frac{q^A}{2} g_{t+1}^A \left(g_{t+1}^A + 2\right) \Big\} + \mu_{t+1}^K \tilde{\Delta}^z h_A(K_{t+1}, A_{t+1}) K_{t+1} \\
&\quad - \mu_{t+1}^A \left\{ \big(1 - \delta - \tilde{\Delta}^z h(K_{t+1}, A_{t+1})\big) - \tilde{\Delta}^z h_A(K_{t+1}, A_{t+1}) A_{t+1} \right\} \Bigg]
\end{aligned} \tag{2.23}$$

[12] Similar discussion about the functional properties of an adaptation function can be found in Bréchet, Hritonenko, and Yatsenko (2013) and Millner and Dietz (2015). Note that in both of these papers, adaptive capital stock reduces the negative damage on the production function.

Again note that $g_t^K := \frac{K_{t+1}}{K_t} - 1$ is the growth rate of productive capital stock and $g_t^A := \frac{A_{t+1}}{A_t} - 1$ is the growth rate of adaptive capital stock. Our system of non-linear equilibrium conditions consist of the two Euler equations in Eqs. (2.22) and (2.23), the two occasionally binding investment constraints in Eqs. (2.12) and (2.13) and the resource constraint in Eq. (2.14).

I need to specify the functional form of $h(\cdot)$ that satisfies Eqs. (2.17), (2.18), (2.20) and (2.21). As far as I have realized, there is no common functional form for this kind of adaptation function in the previous literature.[13] One appealing but straightforward option is an exponential function with a negative argument. Moreover, the exponential function is strictly convex, which is convenient for a numerical solver. I assume the adaptive function to take the form:

$$h(K_t, A_t) = \exp\left(\frac{-\phi A_t}{K_t}\right) \tag{2.24}$$

where ϕ measures the technology level of adaptation.

2.2.4 Time iteration collocation with adaptive sparse grid

In this chapter, our main challenges involve: (1) solving an infinite-time horizon dynamic stochastic model under uncertainty and (2) to approximate stochastic policy functions that exhibit relatively high dimensional hypercubes with possibly non-smooth regions. I rely on the time iteration collocation with an adaptive sparse grid to overcome the two main difficulties (Brumm and Scheidegger, 2017, and references therein.). In the following, I briefly justify why the approach by Brumm and Scheidegger (2017) is suitable for my applications.

The time iteration collocation and the value function iteration method are two mainstreams to solve dynamic stochastic models globally. The value function iteration is relatively easy to implement, however, from a numerical point of view, the time iteration collocation method has the following advantages.[14] The time iteration collocation method, introduced to economics by Judd (1992), solves the system of (non-) linear equations to approximate policy functions directly. Whereas the value function iteration computes a value function involving a numerically expensive non-linear optimization problem. Another advantage of time iteration collocation over a value function iteration is the approximation or interpolation step. A value function is known for exhibiting a higher degree of curvature than a policy function, which sometimes causes numerical problems.

Another issue is the curse of dimensionality. The tensor-product based approximation methods are known to suffer from the curse of dimensionality. One of the most practical methods to handle this problem is a sparse grid (SG), also known as the Smolyak method. It was firstly introduced in economics by Krueger and Kubler (2004, and references therein). SG can interpolate multidimensional hypercubes with a moderate computational expense, if

[13] de Bruin, Dellink, and Tol (2009) and Millner and Dietz (2015) are an early example of adaptation literature, however, their adaptation function works on production function and differs from what I discuss here.

[14] Stachurski (2009) provides the theoretical and computational treatments of value function iteration.

the interpolant is smooth enough, and some refinement algorithms have been proposed, for instance, by Judd et al. (2014).

SG employs global polynomials such as Chebychev or Lagrange in the approximation step. It means that if an interpolated function poses non-smoothness, as Judd, Kubler, and Schmedders (2000) point out, it would become numerically very challenging to approximate the non-smooth functions within a tolerated error, since a standard numerical solver requires the Jacobian or Hessian information. Adaptive sparse grid (ASG) with local polynomial is a practical extension of SG to overcome the main numerical difficulties. ASG algorithm detects the kink in interpolant and adds the necessary amounts of interpolated nodes around the kink. ASG enables approximating non-smooth hypercubes with tolerated approximation error.

As discussed above, ASG is superior to other global methods in the following points: (1) The algorithm can detect a non-smooth region of a policy or value function. (2) It can alleviate the curse of dimensionality. (3) It can deal with high dimensionality and can be parallelized with a moderate computational effort. For my specific problem, point (1) and (3) are particularly important. There are some computational methods to deal with non-smooth functions such as Carroll (2006) and Brumm and Grill (2014), but as far as I understand, the adaptive sparse grid is more suitable in a parallelization than other methods.

I present the pseudocode in Algorithm 2, where IPOPT, a large scale non-linear optimization solver, is used to solve the system of equations (Wächter and Biegler, 2006). To stabilize each optimization iteration, I provide a starting point from the previous iteration, if available. If IPOPT still cannot find the root of the system, neighboring points from the previous iteration are provided as a starting point. We need to ensure IPOPT to solve the system of equations correctly in every iteration. The TASMANIAN sparse grid library is used to generate sparse grids and to adapt or refine sparse grids with local polynomials (Stoyanov, 2015). Finally, I implement all the numerical steps in Python 3.6 with its scientific libraries.

2.2.5 Integration

Due to the two sources of uncertainty, namely rare natural disasters and the first-order autoregressive (AR(1)) productivity shock, I need to evaluate the expectation operator that appears in the Euler equations by using integration procedures. Rare natural disasters are a discrete event; thus, our primary task is to evaluate the integration of continuous AR(1) productivity shock. Since the AR(1) process involves Normal random variables, the Gauss-Hermite quadrature reasonably approximates the expectation of a function of a normal random variable (Judd, 1998, see, p. 261). We adopt the degree five Gauss-Hermite quadrature.[15] I construct multidimensional nodes as a tensor product of the discrete rare disaster nodes and the five nodes Gauss-Hermite quadrature.

[15] Juillard and Villemot (2011) implement three numerical integration methods (the degree four Gauss-Hermite quadrature, the degree five monomial formula and quasi-Monte Carlo integration) on a multi-country real business cycle model and compare the accuracy of each integration method.

Algorithm 2: Time iteration collocation with adaptive sparse grid

Initialization:

Define state s such that $s = (\kappa, \zeta) = \left(\kappa_1, \cdots \kappa_{N_\kappa}, \zeta_1, \cdots, \zeta_{N_\zeta}\right)$, where κ stands for the vector of endogenous state variables and ζ stand for the vector of exogenous state variables.

Solve the certainty equivalent deterministic model to compute the state variables in the steady state s^{SS}.

Define the range of each state variable such that $s \in \left[\underline{s}, \overline{s}\right] \subset S \subset \mathbb{R}^{N_\kappa + N_\zeta}$.

Set a refinement threshold ϵ. Set the maximum level of refinement L_{max} and the initial level of sparse grid L_0. Set $l = L_0$.

Set an approximation tolerance $\overline{\eta}$.

Solve for the optimal policy function $p^* : S \mapsto \mathbb{R}^{2N_\kappa+1}$.

Initial guess for the policy function in the next period:

$p^+ : S \mapsto \mathbb{R}^{2N_\kappa+1} : p^+\left(s^+\right) = \left(\kappa_1^{++}\left(s^+\right), \cdots, \kappa_{N_\kappa}^{++}\left(s^+\right), \mu_1^+\left(s^+\right), \cdots, \mu_{N_\kappa}^+\left(s^+\right), \lambda^+\left(s^+\right)\right)$

while $\eta > \overline{\eta}$ **do**

    Select a course grid $G_{old} \subset S$; Generate level L_0 sparse grid G by adding $2n$ neighborhood points of $x \in G_{old}$ for each policy function

    while $l \leq L_{max}$ **do**

        if $l = L_0$, **then**

            For each grid point $(\kappa, \zeta) \in G$, solve the system of non-linear equilibrium conditions given the next period's policy functions $p^+\left(s^+\right)$.

        else

            For each grid point $(\kappa, \zeta) \in G \setminus G_{old}$, solve the system of non-linear equilibrium conditions given the next period's policy functions $p^+\left(s^+\right)$.

        Generate G_{new} from G by adding the $2d$ points of $x \in G \setminus G_{old}$, if $||p(x) - \hat{p}(x)||_\infty > \epsilon$
        where $\hat{p}(x)$ is given by interpolating $\left\{p(x)\right\}_{x \in G_{old}}$ with the local hierarchical polynomial basis functions.

        if $G_{new} = G$ *or* $l = L_{max}$, **then**

            Set $G = G_{new}$ and break.

        else

            Set $G_{old} = G$, $G = G_{new}$ and $l+ = 1$

    Define policy function $\left\{p\right\}_{x \in G}$.

    Calculate an approximation error $\eta = ||p - p^+||_\infty$.

    if $\eta > \overline{\eta}$, **then**

        Set $p^+ = p$

    else

        break

Derive the optimal policy function $p^* = p$.

2.3 Result

In this section, I present the stochastic policy functions and the Monte-Carlo simulated economic growth paths implemented with either spatial adaptation in Section 2.3.1 and adaptive capital stock Section 2.3.2.

Table 2.1 – Value of parameters

Symbol	Description	Value	Reference
β	Time preference factor	0.99	Juillard and Villemot (2011)
δ	Capital depreciation rate	0.025	Juillard and Villemot (2011)
η	Constant relative risk aversion	2	Millner and Dietz (2015)
α	Capital input share	0.36	Juillard and Villemot (2011)
σ	Elasticity of substitution between KV and KNV	1.5	
θ	Vulnerable capital share	0.25	Dumas and Ha-Duong (2013)
q^{KV}	Coefficient of the quadratic adjustment cost of KV	0.5	Juillard and Villemot (2011)
q^{KNV}	Coefficient of the quadratic adjustment cost of KNV	0.5	Juillard and Villemot (2011)
q^{K}	Coefficient of the quadratic adjustment cost of K	0.5	Juillard and Villemot (2011)
q^{A}	Coefficient of the quadratic adjustment cost of A	0.5	Juillard and Villemot (2011)
ρ	Autocorrelation coefficient	0.95	Juillard and Villemot (2011)
s	Standard deviation in the productivity shock	0.01	Juillard and Villemot (2011)
ϕ	Technology level of the adaptation function $h(\cdot)$	10	
Δ^1	Damage scale of a frequent but moderate shock	0.025	
Δ^2	Damage scale of a rare disaster	0.20	
$\pi(1)$	Annual probability of a frequent but moderate shock	0.10	
$\pi(2)$	Annual probability of a rare disaster	0.01	

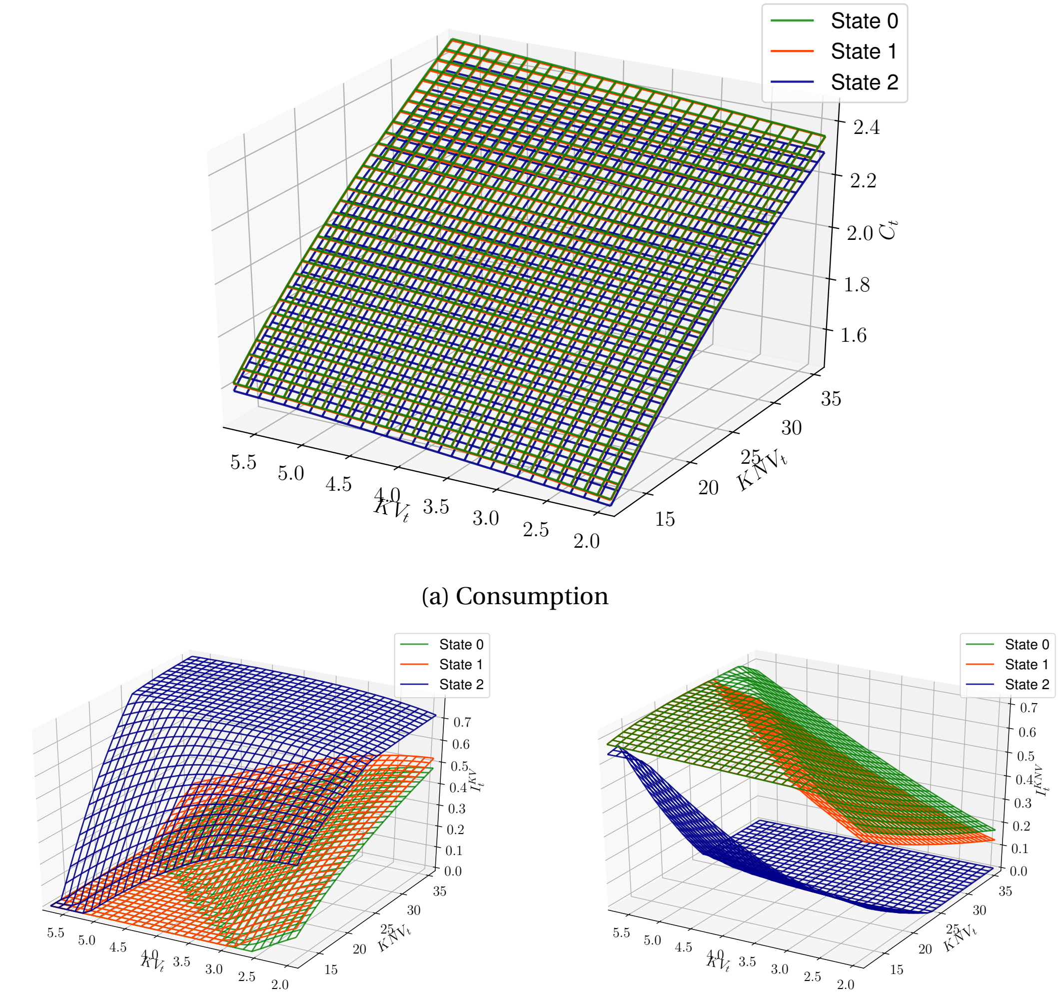
2.3.1 Spatial adaptation

Stochastic policy functions

Optimal stochastic policy functions are summarized in Figure 2.1 when the AR(1) productivity shock a_t is fixed at one. The numerically approximated policy function returns the optimal consumption level, corresponding to the endogenous state variables and the exogenous state variable.

Figure 2.1a shows the consumption policy function. The consumption policy function is sufficiently smooth and increases in both productive capital stocks, vulnerable and non-vulnerable capital. Both capital stocks are productive and substituted based on the CES production function as shown in Eq. (2.8). Figure 2.1a can be interrupted in the following way: When the rare disaster occurs (in state 2), the optimal consumption level is shown to be smaller than that in state 0 (no shock occurs) and in state 1 (frequent but moderate shock occurs). When a rare natural disaster occurs, the vulnerable capital stock is largely damaged, and the associated productivity level is decreased. Additionally, the more investment is allocated to vulnerable capital stock to compensate for the damage, see Figure 2.1b for the optimal investment to vulnerable capital stock. The resource constraint is always strictly binding; thus, the consumption level decreases.

Figure 2.1b displays the optimal stochastic investment policy functions for vulnerable capital stock and Figure 2.1c is for non-vulnerable capital stock. We interpret the optimal stochastic



(b) Investment in vulnerable capital stock

(c) Investment in non-vulnerable capital stock

Figure 2.1 – Stochastic policy functions with spatial adaptation when $a_t = 1$

investment policy functions in the same way as we do for the optimal stochastic consumption policy function in Figure 2.1a. In some specific domains, the curvature of the optimal investment policy functions is non-smooth, which would not be numerically suitable for derivative-based optimization solvers. As mentioned in Section 2.2.4, the ASG algorithm successfully detects kinks and adds the necessary amount of interpolated points to refine the SG around the kinks, which conventional tensor-product multidimensional global polynomial approximations may fail.

The optimal investment decision-making between the vulnerable and non-vulnerable capital stock is very intuitive but offers rich insights. When there is a rich amount of one capital stock, for instance, the amount of vulnerable capital stock in the left domain of Figure 2.1b is large enough, a large fraction of saving is allocated to the other capital stock, see Figure 2.1b and Figure 2.1c. The main reasoning of this observation is that the investment is irreversible, and the resource constraint is strictly binding. Finally, if the rare disaster occurs (State 2 is realized), the amount of investment in vulnerable capital stock is more significant than that in state 0 and 1, as shown in Figure 2.1b. The main reason for this behavior is due quadratic adjustment cost and the strictly binding resource constraint. We need to recover the damage to vulnerable capital stock to avoid paying adjustment costs that are quadratically increased. We need to decrease the level of consumption and investment to non-vulnerable capital stock to allocate more investment to vulnerable capital stock because the resource constraint is strictly binding.

Ex-post Monte-Carlo simulations

I perform the Monte-Carlo simulations based on the computed stochastic policy functions in Section 2.3.1. Instead of showing each simulated path, I evaluate the uncertainty around rare natural disasters and the AR(1) productivity shock based on a 10,000 Monte-Carlo simulations. I classify the Monte-Carlo simulations in two ways. One is based on the stochastic policy functions that internalize the stochastic elements in the *ex-post* way and I call this type of Monte-Carlo simulations an *ex-post* Monte-Carlo simulations. Second is based on the deterministic policy functions with which uncertainty is still treated as the *ex-ante* manner and I call this sort of Monte-Carlo simulations as *ex-ante* Monte-Carlo simulations. *Ex-ante* Monte-Carlo simulations prevail to tackle uncertainty with a given probability distribution, especially in the IAM literature. Lontzek and Narita (2011) suggest that the *ex-post* Monte-Carlo simulation is substantially different from *ex-ante* Monte-Carlo simulation.[16]

We examine stochastic growth paths of some critical variables for two models with either spatial adaptation or adaptive capital stock. In the initial state, the amount of each capital stock is 80% of its certainty-equivalent steady state level. We mainly report the mean, 25% and 75% quartile and the possible simulated range situated between 5% and 95% quartile.

Figure 2.2 presents the consumption profile with spatial adaptation with the best and the worst consumption trajectories. On average, the consumption grows to its deterministic steady state level, and the marginal utility strictly decreases. The mean is nicely situated within the 25% and 75% quartiles. The best and the worst two trajectories are reported. These four trajectories are very unlikely but still possible; however, one should note that all possible consumption paths are situated in the gray cloud with 90% probability.

Figure 2.3 summarizes the growth path of the vulnerable capital stock in Figure 2.3a and the non-vulnerable capital stock in Figure 2.3b. Both capital stocks show a strictly increasing accumulation trend toward their certainty-equivalent steady state levels. If a rare disaster frequently hits the economy, as illustrated in Figure 2.3a with red color, vulnerable capital stocks largely deviate from their average growth trends and cannot return to them (see, the two sample trajectories in red). In these two very unlikely but catastrophic cases, even the

[16] I follow Lontzek and Narita (2011) for the terminology *ex-post* and *ex-ante* Monte-Carlo simulations.

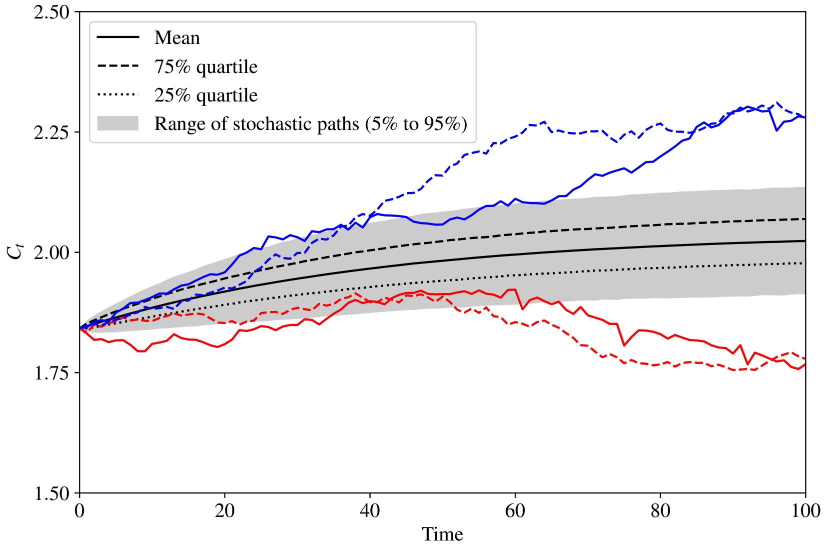


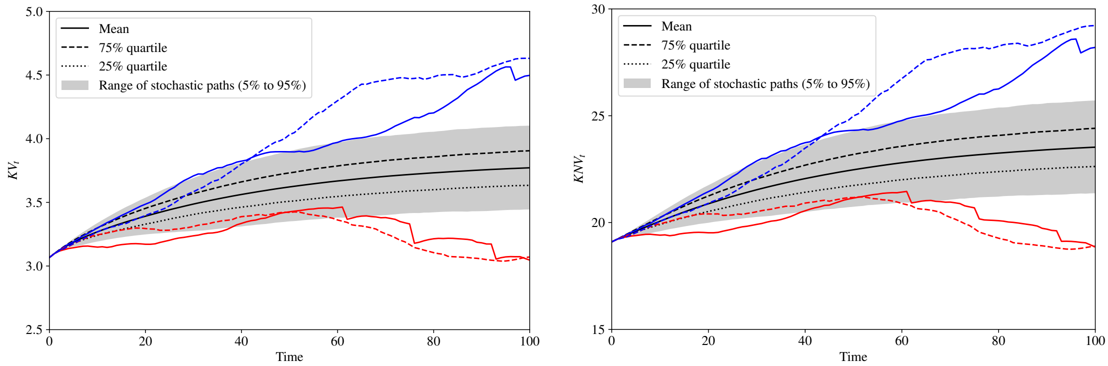
Figure 2.2 – Consumption profile with spatial adaptation. The best and the worst two trajectories are reported in blue and red, respectively.

level of non-vulnerable capital stock exhibits a decreasing trend. Capital stock faces a constant depreciation; therefore, the capital stock can depreciate if the amount of investment is not enough to compensate. Plus the total production can decrease if the amount of capital stock decreases. The consumer requires some amount of consumption to achieve smooth consumption, and in this case, the possible amount of investment allocated in non-vulnerable capital stock becomes limited and it, in turn, accelerates depreciation in non-vulnerable capital stock.

Finally, Figure 2.4 plots the ratio of vulnerable capital stock to non-vulnerable capital stock. We take the first derivative of the capital ratio, as in Eq. (2.25):

$$\frac{d}{dt}\left(\frac{KV_t}{KNV_t}\right) = \frac{KV_t}{KNV_t}\left(\frac{\dot{KV}_t}{KV_t} - \frac{\dot{KNV}_t}{KNV_t}\right) = \frac{KV_t}{KNV_t}\left(g_{KV} - g_{KNV}\right) \tag{2.25}$$

g_{KV} stands for the growth rate of vulnerable capital stock and g_{KNV} is the growth rate of non-vulnerable capital stock. Eq. (2.26) argues that when the curve of the capital ratio slopes upward, the growth rate of the vulnerable capital stock exceeds that of non-vulnerable capital stock. It is the case for roughly the first 10 years. The reason for this is that the marginal productivity of vulnerable capital stock is higher than that of non-vulnerable capital stock in equilibrium. The social planner aims to accumulate vulnerable capital stock to have enough resources for economic growth. Once the economy has enough amount of vulnerable capital stock, the slope of the capital ratio trends downward, meaning that the growth rate of the non-vulnerable capital stock exceeds that of vulnerable capital stock. Why is it the socially optimal decision to invest in less productive but risk-free capital stock? The representative agent is



(a) Vulnerable capital stock

(b) Non-vulnerable capital stock

Figure 2.3 – Capital accumulation profile with spatial adaptation. The best and worst two trajectories are reported in blue and red, respectively.

risk-averse and precautionary saves for future uncertainty. Investment in non-vulnerable capital stock can be understood as precautionary saving in this context.

2.3.2 Adaptive capital stock

Stochastic policy functions

The optimal stochastic policy functions with adaptive capital stock are presented in Figure 2.5 when the AR(1) productivity shock is fixed at one.[17] In Figure 2.5a, the optimal stochastic consumption policy function is smooth enough and strictly increases in the productive capital stock K_t. Remember that the adaptive capital stock A_t is non-productive. It is why we can observe a weak relationship between the level of A_t and consumption. When state 2 is realized where a rare disaster occurs; we observe a substantial consumption loss caused by the shock in the consumption policy function.

Figures 2.5b and 2.5c present the optimal stochastic investment policy functions for productive capital stock and for adaptive capital stock, respectively. As seen in Figure 2.5b, productive capital stock requires almost a constant level of investment throughout the whole domain. When the large shock occurs (state 2), the productive capital stock requires more investment to recover the damage, mainly to avoid paying quadratic adjustment costs. The optimal adaptive investment function in Figure 2.5c presents interesting implications. When the economy is still developing, i.e., the level of productive capital stock is small enough, the social planner does not allocate any savings to adaptive capital stock. The main reason to do this is to ensure resources are available for continuous development. Once the level of the productive capital stock achieves a certain level, the investment in adaptive capital stock starts. In state 2, the social planner saves more than the other states. It is another reason why the level of consumption in state 2 is largely limited over the other state in Figure 2.5a.

[17] Again I arbitrary fix AR(1) shock at one to illustrate a policy function in a 3D format.

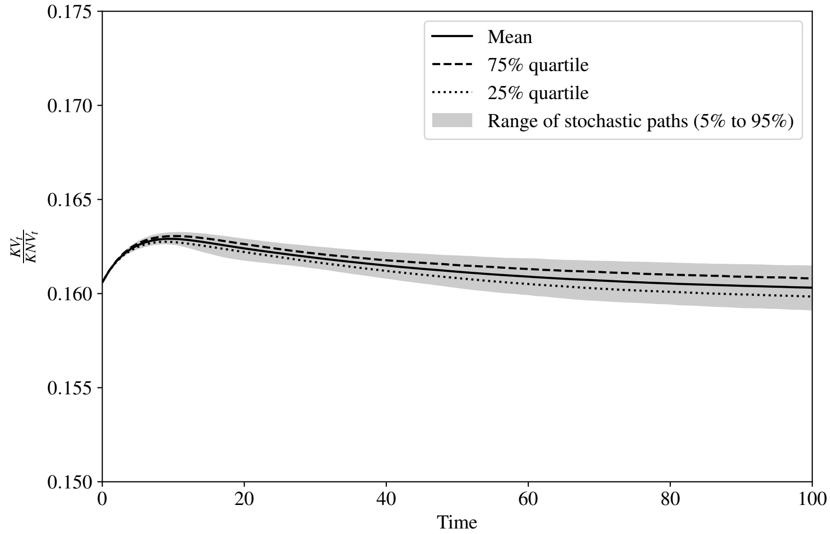


Figure 2.4 – Ratio of vulnerable to non-vulnerable capital stock

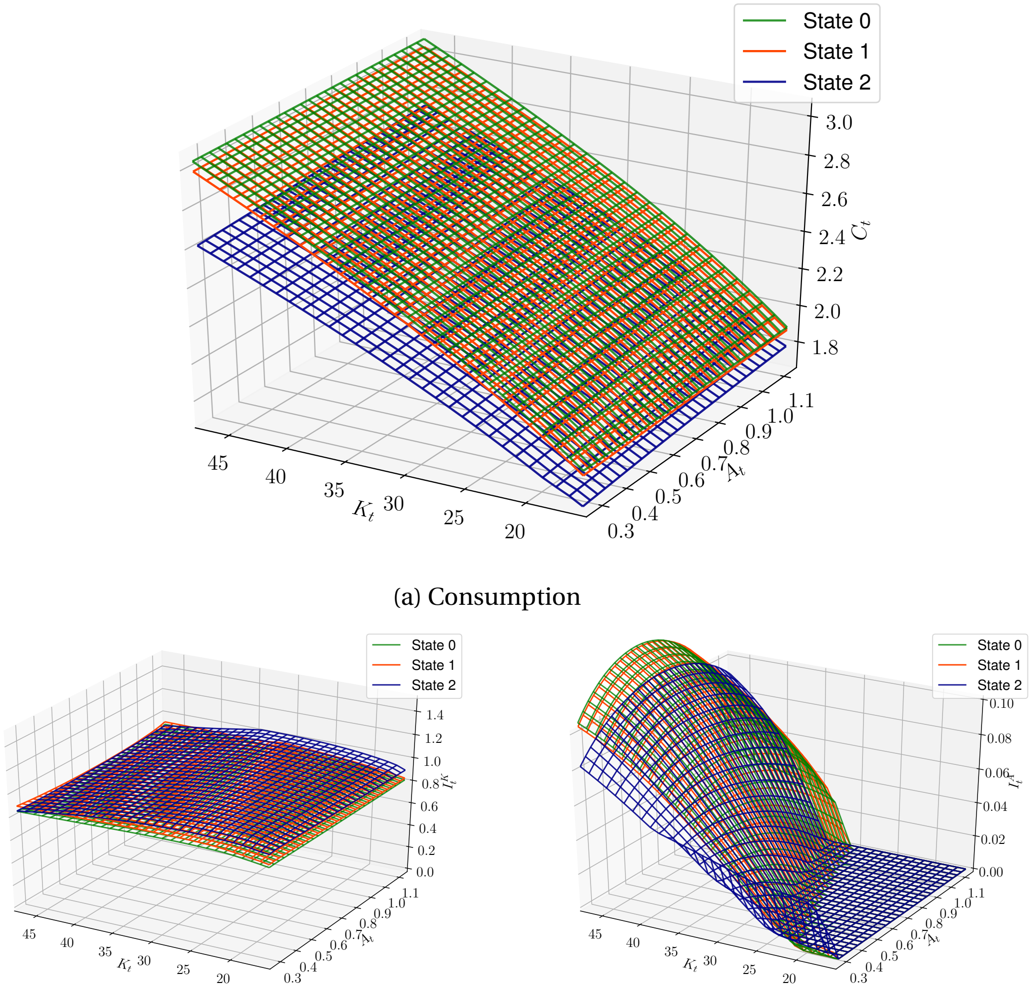
By comparing the optimal policy functions for investment in productive and adaptive capital stock, we can find an interesting observation. In state 2, as shown in Figure 2.5b, the optimal level of investment is slightly greater compared to the other states, but the level of investment to adaptive capital stock is slightly less, as seen in Figure 2.5c. This observation is consistent with the fact that the social planner prioritizes productive capital stock over adaptive capital stock when a rare disaster occurs. This investment behavior is justified because (1) the agent is risk-averse; therefore, he is eager to have smooth consumption (2) productive capital stock contributes to production activity.

Ex-post Monte-Carlo simulations

The adaptive capital stock is not productive and becomes only effective when a shock occurs. The social planner solves an intertemporal optimization problem hedging future uncertainties. Therefore, the investment in the non-productive but adaptive capital stock has a precautionary sense.

Figure 2.6 illustrates the mean consumption and its distribution when adaptive capital stock is available. The blue lines indicate the best two trajectories, whereas the red ones show the most catastrophic pathways. The two red trajectories show the case when the economy has experienced multiple disasters. Note that, even the red lines are very catastrophic, the possible consumption path is situated in the gray region with a probability of 90%. The blue consumption paths present the very optimistic pathways but note that those two are also exceptional.

Figure 2.7 summarizes the growth path of productive and adaptive capital stock. As shown in Figure 2.7a, the productive capital stock grows to its certainty-equivalent steady state level.



(a) Consumption

(b) Investment in productive capital stock

(c) Investment in adaptive capital stock

Figure 2.5 – Stochastic policy functions with adaptive capital stock when $a_t = 1$

As the two red-colored trajectories illustrate, when a rare disaster occurs, a large amount of productive capital stock is damaged. Adaptive capital stock provided in Figure 2.7b grows from the initial period. Adaptive capital stock faces the risk of uncertain disasters; therefore, when a disaster occurs, the adaptive capital stock sustains a loss. I assume that both capital stocks are equally damaged when a disaster occurs. This is why we can observe a similar trend in Figure 2.7a and Figure 2.7b.

We plot the ratio of productive capital to adaptive capital stock in Figure 2.8. In order to interpret the figure, Eq. (2.26) is helpful:

$$\frac{d}{dt}\left(\frac{K_t}{A_t}\right) = \frac{K_t}{A_t}\left(\frac{\dot{K}_t}{K_t} - \frac{\dot{A}_t}{A_t}\right) = \frac{K_t}{A_t}\left(g_K - g_A\right) \tag{2.26}$$

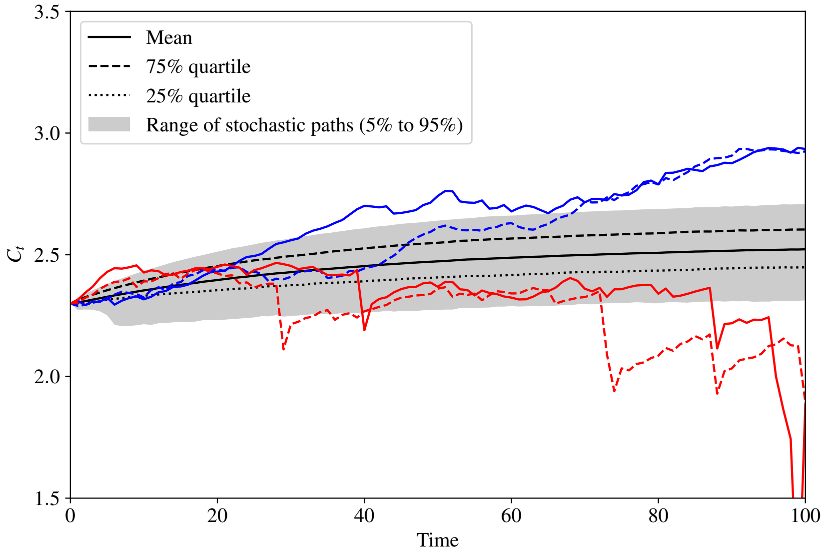


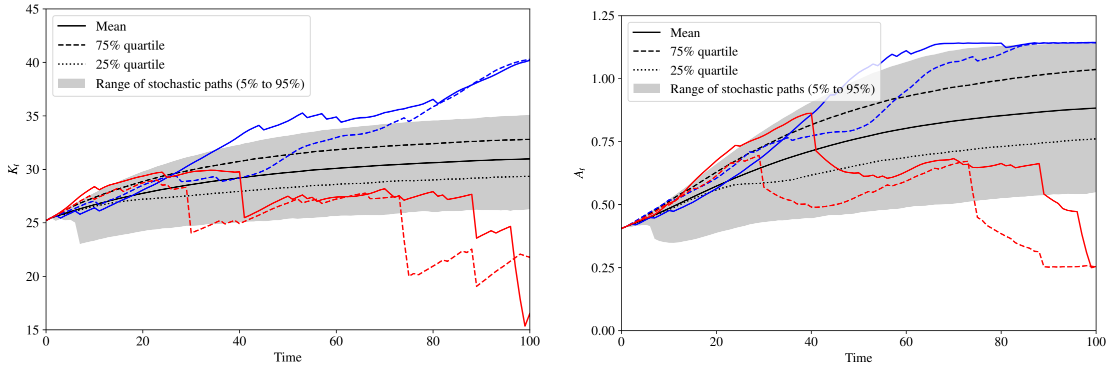
Figure 2.6 – Consumption profile with adaptive capital stock. The best and worst two trajectories are reported in blue and red, respectively.

g_K is the growth rate of the productive capital stock and g_A stands for that of the adaptive capital stock. Note that by definition, $K_t, A_t > 0,\ \forall t$, when the time derivative in Eq. (2.26) is negative, the growth rate of the adaptive capital stock exceeds that of the productive capital stock.

Figure 2.8 shows that, in its mean, $\frac{K_t}{A_t}$ is negative in the first decades and becomes gradually flat. Considering Eq. (2.26), we observe that $g_K < g_A$ holds in the first decades. The adaptive capital stock optimally accumulates at a higher rate than the productive capital stock. When the slope is approaching to flat, the growth rate of both capital stocks is the same. It suggests that the economy is on a balanced growth path.

2.3.3 Euler error

To evaluate the accuracy of our approximation, following Judd (1992) for instance, I calculate the relative error in the Euler equations and call it as an Euler error (EE). We can define for example, Euler errors for spatial adaptation in Section 2.2.2 such that:



(a) Productive capital stock

(b) Adaptive capital stock

Figure 2.7 – Capital accumulation profile with adaptive capital stock. The best and worst two trajectories are reported in blue and red, respectively.

Table 2.2 – Relative error in the Euler equations in $\log_{10}$, where $L_0 = 5$, $L_{max} = 10$ and $\epsilon = 0.001$.

	Spatial adaptation	Adaptive capital
Max error	−2.42	−2.39
Average error	−5.43	−5.70

$$EE_t^{KV} = \left|\beta\mathbb{E}_t\left[\lambda_{t+1}\left\{a_{t+1}f_{KV}\left(KV_{t+1}, KNV_{t+1}\right) + \left(1-\delta-\tilde{\Delta}^z\right) + \frac{q^{KV}}{2}g_{t+1}^{KV}\left(g_{t+1}^{KV}+2\right)\right\}\right.\right.$$
$$\left.\left. -\mu_{t+1}^{KV}\left(1-\delta-\tilde{\Delta}^z\right)\right]\left\{\lambda_t\left[1+q^{KV}g_t^{KV}\right]-\mu_t^{KV}\right\}^{-1}-1\right| \tag{2.27}$$
$$EE_t^{KNV} = \left|\beta\mathbb{E}_t\left[\lambda_{t+1}\left\{a_{t+1}f_{KNV}\left(KV_{t+1}, KNV_{t+1}\right) + (1-\delta) + \frac{q^{KNV}}{2}g_{t+1}^{KNV}\left(g_{t+1}^{KNV}+2\right)\right\}\right.\right.$$
$$\left.\left. -\mu_{t+1}^{KNV}(1-\delta)\right]\left\{\lambda_t\left[1+q^{KNV}g_t^{KNV}\right]-\mu_t^{KNV}\right\}^{-1}-1\right| \tag{2.28}$$

Euler error measures the period-to-period relative deviation the computed policy functions are used. Euler equations should be strictly binding; therefore, the Euler error should be very small, if we reasonably approximate the policy functions.

I evaluate the Euler error at many points in the state space. Evaluation points are given as 1,000 random points drawn from the uniform distribution of the entire state space. Table 2.2 summarizes the maximum and average error for the three models we have developed. All values are reported in $\log_{10}$ scale.

The reported errors are reasonably small compared to existing literature such as Brumm and Grill (2014) and Brumm and Scheidegger (2017). My modeling exercises are different from those of two literature; thus I cannot just conclude the superiority of the adaptive sparse

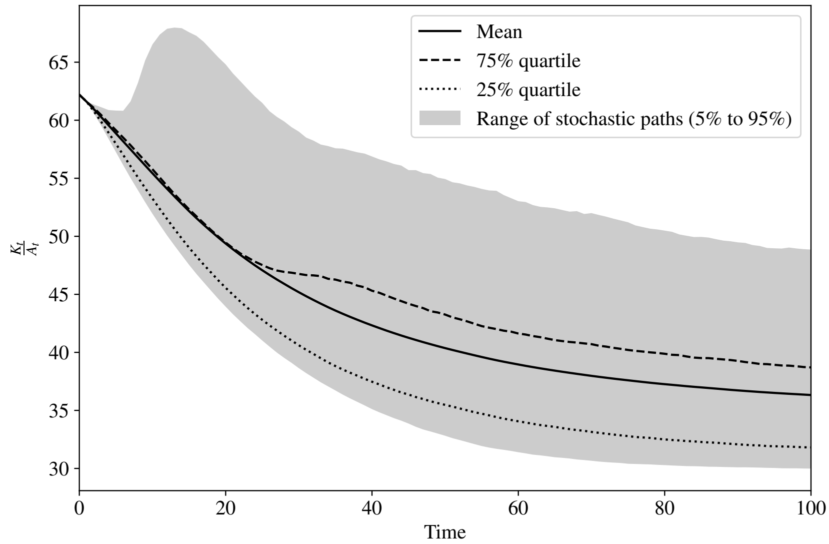


Figure 2.8 – Ratio of productive to adaptive capital stock

grid with a hierarchical local polynomial over other interpolated techniques. I can conclude that the adaptive sparse grid interpolation correctly detects non-smooth regions in policy functions and reasonably approximates them with a tolerated approximation error.

2.4 Uncertainty qualification

In this section, I aim to numerically demonstrate the applicability of stochastic modeling over a deterministic approach, especially in the context of fat-tailed distributed rare natural disasters. We have studied the stochastic policy functions in Section 2.2.2 for spatial adaptation and in Section 2.2.3 for adaptive capital stock. Firstly, the two stochastic models need to be converted to certainty-equivalent deterministic ones.

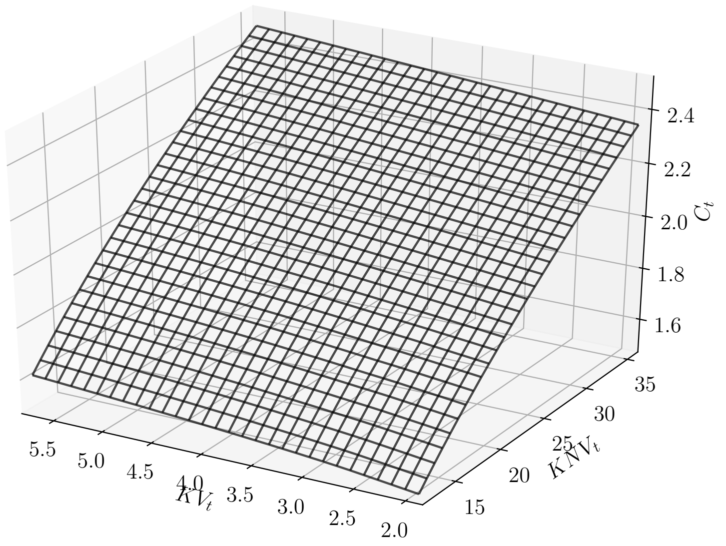
The most convenient way to transform a stochastic model to a certainty-equivalent deterministic model might be to take an expected value of uncertain parameters.[18] In our example, I replace the stochastic damage scale $\tilde{\Delta}^z$ and AR(1) productivity shock by its expected value:[19]

$$\overline{\Delta} = \mathbb{E}\left[\tilde{\Delta}^z\right] = \sum_z \pi(z) \times \Delta^z \tag{2.29}$$

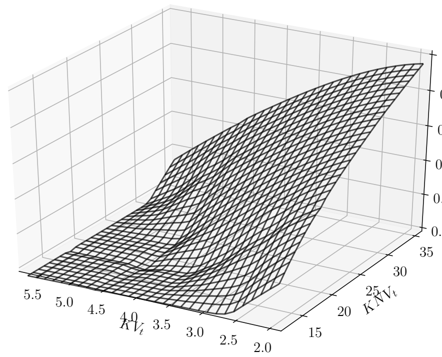
$$\overline{a} = \mathbb{E}\left[a_t\right] = 1 \tag{2.30}$$

[18] Weitzman (2009, 2011) criticize that expected damage may be inappropriate to represent rare but catastrophic events. The main purpose of Section 2.4 is to quantitatively demonstrate the applicability of stochastic modeling over a deterministic model, especially for rare but catastrophic events. I conjecture that a certainty-equivalent deterministic model with expected values is good enough to support my arguments.

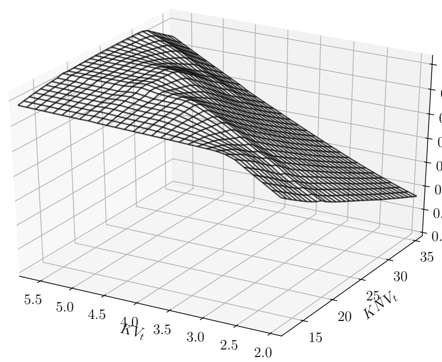
[19] In this transformation, we implicitly assume a risk-neutral economic agent, unless we assume a risk-averse economic agent.



(a) Consumption



(b) Investment to vulnerable capital stock

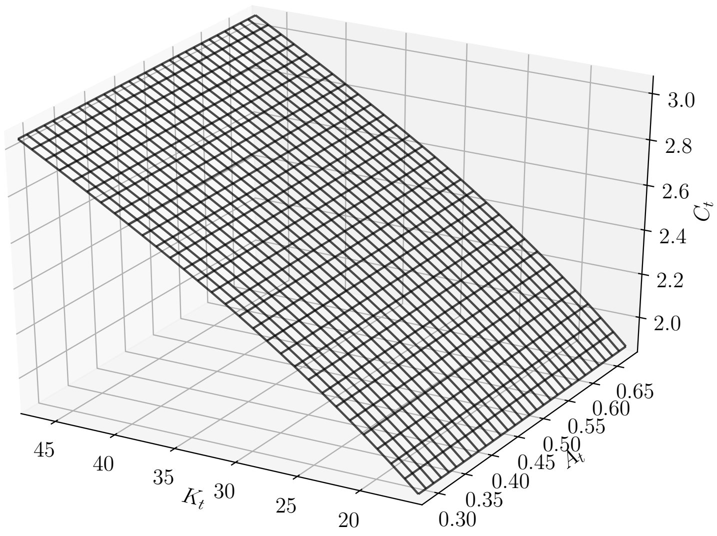


(c) Investment to non-vulnerable capital stock

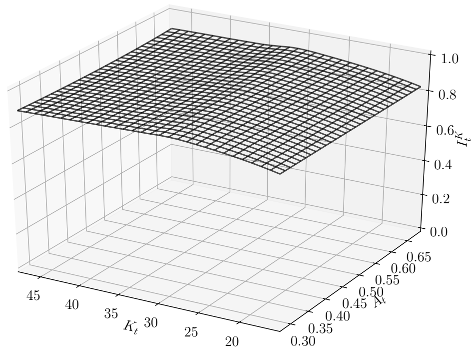
Figure 2.9 – Deterministic policy functions with spatial adaptation

We solve certainty-equivalent deterministic models with either spatial adaptation or adaptive capital stock by the time iteration collocation with ASG. The resulted policy functions with the spatial adaptation are presented in Figure 2.9, and Figure 2.10 shows the results with the adaptive capital stock. Comparison between the stochastic and the deterministic policy functions are provided in Appendix F.

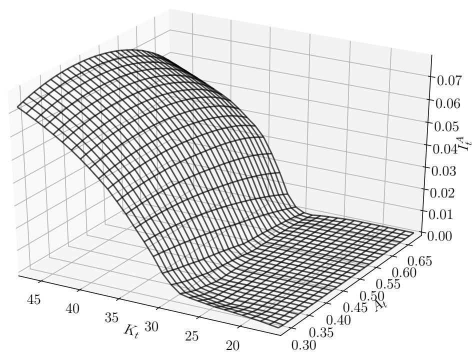
Based on the stochastic policy functions with either spatial adaptation or adaptive capital stock as well as the deterministic policy functions with either spatial adaptation or adaptive capital stock, I perform 10,000 Monte-Carlo simulations. The initial state variables are assumed to be 80% of each certainty-equivalent steady state level. Stochastic policy functions, provided in Figures 2.1 and 2.5, internalize the stochastic damage on a capital stock in the *ex-post* way. Therefore we regard the Monte-Carlo simulations based on the stochastic policy function as an *ex-post* Monte-Carlo simulation. In contrast, the deterministic policy functions in Figures 2.9



(a) Consumption



(b) Investment to productive capital stock

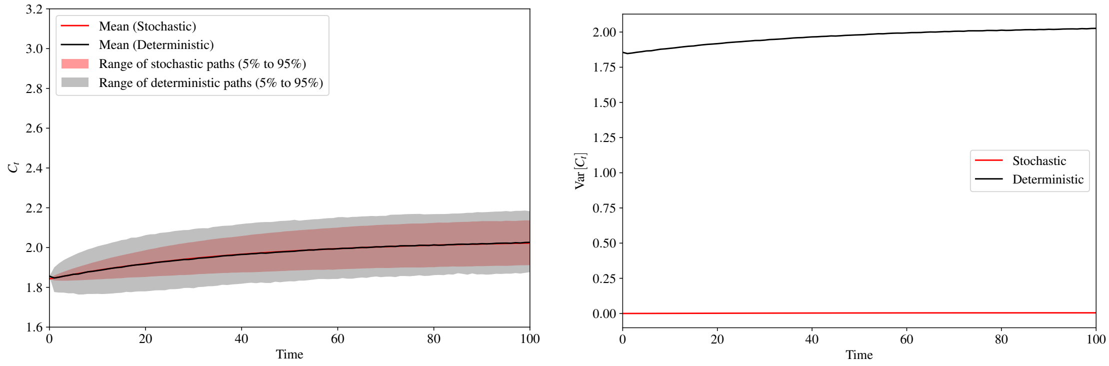


(c) Investment to adaptive capital stock

Figure 2.10 – Deterministic policy functions with adaptive capital stock

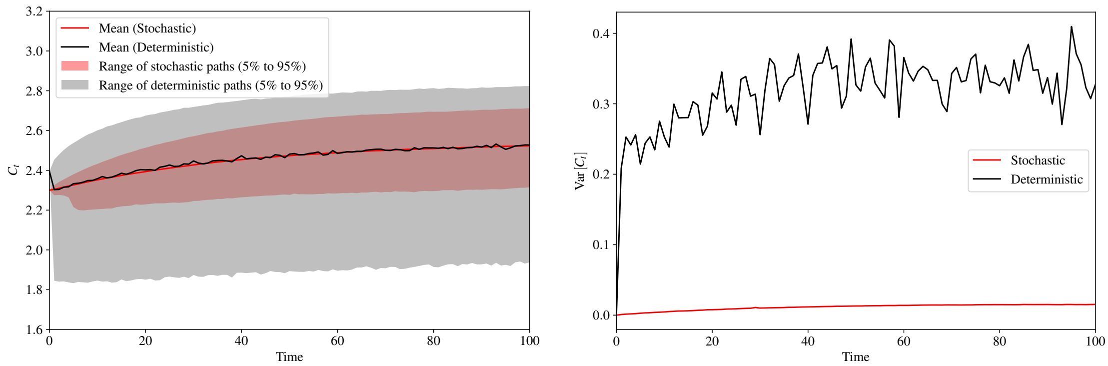
and 2.10 fail to capture uncertainty. Thus, the Monte-Carlo simulations with deterministic policy functions treat uncertainty in the *ex-ante* way.

Figure 2.11 summarizes the consumption paths of the *ex-post* and *ex-ante* Monte-Carlo simulations with spatial adaptation. I plot the mean and the possible simulation range in Figure 2.11a and the variance of 10,000 simulated paths in Figure 2.11b. We cannot observe a clear difference between the *ex-post* and the *ex-ante* Monte-Carlo simulation in their means. This observation is intuitive in the context of rare disasters. The probability of having a shock is small, thus state 0 dominates the other states. Moreover, The expected value computed in following Eq. (2.29) is close to 0. We underestimate the risk of rare disasters in the certainty-equivalent deterministic model. The 5% to 95% simulation range of deterministic paths is much wider than that of stochastically simulated paths and consequently the variance of the *ex-ante* Monte-Carlo simulated path is bigger than that of the *ex-post* simulated path as shown



(a) Mean and possible range (b) Variance

Figure 2.11 – Stochastic and deterministic consumption paths with spatial adaptation



(a) Mean and possible range (b) Variance

Figure 2.12 – Stochastic and deterministic consumption paths with adaptive capital stock

in Figure 2.11b. In the *ex-ante* Monte Carlo simulation, the policy function is deterministic and fails to capture uncertainty. Therefore, an agent based on the deterministic policies does not precautionary save for uncertain shocks in the future. On the other hand, policy functions based on stochastic programming correctly hedge the risk of uncertain damage caused by a shock.

Figure 2.12 reports the statistical information of the *ex-post* and the *ex-ante* Monte-Carlo simulations with an adaptive capital stock. Main observations are the same as what we discuss Figure 2.11.

We demonstrate the advantage of stochastic programming over the certainty-equivalent deterministic programming.[20] We aim to present the optimal adaptation decision to a rare disaster. Especially in the nature of adaptation, if the resulting policy functions induce a random trend

[20] Cai, Judd, and Lontzek (2015) achieve the same conclusion. They solve a dynamic stochastic integration of climate and the economic model to compute the social cost of carbon (SCC). One of their main conclusion is that the mean of the SCC is close to that implied by deterministic models, but the path of SCC is a very random walk with substantial variance.

with substantial variance, these policies lead to an unstable consequence and are inappropriate for adapting to rare disasters. Climate-related events, including rare disasters, are known to pose a fat-tailed probability distribution with sometimes catastrophic consequences. In this case, taking an expected value tends to underestimate the risk of disasters (Weitzman, 2009, 2011). Adaptation decisions recognize and hedge future uncertainties correctly, therefore, policies based on the deterministic models might be misleading.

2.5 Conclusion

In this chapter, I discuss the optimal adaptation decisions to a rare natural disaster. I implement either spatial adaptation or adaptive capital stock on a dynamic stochastic equilibrium model and solve them as a social planner's problem under uncertainty. The time iteration collocation with an adaptive sparse grid algorithm numerically approximate the system of non-linear equilibrium conditions for the optimal policy functions (Brumm and Scheidegger, 2017). Each time iteration and an adaptive step is massively parallelized via MPI and implemented on a high-end computing cluster.

I demonstrate that the time iteration collocation with an adaptive sparse grid correctly detects non-smooth regions in an interpolant and overcomes the related numerical difficulty with a tolerated computing expense. I numerically demonstrate why the algorithm is appealing in the set of modeling exercises in this chapter.

Spatial adaptation and adaptive capital stock take the form of stock adaptation; therefore, it is practically essential to gain insights about the optimal investment decisions between vulnerable and non-vulnerable capital stock for spatial adaptation and productive and adaptive capital stock for the introduction of adaptive capital stock. Our optimal policy functions with spatial adaptation suggest that once a disaster occurs, we need to allocate more investment to rebuild vulnerable capital stock, mainly securing enough production while avoiding paying quadratic adjustment costs. Policy functions reveal that, after a disaster, the social planner should prioritize productive capital stock over adaptive capital stock.

The advantage of stochastic policy functions over the conventional deterministic approach is in the correct internalization of uncertainty around rare disasters and AR(1) productivity shock. Monte-Carlo simulated paths based on the certainty-equivalent deterministic model show a random trend with substantially larger variance than those based on the stochastic policy functions. Any policies that lead to unstable consequences are not appealing, especially for adaptation to natural disasters. Furthermore, as Weitzman (2009) claims, rare natural disasters follow fat-tailed distributions with catastrophic consequences. Conventional deterministic approach underestimates the risk of uncertain disasters.

We quantitatively compare Monte-Carlo simulated paths either with a stochastic or a certainty-equivalent deterministic model. I numerically confirm that if an economic agent relies on deterministic policies, his economic paths show random walks with larger variance than those of stochastic models. My finding is consistent with Cai, Judd, and Lontzek (2015), where they present the same conclusion in the estimation of the social cost of carbon.

3 Spatial adaptation and adaptive capital stock

3.1 Introduction

Adaptation is more likely to be implemented in a highly regional and specific environment (Tol, 2005). The stakeholders of adaptation measures usually are regional or national authorities. Several adaptation measures have been investigated mainly using the approach based on a cost-benefit analysis. Developed countries have realized the importance of adaptation to limit nearer-term impacts from natural hazards. de Bruin, Agrawala, and Dellink (2009) studies a variety of adaptation measures for the Netherlands. Ranger et al. (2010) demonstrate a framework for adaptation decision-making and include some case studies for a high impact flood in the UK. Millner and Dietz (2015) is one of the first to study the optimal adaptation decisions for a developing economy (sub-Saharan Africa) and suggest that a developing country tends to be more vulnerable to exogenous shocks than a developed country. Optimal adaptation decisions should highly depend on the state of current economic development, however, there is no criteria to know how much a developed (or developing) economy invests in adaptation a priori.

Schelling (1992, Section IV) qualitatively claims in his essay that developing countries should not allocate resources to slow or to adapt to climate changes, and that “their best defense against climate change may be their own continued development”. I quantitatively approach this argument in this chapter by extending a dynamic stochastic equilibrium model developed in Chapter 2.

I develop a dynamic stochastic equilibrium model with spatial adaptation and adaptive capital stock.[1] The representative agent decides the allocation of investment between three types of capital stock: productive but vulnerable capital stock (KV), productive and non-vulnerable capital stock (KNV), and non-productive but adaptive capital stock (A). Adaptive capital stock is assumed to be installed only on vulnerable capital stock, to alleviate the damage caused by a rare natural disaster. Non-vulnerable capital stock is risk-free but less productive than

[1] Note that in Chapter 2, I developed two models in which spatial adaptation and adaptive capital stock are introduced independently.

vulnerable capital stock in the equilibrium. The time iteration collocation with an adaptive sparse grid solves the developed model. The model is massively parallelized on the university's computing cluster via message passing interface (MPI).

The main contribution of this chapter is to present the optimal adaptation decisions for the developed and developing economy. I demonstrate that the developed economy precautionary invests in adaptive capital stock to prepare for the uncertain catastrophic shocks from the initial state. In contrast, the developing economy at first invests in productive but vulnerable capital stock, to secure enough available resources for future adaptation. My main findings in this chapter quantitatively support the claim by Schelling (1992, Section IV).

The remainder of this chapter is organized as follows. Section 3.2.1 presents the social planner's problem with two adaptation measures. Section 3.3 shows the optimal economic growth paths for the developed and developing economy. Section 3.4 concludes this chapter with possible extensions for future study.

3.2 The model

3.2.1 Social planner's problem with two adaptation measures

I formulate the social planner's problem with spatial adaptation and adaptive capital stock under uncertainty. The objective function is to maximize the sum of discounted time-separable utility in Eq. (3.1). Constraints involve the three irreversible investment constraints for each capital stock in Eqs. (3.2) to (3.4) and the resource constraint in Eq. (3.5). The adaptive function $h(\cdot)$ defines how much the damage caused by rare disasters can be alleviated by accumulating the adaptive capital stock relative to the vulnerable capital stock. In the resource constraint Eq. (3.5), a convex quadratic adjustment cost is introduced where q^{KV}, q^{KNV} and q^{A} control the intensity of a quadratic cost. Eq. (3.6) defines the standard AR(1) productivity shock.

$$\max_{\{C_t, KV_{t+1}, KNV_{t+1}, A_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(C_t) \right] \tag{3.1}$$

$$\text{s.t.} \quad I_t^{KV} = KV_{t+1} - \left(1 - \delta - \tilde{\Delta}^z h(KV_t, A_t)\right) KV_t \geq 0 \quad \perp \quad \mu_t^{KV} \geq 0 \tag{3.2}$$

$$I_t^{KNV} = KNV_{t+1} - (1 - \delta) KNV_t \geq 0 \quad \perp \quad \mu_t^{KNV} \geq 0 \tag{3.3}$$

$$I_t^{A} = A_{t+1} - \left(1 - \delta - \tilde{\Delta}^z h(KV_t, A_t)\right) A_t \geq 0 \quad \perp \quad \mu_t^{A} \geq 0 \tag{3.4}$$

$$\begin{aligned} & a_t f(KV_t, KNV_t) - I_t^{KV} - \frac{q^{KV}}{2} KV_t \left(\frac{I_t^{KV}}{KV_t} - \delta - \tilde{\Delta}^z h(KV_t, A_t) \right)^2 - I_t^{KNV} \\ & - \frac{q^{KNV}}{2} KNV_t \left(\frac{I_t^{KNV}}{KNV_t} - \delta \right)^2 - I_t^{A} - \frac{q^{A}}{2} A_t \left(\frac{I_t^{A}}{A_t} - \delta - \tilde{\Delta}^z h(KV_t, A_t) \right)^2 \\ & - C_t \geq 0 \quad \perp \quad \lambda_t \geq 0 \quad \text{where} \begin{cases} \tilde{\Delta}^{z=0} = 0, & \Pr[z = 0] = \pi(0) \\ \tilde{\Delta}^{z=1} = \Delta^1, & \Pr[z = 1] = \pi(1) \\ \tilde{\Delta}^{z=2} = \Delta^2, & \Pr[z = 1] = \pi(2) \end{cases} \end{aligned} \tag{3.5}$$

$$\ln a_{t+1} = \rho \ln a_t + \epsilon_{t+1}, \quad \epsilon_{t+1} \sim N\left(0, s^2\right) \tag{3.6}$$

where, β stands for a discount factor. The utility function $u(\cdot)$ is the standard CRRA utility function as in Eq. (2.2). The CES production function $f(\cdot)$ is same as in Eq. (2.8) and the adaptive function is same as in Eq. (2.24). We solve the social planner's problem for the first-order conditions. We need to consider the associated Karush-Kuhn-Tucker (KKT) multipliers for the occasionally binding irreversible investment constraints in Eqs. (3.2) to (3.4), and the Lagrange multiplier for the strictly binding resource constraint, as in Eq. (3.5). Symbol $\perp$ stands for a complementarity slackness.

We formulate Lagrangian and solve it for the first-order conditions. Eqs. (3.7) to (3.9) are the deriving Euler conditions. More details about the Lagrangian is deferred in Section C.3. The Euler equations characterize the equilibrium conditions, where the operator $\mathbb{E}_t$ shows an expectation from time t.

$$
\begin{aligned}
&\lambda_t\left[1+q^{KV}g_t^{KV}\right]-\mu_t^{KV}= \\
&\quad \beta\mathbb{E}_t\Bigg[\lambda_{t+1}\Big\{a_{t+1}f_{KV}\left(KV_{t+1},KNV_{t+1}\right)+\left(1-\delta-\tilde{\Delta}^z h\left(KV_{t+1},A_{t+1}\right)\right) \\
&\quad -\tilde{\Delta}^z h_{KV}\left(KV_{t+1},A_{t+1}\right)\left(KV_{t+1}+A_{t+1}\right) \\
&\quad +\frac{q^{KV}}{2}g_{t+1}^{KV}\left(g_{t+1}^{KV}+2\right)\Big\}-\mu_{t+1}^{KV}\left\{\left(1-\delta-\tilde{\Delta}^z h\left(KV_{t+1},A_{t+1}\right)\right)-\tilde{\Delta}^z h_{KV}\left(KV_{t+1},A_{t+1}\right)KV_{t+1}\right\} \\
&\quad +\mu_{t+1}^{A}\tilde{\Delta}^z h_{KV}\left(KV_{t+1},A_{t+1}\right)A_{t+1}\Bigg] \qquad (3.7) \\
&\lambda_t\left[1+q^{KNV}g_t^{KNV}\right]-\mu_t^{KNV}= \\
&\quad \beta\mathbb{E}_t\Bigg[\lambda_{t+1}\left\{a_{t+1}f_{KNV}\left(KV_{t+1},KNV_{t+1}\right)+(1-\delta)+\frac{q^{KNV}}{2}g_{t+1}^{KNV}\left(g_{t+1}^{KNV}+2\right)\right\} \\
&\quad -\mu_{t+1}^{KNV}(1-\delta)\Bigg] \qquad (3.8) \\
&\lambda_t\left[1+q^{A}g_t^{A}\right]-\mu_t^{A}= \\
&\quad \beta\mathbb{E}_t\Bigg[\lambda_{t+1}\Big\{\left(1-\delta-\tilde{\Delta}^z h\left(KV_{t+1},A_{t+1}\right)\right)-\tilde{\Delta}^z h_{A}\left(KV_{t+1},A_{t+1}\right)\left(KV_{t+1}+A_{t+1}\right) \\
&\quad +\frac{q^{A}}{2}g_{t+1}^{A}\left(g_{t+1}^{A}+2\right)\Big\}+\mu_{t+1}^{KV}\tilde{\Delta}^z h_{A}\left(KV_{t+1},A_{t+1}\right)KV_{t+1} \\
&\quad -\mu_{t+1}^{A}\left\{\left(1-\delta-\tilde{\Delta}^z h\left(KV_{t+1},A_{t+1}\right)\right)-\tilde{\Delta}^z h_{A}\left(KV_{t+1},A_{t+1}\right)A_{t+1}\right\}\Bigg] \qquad (3.9)
\end{aligned}
$$

Our system of equations consists of the three Euler equations in Eqs. (3.7) to (3.9), three occasionally binding irreversible investment constraints in Eqs. (3.2) to (3.4) and one resource constraint in Eq. (3.5). I solve the system of equations using the time iteration collocation with adaptive sparse grid, which is presented in Algorithm 2.

3.2.2 Developed and developing economy

To provide further insight into the general behavior of the model, I define the developed and developing economy, depending on how much capital stock they endow in the initial state. I at first solve the certainty-equivalent deterministic version of the model in Eqs. (3.1) to (3.6).[2] By solving the deterministic model for the certainty-equivalent steady state, I can define the level of state variables in the steady state.

In the time iteration collocation algorithm, I aim to find the optimal policies for the given state space. State space, in this chapter, is the union of the endogenous states (three capital stocks) and the exogenous states (rare natural disasters and AR(1) productivity shock). In this context, it is one option to define the level of economic development by the endogenous state in the

[2]More details on the certainty-equivalent deterministic models are found in Appendix E.

Table 3.1 – Relative error in the Euler equations in $\log_{10}$, where $L_0 = 4$, $L_{max} = 9$ and $\epsilon = 0.001$.

	Spatial adaptation and adaptive capital stock
Max error	−1.67
Average error	−2.07

initial period.[3] I define the developed and developing economy based on how far the three stock variables in each economy are from the levels in the certainty-equivalent deterministic steady state. The initial state variables in the developed economy are assumed to be 90% of each state variable in the certainty-equivalent steady state. The developing economy is assumed to reach 60% of each state variable in the deterministic steady state.

3.3 Result

3.3.1 Approximation errors

Table 3.1 presents the approximation error in the Euler equation in Eqs. (3.7) to (3.9). Compared to the cases in Table 2.2, the approximation error is relatively large. There are three capital stocks; therefore, the occasionally binding constraints become continuously harder to approximate.

The number of state variables is increased by one, but this makes computations much more expensive than the other models in Chapter 2. It significantly increases the number of optimization problems that need to be solved in the time iteration collocation and the adaptive steps. I should increase the maximum level of refinement (L_{max}) to further refine computational grids to achieve a smaller approximation error, however, the computation is already costly with the presented refinement strategy. Interpolation with adaptive sparse grid requires very dense arithmetic operations, and I speed up this process by utilizing the conventional BLAS library. This bottleneck is already reported in Brumm and Scheidegger (2017), who speed up the interpolation part by adopting GPUs.

3.3.2 Monte-Carlo simulations

Monte-Carlo simulations provide more general and graphical representations of my dynamic stochastic modeling approach. I perform 10,000 Monte-Carlo simulations with the stochastic policy functions that are computed in Section 3.2.1. I mainly plot the mean, 75% and 25% quantile, as well as present the 5% to 95% possible simulated range. The developed and developing economy starts from their initial state levels. I run the simulations for 100 periods.

[3] However, it involves an extreme assumption. This assumption implies that I assume, for instance, the same utility function, risk-aversion parameter, production function, for the two very different economies. I intend to do so to make discussions easy to follow. Detail discussions about the definition of the developed and developing economy are left for future analyses.

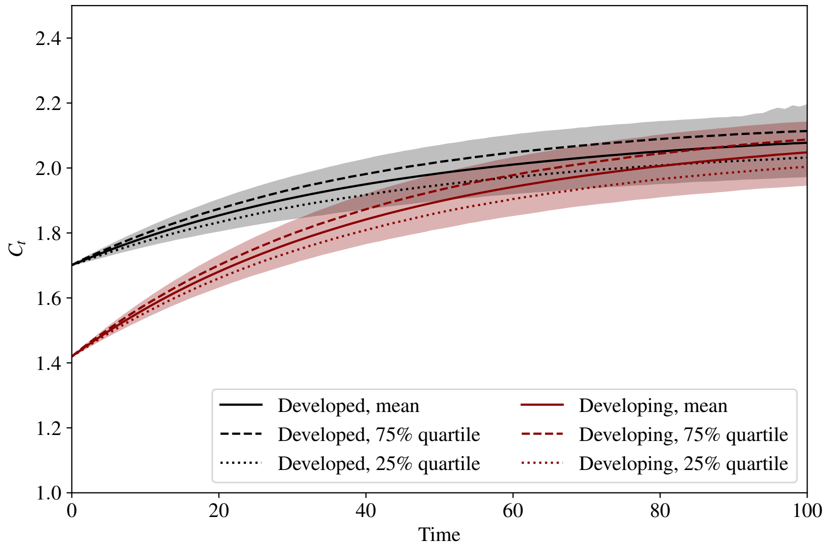
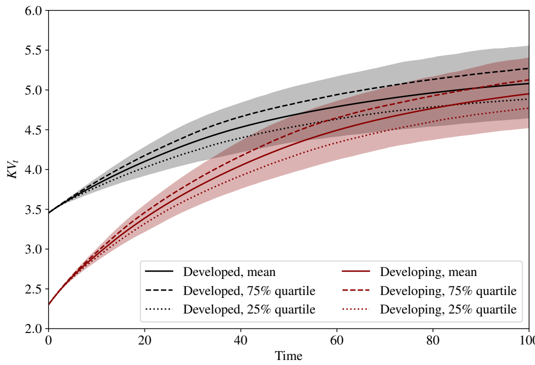


Figure 3.1 – Consumption profile with spatial adaptation and adaptive capital stock

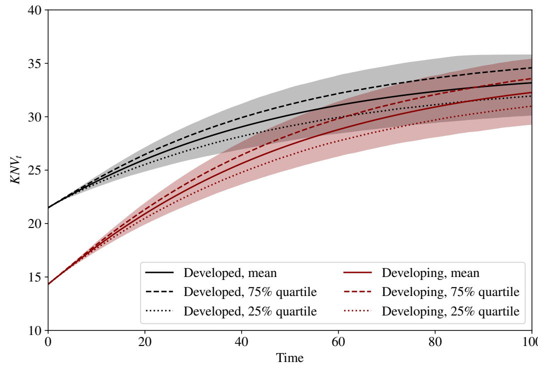
Figure 3.1 shows the consumption paths for the two economies. Developed and developing economies grow constantly. The growth rate of a developing economy exceeds that of a developed economy because the marginal utility decreases as it approaches the deterministic steady state.

Figure 3.2 summarizes the development paths of the three capital stock in the developed and developing economy. Vulnerable capital stock (Figure 3.2a) and non-vulnerable capital stock (Figure 3.2b) continue to grow toward their certainty-equivalent deterministic steady state levels. In contrast, adaptive capital stock in Figure 3.2c presents different accumulation patterns for the two economies. The developed economy starts to invest in adaptive capital stock from the initial state; however, the level of adaptive capital stock in the developing economy depreciates in the first few periods. The developing economy requires more investment in productive capital stock to secure resources for future adaptation with the adaptive capital stock. In both economies, there is an upper limit for the adaptive capital stock. This upper limit is determined by assumptions such as the production technology, the utility function, and the adaptation function. The scale of the ratio of productive capital stock to adaptive capital stock is similar to Millner and Dietz (2015).

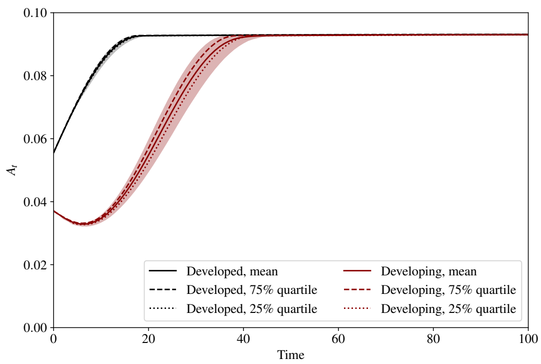
Since I measure the level of the economic development by the stock of capital variables, I focus on the growth rates of each capital stock, specifically vulnerable, non-vulnerable and adaptive capital stock. Figure 3.3 summarizes the growth path of two capital ratios. Figure 3.3a presents the ratio of vulnerable to non-vulnerable capital stock. Note that both capital stocks are productive. Figure 3.3b shows the ratio of a productive capital stock, which is the sum of vulnerable and non-vulnerable capital stock, to adaptive capital stock.



(a) Vulnerable capital stock



(b) Non-vulnerable capital stock



(c) Adaptive capital stock

Figure 3.2 – Capital accumulation profile with spatial adaptation and adaptive capital stock.

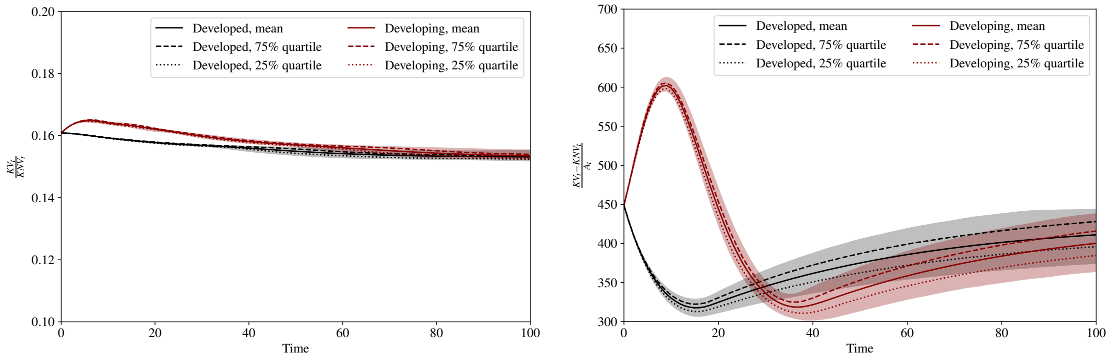
In order to interpret Figure 3.3, it is beneficial to note that:

$$\frac{d}{dt}\left(\frac{KV_t}{KNV_t}\right) = \frac{KV_t}{KNV_t}\left(\frac{\dot{KV}_t}{KV_t} - \frac{\dot{KNV}_t}{KNV_t}\right) = \frac{KV_t}{KNV_t}\left(g_{KV} - g_{KNV}\right) \tag{3.10}$$

$$\frac{d}{dt}\left(\frac{KV_t + KNV_t}{A_t}\right) = \frac{KV_t + KNV_t}{A_t}\left(\frac{\dot{KV_t + KNV_t}}{KV_t + KNV_t} - \frac{\dot{A}_t}{A_t}\right) = \frac{KV_t + KNV_t}{A_t}\left(g_{KV+KNV} - g_A\right) \tag{3.11}$$

where g_{KV} is the growth rate of the vulnerable capital stock, g_{KNV} is the growth rate of the non-vulnerable capital stock, and g_A is the growth rate of the adaptive capital stock. The interpretation of Eqs. (3.10) and (3.11) is the same as in Eq. (2.26).

Figure 3.3 summarizes the two capital ratios. In Figure 3.3a, we notice that the capital ratios of vulnerable capital to non-vulnerable capital stock in both economies are almost constant over time, even though the fact that the vulnerable capital stock faces the risk of large depreciation. This observation is explainable by the difference in the marginal productivity of the vulnerable and the non-vulnerable capital stock. In the equilibrium, the difference between the marginal productivity of the vulnerable and the non-vulnerable capital stock is equal to the difference in their depreciation rates (Acemoglu, 2009, Section 10.4). In our example, it is equal to $\tilde{\Delta}_t^z$.



(a) Ratio of vulnerable to non-vulnerable capital stock

(b) Ratio of productive to adaptive capital stock

Figure 3.3 – Capital ratio with spatial adaptation and adaptive capital stock

The risk of large depreciation by a disaster can be compensated for by the higher marginal productivity in the vulnerable capital stock. The slope of both of the capital share curves eventually flattens out. It suggests that the economy is shifting towards a balanced growth path in the long run.

Figure 3.3b shows the ratio of the productive to the adaptive capital stock. In the developing economy, the growth rate of the productive capital stock exceeds that of the adaptive capital stock in the first few decades. The developing economy prioritizes the development of productive capital stock over adaptive capital stock. Once the developing economy secures available enough resource available to adaptation, the growth rate of the adaptive capital stock becomes more significant than that of the productive capital stock. On the other hand, the developed economy accumulates adaptive capital stock faster than productive capital stock in the first few decades. The developed economy has some resources to allocate non-productive but adaptive capital stock as a precaution to secure itself from the risk of uncertain natural disasters. When enough adaptive capital stock has been prepared, investment in productive capital stock is again prioritized. Our finding quantitatively supports the qualitative claim of Thomas Schelling that the “best defense against climate change may be their own continued development” (Schelling, 1992, Section IV).

3.4 Conclusion

In this chapter, I address the optimal adaptation decisions for spatial adaptation with adaptive capital stock by using an approach based on a stochastic dynamic equilibrium model. The model is formulated as a social planner’s problem under uncertainty and is solved numerically, using the time iteration collocation with adaptive sparse grid algorithm (Brumm and Scheidegger, 2017), for the optimal policy functions. In addition to showing the optimal adaptation profile with spatial adaptation and adaptive capital stock, I quantitatively show that the optimal adaptation decisions depend on the initial state of economic development.

Table 3.2 – Wealth, by type of asset and region, adopted from Lange, Wodon, and Carey (2018)

Type of asset	Low-income	Lower-middle-income	Upper-middle-income	High-income Non-OECD	High-income OECD	World
Produced capital [%]	14	25	25	22	28	27
Natural capital [%]	47	27	17	30	3	9
Human capital [%]	41	51	58	42	70	64
Net foreign asset [%]	−2	−3	0	5	−1	0
Total wealth [10^9]	$7,161	$70,718	$247,793	$76,179	$741,398	$1,143,249
Total wealth per capita	$13,629	$25,948	$112,798	$264,998	$708,389	$168,580

The developed economy starts to invest in adaptive capital stock, which is non-productive capital stock. The developing economy at first accumulates productive capital stock, especially vulnerable capital stock, as a priority to develop over adaptive capital stock. A risk-averse agent in the developed economy saves for future uncertainty as a precaution, on the other hand, the economic agent in the developing economy prioritizes today's development, even I assume the same degree of risk-aversion. Schelling (1992, Section IV) claimed that the developing countries' "best defense against climate change may be their own continued development". My finding quantitatively supports his argument.

In this study, I adopt rather stylized assumptions to distinguish developed economy from developing one. Besides initial capital endowment, there are possibly many sources that are desirable to be considered. In general, a production function in the developed economy is more capital intensive one than that in the developing economy. Representative agent in the developed economy may be more risk-averse than those in the developing economy. Benchmark share of vulnerable capital stock in the developed economy may be smaller than that in the developing economy. More detailed analyses with the above parameter assumptions are desired, but I would leave these for future research.

Finally, Lange, Wodon, and Carey (2018) suggested that the main difference between developed and developing economy is in the importance of human capital, as presented in Table 3.2. Besides production and natural capital, we can observe the considerable difference in human capital between developed and developing economy, though I have not introduced the human capital yet. In economics, human capital refers to intangible assets, such as the stock of skills, knowledge and education, that potentially increase worker's productivity. According to Acemoglu (2009), the seminal contributions by Becker (1965) and Mincer (1974) are noticeable to formalize an idea of human capital in economics. It is a desirable future research topic to endogenize the role of human capital in the presented framework.

There are several possible extensions. First, it would be interesting to introduce heterogeneous agents in the model. After a seminal contribution by Krusell and Smith, Jr. (1998), there is a growing demand to model heterogeneous agents within the general equilibrium framework.

Perspective for uncertainty involves a high degree of heterogeneity; therefore, I can enrich our discussions by adding this point. Second, it would be interesting to investigate the learning effect with which economic agents might reduce or resolve uncertainties. As shown in Kelly and Kolstad (1999), implementing the learning effect in a dynamic setting can provide more reliable policy recommendations, especially in the context of climate change. Finally, my model should be applied to real economies to provide policy recommendations for countries. My modeling framework is scalable to include several sectors. Empirical policy simulations are considerably demanding; however, many of the critical parameters that characterize adaptation decisions are still highly uncertain at this time.

General conclusion

This dissertation aims at opening the way toward a better consideration of uncertainties, in particular, rare natural disasters, and at addressing the adaptation decisions by solving computational economic models. Throughout the thesis, I pursue the research question: How do we optimally adapt or alleviate the risk of uncertain rare natural disasters? In this chapter, I briefly summarize the main modeling lessons, findings and policy recommendations from each chapter, and present the desired research extensions for the future.

First, I closely overview the sources of uncertainty and review related literature to shed light on research gaps between existing literature and the presented dissertation. Among many sources of uncertainty, I ascertain that rare natural disasters and their adaptation decisions are worth to be further investigated since the conventional deterministic approaches underestimate the risk of rare disasters, for instance, see Cai, Judd, and Lontzek (2015). Furthermore, after a seminal contribution by Weitzman (2009), there is no academic consensus on how to model rare natural disasters and to quantify the risk of catastrophic events with a low probability, especially employing a computational model, as far as I have realized. The dissertation aims to fulfill these research gaps in the subsequent chapters.

In Chapter 1 I extend the existing computational general equilibrium model for Switzerland, GENESwIS, to simulate adaptation decisions for the significant empirical floods in Switzerland. Although I developed a new simulation approach called hazard myopia to correctly introduce a subjective estimation of the risk of future floods in intertemporal decision processes, my simulation results contradict the real Swiss situations. Rare natural disasters require a different treatment of uncertainty from that of small but frequent shocks. To correctly quantify the risk of rare disasters, the better consideration of uncertainty is demanding.

In Chapter 2 I develop two dynamic stochastic economic models to better address the uncertainty and study the optimal adaptation decisions for rare natural disasters. I consider spatial adaptation and the introduction of adaptive capital stock in a stylized but rather reproducible setting. The numerically approximated policy functions contain non-smooth regions that are correctly detected by the employed numerical algorithm, namely adaptive sparse grid (Brumm and Scheidegger, 2017). The policy functions with spatial adaptation claim that we need to prioritize vulnerable capital over non-vulnerable capital stock after a disaster, mainly securing enough production in the subsequent periods. The policy function with adaptive capital stock presents that we should start to invest to adaptive capital stock only if we secure enough resources for productive capital stock to prepare for future uncertainty. I confirm that a certainty-equivalent deterministic model underestimates the risk of rare natural disasters. Stochastic policy functions presented in this chapter internalize and hedge the risk of future catastrophes in an *ex-ante* way, whereas a certainty-equivalent deterministic model treats it in an *ex-post* way. I conclude that a dynamic stochastic approach can propose a better adaptation policy to a rare natural disaster than a certainty-equivalent deterministic model as Cai, Judd, and Lontzek (2015) demonstrated in the calculation of the social cost of carbon.

In Chapter 3 I quantitatively approach the classical argument established by Schelling (1992, Section IV). I demonstrate that optimal adaptation decisions depend on the initial state of economic development. As Schelling (1992, Section IV) claimed, my numerical simulations show that the optimal adaptation in the developing economy is in their continued economic development. On the other hand, the developed economy should start to invest in adaptation activities to prepare for future uncertainties.

Using the implementation of adaptation measures on computational economic models, I present stylized but reproducible adaptation decisions to rare natural disasters in Chapters 2 and 3. The modeling of rare natural disasters requires the special treatment of catastrophic outcomes with a low probability. I demonstrate that the conventional (certainty-equivalent) deterministic model underestimates the risk of rare natural disasters. The optimal adaptation strategy depends on the current state of economic development. Throughout Chapters 2 and 3, my implementations are massively parallelized on a high-end computing cluster to speedup the solving processes. The efficient usage of high-end computing facilities is an emerging field in economics. The applicability of the massive power of modern computing resources in economics is demonstrated in this dissertation.

Much work should be pursued based on the presented dissertation in the future. This thesis pays particular attention to the modeling of stochastic but catastrophic shocks; however, we should systematically quantify parametric uncertainty following Harenberg et al. (2018). Economic models, especially in the field of environmental economics, deal with long time horizon. Therefore, the impact of learning should be correctly endogenized, for instance following the classical literature Kelly and Kolstad (1999). The recursive framework adopted in Kelly and Kolstad (1999) can be naturally extended and implemented within the presented modeling approach.

In Switzerland, the scientific community has realized moreover, warned that the frequency of great floods is projected to increase mainly due to climate change (CH2018, 2018). It would accelerate the research and development (R&D) activities in the development of various adaptation technologies. Detailed discussions and the implementations of technological development and innovation would be interesting to provide more elaborate policy recommendations for a specific context.

Adaptation to rare natural disasters does not aim at only alleviating direct damage on capital stock, but also transforming the economy more robust and resilient toward uncertain catastrophic shocks. The thesis strictly discusses the role of non-vulnerable or adaptive capital stock in the context of adaptation. However, it would be desirable to better situate the role of these capital stocks in the discussion of robustness and resilience.

Although economic damage caused by natural disasters are calculated based on insurance claims, especially in the private sector, the insurance system is missing in this dissertation. The optimal adaptation decisions of a risk-averse agent with and without access to an insurance lie outside the scope of the thesis, but I have noticed that it would be an interesting extension and should be pursued in the future.

A More details on MCP formulations

A.1 Zero-profit conditions

The unit cost function of the domestic production sector s is in Eq. (A.1):

$$C_{s,t}^{Y}\left(r_t^K, w_t, p_{i,t}^A\right) = \left[\theta_{KL,s,t}^{Y}\left\{\left[\theta_{K,s,t}^{KL}\left(\frac{\left(1+\overline{t}^K\right)r_t^K}{\overline{r}_t^K}\right)^{1-\sigma_s^{KL}} + \theta_{L,s,t}^{KL}\left(\frac{\left(1+\overline{\tau^L}\right)w_t}{\overline{w}_t}\right)^{1-\sigma_s^{KL}}\right]^{\frac{1}{1-\sigma_s^{KL}}}\right\}^{1-\sigma_s^Y} + \theta_{A,s,t}^{Y}\left\{\left[\sum_i \theta_{i,s,t}^{A}\left(\frac{\left(1+\overline{\tau}_{i,s,t}^A\right)p_{i,t}^A}{\overline{p}_{i,t}^A}\right)^{1-\sigma_s^A}\right]^{\frac{1}{1-\sigma_s^A}}\right\}^{1-\sigma_s^Y}\right]^{\frac{1}{1-\sigma_s^Y}} \tag{A.1}$$

The produced good in sector s is allocated as a domestic usage and export based on the constant elasticity of transformation function. In this case, the unit revenue function is in Eq. (A.2), where η_s^Y measures the constant elasticity of transformation:

$$R_{s,t}^{Y}\left(p_{s,t}^Y, p_t^{FX}\right) = \left[\theta_{Y,s,t}^{Y}\left(\frac{\left(1-\overline{t}_s^Y\right)p_{s,t}^Y}{\overline{p}_{s,t}^Y}\right)^{1+\eta_s^Y} + \theta_{FX,s,t}^{FX}\left(\frac{\left(1-\overline{t}_s^Y\right)p_t^{FX}}{\overline{p}_t^{FX}}\right)^{1+\eta_s^Y}\right]^{\frac{1}{1+\eta_s^Y}} \tag{A.2}$$

Based on the Armington assumption, each product is produced based on the CES function where the domestic and imported product are inputted.

$$C_{i,t}^{A}\left(p_{i,t}^Y, p_t^{FX}\right) = \left[\theta_{i,t}^{A}\left(\frac{p_{i,t}^Y}{\overline{p}_{i,t}^Y}\right)^{1-\sigma_i^A} + \theta_{FX,t}^{A}\left(\frac{p_t^{FX}}{\overline{p}_t^{FX}}\right)^{1-\sigma_i^A}\right]^{\frac{1}{1-\sigma_i^A}} \tag{A.3}$$

I assume the consumption function of the representative household is the sum of the leisure and goods consumption based on the CES function, as in Eq. (A.4).

$$C_t^{C}\left(w_t, p_{i,t}^Y\right) = \left[\theta_{L,t}^{C}\frac{w_t}{\overline{w}_t} + \sum_i \theta_{i,t}^{C}\frac{\left(1+\overline{t}^C+\overline{\tau}_{i,t}^C\right)p_{i,t}^Y}{\overline{p}_{i,t}^Y}\right] \tag{A.4}$$

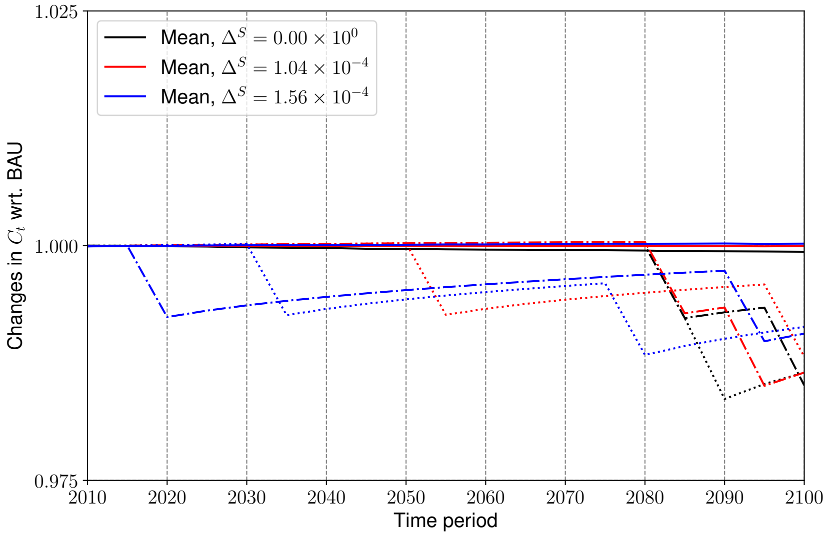


Figure B.1 – The mean of consumption for the flood occurred in 2007. Dashed trajectories show the possible two worst paths.

B Further empirical floods analysis

Flood in August 2007

The flooding process in 2007 is very similar to that in 2005. The flood catchment and the inundated regions are similar, but the reported monetary damage was greatly alleviated. After the flood in 2005, early warning systems as well as physical instruments were introduced by the national and local authorities and had worked correctly (NZZ, 2007a,b).

I developed three scenarios concerning the choice of the subjectively formed damage expectation Δ^S_{2007}. In the first scenario, the representative agent neglects the risk of future flooding by assuming $\Delta^S_{2007} = 0$. In the second scenario, the representative agent forms an expected value of the flood in 2007, and he forms $\Delta^S_{2007} = \mathbb{E}\left[\tilde{\Delta}^O_{2007}\right] = 1.04 \times 10^{-4}$. It is equivalent to the certainty-equivalent deterministic model. In the third scenario, the representative agent is supposed to be more conscious of the risk of flooding. He subjectively forms a higher damage expectation such as $\Delta^S_{2007} = \mathbb{E}\left[\tilde{\Delta}^O_{2007}\right] \times 1.5 = 1.56 \times 10^{-4}$.

In Figure B.1, the mean of 1,000 Monte-Carlo simulated consumption paths with three different subjective expectations of damage scale Δ^S_{2005} are presented. Again, the main result is that the higher expectation of damage the representative agent forms, the higher consumption profile he has. The trajectories show the possible worst cases. One single flood is big enough to deviate from the pre-flooding economic path. However, the probability of flood is $\frac{1}{100}$, and the representative agent does not take precautionary action.

The distribution of possible consumption paths with the flood in 2007 are shown in Figure B.2. With a probability of at least more than 75%, the consumption path is the same as one without

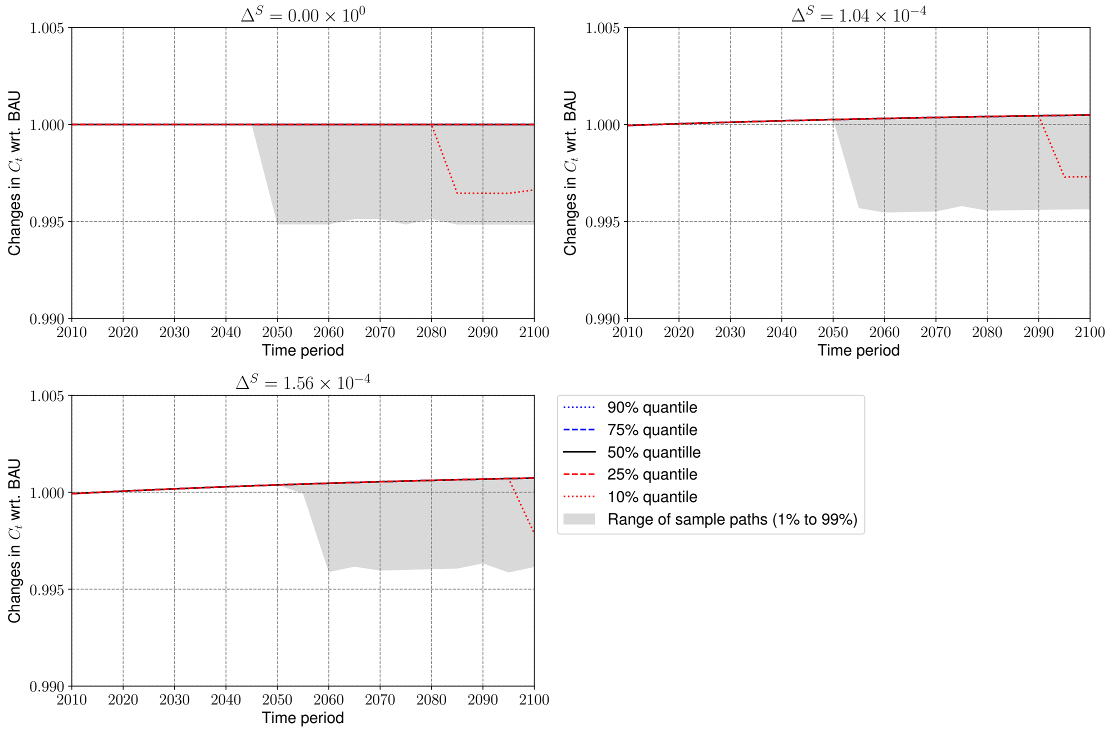


Figure B.2 – Monte-Carlo simulated results for the flood occurred in 2007 when the subjectively formed depreciation rate Δ^S is changed.

flooding. The gray clouds show the possible simulated path from 1% to 99% quantile. The distribution shows that some flooding might occur, however, the probability is not big enough to offer an incentive for the representative agent to allocate investment to less productive but risk-free capital stock: non-vulnerable capital stock.

B.1 Flood in May 1999

The Northern part of Switzerland experienced extremely heavy precipitation from 4 to 22 May 1999. The river Thur, Aare, Linth and the lake Bodensee flooded, and Canton Zürich and Bern were affected. The event was caused by long-lasting precipitation, and it caused damage to a large spatial extent; therefore, the event is characterized as a particular case of flooding. It caused expensive damage costs, see Table 1.1, in these two regions.

In the first scenario, the representative agent completely neglects the risk of flooding $\Delta^S_{1999} = 0$. In the second scenario, the agent correctly observes the risk of flooding and his subjective expectation is equal to the expected value $\Delta^S_{1999} = \mathbb{E}\left[\tilde{\Delta}^O_{1999}\right] = 1.04 \times 10^{-4}$. In the third scenario, I assume that the representative agent is more concerned about the risk of future floods and he sets $\Delta^S_{1999} = \mathbb{E}\left[\tilde{\Delta}^O_{1999}\right] \times 1.5 = 1.56 \times 10^{-4}$.

Figure B.3 summarizes the mean Monte-Carlo simulated consumption paths when the repre-

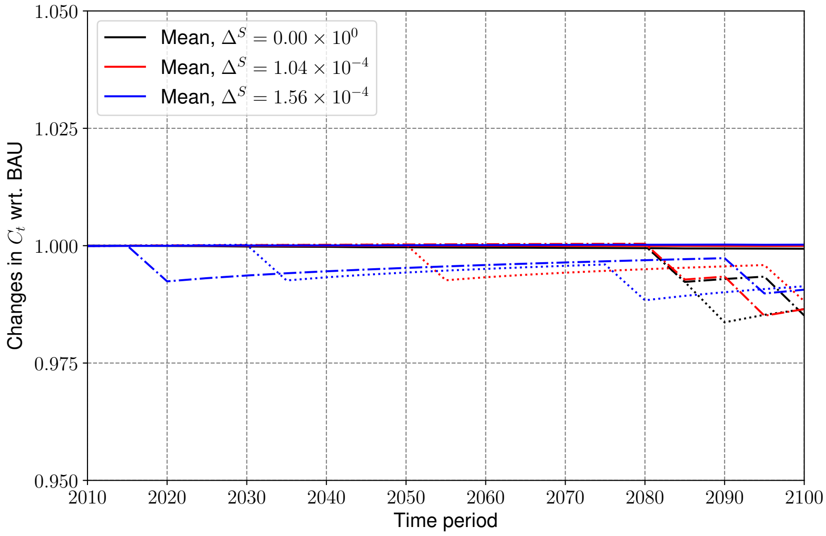


Figure B.3 – The mean consumption during the flood in 1999. Dashed trajectories show the possible two worst paths.

sentative agent changes his subjective belief for future uncertainty. I can develop the same observations as I did in Section 1.4.2.

Figure B.4 presents the distribution of simulated consumption paths. The main results in Figure B.4 are the same in Section 1.4.2.

B.2 Flood in October 2000

There was a severe storm in Canton Valais on October 14. It induced a grave landslide, resulting in water, soil and mud destroying one-third of the village Gondo within seconds.

In the first scenario, the representative agent completely neglects the risk of flooding $\Delta^S_{2000} = 0$. In the second scenario, the agent correctly observes the risk of flooding and his subjective expectation is equal to the expected value $\Delta^S_{2000} = \mathbb{E}\left[\tilde{\Delta}^O_{2000}\right] = 1.79 \times 10^{-4}$. In the third scenario, I assume that the representative agent is more concerned the risk of future floods and he sets $\Delta^S_{2000} = \mathbb{E}\left[\tilde{\Delta}^O_{2000}\right] \times 1.5 = 2.68 \times 10^{-4}$.

Figure B.5 presents the mean Monte-Carlo simulated consumption paths. The main findings are the same as in Section 1.4.2.

Figure B.6 show the distribution of simulated consumption paths. The intuition is the same as what I developed in Section 1.4.2.

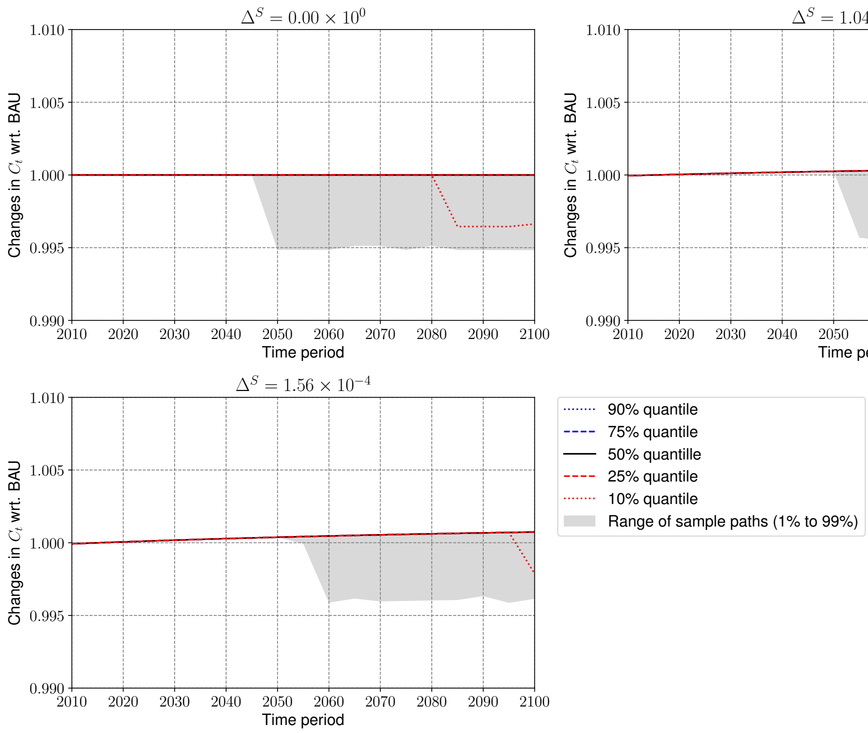


Figure B.4 – Monte-Carlo simulated results for the flood in 1999 when the subjectively formed depreciation rate Δ^S is changed.

B.3 Flood in October 2011

The primary catchment of the flood in October 2011 was the river of Kander and the region Bernese Oberland in the Caton Bern, the Canton Valais and the Canton Glarus were also affected. Heavy precipitation between October 6 and 10 as well as the volume of new snow caused the floods. It is regarded as a 100 years flood.

In the first scenario, the representative agent completely neglects the risk of flooding $\Delta^S_{2011} = 0$. In the second scenario, the agent correctly observes the risk of flooding and his subjective expectation is equal to the expected value $\Delta^S_{2011} = \mathbb{E}[\tilde{\Delta}^O_{2011}] = 2.12 \times 10^{-5}$. In the third scenario, I assume that the representative agent is more concerned about the risk of future floods and he sets $\Delta^S_{2011} = \mathbb{E}[\tilde{\Delta}^O_{2011}] \times 1.5 = 3.18 \times 10^{-5}$.

Figure B.7 summarizes the Monte-Carlo simulated consumption paths. Figure B.8 shows the distribution of simulated consumption paths. Both figures are regarded similarly to the other selected floods.

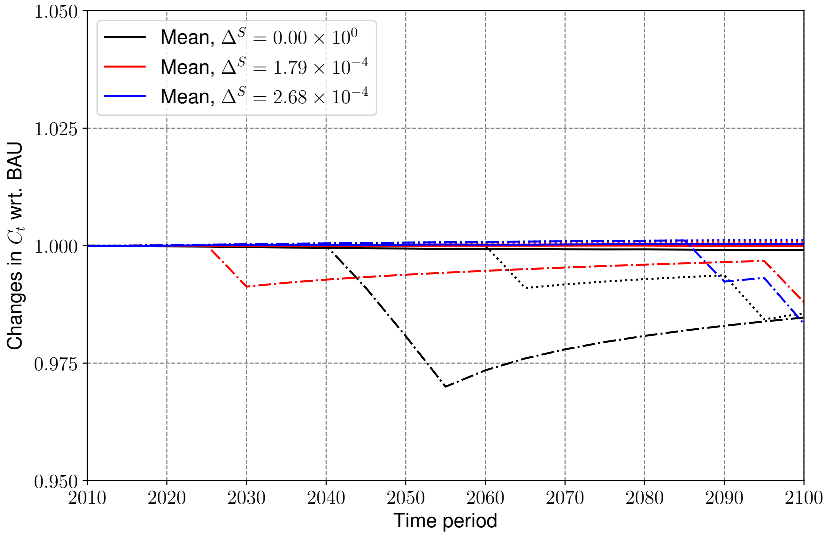


Figure B.5 – The mean consumption during the flood in 2000. Dashed trajectories show the possible two worst paths.

C More details on the Euler equations

In this appendix, I present the Lagrangian for each social planner's problem that I discussed in Section 2.2.2 for spatial adaptation, in Section 2.2.3 for adaptive capital stock and in Section 3.2.1 for spatial adaptation and adaptive capital stock.

C.1 Spatial adaptation

I maximize the discounted sum of the time-separable utility given in Eq. (2.3) subject to the two occasionally binding irreversible investment constraints in Eqs. (2.4) and (2.5) and the strictly binding resource constraint Eq. (2.6). The Lagrangian for the problem is defined such that:

$$\begin{aligned}
\mathscr{L} = {} & \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u(C_t)\right] + \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \mu_t^{KV}\left\{KV_{t+1} - \left(1-\delta-\tilde{\Delta}^z\right)KV_t\right\}\right] \\
& + \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \mu_t^{KNV}\left\{KNV_{t+1} - (1-\delta)KNV_t\right\}\right] \\
& + \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \lambda_t\left\{a_t f(KV_t, KNV_t) + \left(1-\delta-\tilde{\Delta}^z\right)KV_t - KV_{t+1} - \frac{q^{KV}}{2}KV_t\left(\frac{KV_{t+1}}{KV_t}-1\right)^2\right.\right. \\
& \left.\left. + (1-\delta)KNV_t - KNV_{t+1} - \frac{q^{KNV}}{2}KNV_t\left(\frac{KNV_{t+1}}{KNV_t}-1\right)^2 - C_t\right\}\right] \qquad \text{(C.1)}
\end{aligned}$$

Note that μ_t^{KV} and μ_t^{KNV} are a KKT multiplier associated with Eqs. (2.4) and (2.5) respectively. λ_t is a Lagrangian multiplier for the resource constraint in Eq. (2.6).

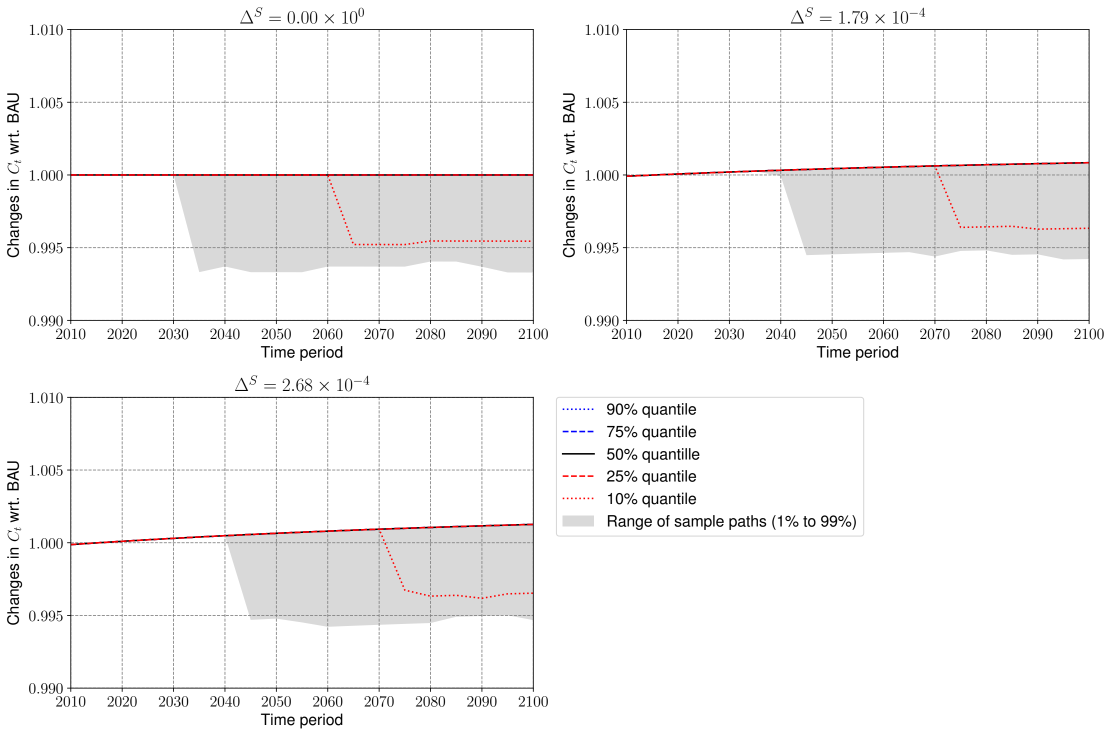


Figure B.6 – Monte-Carlo simulated results for the flood in 2000 when the subjectively formed depreciation rate Δ^S is changed.

The first-order conditions of Eq. (C.1) are in following:

$$\frac{\partial \mathscr{L}}{\partial C_t} = 0 \quad \Leftrightarrow u'(C_t) = \lambda_t \tag{C.2}$$

$$\frac{\partial \mathscr{L}}{\partial KV_t} = 0 \quad \Leftrightarrow \lambda_t \left[1 + q^{KV} \left(\frac{KV_{t+1}}{KV_t} - 1 \right) \right] - \mu_t^{KV} =$$
$$\beta \mathbb{E}_t \left[\lambda_{t+1} \left\{ a_{t+1} f_{KV} (KV_{t+1}, KNV_{t+1}) + \left(1 - \delta - \tilde{\Delta}^z\right) + \frac{q^{KV}}{2} \left(\frac{KV_{t+2}}{KV_{t+1}} - 1 \right) \left(\frac{KV_{t+2}}{KV_{t+1}} + 1 \right) \right\} - \mu_{t+1}^{KV} \left(1 - \delta - \tilde{\Delta}^z\right) \right] \tag{C.3}$$

$$\frac{\partial \mathscr{L}}{\partial KNV_t} = 0 \quad \Leftrightarrow \lambda_t \left[1 + q^{KNV} \left(\frac{KNV_{t+1}}{KNV_t} - 1 \right) \right] - \mu_t^{KNV} =$$
$$\beta \mathbb{E}_t \left[\lambda_{t+1} \left\{ a_{t+1} f_{KNV} (KV_{t+1}, KNV_{t+1}) + (1 - \delta) + \frac{q^{KNV}}{2} \left(\frac{KNV_{t+2}}{KNV_{t+1}} - 1 \right) \left(\frac{KNV_{t+2}}{KNV_{t+1}} + 1 \right) \right\} - \mu_{t+1}^{KV} (1 - \delta) \right] \tag{C.4}$$

Eq. (C.3) are the Euler equations corresponding to Eqs. (2.9) and (2.10).

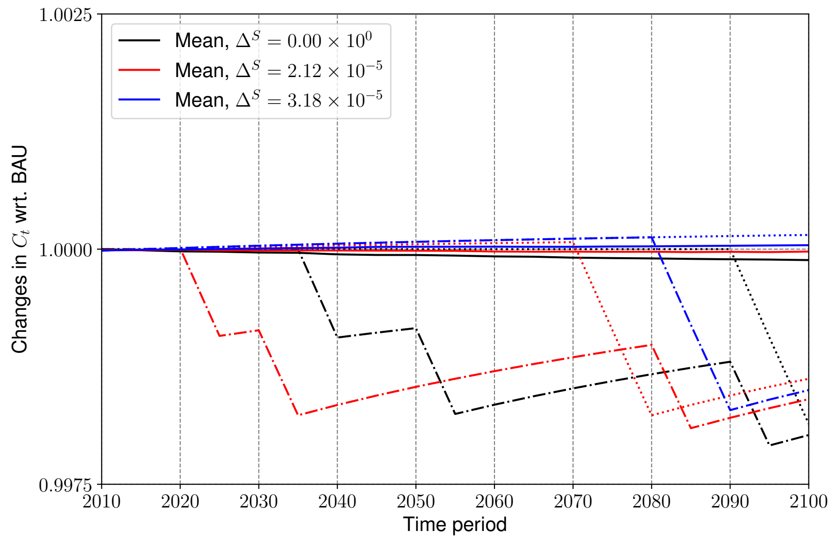


Figure B.7 – The mean consumption during the flood in 2011. Dashed trajectories show the possible two worst paths.

C.2 Adaptive capital stock

Social planner solves an infinite-time dynamic stochastic optimization problem Eq. (2.11) subject to Eqs. (2.12) to (2.14). The Lagrangian of the problem is formulated as in Eq. (C.5):

$$
\begin{aligned}
\mathscr{L} = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(C_t) \right] + \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \mu_t^K \left\{ K_{t+1} - \left(1 - \delta - \tilde{\Delta}^z h(K_t, A_t)\right) K_t \right\} \right] \\
+ \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \mu_t^A \left\{ A_{t+1} - \left(1 - \delta - \tilde{\Delta}^z h(K_t, A_t)\right) A_t \right\} \right] \\
+ \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \lambda_t \left\{ a_t f(K_t) + \left(1 - \delta - \tilde{\Delta}^z h(K_t, A_t)\right) K_t - K_{t+1} - \frac{q^K}{2} K_t \left(\frac{K_{t+1}}{K_t} - 1 \right)^2 \right. \right. \\
\left. \left. + \left(1 - \delta - \tilde{\Delta}^z h(K_t, A_t)\right) A_t - A_{t+1} - \frac{q^A}{2} A_t \left(\frac{A_{t+1}}{A_t} - 1 \right)^2 - C_t \right\} \right] \quad \text{(C.5)}
\end{aligned}
$$

Note that μ_t^K and μ_t^A are a KKT multiplier for Eqs. (2.12) and (2.13) and λ_t is a Lagrange multiplier for the resource constraint in Eq. (2.14).

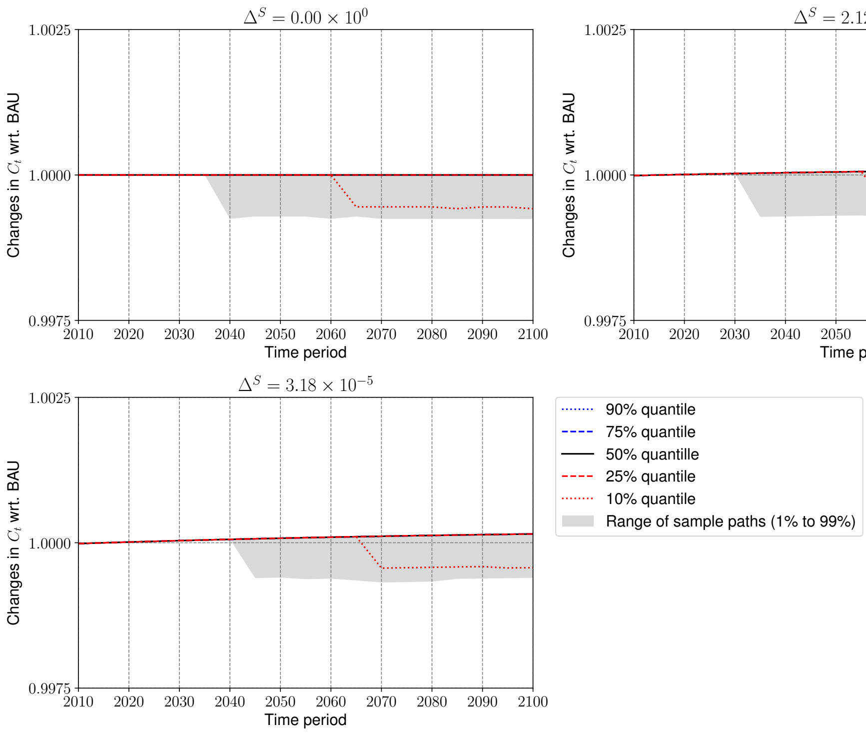


Figure B.8 – Monte-Carlo simulated results for the flood in 2011 when the subjectively formed depreciation rate Δ^S is changed.

Computing Eq. (C.5) for the first-order conditions, I derive:

$$\frac{\partial \mathcal{L}}{\partial C_t} = 0 \quad \Leftrightarrow u'(C_t) = \lambda_t \tag{C.6}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial K_t} = 0 \quad &\Leftrightarrow \lambda_t \left[1 + q^K \left(\frac{K_{t+1}}{K_t} - 1\right)\right] - \mu_t^K = \\
&\beta \mathbb{E}_t \Bigg[\lambda_{t+1} \Big\{a_{t+1} f_K(K_{t+1}) + \left(1 - \delta - \tilde{\Delta}^z h(K_{t+1}, A_{t+1})\right) - \tilde{\Delta}^z h_K(K_{t+1}, A_{t+1})(K_{t+1} + A_{t+1}) \\
&+ \frac{q^K}{2}\left(\frac{K_{t+2}}{K_{t+1}} - 1\right)\left(\frac{K_{t+2}}{K_{t+1}} + 1\right)\Big\} - \mu_{t+1}^K \left\{\left(1 - \delta - \tilde{\Delta}^z h(K_{t+1}, A_{t+1})\right) - \tilde{\Delta}^z h_K(K_{t+1}, A_{t+1}) K_{t+1}\right\} \\
&+ \mu_{t+1}^A \tilde{\Delta}^z h_K(K_{t+1}, A_{t+1}) A_{t+1}\Bigg]
\end{aligned} \tag{C.7}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial A_t} = 0 \quad &\Leftrightarrow \lambda_t \left[1 + q^A \left(\frac{A_{t+1}}{A_t} - 1\right)\right] - \mu_t^A = \\
&\beta \mathbb{E}_t \Bigg[\lambda_{t+1} \Big\{\left(1 - \delta - \tilde{\Delta}^z h(K_{t+1}, A_{t+1})\right) - \tilde{\Delta}^z h_A(K_{t+1}, A_{t+1})(K_{t+1} + A_{t+1}) \\
&+ \frac{q^A}{2}\left(\frac{A_{t+2}}{A_{t+1}} - 1\right)\left(\frac{A_{t+2}}{A_{t+1}} + 1\right)\Big\} + \mu_{t+1}^K \tilde{\Delta}^z h_A(K_{t+1}, A_{t+1}) K_{t+1} \\
&- \mu_{t+1}^A \left\{\left(1 - \delta - \tilde{\Delta}^z h(K_{t+1}, A_{t+1})\right) - \tilde{\Delta}^z h_A(K_{t+1}, A_{t+1}) A_{t+1}\right\}\Bigg]
\end{aligned} \tag{C.8}$$

Eqs. (C.7) and (C.8) are corresponding to Eqs. (2.22) and (2.23).

C.3 Spatial adaptation and adaptive capital stock

The social planner's objective function that is given in Eq. (3.1) is maximized subject to the irreversible investment constraints in Eqs. (3.2) to (3.4) and the resource constraint in Eq. (3.5). The Lagrangian is formulated as in Eq. (C.9):

$$\begin{aligned}
\mathscr{L} &= \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u(C_t)\right] + \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \mu_t^{KV}\left\{KV_{t+1} - \left(1-\delta-\tilde{\Delta}^z h(KV_t, A_t)\right)KV_t\right\}\right] \\
&+ \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \mu_t^{KNV}\left\{KNV_{t+1} - (1-\delta)KNV_t\right\}\right] \\
&+ \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \mu_t^{A}\left\{A_{t+1} - \left(1-\delta-\tilde{\Delta}^z h(KV_t, A_t)\right)A_t\right\}\right] \\
&+ \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \lambda_t\left\{a_t f(KV_t, KNV_t) + \left(1-\delta-\tilde{\Delta}^z h(KV_t, A_t)\right)KV_t - KV_{t+1} - \frac{q^{KV}}{2}KV_t\left(\frac{KV_{t+1}}{KV_t}-1\right)^2\right.\right. \\
&\quad + (1-\delta)KNV_t - KNV_{t+1} - \frac{q^{KNV}}{2}KNV_t\left(\frac{KNV_{t+1}}{KNV_t}-1\right)^2 \\
&\quad \left.\left. + \left(1-\delta-\tilde{\Delta}^z h(KV_t, A_t)\right)A_t - A_{t+1} - \frac{q^{A}}{2}A_t\left(\frac{A_{t+1}}{A_t}-1\right)^2 - C_t\right\}\right]
\end{aligned} \tag{C.9}$$

In Eq. (C.9), μ_t^{KV}, μ_t^{KNV} and μ_t^{A} are KKT multipliers for the corresponding occasionally binding investment constraints. λ_t is a Lagrange multiplier.

When I solve Eq. (C.9) for the first-order conditions, I have the following intertemporal equilibrium conditions.

$$\frac{\partial \mathcal{L}}{\partial C_t} = 0 \qquad \Leftrightarrow u'(C_t) = \lambda_t \tag{C.10}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial KV_t} = 0 \qquad & \Leftrightarrow \lambda_t \left[1 + q^{KV}\left(\frac{KV_{t+1}}{KV_t} - 1\right)\right] - \mu_t^{KV} = \\ & \beta \mathbb{E}_t \Bigg[\lambda_{t+1} \Big\{ a_{t+1} f_{KV}(KV_{t+1}, KNV_{t+1}) + \left(1 - \delta - \tilde{\Delta}^z h(KV_{t+1}, A_{t+1})\right) \\ & - \tilde{\Delta}^z h_{KV}(KV_{t+1}, A_{t+1})(KV_{t+1} + A_{t+1}) + \frac{q^{KV}}{2}\left(\frac{KV_{t+2}}{KV_{t+1}} - 1\right)\left(\frac{KV_{t+2}}{KV_{t+1}} + 1\right)\Big\} \\ & - \mu_{t+1}^{KV}\left\{\left(1 - \delta - \tilde{\Delta}^z h(KV_{t+1}, A_{t+1})\right) - \tilde{\Delta}^z h_{KV}(KV_{t+1}, A_{t+1}) KV_{t+1}\right\} \\ & + \mu_{t+1}^{A} \tilde{\Delta}^z h_{KV}(KV_{t+1}, A_{t+1}) A_{t+1} \Bigg]\end{aligned} \tag{C.11}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial KNV_t} = 0 \qquad & \Leftrightarrow \lambda_t \left[1 + q^{KNV}\left(\frac{KNV_{t+1}}{KNV_t} - 1\right)\right] - \mu_t^{KNV} = \\ & \beta \mathbb{E}_t \Bigg[\lambda_{t+1} \Big\{ a_{t+1} f_{KNV}(KV_{t+1}, KNV_{t+1}) + (1 - \delta) + \frac{q^{KNV}}{2}\left(\frac{KNV_{t+2}}{KNV_{t+1}} - 1\right)\left(\frac{KNV_{t+2}}{KNV_{t+1}} + 1\right)\Big\} \\ & - \mu_{t+1}^{KV}(1 - \delta) \Bigg]\end{aligned} \tag{C.12}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial A_t} = 0 \qquad & \Leftrightarrow \lambda_t \left[1 + q^{A}\left(\frac{A_{t+1}}{A_t} - 1\right)\right] - \mu_t^{A} = \\ & \beta \mathbb{E}_t \Bigg[\lambda_{t+1} \Big\{ \left(1 - \delta - \tilde{\Delta}^z h(KV_{t+1}, A_{t+1})\right) - \tilde{\Delta}^z h_A(KV_{t+1}, A_{t+1})(KV_{t+1} + A_{t+1}) \\ & + \frac{q^{A}}{2}\left(\frac{A_{t+2}}{A_{t+1}} - 1\right)\left(\frac{A_{t+2}}{A_{t+1}} + 1\right)\Big\} + \mu_{t+1}^{KV} \tilde{\Delta}^z h_A(KV_{t+1}, A_{t+1}) KV_{t+1} \\ & - \mu_{t+1}^{A}\left\{\left(1 - \delta - \tilde{\Delta}^z h(KV_{t+1}, A_{t+1})\right) - \tilde{\Delta}^z h_A(KV_{t+1}, A_{t+1}) A_{t+1}\right\} \Bigg]\end{aligned} \tag{C.13}$$

First-order conditions Eqs. (C.11) to (C.13) are corresponds to Eqs. (3.7) to (3.9) respectively.

D The system of equations compatible with Algorithm 2

In this appendix, I present the system of equations in a more compatible form with Algorithm 2. I evaluate the expectation operator around the integral of continuous AR(1) productivity shock by applying the five-nodes Gauss-Hermite quadrature with the weight $\omega(x)$ for each node x (Judd, 1998, see, p. 261).

D.1 Spatial adaptation

Following Algorithm 2, I define the policy function p that maps from the given state space (s, z) to the five optimal policy functions, where s is the vector of the endogenous state variables (capital variables) and z is the vector of the exogenous state variables.[4]

$$p : S \mapsto \mathbb{R}^5, \quad p(s,z) = \left(KV^+(s,z), KNV^+(s,z), \lambda(s,z), \mu^{KV}(s,z), \mu^{KNV}(s,z)\right) \tag{D.1}$$

Substituting Eq. (D.1) into the system of equations, I have:

$$\begin{aligned}
&p_2\left[1+q^{KV}\left(\frac{p_0}{s_0}-1\right)\right]-p_3 = \\
&\quad \beta\sum_{z^+}\sum_{x^+}\pi(z^+)\omega\left(x^+\right)\left[p_2^+\left\{a^+\left(x^+\right)f_{KV}\left(p_0,p_1\right)+\left(1-\delta-\tilde{\Delta}^{z+}\right)+\frac{q^{KV}}{2}\left(\frac{p_0^+}{p_0}-1\right)\left(\frac{p_0^+}{p_0}+1\right)\right\}\right. \\
&\quad \left.-\,p_3^+\left(1-\delta-\tilde{\Delta}^{z+}\right)\right] &\text{(D.2)} \\
&p_2\left[1+q^{KNV}\left(\frac{p_1}{s_1}-1\right)\right]-p_4 = \\
&\quad \beta\sum_{z^+}\sum_{x^+}\pi(z^+)\omega\left(x^+\right)\left[p_2^+\left\{a^+\left(x^+\right)f_{KNV}\left(p_0,p_1\right)+(1-\delta)+\frac{q^{KNV}}{2}\left(\frac{p_1^+}{p_1}-1\right)\left(\frac{p_1^+}{p_1}+1\right)\right\}\right. \\
&\quad \left.-\,p_4^+\left(1-\delta\right)\right] &\text{(D.3)} \\
&p_0-\left(1-\delta-\tilde{\Delta}^z\right)s_0 \geq 0 \quad \perp \quad p_3 \geq 0 &\text{(D.4)} \\
&p_1-(1-\delta)\,s_1 \geq 0 \quad \perp \quad p_4 \geq 0 &\text{(D.5)} \\
&af\left(s_0,s_1\right)+\left(1-\delta-\tilde{\Delta}^z\right)s_0-p_1-\frac{q^{KV}}{2}s_0\left(\frac{p_0}{s_0}-1\right)^2 \\
&\quad +(1-\delta)\,s_1-p_1-\frac{q^{KNV}}{2}s_0\left(\frac{p_1}{s_1}-1\right)^2-u^{-1}\left(p_2\right)\geq 0 \quad \perp \quad p_2 \geq 0 &\text{(D.6)}
\end{aligned}$$

In Eqs. (D.2) to (D.6), $p_0 = KV^+(s,z)$, $p_1 = KNV^+(s,z)$, $p_2 = \lambda(s,z)$, $p_3 = \mu^{KV}(s,z)$ and $p_4 = \mu^{KNV}(s,z)$. Similarly for the exogenous state variables, $s_0 = KV$ and $s_1 = KNV$.

D.2 Adaptive capital stock

Following our Algorithm 2, the policy function can be defined as:

$$p : S \mapsto \mathbb{R}^5, \quad p(s,z) = \left(K^+(s,z), A^+(s,z), \lambda(s,z), \mu^K(s,z), \mu^A(s,z)\right) \tag{D.7}$$

In Eq. (D.7), s is the vector of endogenous state variables and z is the vector of exogenous state variables. Our system of non-linear equilibrium conditions consists of the two Euler equations Eqs. (D.8) and (D.9), the two occasionally binding investment constraints Eqs. (D.10)

[4] I drop off the time notation t and indicate the next period activities with the symbol +

and (D.11) and the resource constraint Eq. (D.12).

$$
\begin{aligned}
& p_2\left[1+q^K\left(\frac{p_0}{s_0}-1\right)\right]-p_3= \\
& \beta\sum_{z^+}\sum_{x^+}\pi(z^+)\left[p_2^+\left\{a^+\left(x^+\right)f_K\left(p_0\right)+\left(1-\delta-\tilde{\Delta}^{z+}h\left(p_0,p_1\right)\right)-\tilde{\Delta}^{z+}h_K\left(p_0,p_1\right)\left(p_0+p_1\right)\right.\right. \\
& \left.+\frac{q^K}{2}\left(\frac{p_0^+}{p_0}-1\right)\left(\frac{p_0^+}{p_0}+1\right)\right\}-p_3^+\left\{\left(1-\delta-\tilde{\Delta}^{z+}h\left(p_0,p_1\right)\right)-\tilde{\Delta}^{z+}h_K\left(p_0,p_1\right)p_0\right\} \\
& \left.+p_4^+\tilde{\Delta}^{z+}h_K\left(p_0,p_1\right)p_1\right] \qquad \text{(D.8)}
\end{aligned}
$$

$$
\begin{aligned}
& p_2\left[1+q^A\left(\frac{p_1}{s_1}-1\right)\right]-p_4= \\
& \beta\sum_{z^+}\pi(z^+)\left[p_2^+\left\{\left(1-\delta-\tilde{\Delta}^{z+}h\left(p_0,p_1\right)\right)-\tilde{\Delta}^{z+}h_A\left(p_0,p_1\right)\left(p_0+p_1\right)+\frac{q^A}{2}\left(\frac{p_1^+}{p_1}-1\right)\left(\frac{p_1^+}{p_1}+1\right)\right\}\right. \\
& \left.+p_3^+\tilde{\Delta}^{z+}h_A\left(p_0,p_1\right)p_1-p_4^+\left\{\left(1-\delta-\tilde{\Delta}^{z+}h\left(p_0,p_1\right)\right)-\tilde{\Delta}^{z+}h_A\left(p_0,p_1\right)p_1\right\}\right] \qquad \text{(D.9)}
\end{aligned}
$$

$$p_0-\left(1-\delta-\tilde{\Delta}^z h\left(s_0,s_1\right)\right)s_0\geq 0 \quad \perp \quad p_3\geq 0 \qquad \text{(D.10)}$$

$$p_1-\left(1-\delta-\tilde{\Delta}^z h\left(s_0,s_1\right)\right)s_1\geq 0 \quad \perp \quad p_4\geq 0 \qquad \text{(D.11)}$$

$$
\begin{aligned}
& f\left(s_0\right)+\left(1-\delta-\tilde{\Delta}^z h\left(s_0,s_1\right)\right)s_0-p_0-\frac{q^K}{2}s_0\left(\frac{p_0}{s_0}-1\right)^2 \\
& +\left(1-\delta-\tilde{\Delta}^z h\left(s_0,s_1\right)\right)s_1-p_1-\frac{q^A}{2}s_1\left(\frac{p_1}{s_1}-1\right)^2-u^{-1}\left(p_2\right)\geq 0 \quad \perp \quad p_2\geq 0 \qquad \text{(D.12)}
\end{aligned}
$$

In Eqs. (D.8) to (D.12), the vector of policy functions are $p_0 = K^+(s,z)$, $p_1 = A^+(s,z)$, $p_2 = \lambda(s,z)$, $p_3 = \mu^K(s,z)$ and $p_4 = \mu^A(s,z)$. Similarly for the endogenous state variable s, $s_0 = K$ and $s_1 = A$.

D.3 Spatial adaptation and adaptive capital stock

Following our Algorithm 2, the policy function can be defined as:

$$p: S \mapsto \mathbb{R}^7, \quad p\left(s,z\right)=\left(KV^+\left(s,z\right),KNV^+\left(s,z\right),A^+\left(s,z\right),\lambda\left(s,z\right),\mu^{KV}\left(s,z\right),\mu^{KNV}\left(s,z\right),\mu^A\left(s,z\right)\right) \qquad \text{(D.13)}$$

By applying Eq. (D.13), I transform the three Euler equations in Eqs. (3.7) to (3.9) and the three occasionally binding irreversible investment constraints in Eqs. (3.2) to (3.4) and one strictly binding resource constraint in Eq. (3.5).

Our resulting system of non-linear equations are as follows:

$$p_3\left[1+q^{KV}\left(\frac{p_0}{s_0}-1\right)\right]-p_4=$$
$$\beta\sum_{z^+}\sum_{x^+}\pi\left(z^+\right)\omega\left(x^+\right)\Bigg[p_3^+\Big\{a^+\left(x^+\right)f_{KV}\left(p_0,p_1\right)+\left(1-\delta-\tilde{\Delta}^{z+}h\left(p_0,p_2\right)\right)-\tilde{\Delta}^{z+}h_{KV}\left(p_0,p_2\right)\left(p_0+p_2\right)$$
$$+\frac{q^{KV}}{2}\left(\frac{p_0^+}{p_0}-1\right)\left(\frac{p_0^+}{p_0}+1\right)\Big\}-p_4^+\left\{\left(1-\delta-\tilde{\Delta}^{z+}h\left(p_0,p_2\right)\right)-\tilde{\Delta}^{z+}h_{KV}\left(p_0,p_2\right)p_0\right\}$$
$$+p_6\tilde{\Delta}^{z+}h_{KV}\left(p_0,p_2\right)p_2\Bigg] \tag{D.14}$$

$$p_3\left[1+q^{KNV}\left(\frac{p_1}{s_1}-1\right)\right]-p_5=$$
$$\beta\sum_{z^+}\sum_{x^+}\pi\left(z^+\right)\omega\left(x^+\right)\Bigg[p_3^+\left\{a^+\left(x^+\right)f_{KNV}\left(p_0,p_1\right)+(1-\delta)+\frac{q^{KNV}}{2}\left(\frac{p_1^+}{p_1}-1\right)\left(\frac{p_1^+}{p_1}+1\right)\right\}$$
$$-p_4\left(1-\delta\right)\Bigg] \tag{D.15}$$

$$p_3\left[1+q^{A}\left(\frac{p_2}{s_2}-1\right)\right]-p_6=$$
$$\beta\sum_{z^+}\pi\left(z^+\right)\Bigg[p_3^+\Big\{\left(1-\delta-\tilde{\Delta}^{z+}h\left(p_0,p_2\right)\right)-\tilde{\Delta}^{z+}h_A\left(p_0,p_2\right)\left(p_0+p_2\right)+\frac{q^A}{2}\left(\frac{p_2^+}{p_2}-1\right)\left(\frac{p_2^+}{p_2}+1\right)\Big\}$$
$$+p_4\tilde{\Delta}^{z+}h_A\left(p_0,p_2\right)p_0-p_6\left\{\left(1-\delta-\tilde{\Delta}^{z+}h\left(p_0,p_2\right)\right)-\tilde{\Delta}^{z+}h_A\left(p_0,p_2\right)p_2\right\}\Bigg] \tag{D.16}$$

$$p_0-\left(1-\delta-\tilde{\Delta}^{z}h\left(s_0,s_2\right)\right)s_0\geq 0\quad\perp\quad p_4\geq 0 \tag{D.17}$$
$$p_1-\left(1-\delta\right)s_1\geq 0\quad\perp\quad p_5\geq 0 \tag{D.18}$$
$$p_2-\left(1-\delta-\tilde{\Delta}^{z}h\left(s_0,s_2\right)\right)s_2\geq 0\quad\perp\quad p_6\geq 0 \tag{D.19}$$

$$af\left(s_0,s_1\right)+\left(1-\delta-\tilde{\Delta}^{z}h\left(s_0,s_2\right)\right)s_0-p_0-\frac{q^{KV}}{2}s_0\left(\frac{p_0}{s_0}-1\right)^2$$
$$+\left(1-\delta\right)s_1-p_1-\frac{q^{KNV}}{2}s_1\left(\frac{p_1}{s_1}-1\right)^2$$
$$+\left(1-\delta-\tilde{\Delta}^{z}h\left(s_0,s_2\right)\right)s_1-p_2-\frac{q^{A}}{2}s_2\left(\frac{p_2}{s_2}-1\right)^2-u^{-1}\left(p_3\right)\geq 0\quad\perp\quad\lambda_t\geq 0 \tag{D.20}$$

In Eqs. (D.14) to (D.20), $p_0 = KV^+(s,z)$, $p_1 = KNV^+(s,z)$, $p_2 = A^+(s,z)$, $p_3 = \lambda(s,z)$, $p_4 = \mu^{KV}(s,z)$, $p_5 = \mu^{KNV}(s,z)$ and $p_6 = \mu^A(s,z)$. Similarly for the exogenous state variables, $s_0 = KV$ and $s_1 = KNV$.

E Certainty-equivalent steady states

I can convert dynamic stochastic models to the certainty-equivalent deterministic models by taking an expected value of the two uncertain parameters:

$$\overline{\Delta} = \mathbb{E}\left[\tilde{\Delta}^z\right] = \sum_z \pi(z) \times \Delta^z \tag{E.1}$$

$$\overline{a} = \mathbb{E}\left[a_t\right] = 1 \tag{E.2}$$

E.1 Spatial adaptation

I solve the certainty-equivalent deterministic model for its steady state. The following equilibrium conditions characterize the steady state, where the superscript ss over each variable stands for steady state, and I drop off the time notation t throughout this appendix.

$$\lambda^{ss} - \mu^{KV,ss} = \beta\left[\lambda^{ss}\left\{\overline{a} f_{KV}\left(KV^{ss}, KNV^{ss}\right) + \left(1 - \delta - \overline{\Delta}\right)\right\} - \mu^{KV,ss}\left(1 - \delta - \tilde{\Delta}^z\right)\right] \tag{E.3}$$

$$\lambda^{ss} - \mu^{KNV,ss} = \beta\left[\lambda^{ss}\left\{\overline{a} f_{KNV}\left(KV^{ss}, KNV^{ss}\right) + (1 - \delta)\right\} - \mu^{KV,ss}(1 + \delta)\right] \tag{E.4}$$

$$\mu^{KV,ss}\left(\delta + \overline{\Delta}\right) KV^{ss} = 0 \tag{E.5}$$

$$\mu^{KNV,ss} \delta KNV^{ss} = 0 \tag{E.6}$$

$$\overline{a} f\left(KV^{ss}, KNV^{ss}\right) - \left[\left(\delta + \overline{\Delta}\right) KV^{ss} + \delta KNV^{ss} + \left(\lambda^{ss}\right)^{-\frac{1}{\eta}}\right] = 0 \tag{E.7}$$

The system of non-linear equations (Eqs. (E.3) to (E.7)) gives us the certainty-equivalent steady state values for KV^{ss}, KNV^{ss}, $\mu^{KV,ss}$, $\mu^{KNV,ss}$ and λ^{ss}.

E.2 Adaptive capital stock

The following system of non-linear equations (Eqs. (E.8) to (E.12)) gives us the certainty-equivalent deterministic steady state values for K^{ss}, A^{ss}, $\mu^{K,ss}$, $\mu^{A,ss}$ and λ^{ss}.

$$\begin{aligned}\lambda^{ss} - \mu^{K,ss} = \beta\Big[&\lambda^{ss}\left\{\overline{a} f_{KV}\left(KV^{ss}\right) + \left(1 - \delta - \overline{\Delta} h\left(K^{ss}, A^{ss}\right)\right) - \overline{\Delta} h_K\left(K^{ss}, A^{ss}\right)\left(K^{ss} + A^{ss}\right)\right\} \\ &- \mu^{K,ss}\left\{\left(1 - \delta - \tilde{\Delta} h\left(K^{ss}, A^{ss}\right)\right) - \overline{\Delta} h_K\left(K^{ss}, A^{ss}\right) K^{ss}\right\} + \mu^{A,ss} \overline{\Delta} h_K\left(K^{ss}, A^{ss}\right) A^{ss}\Big]\end{aligned} \tag{E.8}$$

$$\begin{aligned}\lambda^{ss} - \mu^{A,ss} = \beta\Big[&\lambda^{ss}\left\{\left(1 - \delta - \overline{\Delta} h\left(K^{ss}, A^{ss}\right)\right) - \overline{\Delta} h_A\left(K^{ss}, A^{ss}\right)\left(K^{ss} + A^{ss}\right)\right\} \\ &+ \mu^{K,ss} \overline{\Delta} h_A\left(K^{ss}, A^{ss}\right) K^{ss} - \mu^{A,ss}\left\{\left(1 - \delta - \tilde{\Delta} h\left(K^{ss}, A^{ss}\right)\right) - \overline{\Delta} h_A\left(K^{ss}, A^{ss}\right) A^{ss}\right\}\Big]\end{aligned} \tag{E.9}$$

$$\mu^{K,ss}\left(\delta + \overline{\Delta} h\left(K^{ss}, A^{ss}\right)\right) K^{ss} = 0 \tag{E.10}$$

$$\mu^{A,ss}\left(\delta + \overline{\Delta} h\left(K^{ss}, A^{ss}\right)\right) A^{ss} = 0 \tag{E.11}$$

$$\overline{a} f\left(K^{ss}\right) - \left[\left(\delta + \overline{\Delta} h\left(K^{ss}, A^{ss}\right)\right) K^{ss} + \left(\delta + \overline{\Delta} h\left(K^{ss}, A^{ss}\right)\right) A^{ss} + \left(\lambda^{ss}\right)^{-\frac{1}{\eta}}\right] = 0 \tag{E.12}$$

E.3 Spatial adaptation and adaptive capital stock

The following system of non-linear equations (Eqs. (E.13) to (E.19)) gives us the certainty-equivalent deterministic steady state values for KV^{ss}, KNV^{ss}, A^{ss}, $\mu^{KV,ss}$, $\mu^{KNV,ss}$, $\mu^{A,ss}$ and λ^{ss}.

$$\begin{aligned}\lambda^{ss} - \mu^{KV,ss} = \beta\Big[&\lambda^{ss}\Big\{\overline{a} f_{KV}\left(KV^{ss}, KNV^{ss}\right) + \Big(1-\delta-\overline{\Delta} h\left(KV^{ss}, A^{ss}\right)\Big)\\ &- \overline{\Delta} h_{KV}\left(KV^{ss}, A^{ss}\right)\left(KV^{ss}+A^{ss}\right)\Big\}\\ &- \mu^{KV,ss}\Big\{\Big(1-\delta-\overline{\Delta} h\left(KV^{ss}, A^{ss}\right)\Big) - \overline{\Delta} h_{KV}\left(KV^{ss}, A^{ss}\right)KV^{ss}\Big\}\\ &+ \mu^{A,ss}\overline{\Delta} h_{KV}\left(KV^{ss}, A^{ss}\right)A^{ss}\Big]\end{aligned} \tag{E.13}$$

$$\lambda^{ss} - \mu^{KNV,ss} = \beta\Big[\lambda^{ss}\left\{\overline{a} f_{KNV}\left(KV^{ss}, KNV^{ss}\right) + (1-\delta)\right\} - \mu^{KNV,ss}(1-\delta)\Big] \tag{E.14}$$

$$\begin{aligned}\lambda^{ss} - \mu^{A,ss} = \beta\Big[&\lambda^{ss}\Big\{\Big(1-\delta-\overline{\Delta} h\left(KV^{ss}, A^{ss}\right)\Big) - \overline{\Delta} h_{A}\left(KV^{ss}, A^{ss}\right)\left(KV^{ss}+A^{ss}\right)\Big\}\\ &+ \mu^{KV,ss}\overline{\Delta} h_{A}\left(KV^{ss}, A^{ss}\right)KV^{ss}\\ &- \mu^{A,ss}\Big\{\Big(1-\delta-\overline{\Delta} h\left(KV^{ss}, A^{ss}\right)\Big) - \overline{\Delta} h_{A}\left(KV^{ss}, A^{ss}\right)A^{ss}\Big\}\Big]\end{aligned} \tag{E.15}$$

$$\mu^{KV,ss}\Big(\delta+\overline{\Delta} h\left(KV^{ss}, A^{ss}\right)\Big)KV^{ss} = 0 \tag{E.16}$$

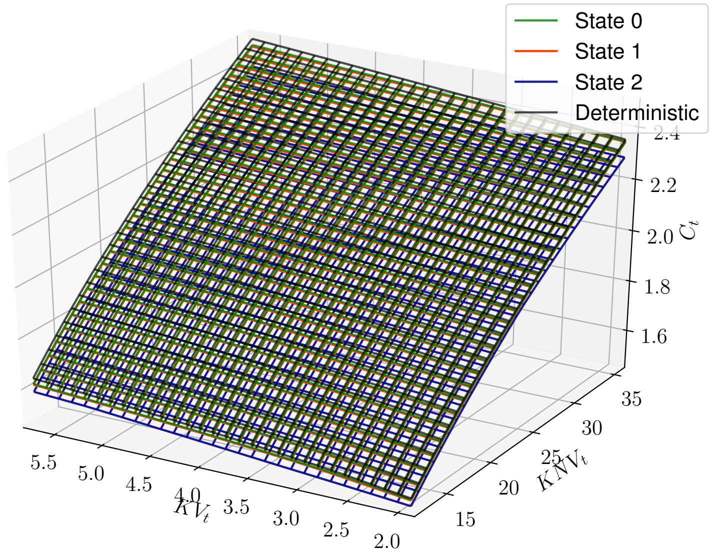
$$\mu^{KNV,ss}\delta KNV^{ss} = 0 \tag{E.17}$$

$$\mu^{A,ss}\Big(\delta+\overline{\Delta} h\left(KV^{ss}, A^{ss}\right)\Big)A^{ss} = 0 \tag{E.18}$$

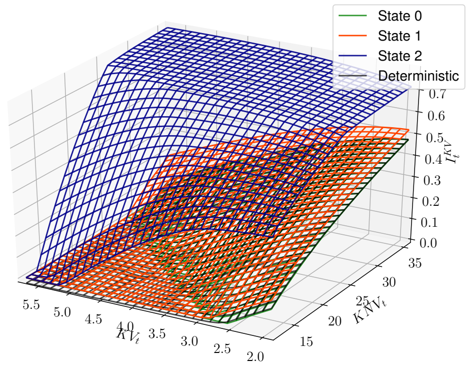
$$\begin{aligned}&\overline{a} f\left(KV^{ss}, KNV^{ss}\right)\\ &- \Big[\Big(\delta+\overline{\Delta} h\left(KV^{ss}, A^{ss}\right)\Big)KV^{ss} + \delta KNV^{ss} + \Big(\delta+\overline{\Delta} h\left(KV^{ss}, A^{ss}\right)\Big)A^{ss} + \left(\lambda^{ss}\right)^{-\frac{1}{\eta}}\Big] = 0\end{aligned} \tag{E.19}$$

F Comparison between the stochastic and the deterministic policy functions

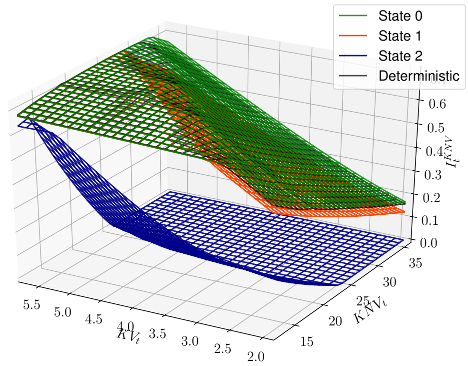
I compare the stochastic and the deterministic policy functions with spatial adaptation in Figure F.1 and with adaptive capital stock in Figure F.2. In both figures, I fix AR(1) productivity shock at one. I observe that, in both figures, the deterministic policy functions are almost the same as the stochastic policy functions in state 0 where no shock occurs. It is an intuitive result. In the deterministic models, I take an expected value of a damage scale for the uncertain shocks. Weitzman (2009) claims that many economic and climate events generally have a fat-tailed (or a heavy-tailed) probability distribution, and in this context, taking an expected value becomes an inappropriate way. The modeling of rare disasters is a typical example of Weitzman (2009)'s claim. Deterministic models underestimate the risk of rare disasters, and the fact makes the policy recommendations based on the deterministic misleading, especially



(a) Consumption



(b) Investment to vulnerable capital stock



(c) Investment to non-vulnerable capital stock

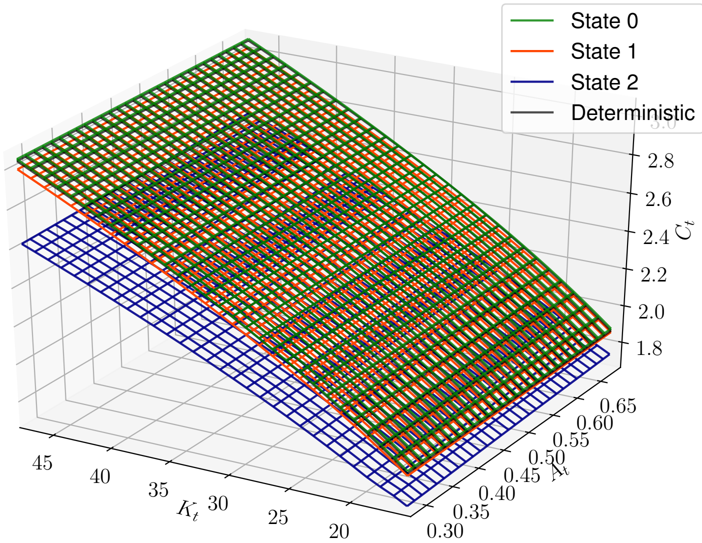
Figure F.1 – Stochastic and deterministic policy functions with spatial adaptation when $a_t = 1$

in the framework of rare disasters.

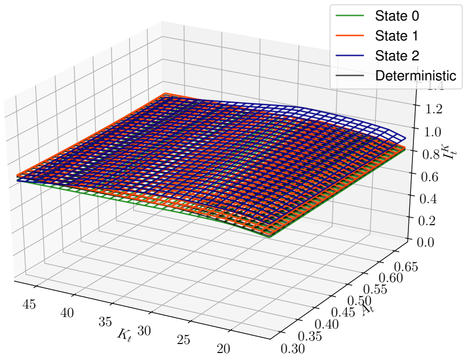
G Policy functions with a zero social rate of pure time preference

In the main numerical exercises presented in Chapter 2, I adopt the discount factor β equal to 0.99 following Juillard and Villemot (2011, for instance). Ramsey (1928) suggests that a social rate of pure time preference should be zero, claiming that there should be no discount. In this appendix, I set the discount factor equal to 1 and show the corresponding policy functions for spatial adaptation.

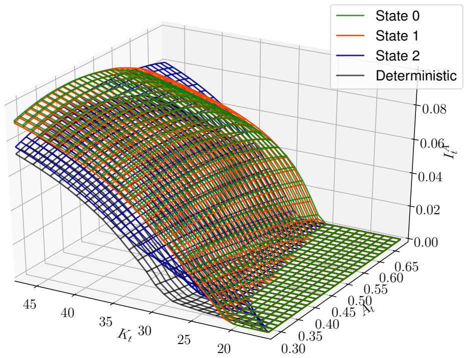
Figure G.1 summarizes sensitivity to the choice of the discount factor among 0.99 and 1. In general, we can observe that the social planner saves more when $\beta = 1$ than the case when



(a) Consumption



(b) Investment to productive capital stock



(c) Investment to adaptive capital stock

Figure F.2 – Stochastic and deterministic policy functions with adaptive capital stockwhen $a_t = 1$

$\beta = 0.99$. The level of investment to both types of capital stock with $\beta = 1$ is greater than those with $\beta = 0.99$, as illustrated. This is an intuitive result. When the social planner choices $\beta = 1$, he treats the value in t and $t+1$ equally in the Euler equation, and he would save more for the period $t+1$.

As shown above, the choice of the discount factor is crucial. Arrow et al. (2004) suggest that the discount factor should be in the range of $0.995 \geq \beta \geq 1$ to achieve sustainable development, but there is no scientific consensus. The choice of the discount factor is an active research field, especially under uncertainty.

H Ergodic set in the model with spatial adaptation or adaptive capital stock

It is particularly important, especially for global methods, to ensure that the possible economic paths should be inside of the *ex-ante* selected approximation domain. For instance, Judd, Maliar, and Maliar (2011) at first compute the possible ergodic set with a closed-form solution and efficiently determine an approximation domain for subsequent analysis with a global method. In this appendix, I illustrate the ergodic set of the two models, spatial adaptation and adaptive capital, and check whether a long time horizon simulation results are inside of the approximation domain or not.

Figure H.1 illustrated the ergodic set of the simulated results where I set the $10,000$ time horizon. We can confirm twofold in Figure H.1. First, when plotting the possible path of the three state variables, we confirm that all possible paths are the inside of the selected approximation domain. Second, we selected a much broader approximation domain for the spatial planning model as shown in Figure H.1a. When we restrict the approximation domain for the spatial planning model, we might achieve a smaller approximation error in the Euler equation because we can approximate policy functions with a finer approximated domain, but there is no way to know enough size of an approximation domain *ex-ante*.

I Scalability and efficiency of the parallelized adaptive sparse grid

In this appendix, I report the strong scaling efficiency of the parallelized adaptive sparse grid. More detailed discussions about the specifications of the computing cluster installed in EPFL is in Appendix K.

In Figure I.1, I define the speed up S_m and the efficiency E_m of the code as in Eq. (I.1).

$$S_m = \frac{T_1}{mT_m}, \quad E_m = \frac{T_1}{mT_m} \tag{I.1}$$

where T_1 is the benchmark execution time (number of available CPU is one) and T_m is the execution time with the m number of CPUs. I report the execution time and the ideal speedup when I change the level of the sparse grid in Figure I.1a and Figure I.1b. Both figures show a nice scaling effect for most regions. For instance, if I employ the level 4 sparse grid (177 grid points) with 224 CPUs, the number of CPUs is higher than that of grid points. However, except these cases, we can achieve almost more than 65% parallelization efficiency, and it is acceptable compared with Brumm and Scheidegger (2017) and Scheidegger et al. (2018) for instance.

J Parallelization via MPI

High-performance computing (HPC) enables us to access the massive computing power of the high-end system. A clock speed of a single CPU is reaching its transition limit and to make a single CPU faster is incredibly expensive. Serial computing is no longer suitable to solve a large or numerically expensive model within a tolerable amount of time; therefore, there is a growing demand to solve a model in a concurrent way: parallel computing.

HPC architecture has become a workhouse in many areas of science and engineering, and applications in economics are emerging. Aldrich et al. (2011) solve a basic RBC model by massively parallelizing the value function iteration steps with graphical processing units (GPU). Cai et al. (2015) demonstrate the applicability of high throughput computing (HTC) parallelization scheme. In the HTC scheme, one Master node decomposes a problem into small tasks and deploys them to workstations on the network when they are not occupied for another purpose. Brumm and Scheidegger (2017) combines several parallelization schemes to solve high dimensional economic models efficiently. The first parallelization is realized by the message passing interface (MPI). Subsequently in each computing node, further parallelization is done by threading building blocks (TBB). A high intensity of arithmetic computations is accelerated by using GPUs.

Given that I can access a high-end computing cluster, MPI is suitable for the first parallelization. The dimension of my models is not so "high"; therefore, the benefit from further parallelization might be insufficient.

My parallelization strategy is illustrated in Figure J.1. Before entering a time iteration collocation step, I initialize the model in the root MPI process (or rank in the MPI language) and broadcast the initialized information to the rest of the ranks. I distribute the set of grid points to each rank by MPI. Each rank is independent, and the optimal controls are derived based on the policy functions from the previous iteration. The derived optimal controls are gathered from each rank to the root rank. I interpolate policy functions in the root rank and refine the sparse grid by one. The newly generated grid points are distributed among multiple MPI processes and in each rank, I solve for the optimal policy functions given the policy functions in the next period.

Massive parallelization substantially speeds up the computing processes. I can assess 8 computing nodes with 28 cores each, thus 224 CPUs at the same time. All of these processes are implemented on an Intel Broadwell based cluster on site at the university on-site.

Finally, I report how the ASG algorithm detects kinks and adds a necessary amount of points to approximate a non-smooth function with an acceptable approximation error. Table J.1 shows the number of grid points with the classical sparse grid (the SG column) and that of an adaptive sparse grid with two adaptation measures when I adopt the benchmark parameters for the adaptive sparse grid. As seen, the finer level of the sparse grid I assume, the more grid points for the three cases. However, the adaptive sparse grid algorithm successfully and

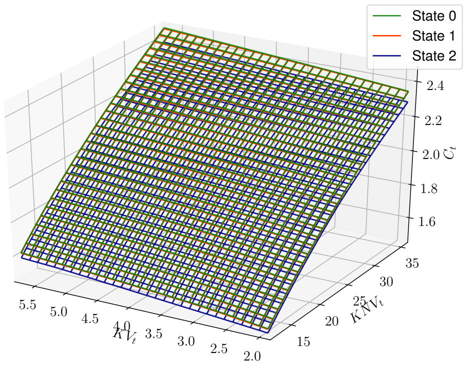
Table J.1 – Number of grid points where $L_0 = 5$, $L_{max} = 10$ and $\epsilon = 0.001$.

Level	SG	Spatial planning, ASG	Adaptive capital, ASG
5	441	441	441
6	1073	820	759
7	2561	1440	1251
8	6017	2400	1913
9	13953	3776	2836
10	32001	5821	4132

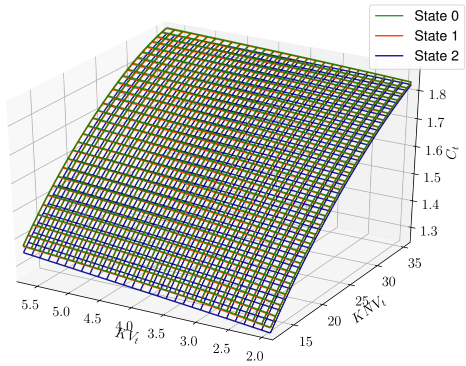
efficiently reduces the number of grid points compared to the classical sparse grid. It is the advantage to adopt the adaptive sparse grid.

K Detailed EPFL cluster specifications

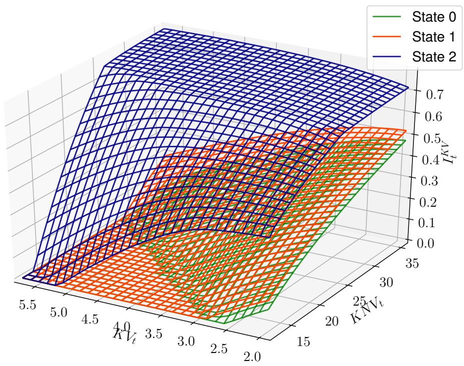
My MPI parallelization is implemented on the Intel Broadwell based computing cluster at EPFL named Fidis. Fidis have started its service from May, 2017. Fidis achieves 401.3 TFLOPs Linpack performance and equips 61 TB RAM as well as 350 TB storage in total. The cluster equips, in total, 408 computing nodes, and each has 2 Intel Broadwell processors running at 2.6 GHz. Each processor has 14 cores, which means that 28 cores per each node. I access to at most 8 computing nodes, which means that my codes are parallelized with $224(= 28 \times 8)$ MPI processes.



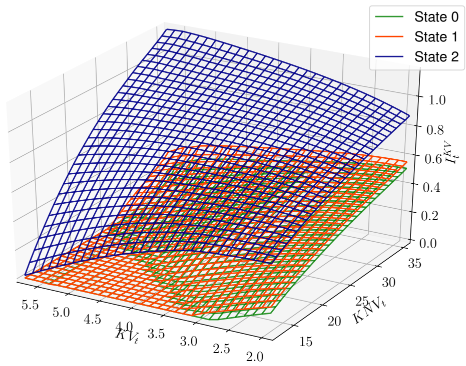
(a) Consumption, $\beta = 0.99$



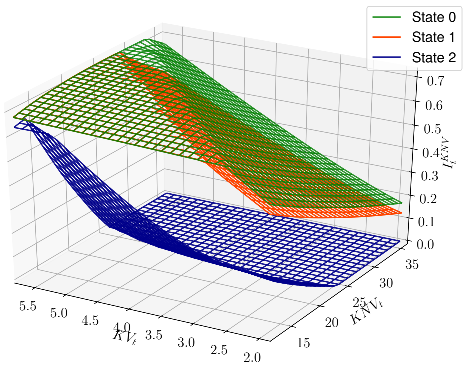
(b) Consumption, $\beta = 1$



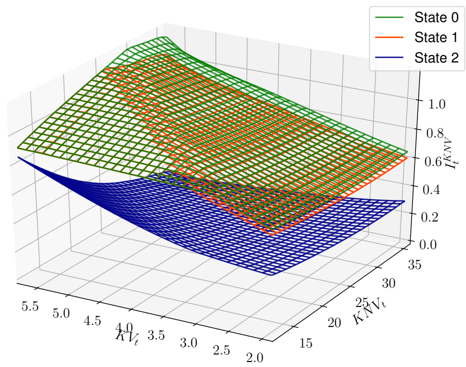
(c) Investment to vulnerable capital stock, $\beta = 0.99$



(d) Investment to vulnerable capital stock, $\beta = 1$

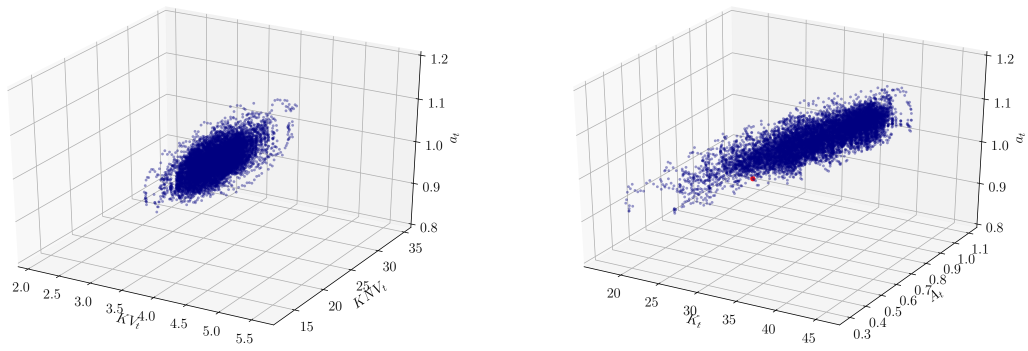


(e) Investment to non-vulnerable capital stock, $\beta = 0.99$



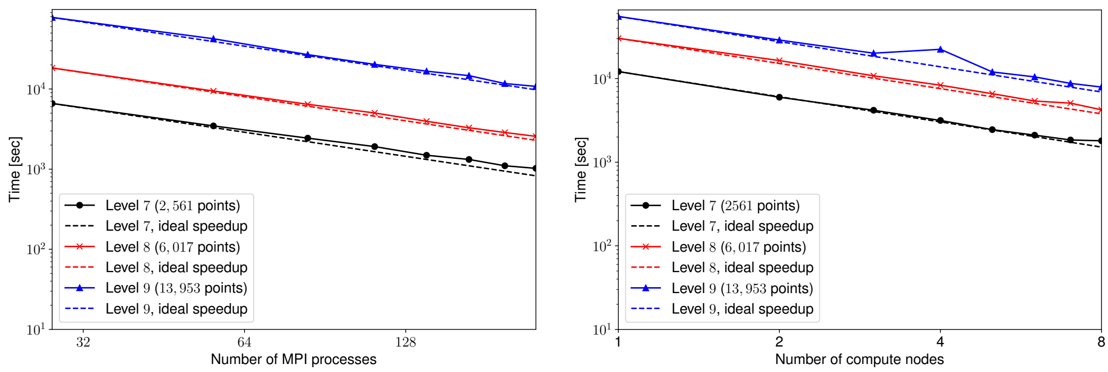
(f) Investment to non-vulnerable capital stock, $\beta = 1$

Figure G.1 – Stochastic policy functions with the discount factor $\beta = 0.99$ and $\beta = 1$



(a) Spatial adaptation (b) Adaptive capital stock

Figure H.1 – Ergodic set in the models with the derived policy functions



(a) Spatial adaptation (b) Adaptive capital stock

Figure I.1 – Scaling effect of the parallelized adaptive sparse grid

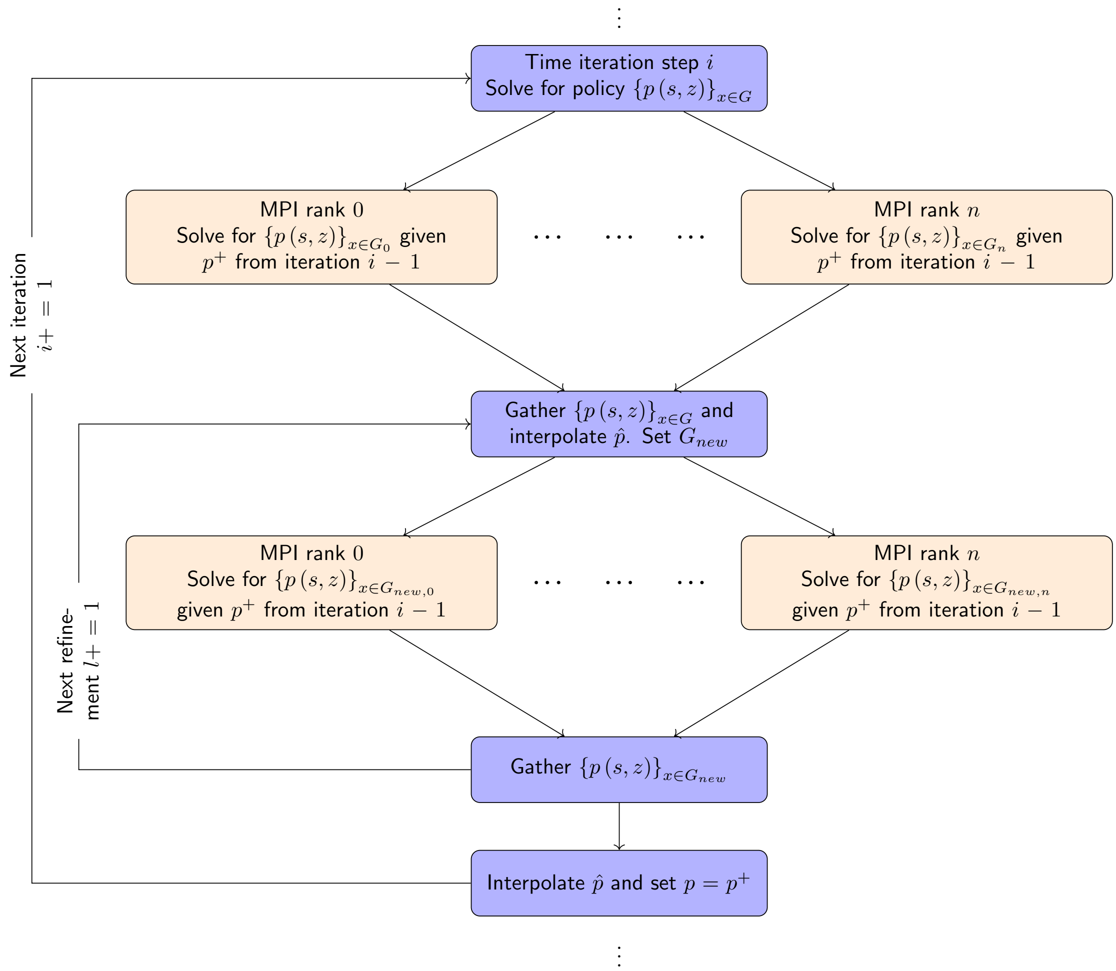


Figure J.1 – Schematic overview of parallelization via MPI

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Short-term Master project, Host supervisor: Prof. Thomas F. Rutherford

RESEARCH

Working paper

Takafumi Usui. Adaptation to rare natural disasters in a dynamic stochastic equilibrium model. 2018

Publications († in Japanese)

Takafumi Usui, Takaaki Furubayashi, and Toshihiko Nakata. Induced technological change and the timing of public R&D investment in the Japanese electricity sector considering a two-factor learning curve. *Clean Technologies and Environmental Policy*, 19(5):1347–1360, jul 2017. ISSN 1618-954X. doi: 10.1007/s10098-017-1333-1

† Takafumi Usui, Takaaki Furubayashi, and Toshihiko Nakata. The Impact of Learning-by-doing and Learning-by-searching on the Diffusion of Renewable Energy Technologies in the Japanese Electricity Sector. *Environmental Economics and Policy Studies*, 8(1):19–36, 2015. doi: 10.14927/reeps.8.1_19

Tomoya Kusunoki, Takaaki Furubayashi, Toshihiko Nakata, and Takafumi Usui. Development of an Energy-Economic Model with Endogenous Technological Progress and Feasibility Study of CCS Systems. *Heat Transfer-Asian Research*, 43(4):332–351, jun 2014. ISSN 10992871. doi: 10.1002/htj.21078

Conferences († in Japanese)

2016

2nd Simlab educational workshop (ETH Zürich, Oral), 15th International Swiss Climate Summer School (Grindelwald, Switzerland, Poster).

2015

14thInternational Swiss Climate Summer School(Ascona, Switzerland, Poster).

2014

†30th Energy system-Economics-Environmental Conference (Tokyo, Japan, Oral).

2013

†29th Energy system-Economics-Environmental Conference (Tokyo, Japan, Oral).

2011

10th International Gas Turbine Congress (Osaka, Japan, Oral).

Memberships

Swiss Society of Economics and Statistics (SSES)
International Association for Energy Economics (IAEE)
Society for Environmental Economics and Policy Studies (SEEPS)
Japan Society of Energy and Resources

Honors, Awards, and Fellowships

Financial award for a graduate student, Aoba foundation for the promotion of engineering, 2013.

Exchange study fellowship, Tohoku University, 2012.

Complementary Skills

Computational expertise

Python: scientific computing, numerical analysis in economics
C/C++: scientific computing
Julia: scientific computing, numerical analysis in economics
GAMS and MPSGE: numerical optimization, equilibrium analysis
R: statistical analysis
MPI: parallel computing
LaTeX: documentation, presentation, poster creation
Git: version control
MS Office: office software
Mac OS X, Linux (Ubuntu), Windows: operating system

Language

English: fluent
Japanese: native

Reference

Prof. Dr. Philippe Thalmann

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